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A Geometric Model of Information Retrieval Systems

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ABSTRACT

This decade has seen a great deal of progress in the development of information retrieval systems. Unfortunately, we still lack a systematic understanding of the behavior of the systems and their relationship with documents. In this paper we present a completely new approach towards the understanding of the information retrieval systems. Recently, it has been observed that retrieval systems in TREC 6 show some remarkable patterns in retrieving relevant documents. Based on the TREC 6 observations, we introduce a geometric linear model of information retrieval systems. We then apply the model to predict the number of relevant documents by the retrieval systems. The model is also scalable to a much larger data set. Although the model is developed based on the TREC 6 routing test data, I believe it can be readily applicable to other information retrieval systems. In Appendix, we explained a simple and efficient way of making a better system from the existing systems.

1 Introduction

Presently, internet is being more and more frequently used for information retrieval for a wide variety of activities ranging from routine tasks such as shopping and reading newspaper to esoteric researches. As the internet was about to become ubiquitous and grow at an explosive rate, in 1992, National Institute of Standards & Technology, Information Access & User Interfaces Division(IAUI) and Defense Advanced Research Projects Agency (DARPA), joined to start a conference named "TREC (Text Retrieval Conference)" for exchanging technology and research. One of the most exciting events in TREC conference is the performance (of retrieving relevant documents) competition of participating institutes all over the countries. To get some idea, we need to know the following competition procedure.

Step 1. To thirty-one participating systems, TREC 6 provided

i) 47 topics

ii) 120,653 documents, which each system was supposed to search through for picking up relevant documents, for each of 47 topics

Step 2. Thirty-one computer systems submitted 1000 ranked retrieved documents for each topic (a document of rank 1 is considered as the most relevant by a system). The total is 31 x 47 x 1000 lists of pairs of topic and document id.

Step 3. For each topic, TREC collected top 100 documents for each system and judged to their relevance to the given topic. For example, for topic 1, if F128 is retrieved by system att97re as 20th document, it judged whether the document for topic 1 was relevant. The performance results of 31 systems were announced. This is the data I worked on and analyzed.

For some topics, most of systems did not do well in retrieving relevant documents. As a matter of fact, there were a few relevant documents to be retrieved and it is natural for systems to choose sufficient number of relevant documents for determining a certain pattern of behavior. In other words, the collection of documents is not suitable for some topics. For testing the model, this is the reason why we chose the best five systems and six topics.(See Section 4).

2 A Geometric Linear Model

For a set of retrieval systems and a set of relevant documents for a topic, we describe a geometric approximation model. Before we go into abstract setup, it is good to see the overall picture: For example, suppose that there are three systems, A, B, C and a topic, "car". And suppose that there is a collection of 1000 documents. Let those A, B, and C retrieve 100 documents, about car, from the collection. Then by the lemmas below, we can calculate all the angles, in this case, three angles and three ratios (a kind of relative detection power for each pair of systems). Now, if we have the same three systems and a collection of one million documents (Remark: This collection has nothing to do with the collection of one thousand documents), choose any one of those A, B, C system and let it retrieve relevant documents from those million documents. Let the number of relevant documents retrieved be n. Then we may estimate all the numbers of relevant documents retrieved by the remaining two systems, by multiplying ratios calculated in the previous steps. Moreover, we may estimate the total number of relevant documents retrieved by three systems.

Now, we are ready for the model. Let R^n be the Euclidean n-dimensional space with the usual inner product, i.e.,

$$v \cdot w = \sum_{i=1}^{n} v_i w_i$$

where $v, w \in R^n$ and n is an integer.

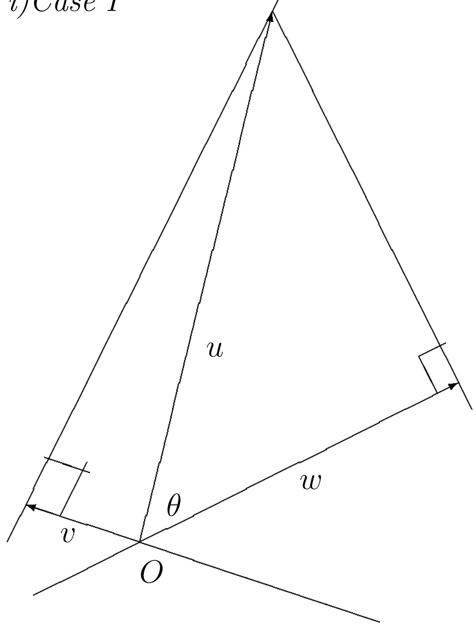
Assumption 1. For each topic, in R^n each retrieval system is represented by a line passing through the origin and a set of all lines(i.e. systems) are linearly independent.

