DIMACS Technical Report 2003-03
January 2003

Chordal Bipartite Graphs of Bounded Tree- and Clique-width

by

Vadim Lozin
RUTCOR, Rutgers University
640 Bartholomew Rd., Piscataway
NJ 08854-8003, USA
lozin@rutcor.rutgers.edu

Dieter Rautenbach[1]
Lehrstuhl II für Mathematik
RWTH-Aachen, 52056 Aachen, Germany
rauten@math2.rwth-aachen.de

[1]This research has been carried out during the visit of Dieter Rautenbach to Rutgers University: assistance of RUTCOR and DIMACS is gratefully acknowledged.

DIMACS is a partnership of Rutgers University, Princeton University, AT&T Labs-Research, Bell Labs, NEC Research Institute and Telcordia Technologies (formerly Bellcore).

DIMACS was founded as an NSF Science and Technology Center, and also receives support from the New Jersey Commission on Science and Technology.

ABSTRACT

A bipartite graph is chordal bipartite if every cycle of length at least six has a chord. In the class of chordal bipartite graphs the tree-width and the clique-width are unbounded.

Our main results are that chordal bipartite graphs of bounded vertex degree have bounded tree-width and that k-fork-free chordal bipartite graphs have bounded clique-width, where a k-fork is the graph arising from a $K_{1,k+1}$ by subdividing one edge once. This implies polynomial-time solvability for a variety of algorithmical problems for these graphs.

1 Introduction

The tree-width and clique-width are two graph parameters which are of interest due to the fact that many problems being NP-hard in general graphs become polynomial time solvable when restricted to graphs, where one of these parameters is bounded. Among graphs with bounded clique-width are distance hereditary graphs [19], cographs and some of their extensions [25], bi-cographs (bipartite analog of cographs) [17] and several of their generalizations [15, 23, 24]. On the other hand, the clique-width is known to be unbounded in the class of split graphs [25], unit interval and permutation graphs [19] and even for bipartite permutation graphs [8]. In this paper we focus on the class of chordal bipartite graphs, where both parameters are unbounded, and detect some of its subclasses with bounded tree- or clique-width.

The class of chordal bipartite graphs is the proper subclass of bipartite graphs in which every cycle of length at least six has a chord. This class was introduced by Golumbic and Goss [18] in 1978 and has received a lot of attention.

Chordal bipartite graphs are useful in the study of linear programming, since the bipartite adjacency matrix of any graph in this class is totally balanced [3]. The class of chordal bipartite graphs includes many interesting subclasses such as forests, bipartite permutation graphs [31], convex and biconvex graphs [1], bipartite distance hereditary [4] and difference graphs [16]. On the other hand, the class of chordal bipartite graphs differs essentially from the listed subclasses, because all of them are factorial, in the terminology of [30], while the chordal bipartite graphs constitute a superfactorial class [32] (see Figure 2 for the inclusion relationship between classes). It has been shown in [6] that the clique-width is bounded only in factorial classes of graphs.

Several important problems remain NP-hard when restricted to chordal bipartite graphs. This is the case for HAMILTONIAN CYCLE [28], JUMP NUMBER [27], STEINER TREE and DOMINATING SET [26] (with variations such as CONNECTED DOMINATING SET and INDEPENDENT DOMINATING SET [14]). Moreover, many of these problems are NP-hard also for bipartite graphs with vertex degree at most 3 (cf. [7, 22]).

In contrast to these facts, we prove as our main results that chordal bipartite graphs of bounded vertex degree have bounded tree-width and that k-fork-free chordal bipartite graphs have bounded clique-width (see Figure 1 for a k-fork). Hence all problems expressible in monadic second order logic become polynomial time solvable when restricted to chordal bipartite graphs of bounded vertex degree [9] and all problems expressible in monadic second order logic using quantifiers on vertices but not on edges become polynomial time solvable when restricted to k-fork-free chordal bipartite graphs [11, 12].

