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Efficient Dominating Sets in Cayley Graphs

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ABSTRACT

An independent set C of vertices in a graph is an efficient dominating set when each vertex not in C is adjacent to exactly one vertex in C.

A countable family of nested graphs, each of which has at least one efficient dominating set, is produced for each real number in the unit interval [0,1].

These families range from star graphs, for which the efficient domination property was proved by Arumugam and Kala, to pancake graphs.

The families obtained are extended to partitions of the vertex sets into efficient dominating sets. Uniqueness holds for the given partitions whenever the countable family is not that of the star graphs.

For star graphs, all the possible ways in which such a partition occurs are established.

1 Introduction and Notation

Let $G = (V, E)$ be a finite undirected graph with no loops and multiple edges. We follow the terminology of [3]. Given $C \subseteq V$, let the *open neighborhood* $N(C)$ of C in G be the subset of vertices in $V \setminus C$ adjacent to some vertex in C, and let the corresponding *closed neighborhood* be $N[C] = N(C) \cup C$. A set $C \subset V$ is a *dominating set* if $N[C] = V$, that is, every vertex in G is adjacent to some vertex in C. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set in G. If the dominating set C is a stable set of G, then C is an *independent dominating set*. Also, when every vertex in $V \setminus C$ is adjacent to exactly one vertex in C then C is a *perfect dominating set*. A dominating set C which is both independent and perfect is an *efficient* dominating set. In what follows we may refer to an efficient dominating set as an E-set for short.

E-sets correspond to perfect 1-correcting codes in G, as treated by J. Kratochvil in [5]. Equivalently, they provide a perfect packing of G by balls of radius 1. When G is r-regular, the so-called *sphere packing* condition

$$|V| = (r + 1)|C|$$

is trivially a necesary condition for C to be an E-set of G.

When the set of vertices is $V = F^n$, where F is an alphabet of q symbols, and two n-tuples in V are adjacent whenever they agree in all but one coordinate, E-sets are the standard 1-perfect codes. For any group structure given to F, the resulting graph can be viewed as a Cayley graph, which we denote by $F(q, n)$. When q is a prime power, the classical Hamming codes show that the sphere packing condition, expressible now as

$$q^n = 0 \pmod{n(q - 1) + 1},$$

is also sufficient for the existence of E-sets in $F(q, n)$.

We say that a countable family of graphs

$$\mathcal{G} = \{G_1 \subset G_2 \subset \ldots \subset G_i \subset G_{i+1} \subset \ldots\}$$

is an E-*chain* if every G_i is an induced subgraph of G_{i+1} and each G_i has an E-set C_i.

Let κ_i stand for the embedding inclusion of G_i into G_{i+1} induced by $\mathcal{G}$, for $i \geq 1$. If $C_{i+1} = N(\kappa_i(G_i))$ then we say that the E-chain $\mathcal{G}$ is a *neighborly* E-chain.

If there exists a one-to-one graph map $\zeta_i : G_i \to G_{i+1}$ such that $\zeta_i(C_i) \subset C_{i+1}$, then we say that ζ_i and G_i are *inclusive*. If they are inclusive for every index i, then the E-chain $\mathcal{G}$ is also said to be *inclusive*. Notice that an inclusive neighborly E-chain has $\kappa_i \neq \zeta_i$, for every positive integer i.

A particular case of inclusive G_i is the one in which C_{i+1} has a partition into images $\zeta_i^j(C_i)$ of C_i through respective inclusive maps ζ_i^j, where j varies on a suitable finite indexing set. In such a case, $\mathcal{G}$, G_{i+1} and C_{i+1} are said to be *segmental*.