Assumption 2. The set of relevant documents is represented as a vector, which we call the *relevant set vector* . Moreover, consider the projection of the relevant set vector to the line represented by a system. We denote it by $Pr(v)$,where v is the *relevant set vector* and

Assumption 3. Assume its length (the absolute value of $Pr(v)$, |Pr(v)|) as an approximation of the number of relevant documents retrieved by the system. Before we make the fourth assumption in section 3, we need to mention the following useful lemma. By using it, given the angle between a pair of projection vectors, we may estimate the number of relevant documents retrieved by the pair.

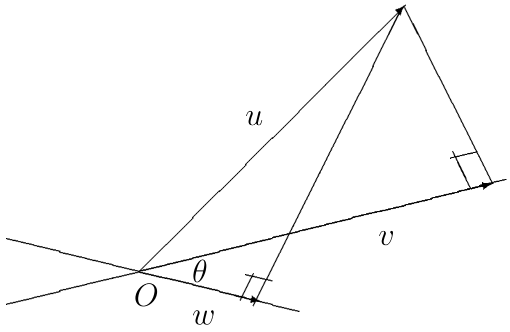
Lemma 1 *Suppose that there are a vector u in a plane and two straight lines through the origin as below. Let two vectors v and w be the projections of u to those two lines and θ be the angle between those vectors v and w . Then u may be expressed as follows*

i)Case I



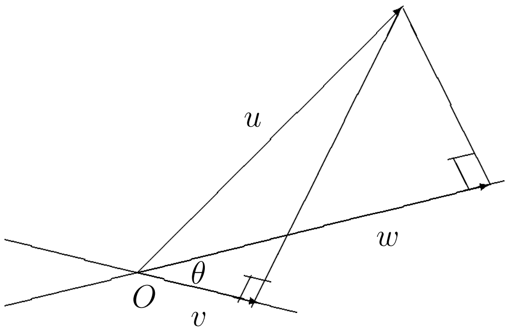
$$u = \frac{\sqrt{|u|^2 - |v|^2}}{\sin\theta}\frac{w}{|w|} + \frac{\sqrt{|u|^2 - |w|^2}}{\sin\theta}\frac{v}{|v|}$$

ii)Case II



$$u = \frac{\sqrt{|u|^2 - |v|^2}}{\sin\theta}\frac{w}{|w|} - \frac{\sqrt{|u|^2 - |w|^2}}{\sin\theta}\frac{v}{|v|}$$

iii)Case III



$$u = -\frac{\sqrt{|u|^2 - |v|^2}}{\sin\theta}\frac{w}{|w|} + \frac{\sqrt{|u|^2 - |w|^2}}{\sin\theta}\frac{v}{|v|}$$

and

$$|u| = \frac{\sqrt{|v|^2 + |w|^2 - 2|v||w|\cos\theta}}{\sin\theta}$$

.

Proof) Refer to any calculus book.

3 Behavior of the systems in TREC 6

According to TREC 6 data, for each topic, it is evident that all systems showed surprising patterns in retrieving relevant documents ([1] and [2]). Here is one of such patterns, which motivated us to write this paper and lead to the last and most important assumption.

Notations : For topic t, let $a_1(r,t)$, $a_2(r,t)$ and $a_{12}(r,t)$ be the accumulated numbers of relevant documents retrieved by system1, system2, and by both, up to certain stage r (In our setting, r represents rank.) More precisely, if we have system1 and system2 retrieved relevant documents, for a topic t, as follows:

r	**system1**		**system2**	
1	F234	1	F345	1
2	F345	1	F1789	0
3	F4	1	F56	1
4	F78	1	F3590	0
5	F1789	0	F23	1
6	F23	1	F4	1
7	F57	0	F983	0

where in the third and fifth columns, 1 means "relevant" and 0 means "irrelevant", then

$a_1(1,t) = 1$, $a_1(2,t)$=2, $a_1(3,t)$=3, $a_1(7,t)$=5, $a_2(1,t)$ =1, $a_2(2,t)$= 1.....$a_2(7,t)$= 4, and $a_{12}(1,t)$=0, $a_{12}(2,t)$=1.......,$a_{12}(7,t)$=3.

Remarkably we have found that the ratio of $a_1(r,t)$ to $a_2(r,t)$ is almost constant over all r and, even so is the ratio of $a_{12}(r,t)$ to $a_2(r,t)$. For example, for the topic 62, if we run a linear regression for the numbers of relevant documents retrieved by two systems -*Cor6R1cc* and *ETH6R2*- up to rank (or r) 100, the graph has the slope 0.95($= a_1/a_2$) and 0.58($= a_{12}/a_2$) with high accuracy, i.e. rsquare 0.9994 and 0.9910 respectively. This implies that the ratio, so called '*relative detection powers*' of those two systems, is almost constant. The results are similar for all topics. From this remarkable observed facts, we assume the most interesting assumption.