All graphs will be finite, undirected, without loops or multiple edges. For a graph G, we denote by $V(G)$ and $E(G)$ the set of vertices and edges of G, respectively. If u is a vertex of G, then $N_G(u)$ stands for the neighborhood of u in G (i.e. the set of vertices adjacent to u in G) and $d_G(u) = |N_G(u)|$ for the degree of u. The maximum degree of a vertex in G is denoted $\Delta(G)$. For a subset of vertices $U \subseteq V(G)$, we denote by $N_G(U)$ the neighborhood of U (i.e. the set of vertices not in U that have a neighbor in U in the graph G) and by

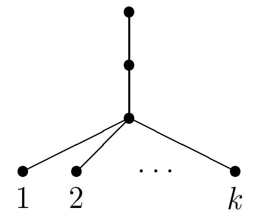

Figure 1: A k-fork F_k

$G - U$ the subgraph of G induced by the vertices in $V(G) \setminus U$. We say that a graph G is H-free if it does not contain the graph H as an induced subgraph. A subset of pairwise non-adjacent vertices is called independent, and a subset of pairwise adjacent vertices a clique. The maximum size of a clique of G is denoted $\omega(G)$.

A graph is chordal if every cycle of length at least 4 has a chord. We denote by $K_{r,s}$ the complete bipartite graph with partite sets of cardinality r and s, and by C_n the chordless cycle on n vertices. The graph that arises from $K_{1,k+1}$ by subdividing one edge once is denoted F_k and is called a k-fork (see Figure 1).

The complement of a graph G is denoted by $\bar{G}$. If G is a bipartite graph with partite sets V_1 and V_2, then the *bipartite complement* $\bar{G}^{\rm bip}$ of G is the graph with vertex set $V(G)$ and edge set $\{uv \mid uv \notin E(G),\, u \in V_1,\, v \in V_2\}$. (Note that the bipartite complement of a disconnected bipartite graph may depend on the choice of the partite sets.)

2 Definitions and preparatory statements

A *tree decomposition* (cf. [29]) of a graph G is a pair $(T, \mathcal{W})$ where T is a tree and $\mathcal{W}$ assigns a set $W_t \subseteq V(G)$ to each vertex t of T such that

(i) $V(G) = \bigcup_{t \in V(T)} W_t$,

(ii) for every edge $uv \in E(G)$, there is some $t \in V(T)$ such that $u, v \in W_t$ and

(iii) for every vertex $u \in V(G)$, the set $\{t \in V(T) \mid u \in W_t\}$ induces a subtree of the tree T.

The *width* of a tree decomposition $(T, \mathcal{W})$ is $\max_{t \in V(T)} |W_t| - 1$ and the *tree-width* $\mathrm{tw}(G)$ of G is the minimum width of a tree decomposition of G.

We will only use the following two elementary and well-known properties of the tree-width (cf. [29]): If G is a graph and $U \subseteq V(G)$, then