The family of Hamming cubes $\{F(2, 2^n - 1),\ n \geq 1\}$ provides an example of a segmental E-chain. An example of a segmental neighborly E-chain is given by the star graphs, (or star-transposition graph), $ST_n,\ n \geq 1$. The star graph ST_n is the Cayley graph on the symmetric

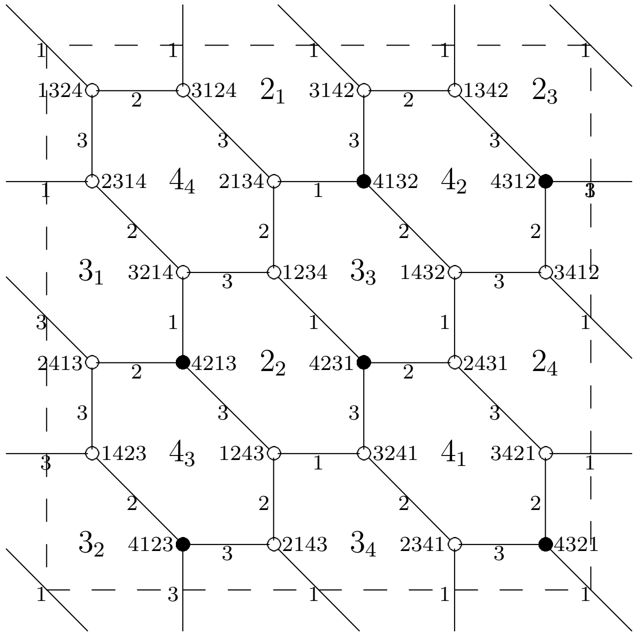

Figure 1: The star graph ST_4 can be drawn in an embedding of the torus.

group on n letters with respect to the set of transpositions $\Sigma = \{(1\ i),\ i = 2, \ldots, n\}$. It was shown by Arumugam and Kala in [1] that the graph ST_n do possess an E-set. Moreover, they showed that the E-set of ST_n contains an E-set of ST_{n-1} for each $n \geq 2$.

Figure 1 shows the graph ST_4 with the vertices of one of its E-sets indicated in black, where vertex notation is shown without parentheses and commas and where edges and copies of ST_3, (these given as hexagonal faces of the shown toral embedding), are labeled as follows:

(a) if $v = v_1v_2v_3v_4$ and $w = w_1w_2w_3w_4$ are the endvertices of an edge e in ST_4, then there is a unique integer entry i with $1 < i \leq 4$ such that $v_i \neq w_i$, which we use as a label for e;

(b) for each $i = 2, 3, 4$ and $j = 1, 2, 3, 4$, a copy of $ST(3)$ in $ST(4)$, that we denote i_j, is given by the set of all vertices $v = v_1v_2v_3v_4$ of $ST(4)$ with $v_i = j$; the 4 6-cycles labeled i_j avoid the edges labeled i, which form themselves a 1-factor.

Items (a) and (b) are extensible to any ST_{n+1}. In particular, the given definition of copies of $ST(3)$ in $ST(4)$ denoted i_j, is immediately extensible to the definition of copies i_j of ST_n in ST_{n+1}, for $i = 2, \ldots, n+1$ and $j = 1, \ldots, n+1$. Then, observe that $N(i_j)$ is a minimum cardinality E-set in ST_{n+1}, in every feasible instance, for $|V(ST_{n+1})| = (n+1)!$ and $|N(i_j)| = n!$, the smallest size for an E-set in ST_{n+1}. Thus, $\gamma(ST_{n+1}) = n!$.

Notice that the E-set shown in Figure 1 can be made segmental in 3 different ways, via 3 respective inclusive maps from ST_3 into ST_4, because for $k = 1, 2, 3$ and $j = 2, 3, 4$, there are embeddings $\zeta_3^{k,j} : ST_3 \to ST_4$ with $\zeta_3^{k,j}(ST_3) = j_k$, such that the black vertices in j_k form the image of an E-set in ST_3 through $\zeta_3^{k,j}$; this way, the 3 maps $\zeta_3^{k,j}$, with j fixed, play the role of the maps $\zeta_i^j = \zeta_3^j$ in the definition of a segmental E-chain. This extends immediately for any ST_{n+1}.

Notice that if $\mathcal{G}$ is an E-chain of regular connected graphs (and G_1 is 1-regular as we may assume without loss of generality), then the sphere packing condition implies $|V(G_n)| \geq (n+1)!$. Therefore, the E-chain of the star graphs $G_n = ST_{n+1}$ is 'dense' as such a chain can be, because $|V(ST_{n+1})| = (n+1)!$, $(n \geq 2)$, whereas the E-chain of the Hamming cubes is rather 'sparse'. Moreover, any corresponding minimum cardinality E-set C_n of $G_n = ST_{n+1}$ satisfies $C_n = n!$. An E-chain $\mathcal{G}$ is said to be *dense* if each of its E-sets G_n has $|V(G_n)| = (n+1)!$ and $|C_n| = n!$.