Assumption 4. For each topic, all the ratios of $a_1(r,t)$ to $a_2(r,t)$ and $a_{12}(r,t)$ to $a_2(r,t)$ are constant over all r.

4 Application

In this section, we apply the model (which consists of 4 **assumptions**) to calculate the total number of relevant documents retrieved by several systems. To get an idea, it is enough to consider three systems, since we may reduce any cases to the case of two systems.

Suppose that there are three systems(i.e. lines) s_1, s_2, s_3 their relative detection powers(i.e. ratios of $a_1(r,t)$ to $a_2(r,t)$ and $a_{12}(r,t)$ to $a_2(r,t)$) are known for each topic. Suppose that a relevant set vector lies in the space spanned by the three systems. Then, given the number of relevant documents retrieved by any one of the three systems for some topic, by applying the model above, we may estimate the total number of relevant documents retrieved by the three systems for the given topic, as follows. To do it, first we need to calculate *cosine* value of the angle between the lines represented by systems(More precisely, the angle between the projection vectors of the given *relevant set vector* to the lines).

Lemma 2 *Let a_1, a_2 and a_{12} be the numbers of relevant documents retrieved by system1, system2 and both of two systems. Let θ be the angle between two systems. And let k and ρ be the ratios $\frac{a_1}{a_2}$ and $\frac{a_{12}}{a_2}$ respectively. Then $a_1 = ka_2$ $a_{12} = \rho a_2$, and we have*

i)Case I

$$\cos\theta = \frac{a_1a_2 - \sqrt{(a_1+a_2-a_{12})^2 - a_1^2}\sqrt{(a_1+a_2-a_{12})^2 - a_2^2}}{(a_1+a_2-a_{12})^2}$$

or

$$= \frac{k - \sqrt{(k+1-\rho)^2 - k^2}\sqrt{(k+1-\rho)^2 - 1}}{(k+1-\rho)^2}$$

ii)Case II and III

$$\cos\theta = \frac{a_1a_2 + \sqrt{(a_1+a_2-a_{12})^2 - a_1^2}\sqrt{(a_1+a_2-a_{12})^2 - a_2^2}}{(a_1+a_2-a_{12})^2}$$

or

$$= \frac{k + \sqrt{(k+1-\rho)^2 - k^2}\sqrt{(k+1-\rho)^2 - 1}}{(k+1-\rho)^2}$$

Proof) It follows from the *cosine* sum formula to two triangles in the figures of **lemma 1**.

Suppose v_i is the projection vectors of systems and n_i is the number of relevant documents retrieved by system s_i for $i = 1, 2, 3$, in other words, $|v_i| = n_i$.As we mentioned in the **section 3, behavior of the systems in TREC 6**, we can obtain all the k and ρ values for all pairs of systems for each topic with the help of a statistical software such as SAS. Suppose only n_1 is known. Then n_2 and n_3 are estimated as $n_1$times proper k. By the **lemma 1**, we get the *relevant sum vector* u (in the plane generated by $v_1$and v_2) of two projection vectors v_1, v_2. Again by applying **lemma 1** to u and v_3, we get the *relevant sum vector* w(in the three dimensional space generated by v_1, v_2, v_3) for the v_1, v_2, v_3 and the absolute value of the sum vector w will be the total number of relevant documents retrieved by the three systems. In this way, this procedure can be applied to any finite number of systems.

To test the geometric model, we chose the best five(Cor6R1cc, ETH6R2, att97re city6r1 pirc7R2) out of 31 systems in TREC 6, and seven topics (189, 161, 111, 10002, 62, 54, 154) of most relevant documents. By assuming *Case I* for all pairs, the following results are obtained.

Estimation by the Geometric Model

Topic	**One**	**Two**	**Three**	**Four**	**Five**
54	89	100.643	106.624	110.110	125.084
62	81	116.067	123.336	181.108	312.631
111	74	137.786	160.335	194.045	263.126
154	83	110.412	113.516	121.442	124.426
161	71	78.726	87.896	102.343	110.606
189	90	150.801	180.346	247.611	286.277
10002	69	102.358	115.578	119.521	120.998

Numbers of True Relevant Documents

Topic	**One**	**Two**	**Three**	**Four**	**Five**
54	89	101	111	113	114
62	81	116	129	170	217
111	74	141	162	180	208
154	83	113	117	128	130
161	71	80	91	102	102
189	90	152	174	200	216
10002	69	103	114	121	126

Here *One* means the total number of documents by Cor6R1cc, *Two* by Cor6R1cc and ETH6R2, *Three* by Cor6R1cc ETH6R2 and att97re, *Four* by Cor6R1cc ETH6R2 att97re and city6r1, *Five* by Cor6R1cc ETH6R2 att97re city6r1 and pirc7R2.