$$\mathrm{tw}(G) \leq |U| + \mathrm{tw}(G - U). \tag{1}$$

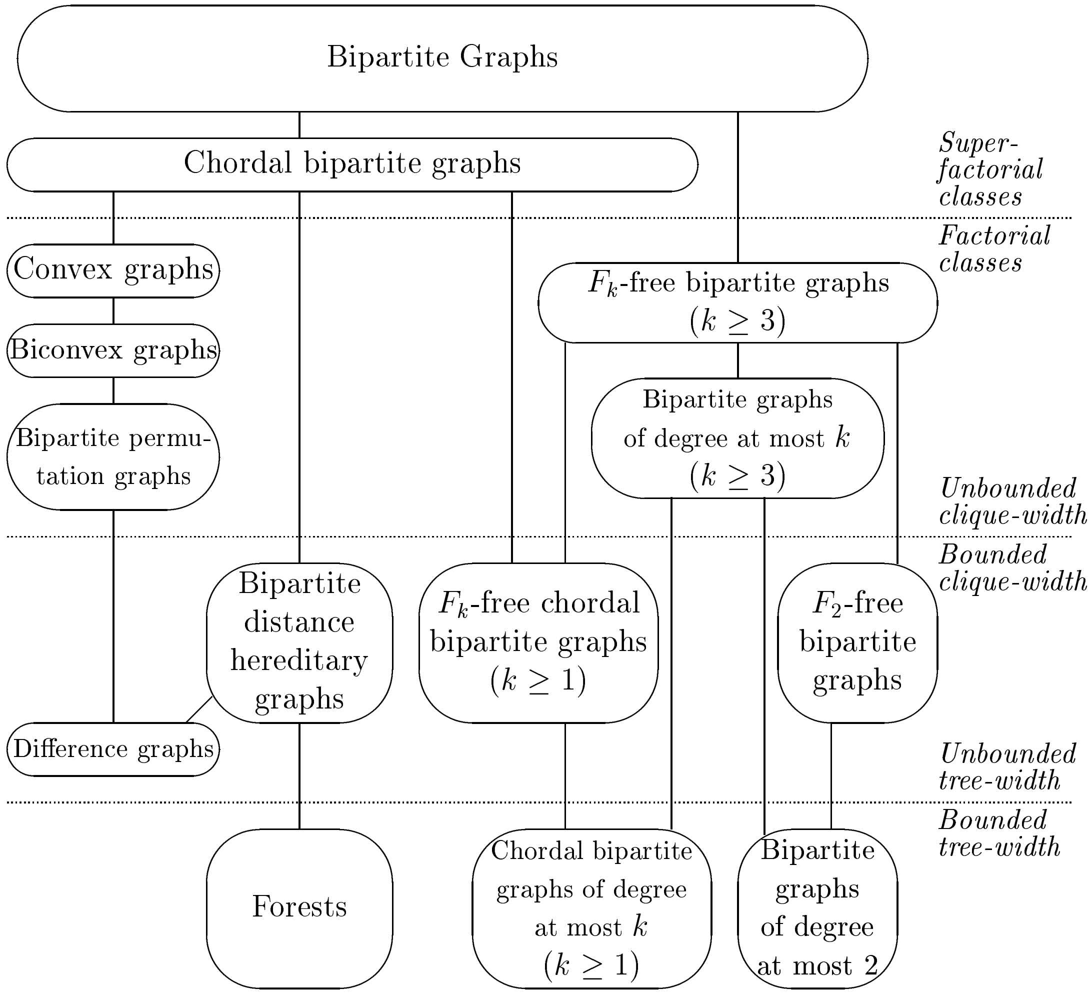

Figure 2: Inclusion relationship between graph classes

Furthermore,

$$\mathrm{tw}(G) = \min\{\omega(H) - 1 \mid H \text{ is chordal and } G \text{ is a subgraph of } H\}. \qquad (2)$$

The *clique-width* $\mathrm{cw}(G)$ [10] of a graph G is the minimum number of labels needed to construct G using the following four operations:

(i) Creation of a new vertex v with label i (denoted $i(v)$).

(ii) Disjoint union of two labeled graphs G and H (denoted $G \oplus H$).

(iii) Connection of all vertices with label i to all vertices with label j ($i \neq j$, denoted $\eta_{i,j}$).

(iv) Renaming label i to j (denoted $\rho_{i \to j}$).

Every graph can be defined by an algebraic expression using these four operations. For instance, the path on four consecutive vertices a, b, c, d can be defined as follows:

$$\eta_{3,2}(3(d) \oplus \rho_{3\to 2}(\rho_{2\to 1}(\eta_{3,2}(3(c) \oplus \eta_{2,1}(2(b) \oplus 1(a)))))).$$

Such an expression is called a *k-expression* if it uses at most k different labels. Thus the clique-width of G is the minimum k for which there exists a k-expression defining G.

For a class of graphs with clique-width at most k, Courcelle et al. presented in [11] a number of optimization problems, which, given a graph G in the class and an $O(f(|V(G)|, |V(E)|))$ algorithm to construct a k-expression defining G, can be solved for G in time $O(f(|V(G)|, |V(E)|))$.

Comparing the tree-width and clique-width of a graph G, Courcelle and Olariu showed in [13] that

$$\mathrm{cw}(G) \leq 2^{2\mathrm{tw}(G)+2} + 1. \qquad (3)$$

Remark. The proof of this inequality is constructive and permits to build an algebraic expression with bounded number of labels if a tree-decomposition of bounded width is available for G. According to [5] such a decomposition can be found in polynomial time.

Furthermore, Courcelle and Olariu proved that

$$\mathrm{cw}(\bar{G}) \leq 2\mathrm{cw}(G) \qquad (4)$$

for any graph G.