In this paper we give a general constructing tool for dense segmental neighborly E-chains, or DSNE-chains. We show that, in addition to the star graphs $\{ST_n,\ n \geq 1\}$, an uncountable collection of other countable families of Cayley graphs, among which the pancake graphs, provide cases of such DSNE-chains.

2 E-chains of Cayley graphs

Recall that the Cayley graph of a group Γ with respect to a subset $\Sigma \subset \Gamma$, which we denote by Γ_Σ or $(\Gamma)_\Sigma$, has the elements of the group as vertices and there is an edge $\{x, y\}$ whenever $x^{-1}y \in \Sigma \cup \Sigma^{-1}$. We will always assume that Σ does not contain the identity element $1 \in \Gamma$, so that the resulting graph has no loops. We also assume that $\Sigma = \Sigma^{-1}$.

Cayley graphs are regular of degree $|\Sigma|$; they are connected if and only if Σ generates Γ and they are vertex-transitive. In particular, the left translations $\phi_a(x) = ax$ are automorphisms of the graph. Notice that, for every subset X of vertices in the Cayley graph Γ_Σ, it holds that $N(X) = X\Sigma \setminus X$ and $N[X] = X\Sigma'$, where $\Sigma' = S \cup \{1\}$.

The construction of E-chains of Cayley graphs is based on the following lemma.

Lemma 1 *Let Σ be a generating set of a finite group Γ such that $s^2 = 1$ for each $s \in \Sigma$.*

Let $u \in \Sigma$ be such that $\Sigma_u = \Sigma \setminus \{u\}$ generates a proper subgroup H of Γ of index $(|\Sigma|+1)$ in Γ.

If $U = H \cap uHu$ is an E-set in H_{Σ_u}, then the open neighborhood $N(H)$ of H is an E-set in Γ_Σ.

Proof. Notice that U is the intersection of H and its conjugate by u. Therefore, U is also a subgroup of Γ and u belongs to the normalizer of U in Γ.

For each $x \in \Gamma$, let H_x be the subgraph of Γ_Σ induced by the vertices in xH. In particular, $H_1 = H_{\Sigma_u}$ and each H_x is isomorphic to H_1 (by the left translation ϕ_x).

Since U is an E-set of H_1, which is an $(|\Sigma|-1)$-regular graph, the sphere packing condition gives $|H| = |\Sigma|.|U|$.

Let $D = N(H)$ be the open neighborhood of H in Γ_Σ. For each $x \in \Gamma$, let $D_x = D \cap xH$. Since $u \notin H$ and $H = \langle \Sigma_u \rangle$, we have $D = Hu$. If $D_x \neq \emptyset$, then there are elements $h_1, h_2 \in H$ such that $x = h_1 u h_2$. Without loss of generality we may assume that $x = hu$ for some $h \in H$. Then,

$$D_x = Hu \cap xH = (hu)(uHu \cap H) = xU.$$

Therefore, D_x is an E-set of H_x for each $x \in \Gamma$ with $D_x \neq \emptyset$. In particular, $D_x\Sigma_u = xH \setminus D_x$.

Notice that, for each pair $h, h' \in H$, hu and $h'u$ belong to the same coset xH if and only if h, h' belong to the same coset of U in H. Therefore, $D = Hu$ intersects $|\Sigma|$ different cosets of H in Γ. Since $|\Gamma| = (|\Sigma| + 1)|H|$, all cosets but H itself are intersected by Hu. Hence

$$N(D) = \cup_{x\in\Gamma\setminus H}(D_x\Sigma_u) \cup Du = \cup_{x\in\Gamma\setminus H}(xH \setminus D_x) \cup H = \Gamma \setminus D.$$

Since D has the right cardinality, $(|\Sigma| + 1)$, it is an E-set of Γ. □

Theorem 2 [1] *The star graphs $\{G_n = ST_{n+1},\ n \geq 1\}$ form a DSNE-chain.*