5 Conclusion

As the table in **section 4** showed, the geometric model works well for three-systems, for all topics (Note that, since we get the angle from the number of relevant documents retrieved by two systems, the model is expected to fit well with two systems). To make this geometric model work better for all topics and more than three systems, further investigation is required to find the best *combination of cases* (See cases in **lemma 1** and recall that in our experiments, we assume *case 1* for all). Note that, to each system, by associating a line in an Euclidean space, a new concept of "independence" of systems was introduced. In other words, if the set of representing lines is linearly independent, then we might say that the corresponding systems are independent. It seems that there is a subtle point in this concept, because all systems have a tendency to resemble each other. In **Appendix**, we present a very simple and interesting scoring method, in other words, describe a way of making a new system from the existing systems. The experimental results say that this method, i.e., new system, performs better than each individual system, the principle of which is widely applicable.

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References

[1] Paul Kantor, Myung Ho Kim, Ulukbek Ibraev, Koray Atasoy(1999). *Estimating the Number of Relevant Documents in Enormous Collections*, Proceedings of the Annual Meeting of the American Society for Information Science, Nov. 1999.

[2] TREC Web Site, http://trec.nist.gov.

A On a Scoring Method of Retrieved Documents by Systems

We introduce a new scoring algorithms based on the ranks given by systems for better results over all individual systems. Since each system has its own scoring method and it needs to be normalized in some way. The ranks given by systems are our natural choice and here is our definition of the new scoring method.

A.1 Definition of scoring and results

Definition 3 *Given a pair of topic and document, first, we interpret 100-rank (or 101-rank) as its score given by each system, if ranks of a pair are less than or equal to 100, otherwise 0. Second, compute the average of the scores and assign it to the score of the document for a given topic. By the definition above, we re-scored all pairs (document id, rank) judged by TREC 6 and chose top100 documents for each topic.*

Here is the table by our new scoring method.
T : topic
f_{31}: the system of combining thirty-one, i.e., by averaging all scores from 31 systems
B_{31} : number of systems which perform better than f_{31} among 31 systems
f_6 : the system of combining the six best systems, i.e., by averaging the scores from only the best six systems
B_6 :number of systems which perform better than f_6
G :number of relevant documents retrieved by the whole 31 systems

T	f_{31}	B_{31}	f_6	B_6	G	**T**	f_{31}	B_{31}	f_6	B_6	G
1	39	0	35	4	51	123	44	2	43	2	62
3	45	3	48	0	70	125	18	3	23	0	27
4	41	4	45	3	70	126	15	1	15	1	19
5	6	0	6	0	7	128	65	2	70	1	281
6	54	5	55	4	146	142	57	1	51	3	200
11	48	7	45	7	150	148	79	11	86	11	241
12	78	6	81	2	270	154	88	6	92	0	168
23	4	3	4	3	6	161	84	6	88	2	118
24	11	14	17	3	34	173	11	3	15	0	16
44	4	0	4	0	4	180	2	9	5	5	17
54	94	1	92	3	164	185	16	2	16	2	18
58	16	2	17	1	18	187	10	7	10	7	19
62	86	6	85	7	368	189	91	4	90	5	667
77	13	1	13	1	16	192	7	0	4	13	7
78	40	1	39	1	45	194	3	0	3	0	4
82	61	0	59	2	80	202	87	5	88	4	534
94	48	3	49	3	174	228	29	4	28	5	58
95	36	5	38	1	123	240	29	0	25	2	121
100	80	4	82	4	179	282	16	0	17	0	28
108	71	5	77	2	293	10001	70	1	66	2	127
111	90	5	91	1	480	10002	73	3	72	3	267
114	37	1	38	1	57	10003	47	3	55	0	72
118	38	0	36	2	83	10004	12	4	14	0	17
119	20	5	22	2	76						

B Implications

First, information retrieval systems are more or less independent, which means they retrieve documents in their own way. Second, more importantly, systems tend to choose relevant documents more than irrelevant ones, i.e., give higher scores to relevant ones (standard deviation are smaller). This observation has an intuitive appeal if we believe that noise, physical or informational, tend to be uncorrelated. This is believed to be the reason that f_{31} and f_6 systems do better than each individual system.