3 Results

In [20], Kloks and Kratsch describe an efficient algorithm to determine the tree-width of chordal bipartite graphs. Their algorithm relies on the analysis of the maximal (with respect to inclusion) complete bipartite subgraphs. We will use some of their statements in the proof of our first main result.

Theorem 1 *Let Δ be a positive integer and let G be a chordal bipartite graph such that $\Delta(G) \leq \Delta$. Then* $\mathrm{tw}(G) \leq \Delta^2$.

Proof. Denote by $\mathcal{M}$ the set of maximal complete bipartite subgraphs H of G with partite sets $V_1(H)$ and $V_2(H)$ such that $|V_1(H)|, |V_2(H)| \geq 2$.

It follows from Theorems 3.1 and 4.1 in [20] that there is a mapping $C : \mathcal{M} \to 2^{V(G)}$ such that $C(H) \in \{V_1(H), V_2(H)\}$ for each $H \in \mathcal{M}$ and the graph G^* with vertex set $V(G)$ and edge set

$$E(G^*) = E(G) \cup \bigcup_{H \in \mathcal{M}} \{uv \mid u, v \in C(H),\, u \neq v\}$$

is chordal.

Let $u \in V(G)$ be such that $d_{G^*}(u) = \Delta(G^*)$. By definition, all neighbors of u in G^* are at distance at most 2 from u in G. Hence $\Delta(G^*) \leq \Delta + (\Delta - 1)\Delta = \Delta^2$. With the help of inequality (2) this implies that $\mathrm{tw}(G) \leq \omega(G^*) - 1 \leq \Delta(G^*) \leq \Delta^2$ and the proof is completed. ■

Combining this theorem with inequality (3) and the remark after it, we obtain the following result.

Corollary 1 *In the class of chordal bipartite graphs of maximum vertex degree at most Δ the clique-width is bounded by a constant c depending on Δ and a c-expression defining a graph in the class can be constructed in time polynomial in the size of the graph.*

We will establish the next main result using a series of statements some of which we found to be quite interesting for their own sake.

Our first result in this series is a relation between the clique-width of a bipartite graph and the clique-width of its bipartite complement, which is similar to inequality (4). Notice, however, that our result is not implied by Proposition 4.3 in [13] that proves inequality (4).

Proposition 1 *If G is a bipartite graph, then* $\mathrm{cw}(\bar{G}^{\mathrm{bip}}) \leq 4\mathrm{cw}(G)$.

Proof. Let G be a bipartite graph with partite sets V_1 and V_2. By the result of Courcelle and Olariu, we know that $\mathrm{cw}(\bar{G}) \leq 2\mathrm{cw}(G)$. We will transform a $\mathrm{cw}(\bar{G})$-expression L of $\bar{G}$ into a $2\mathrm{cw}(\bar{G})$-expression of $\bar{G}^{\mathrm{bip}}$ as follows:

(i) Let I be the set of labels used by L and let $I' = \{i_1, i_2 \mid i \in I\}$.

(ii) Replace every operation of the form $i(u)$ in L by $i_1(u)$ if $u \in V_1$ and $i_2(u)$ if $u \in V_2$.

(iii) Replace every operation of the form $\rho_{i\to j}(\cdot)$ in L by $\rho_{i_1\to j_1}(\rho_{i_2\to j_2}(\cdot))$.

(iv) Replace every operation of the form $\eta_{i,j}(\cdot)$ in L by $\eta_{i_1,j_2}(\eta_{i_2,j_1}(\cdot))$.

Clearly, the $2\mathrm{cw}(\bar{G})$-expression L' obtained in this way produces a graph G' with vertex set $V(G)$ whose edge set consists of all edges uv of $\bar{G}$ such that u and v lie in different partite sets of G. Hence $G' = \bar{G}^{\mathrm{bip}}$ and the proof is completed. ■

Our next result states that the bipartite complement of a chordal bipartite graph is not too far from being chordal bipartite itself.

