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A Minimax Game of Power Control in a Wireless Network under Incomplete Information

by

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ABSTRACT

The purpose of this paper is to study optimal transmit power control and capacity of a wireless multimedia network, where there is incomplete information about the link gain coefficients. Previously, Perron-Frobenius theory has been applied to the study of optimal power control under complete information about the link gain matrix and the interference matrix. This paper derives a characterization of optimal power vector and capacity in terms of an incomplete-information-matrix-game against nature choosing the elements of the link gain and interference matrices. A game theoretical rationale for not using power control is given.

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1 Introduction

Future wireless multimedia networks provide new challenges to radio resource management. The key problems in multimedia-type systems with mobile users having diverse requirements are the time varying data rates, delay constraints and insufficient information. The purpose of this paper is to study the optimal capacity (signal-to-noise ratio) of a wireless network under incomplete information.

The purpose of the current paper is to extend the Perron-Frobenius result of optimum transmit power control [20] to the case of incomplete information about the channel parameters. Previously, [11] suggests the modeling of power control in terms of a minimax matrix game against nature. This paper focuses on extending the results in [11] to the case of incomplete information about the elements of the gain and interference matrices. The model describes the uplink of a mobile radio network that consists of m users transmitting to n receivers. Thus, uncertainty applies to all $m \times mn$ elements of the gain and interference matrices.

The paper is organized as follows. Section 2 discusses the modeling cellular network capacity using game theory. Section 3 presents a model of a multimedia network extending the minimax model due to von Neumann to the case of incomplete information about the matrix game. The model yields a rationale for not using power control when very little is known about the system and thus participates in the discussion about power control vs. no power control (c.f. [4]). Section 4 concludes.

2 Game-theoretical Models of Power Control

2.1 Models of Pricing, Power Control and Resource Management

Game theory due to [18] emerged as the new mathematical language for economics in 1946. The situation prevalent in economic systems is the one where there are multiple agents (i.e. consumers, firms) each individually attempting to maximize the individual payoff functions (utility functions and profits). Game theory extends optimization theory to the study of a multiplicity of (possibly contradictory) payoff functions and possible equilibria, game-theoretical equilibria often being defined in terms of an equilibrium price (i.e fixed point of the excess demand functions). An economics framework for the optimal transmit power control of a wireless network has been studied i.a. by [12]. A recent review of the economics of data networks is in [3]; both noncooperative game theory and cooperative game theory are being applied to the study of pricing and resource allocation in communication networks. Noncooperative game theory seeks to study stable distributed systems where the players are not able to communicate whereas cooperative game theory concentrates on the study of the efficient and fair sharing of the payoffs of various coalitions of players such that the players are allowed to communicate.

In the introduction of "Theory of Games and Economic Behaviour" [18] von Neumann and Morgenstern consider economics as a candidate for natural sciences [19]; the book changed the way economics was done since, and also influenced other discipines. The use of game theory in fields other than economics is in line with the some philosophical ideas of von Neumann, e.g. multidisciplinarity and there being a unifying force behind all manifestations of nature [19]. "The Theory of Games" is mainly concerned with establishing noncooperative game solutions; section III constructs a noncooperative solution to transmit power control such that the noncooperative max-min solution coincides with a Pareto-optimal (cooperative) solution that is both efficient and (max-min)-fair.

This paper focuses on studying a game-theoretical model of power control of a wireless network under incomplete information. The extension of Nash-equilibria to the case of incomplete information is due to Harsanyi [10] whose main idea of nature taking the form of various types is being followed in section 3.

2.2 Games against Nature

Let the coefficient $g_{i,j} = (\mathbf{G})_{i,j}$ denote the fading of transmitted information signals by user i to receiver j that accommodates transmissions of datatype j (where i refers to the row of users and j to the column of cells). The link gain matrix $\mathbf{G}$ thus describes the instantaneous propagation conditions in the system. Denote the sequence of the powers of the transmitted signals by $(x_i)_{i=1}^{nm}$, and thus the sequence of the received signals by $(\sum_k g_{i,k} x_{i,k})_{i=1}^{m}$. Analogously to $\mathbf{G}$, $f_{i,j} = (\mathbf{F})_{i,j}$ denotes the coefficient of the energy of interference received by user i at cell or receiver j.

Formally, the power control problem then is to find powers that satisfy the signal to noise inequalities

$$\mathbf{G}\mathbf{x} \geq \alpha \mathbf{F}\mathbf{x}. \tag{1}$$

where α is the required signal-to-noise-ratio SNR, and where the thermal noise N is neglected.

Allowing each user the have the simultaneous access to a multiplicity of receivers (datatypes) suggests exteding the Perron-Frobenius theory of power control to a more general minimax power control game against nature; the minimax solution is argued to correspond to a particular Perron-Frobenius control vector (see [11]). In other words, behind each optimum power vector and optimal capacity γ^* as derived by Zander [20], there exists a game against nature choosing a state vector $\mathbf{p}$. More specifically, the game is defined by two players: power vector selector choosing the vector of transmit powers $\mathbf{x}$ and nature choosing a distribution $\mathbf{p}$, i.e. a mixed strategy as row player; the unique minimax value of the minimax game is the capacity of the system,

$$\alpha^* = \max_{\mathbf{x}} \min_{\mathbf{p}} \frac{\mathbf{p}'\mathbf{G}\mathbf{x}}{\mathbf{p}'\mathbf{F}\mathbf{x}}. \tag{2}$$