Proof. The proof is by induction. We apply Lemma 1 with $\Gamma = S_n$, the symmetric group on the n letters $1, 2, \ldots, n$ and $u = (1\ n) \in \Sigma = \{(1\ i),\ i = 2, \ldots, n\}$, so that H coincides with the image of S_{n-1} through the homomorphism $\phi : S_{n-1} \to S_n$ given by $\phi(a_1\ a_2\ \ldots\ a_{n-1}) = (a_1\ a_2\ \ldots\ a_{n-1}\ n)$. All permutations $(a_1\ a_2\ \ldots, a_{n-1}\ a_n) \in \phi(S_{n-1})$ with a_{n-1} fixed will have image $(b_1\ b_2\ \ldots\ b_n)$ through the natural mapping $\xi : H \to uHu$ with $b_n = a_{n-1}$ unless $a_{n-1} = 1$, in which case $b_n = n$. Since there are $(n-1)!$ permutations $(b_1\ \ldots\ b_n) \in \Gamma = S_n$ with $b_n = n$ and also in $\phi(S_{n-1})$ and since ξ is one-to-one, we conclude that these $(b_1\ \ldots b_{n-1}\ n) \in S_n$ coincide with $U = H \cap uHu$. On the other hand, we observe that U in H_{S_u} plays the role of $N(H)$ in Γ_S for the previous inductive value of n, namely $n-1$. Because of that, U is an E-set in H_{S_u}. The conclusion of the lemma holds, and so does the assertion of the theorem. Segmentality of this E-chain follows the pattern shown in Section 1, in relation to Figure 1. □

Example. For $n = 4$ we have $u = (1\ 4)$ and

$$H = \{(1\ 2\ 3\ 4), (2\ 1\ 3\ 4), (3\ 1\ 2\ 4), (1\ 3\ 2\ 4), (2\ 3\ 1\ 4), (3\ 2\ 1\ 4)\},$$

giving its elements in the order appearing in the 6-cycle that constitutes H_{S_u}. (In particular, observe that H is the vertex set of 4_4 in Figure 1). Then by yielding the elements of uHu in the same order as the corresponding elements of H, we have that

$$uHu = \{(1\ 4\ 2\ 3), (1\ 2\ 4\ 3), (1\ 3\ 4\ 2), (1\ 4\ 3\ 2), (1\ 2\ 3\ 4), (1\ 3\ 2\ 4)\},$$

so that the first and fourth vertices in the representation of H appear as the fifth and sixth vertices in the representation of uHu. This is due to the fact that the fifth and sixth elements of H have its third component $a_3 = a_{n-1}$ equal to 1, which produces that $b_4 = 4$ in the corresponding elements of uHu through ξ. This way, as a result of Lemma 1, $N(4_4)$, indicated in Figure 1 as the black vertices, is an E-set. □

3 From Star Graphs to Pancake Graphs

We next show that another well-known family of Cayley graphs on permutations forms a DSNE-chain as well.

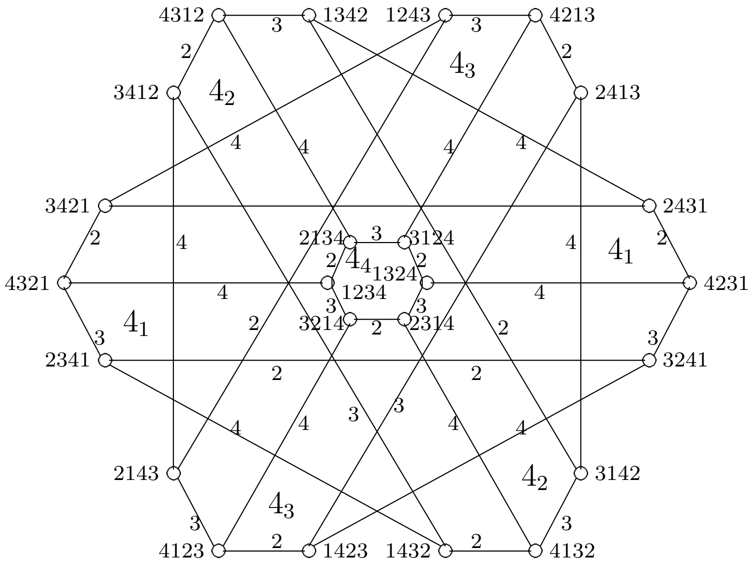

Figure 2: Representation of the pancake graph PC_4.