Proposition 2 *If G is a chordal bipartite graph, then $\bar{G}^{\mathrm{bip}}$ is either chordal bipartite or there is a set $X \subseteq V(G)$ such that X induces a chordless cycles of length six in $\bar{G}^{\mathrm{bip}}$ and $V(G) \setminus (X \cup N_{\bar{G}^{\mathrm{bip}}}(X))$ is an independent set in $\bar{G}^{\mathrm{bip}}$.*

Proof. Let G be a chordal bipartite graph with partite sets V_1 and V_2. Assume that $\bar{G}^{\mathrm{bip}}$ is not chordal bipartite.

Claim 1 $\bar{G}^{\mathrm{bip}}$ is C_{2l}-free for $l \geq 4$.

Proof of Claim 1: Clearly, as $\bar{C_8}^{\mathrm{bip}} = C_8$, the graph $\bar{G}^{\mathrm{bip}}$ is C_8-free. If $x_1y_1x_2y_2\dots x_ly_l$ with $l \geq 5$ is a chordless cycle in $\bar{G}^{\mathrm{bip}}$, then $x_1y_2x_4y_1x_3y_4$ is a chordless C_6 in G which is a contradiction. Hence $\bar{G}^{\mathrm{bip}}$ is C_{2l}-free for $l \geq 4$ and Claim 1 is proved.

Claim 2 $\bar{G}^{\mathrm{bip}}$ does not contain a chordless cycles C of length 6 and an edge uv such that $\{u, v\} \subseteq V(G) \setminus (V(C) \cup N_{\bar{G}^{\mathrm{bip}}}(V(C)))$.

Proof of Claim 2: For contradiction, we assume that $C : x_1y_1x_2y_2x_3y_3x_1$ is a chordless cycle in $\bar{G}^{\mathrm{bip}}$ and that $uv \in E(\bar{G}^{\mathrm{bip}})$ such that $\{u, v\} \subseteq V(G) \setminus (V(C_1) \cup N_{\bar{G}^{\mathrm{bip}}}(V(C_1)))$. We assume that x_1 and u lie in the same partite set of G. Now, $x_1y_2uy_3x_2vx_1$ is a chordless cycles of length 6 in G which is a contradiction and Claim 2 is proved.

Clearly, the two claims imply the desired result and the proof is completed. ■

Our next auxiliary result is a generalization of a recent result of Alekseev [2] which states that for a connected F_2-free bipartite graph G, either $\Delta(G) \leq 2$ or $\Delta(\bar{G}^{\mathrm{bip}}) \leq 1$.

Proposition 3 *For $k \geq 3$ let G be a connected F_k-free bipartite graph, then either $\Delta(G) \leq k(k-1)$ or $\Delta(\bar{G}^{\mathrm{bip}}) \leq (k-1)^3$.*

Proof. We may assume that G is a connected F_k-free bipartite graph such that $\Delta(G) \geq k(k-1) + 1$. Consider a vertex u with $d_G(u) = \Delta(G)$ and for $i = 1, 2, \dots$ let A_i denote the set of vertices of G at distance i from u. Note that $|A_1| = \Delta(G) \geq k(k-1) + 1$ and that for $i \geq 1$, A_i is an independent set of vertices.

Claim 1 Every vertex in A_2 is non-adjacent to at most $k - 1$ vertices of A_1.

Proof of Claim 1: For contradiction we assume that $w \in A_2$, $v_0 \in A_1 \cap N_G(w)$ and $v_1, v_2, \dots, v_k \in A_1 \setminus N_G(w)$. Now the set $\{u, v_0, v_1, \dots, v_k, w\}$ induces a k-fork, which is a contradiction.

Claim 2 Every vertex in A_1 is non-adjacent to at most $k-1$ vertices of A_2.

Proof of Claim 2: For contradiction we assume that $v \in A_1$ and $w_1, w_2, \ldots, w_k \in A_2 \setminus N_G(v)$. Since

$$\left| A_1 \cap \bigcap_{i=1}^{k} N_G(w_i) \right| \geq k(k-1) + 1 - k(k-1) \geq 1,$$

there is a vertex $v' \in A_1 \cap \bigcap_{i=1}^{k} N_G(w_i)$ and hence the set $\{u, v, v', w_1, w_2, \ldots, w_k\}$ induces a k-fork, which is a contradiction.

Claim 3 Every vertex in A_3 is non-adjacent to at most $k-2$ vertices of A_2.

Proof of Claim 3: For contradiction we assume that $x \in A_3$, $w_0 \in A_2 \cap N_G(x)$ and $w_1, w_2, \ldots, w_{k-1} \in A_2 \setminus N_G(x)$. As in the proof of Claim 2 we obtain that

$$\left| A_1 \cap \bigcap_{i=0}^{k-1} N_G(w_i) \right| \geq k(k-1) + 1 - k(k-1) \geq 1.$$

Hence there is a vertex $v \in A_1 \cap \bigcap_{i=0}^{k-1} N_G(w_i)$ and hence the set $\{u, v, w_0, w_1, w_2, \ldots, w_{k-1}, x\}$ induces a k-fork, which is a contradiction.

Claim 4 Every vertex in A_2 is adjacent to at most $k-1$ vertices of A_3.

Proof of Claim 4: For contradiction we assume that $w \in A_2$ and $x_1, x_2, \ldots, x_k \in A_3 \cap N_G(w)$. If $v \in A_1 \cap N_G(w)$, then the set $\{u, v, w, x_1, x_2, \ldots, x_k\}$ induces a k-fork, which is a contradiction.

Claim 5 $A_i = \emptyset$ for $i \geq 4$.

Proof of Claim 5: For contradiction we assume that $y \in A_4$, $x \in A_3 \cap N_G(y)$ and $w \in A_2 \cap N_G(x)$. By Claim 1, we have $|A_1 \cap N_G(w)| \geq k(k-1) + 1 - (k-1) \geq k$. Hence there are vertices $v_1, v_2, \ldots, v_k \in A_1 \cap N_G(w)$ and hence the set $\{y, x, w, v_1, v_2, \ldots, v_k\}$ induces a k-fork, which is a contradiction.

Claim 6 If $|A_2| \geq k(k-2) + 1$, then $|A_3| \leq k-1$.

Proof of Claim 6: For contradiction we assume that $|A_2| \geq k(k-2)+1$ and that $x_1, x_2, \ldots, x_k \in A_3$. By Claim 3, we have

$$\left| A_2 \cap \bigcap_{i=1}^{k} N_G(x_i) \right| \geq k(k-2) + 1 - k(k-2) \geq 1.$$

Hence there is a vertex $w \in A_2 \cap \bigcap_{i=1}^{k} N_G(x_i)$. If $v \in A_2 \cap N_G(w)$, then the set $\{u, v, w, x_1, x_2, \ldots, x_k\}$ induces a k-fork, which is a contradiction.

Now, if $|A_2| \geq k(k-2) + 1$, then, by Claim 6, $|A_3| \leq k-1$ and we obtain

$$\Delta(\bar{G}^{\text{bip}}) \leq 2(k-1) \leq (k-1)^3.$$

Hence we assume that $|A_2| \leq k(k-2)$. Since every vertex in A_3 has a neighbor in A_2, we obtain, by Claim 4, that $|A_3| \leq (k-1)|A_2| \leq k(k-1)(k-2)$. This finally implies that

$$\Delta(\bar{G}^{\mathrm{bip}}) \leq (k-1) + k(k-1)(k-2) = (k-1)^3$$

and the proof is completed. ■

Our last preparatory result concerns chordal bipartite graphs G such that either $\Delta(G)$ or $\Delta(\bar{G}^{\mathrm{bip}})$ is small.

Proposition 4 *Let Δ be a positive integer and let G be a chordal bipartite graph such that either $\Delta(G) \leq \Delta$ or $\Delta(\bar{G}^{\mathrm{bip}}) \leq \Delta$. Then the clique-width of G is bounded by some constant c depending on Δ and a c-expression defining G can be constructed in polynomial time.*

Proof. If $\Delta(G) \leq \Delta$, then the desired result follows from Corollary 1. The same is true if $\Delta(\bar{G}^{\mathrm{bip}}) \leq \Delta$ and $\bar{G}^{\mathrm{bip}}$ is chordal bipartite. Now we consider the case when $\Delta(\bar{G}^{\mathrm{bip}}) \leq \Delta$ and $\bar{G}^{\mathrm{bip}}$ is not a chordal bipartite graph.