For each positive integer n, the pancake graph PC_n is the Cayley graph of the symmetric group on n letters with respect the set of involutions

$$S = \{(1\ i)(2\ i-1)\cdots(\lfloor i/2\rfloor\ \lceil i/2\rceil),\ i=2,\ldots,n\}.$$

The first 3 members of the family, PC_1, PC_2, PC_3, coincide with ST_1, ST_2, ST_3, respectively. The pancake graph PC_4 is represented in Figure 2, where edges are labeled as in Figure 1, but here we are able to represent copies of PC_3 only in 4 instances, which, with a notation similar to that of Figure 1, can be denoted $4_1, 4_2, 4_3, 4_4$, given by the 4 6-cycles whose vertices have their last entry equal to the subindex i of the respective 4_i, for $i = 1, 2, 3, 4$. Observe that these 4 6-cycles avoid the edges labeled 4, which form themselves a 1-factor, and that the neighbors of each 4_i form an E-set, as for example the vertices shown in black, for $i = 4$. This E-set C_3^4 is segmental: we have at our disposition 4 embeddings $\zeta_3^j : PC_3 \to 4_j$, where $j = 1, 2, 3, 4$, and the black vertices in 4_j, for $j = 1, 2, 3$, form the image of an E-set in PC_3 through ζ_3^j, so that the disjoint union of the 3 resulting images yields C_3^4. Also, the neighbors of 4_i, for $i = 1, 2, 3$, form a respective E-set C_3^i, for which the maps ζ_3^j, for $j \in \{1, 2, 3, 4\}\setminus\{i\}$, play a similar role. (We represent these E-sets in PC_4 with subindex 3, as in C_3^i, to follow the notation of E-chains already used for star graphs $G_n = ST_{n+1}$, so now $G_n = PC_{n+1}$).

Theorem 3 *The pancake family* $\{G_n = PC_{n+1},\ n \geq 1\}$ *forms a DSNE-chain.*

Proof. The proof is similar to that of Theorem 2, where now

$$u = (1\ n)(2\ (n-1))\cdots(\lfloor n/2\rfloor\ \lceil n/2\rceil).$$

Segmentality of this E-chain follows the pattern expressed in relation to Figure 1. In the case of Figure 2, this works only for the copies of PC_3 denoted $4_1, 4_2, 4_3$. $\square$

Lemma 1 allows even stronger applications that those given in Theorems 2 and 3 above. As an uncountable collection of examples of countable families of DSNE-chains ranging from the star graphs to the pancake graphs, we offer the following development.

Theorem 4 *There is an uncountable set of DSNE-chains that can be defined over the chain of symmetric groups*

$$S_1 \subset \ldots \subset S_i \subset S_{i+1} \subset \ldots.$$

More specifically, each infinite binary sequence $D = (d_1, d_2, \ldots)$ determines a corresponding unique DSNE-chain $\mathcal{G}^D$ such that if D is null then $\mathcal{G}^D$ is the star-graph E-chain and if D is nowhere-null then $\mathcal{G}^D$ is the pancake-graph E-chain.

Proof. Let us consider an infinite sequence $D = (d_1, d_2, \ldots)$ such that $d_i \in \{0, 1\}$, for each $i = 1, 2, \ldots$. Consider the family of graphs $\mathcal{G}$ whose n-th term is the symmetric group S_n on the n letters $1, 2, \ldots, n$. A corresponding generating set $\Sigma_n = \{s_1, s_2, \ldots, s_n\}$ of S_n may be obtained via D as follows. If $d_n = 0$, let $s_n = (1\ n)$; if $d_n = 1$, let $s_n = (1\ n)(2\ (n-1)) \cdots (\lfloor n/2 \rfloor\ \lceil n/2 \rceil)$. Then Lemma 1 applies with $\Gamma = S_n$, $S = \Sigma_n$ and $u = d_n$. As a result, we get not only an E-set C_n in each corresponding graph $(S_{n+1})_{\Sigma_{n+1}}$ but we also have the corresponding chain of the graphs $(S_n)_{\Sigma_n}$ as an E-chain. The star graphs are obtained in the way indicated above from the constant null sequence, while the pancake graphs are obtained from its complementary sequence, the all-ones sequence. □

4 Extending Results to Partitions

We extend the definitions given in Section 1, to partitions of the Cayley graphs involved into E-sets. Then the results in Section 2 are subsequently extended.