By Proposition 2, there is a set U of at most $6 + 6(\Delta - 2) = 6(\Delta - 1)$ vertices such that $V(G) \setminus U$ is an independent set in $\bar{G}^{\mathrm{bip}}$. Hence, by Theorem 1 and inequality (1),

$$\mathrm{tw}(\bar{G}^{\mathrm{bip}}) \leq |U| + \mathrm{tw}(\bar{G}^{\mathrm{bip}} - U) \leq 6(\Delta - 1) + 0.$$

Now the inequality (3) implies that $\mathrm{cw}(\bar{G}^{\mathrm{bip}})$ is bounded by some constant depending on Δ. Finally, Proposition 1 implies the desired result and the proof is completed. ■

We now combine the above propositions to obtain our second main result.

Theorem 2 *For $k \geq 3$, the clique-width of a k-fork-free chordal bipartite graph G is bounded by a constant c depending on k, and a c-expression defining G can be constructed in time polynomial in the size of G.*

Proof. Let G be a k-fork-free chordal bipartite graph and let $G_1, G_2, \ldots, G_p$ be the connected components of G. Obviously,

$$\mathrm{cw}(G) = \max\{\mathrm{cw}(G_1), \mathrm{cw}(G_2), \ldots, \mathrm{cw}(G_p)\}.$$

Therefore, we may assume that G is connected.

By Proposition 3, we have that either $\Delta(G) \leq k(k-1) \leq (k-1)^3$ or $\Delta(\bar{G}^{\mathrm{bip}}) \leq (k-1)^3$ and Proposition 4 implies the desired result. ■

4 Conclusion and open problems

In this paper we proved that for any positive integer k, chordal bipartite graphs without an induced $K_{1,k+1}$ (i.e. of vertex degree at most k) have bounded tree-width. We extended this result by showing that F_k-free chordal bipartite graphs have bounded clique-width, where F_k is the graph obtained by a single subdivision of exactly one edge of a $K_{1,k+1}$. A natural question arises: Is it possible to extend these results to larger subclasses of chordal bipartite graphs defined by forbidding a graph obtained from $K_{1,k+1}$ by subdividing more than one edge?

Let us denote by $S_{i,j,k}$ a tree with exactly three vertices of degree one of distance i, j, k from the only vertex of degree three. If we subdivide at least three edges of a $K_{1,k+1}$, then we obtain a graph that contains $S_{2,2,2}$ as an induced subgraph. The class of $S_{2,2,2}$-free chordal bipartite graphs contains all bipartite permutation graphs and hence is not of bounded clique-width. From this observation it follows that the above question remains open only if we subdivide exactly two edges of a $K_{1,k+1}$.

Let E_k denote the graph obtained by single subdivisions of two edges of a $K_{1,k+1}$. In particular, $E_2 = S_{1,2,2}$. It is known [21] that for $k = 2$, the clique-width of E_k-free bipartite graphs is bounded. This result cannot be extended to larger values of k without additional restrictions, because the class of E_3-free bipartite graphs contains all bipartite graphs of vertex degree at most three. We conjecture that E_k-free chordal bipartite graphs have bounded clique-width for any particular value of k.

References

[1] N. Abbas and L.K. Stewart, Biconvex graphs: ordering and algorithms, *Discrete Appl. Math.* 103 (2000) 1–19.

[2] V.E. Alekseev, A polynomial algorithm for finding the largest independent sets in fork-free graphs, *Discrete Analysis and Operations Research, Ser.1,* 6 (1999) 3–19 (in Russian).

[3] R.P. Anstee and M. Farber, Characterizations of totally balanced matrices, *J. Algorithms* 5 (1984) 215–230.

[4] H.-J. Bandelt and H.M. Mulder, Distance-hereditary graphs, *J. Combin. Theory Ser. B* 41 (1986) 182–208.

[5] H.L. Bodlaender, A linear-time algorithm for finding tree-decompositions of small treewidth, *SIAM J. Comput.* 25 (1996) 1305–1317.

[6] R. Boliac and V.V. Lozin, On the clique-width of graphs in hereditary classes, *Lecture Notes in Computer Science* 2518 (2002) 44–54.

[7] R. Boliac and V.V. Lozin, Independent domination in finitely defined classes of graphs, *Theoretical Computer Science*, to appear.

[8] A. Brandstädt and V.V. Lozin, On the linear structure and clique-width of bipartite permutation graphs, *Ars Combinatoria*, to appear.

[9] B. Courcelle, The monadic second-order logic of graphs. III. Tree-decompositions, minors and complexity issues, *RAIRO Inform. Théor. Appl.* 26 (1992) 257–286.

[10] B. Courcelle, J. Engelfriet and G. Rozenberg, Handle-rewriting hypergraph grammars, *J. Comput. System Sci.* 46 (1993) 218–270.

[11] B. Courcelle, J.A. Makowsky and U. Rotics, Linear time solvable optimization problems on graphs of bounded clique-width, *Theory Comput. Systems* 33 (2000) 125–150.

[12] B. Courcelle, J.A. Makowsky and U. Rotics, On the fixed parameter complexity of graph enumeration problems definable in monadic second-order logic, *Discrete Appl. Math.* 108 (2001) 23–52.

[13] B. Courcelle and S. Olariu, Upper bounds to the clique width of graphs, *Discrete Appl. Math.* 101 (2000) 77–114.

[14] P. Damaschke, H. Müller and D. Kratsch, Domination in convex and chordal bipartite graphs, *Inform. Process. Lett.* 36 (1990) 231–236.

[15] J-L. Fouquet, V. Giakoumakis and J.M. Vanherpe, Bipartite graphs totally decomposable by canonical decomposition, *International J. Foundations of Computer Science* 10 (1999) 513-533.

[16] P.L. Hammer, U.N. Peled and X. Sun, Difference graphs, *Discrete Appl. Math.* 28 (1990) 35–44.

[17] V. Giakoumakis and J.-M. Vanherpe, Bi-complement reducible graphs, *Advances in Appl. Math.* 18 (1997) 389–402.

[18] M.C. Golumbic and C.F. Goss, Perfect elimination and chordal bipartite graphs, *J. Graph Theory* 2 (1978) 155–163.

[19] M.C. Golumbic and U. Rotics, On the clique-width of some perfect graph classes, *International J. Foundations of Computer Science* 11 (2000) 423–443.

[20] T. Kloks and D. Kratsch, Treewidth of chordal bipartite graphs, *J. Algorithms* 19 (1995) 266–281.

[21] V.V. Lozin, E-free bipartite graphs, *Discrete Analysis and Operations Research, Ser.1,* 7 (2000) 49-66 (in Russian).

[22] V.V. Lozin and M.U. Gerber, On the jump number problem in hereditary classes of bipartite graphs, *Order* 17 (2000) 377–385.

[23] V.V. Lozin, On a generalization of bi-complement reducible graphs, *Lecture Notes in Computer Science* 1893 (2000) 528–538.

[24] V.V. Lozin, Bipartite graphs without a skew star, *Discrete Math.* 257 (2002) 83-100.

[25] J.A. Makowsky and U. Rotics, On the clique-width of graphs with few P_4's, *International J. Foundations of Computer Science* 10 (1999) 329–348.

[26] H. Müller and A. Brandstädt, The NP-completeness of steiner tree and dominating set for chordal bipartite graphs, *Theoretical Computer Science* 53 (1987) 257–265.

[27] H. Müller, Alternating cycle-free matching, *Order* 7 (1990) 11–21.

[28] H. Müller, Hamiltonian circuits in chordal bipartite graphs, *Discrete Math.* 156 (1996) 291–298.

[29] B. Reed, Tree width and tangles: a new connectivity measure and some applications, in *London Math. Soc. Lecture Note Ser.* 241, pages 87–162, Cambridge Univ. Press, Cambridge, 1997.

[30] E.R. Scheinerman and J. Zito, On the size of hereditary classes of graphs, *J. Combin. Theory, Ser. B* 61 (1994) 16–39.

[31] J. Spinrad, A. Brandstädt and L. Stewart, Bipartite permutation graphs, *Discrete Appl. Math.* 18 (1987) 279–292.

[32] J. Spinrad, Nonredundant 1's in Γ-free matrices, *SIAM J. Discrete Math.* 8 (1995) 251–257.