Given $G = (V, E)$, if $C^1, \ldots, C^r$ are pairwise disjoint E-sets of G such that $V = \cup_{i=1}^r C^i$, then we say that $\{C^1, \ldots, C^r\}$ is an E-partition of G. An E-chain $\mathcal{G}$ is said to be *split* if each of its component graphs, say G_i, has a partition into E-sets $C_i^1, C_i^2, \ldots, C_i^{r_i}$. If $r_i = i$, ($r_i = 2^i$), for every positive integer i, then we say that the split E-chain $\mathcal{G}$ is *linearly*, (2-*exponentially*), split. The classical Hamming codes constitute a 2-exponentially split E-chain. However, nonlinear Hamming codes may not yield split E-chains.

Theorem 5 *For every infinite sequence D, the E-chain $\mathcal{G}^D$ is linearly split.*

Proof. ¿From the arguments in Section 2 it is seen that for each positive integer n an E-set $C_n = C_n^{n,D}$ is given by the set of vertices $(a_1, \ldots, a_n)$ of $G_n = G_n^D \in \mathcal{G}^D$ for which $a_n = n$. Similar arguments hold for those vertices $(a_1\ \ldots\ a_n)$ of G_n^D for which a_n is equal to any fixed integer $i \in \{1\ \ldots\ (n-1)\}$, yielding corresponding E-sets $C_n^{i,D}$, for $i = 1, \ldots, (n-1)$. These together with $C_n^{n,D}$ determine the desired partition of $\mathcal{G}^D$. □

Remark 6 Another way to put the last result is to recognize that G_n^D possess a partition into copies $n_1, n_2, \ldots, n_n$ of G_{n-1}^D, expressed by means of the immediate extension of the notation used in Figures 1 and 2. Then it also admits a partition into edgeless (or empty) subgraphs $N(n_1), N(n_2) \ldots, N(n_n)$, which can also be written as $C_n^{1,D}, C_n^{2,D}, \ldots, C_n^{n,D}$. This is due to the neighborliness of $\mathcal{F}^{\mathcal{D}}$. This neighborly effect is produced in the E-chains considered because all of their component graphs are Cayley graphs whose associated group generators are involutions, which divide the edge set in each instance into 1-factors, colored each one by a respective involution generator. Each G_i attaches itself to $G_{i+1} \setminus G_i$ via edges in a common 1-factor, whose endvertices in $G_{i+1} \setminus G_i$ have first coordinate k when $G_i = n_k$, with the notation used in the cited figures. These endvertices are the constituents of a corresponding E-set $C_n^{k,D} = N(n_k)$, a subset in the second partition mentioned above. We can even say that the partitions of G_n^D into $n_1, n_2, \ldots, n_n$ and into $C_n^{1,D}, C_n^{2,D}, \ldots, C_n^{n,D}$ are 2-*orthogonal*, in the sense that each n_j and $C_n^{k,D}$ intersect exactly in 2 elements, if $j \neq k$, and do not intersect, otherwise.

Theorem 7 *For each infinite binary sequence D whose support is nonempty, the E-chain $\mathcal{G}^D$ becomes linearly split in a unique way. Moreover, the only E-chain $\mathcal{G}^D$ that becomes linearly split in more than one way is the star-graph E-chain.*

Proof. If $D = (d_1, d_2, \ldots, d_n, \ldots)$ is the null sequence, then arriving to G_n^D from the sequence of previous terms G_i^D in $\mathcal{G}^D$ could not only be done by means of the successively attaching permutations $u = u_i = (1\ i)$, for increasing values of $i \leq n$, as done in our proofs, but in any arbitrary order, due to the homogeneity in form of these permutations u. In particular, in Figure 1, instead of obtaining the E-partition given by the E-sets formed by the neighbors of $4_1, 4_2, 4_3, 4_4$, respectively, (the latter one being indicated in the figure), we could get the E-partition given by the E-sets formed by the neighbors of i_1, i_2, i_3, i_4, where $i = 2$ or 3. However, if $D = (d_1, d_2, \ldots, d_n, \ldots)$ has a term $d_n = 1$, then each G_i^D with $i \geq n$ cannot have an E-partition different from the one obtained with our arguments above, as can be seen in the particular case of Figure 2, where we are only able to have copies $4_1, 4_2, 4_3, 4_4$ of PC_3. □

5 Tree-Transposition Graphs

The star graphs belong to the general family of tree graphs, (or tree-transposition graphs), whose hamiltonicity was proved in [4, 6]. Since a star is a tree, then a star graph is a tree graph. More specifically, given a tree T on n vertices labeled by the integers $1, 2, \ldots, n$, for each edge $e = ij$ in T let τ_e be the transposition $(i\ j)$ in the symmetric group S_n of permutations on n symbols. It is well-known, ([2]), that the resulting set of transpositions, which we denote by $T_\tau = \{\tau_e,\ e \in E(T)\}$, generates the entire symmetric group S_n. The T-graph, that is the tree graph associated to T, is the Cayley graph $\Gamma_T = \Gamma_{T_\tau}$ on $\Gamma = S_n$ with respect to the transpositions in T_τ. When T is a star with $n-1$ leaves, the resulting graph is the star graph ST_n.

For any sequence $T_1 \subset T_2 \subset \ldots \subset T_n$ of trees such that T_i is obtained from T_{i+1} by deleting one of its leaves, the corresponding tree graphs $\Gamma_{T_1} \subset \Gamma_{T_2} \subset \ldots \subset \Gamma_{T_n}$ form a chain of graphs, each of which verifies the sphere-packing condition.

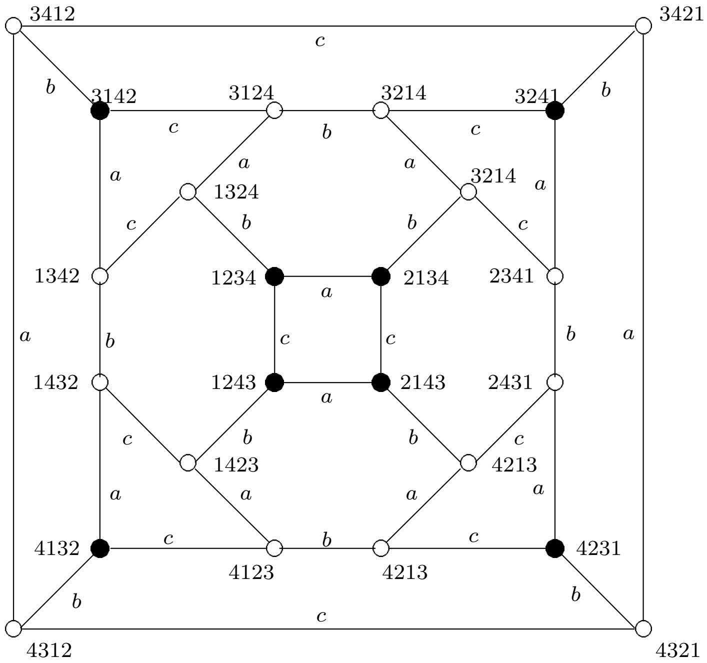

Figure 3: The 3-path graph Γ_{P_3} is a truncated octahedron graph.

However, we suspect that they form an E-chain if and only if T_n is a star. Moreover, we conjecture that given a tree T, the T-graph has an E-set if and only if T is a star.

To support this conjecture, notice that:

1. Every tree T is not a star if and only if contains a 3-path P_3.

2. The P_3-graph Γ_{P_3} coincides with the truncated octahedron graph (see Figure 3, with a, b, c as edge labels for generators $(1\ 2), (2\ 3), (3\ 4)$, respectively) which contains a maximum number of isolated vertices in a perfect dominating set realizing $\gamma(P_3) = 8$, equal to 4; in fact such a perfect dominating set induces forcedly an additional connected component which is a 4-cycle.

The above conjecture, if proved, would show that the star graphs enjoy a relatively rare property.

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