Since both α^* and γ^* correspond to SNR, i.e. capacity-equalization, $\alpha^* = \gamma^*$. The minimax solution to trasnmit power control is one where both nature and power selector will play mixed strategies. Under traditional view, mixed strategies are seen as conscious randomizations [14, 1]. But there is also a different view of mixing, relating the optimality of mixed strategies to uncertainty about the actions chosen by the other players [10, 14]. The mixing then represents this uncertainty and corresponding conjectures about the other players choices. [14]

Suppose we start with a capacity game described by (2) with a unique mixed-strategy equilibrium in which the players actually randomize. Now change the game by introducing many different types of each player such that all types of a player have identical preferences (same utility functions, see III.A). A pure strategy for the power selector in a Bayesian game is a function $a_i(g_i) = x_i$ or decision rule that gives player's strategy choice for each realization of his type g_i. Power selector's pure strategy set is therefore the set of all such functions. A pure strategy Bayesian Nash Equilibrium [8] is then precisely equivalent to a mixed strategy Nash equilibrium of the original complete information game. Section 3.1 proves this in the case of a matrix game representing optimal transmit power control.

3 Power Control of a Multimedia Network under Incomplete Information

Below we extend the minimax model [11] (see eq. (2)) to study the capacity/resource allocation problem as a game under incomplete information. Previously, stochastic approximation techniques have been applied in the analysis of power control under uncertainty [17]. Here, uncertainty is modeled as a minimax matrix game against nature.

3.1 A Mixed-Strategy Solution to Power Control

Define a strategic form game by set of players $M = \{1, ..., m\}$, together with an action set A_i for each player i. Set $A := A_1 \times ... \times A_m$. A mixed strategy of player i assigns to each pure strategy (action) a probability that it will be played (such that the probabilities sum to one). This section seeks to verify that a mixed strategy equilibrium capacity can be interpreted as the optimal outcome from an incomplete information game against nature. I.e, given a mixed strategy Nash equilibrium with complete information, a belief system can be constructed such that the Nash-equilibrium transmit power vector coincides with a Bayesian Nash-equilibrium vector that emerges from a game against nature under incomplete information. This result will be used in III.B to generalize the definition of an optimal capacity to the case of randomly perturbed games, formed by adding a random component to each element of $\mathbf{G}$.

Following [1], define a *belief system* to consist of:

- for each player i, a set S_i (i's types), and for each type s_i of i,
- a probability distribution on the set S^{-i} of (($m-1$)-tuples of) the types of the other player(s) (s_i's theory)
- an action a_i of i (s_i's action) and
- a function u_i, i.e. $u_i : A \to R$ denoting the payoff function of type (user) i.

A function $u : A \to R^m$ (an m-tuple of payoff functions) is called a game. Set $S = S_1 \times ... \times S_m$. Call the members $s = (s_1, ..., s_m)$ of S the states (of the world). s_i's action is denoted by $\mathbf{a}_i(s)$. A function like $\mathbf{a}_i, \mathbf{u}_i$ and $\mathbf{u}$ defined on S can be interpreted as random variables in probability theory. For instance, in terms of the matrix game defined in II.B, $[\mathbf{a}_i = a_i]$ (or a_i) denotes the event $\{s \in S : \mathbf{a}_i(s) = a_i\}$. Let $a_i = x_i^* = \frac{1}{g_{ik}^*}$, where g_{ik}^* denotes $\max\{g_{i1}, .., g_{in}\}$; then the corresponding state of the world $\bar{s}$ is a pair of types of the power selector and nature such that power selector is of type i and nature is of type k. Thus, in equilibrium defined by (2), the state $\bar{s}$ corresponds to the event g_{ik}^*. This allows to interpret $\mathbf{p}$ as a probability distribution on the link gain elements, if each type corresponds to a distinct user. In general, the construction of types can be based on Kolmogorov's existence theorem from the theory of stochastic processes [7, 5].

The types of each player correspond to the different possible outcomes of the mixing of the strategies; each type chooses a different action. It remains to argue that in the SNR-model each type of the power selector corresponds to a user. Suppose a mixed-strategy equilibrium capacity α is given. What determines mixed-strategy equilibrium probabilities is the equilibrium consideration: the need to make the *other* player indifferent over his strategies. Abstract from the interference matrix $\mathbf{F}$ describing externalities not taken into account in individual optimization [14]. Nature is the row player choosing $\mathbf{p}$ such that $\sum_{i=1}^{m} p_i = 1$ in the matrix game defined by $\mathbf{G}$ and the power selector is the column player choosing $\mathbf{x}$ such that $\sum_{i=1}^{mn} x_i = k$, where k is a given constant. Since there are more columns than rows it can be optimal for the column player to skip some columns. Indeed, the various types of the power selector optimally play $a_i = x_i^*$ since this choice of strategies will make nature indifferent between his types; if the received transmit power is the same for all users as implied by the optimal power control rule x_i^*, nature will be indifferent between playing the various rows. Thus, each type i of the power selector can be identified with a row of user i.

Given a mixed strategy equilibrium capacity, the mixed strategies can be interpreted as follows: the power selector has various types, representing the different users. Nature has various types $1, ..., m$ representing the gain coefficients $i = 1, ..., m$ for the different users, with $p(i) > 0 \; \forall \; i = 1, ..., m$. Nature's pure strategy set is therefore the set of all possible coefficients. A pure strategy for the power selector in a Bayesian game is the decision rule, or action a_i, that gives the player's strategy choice a_i for each realization of his type. For the power selector in the SNR-model, $a_i = x_i \; \forall \; i$. Define the power selector's pure strategy set $L = \{x_i = \frac{k}{g_i}\}$. In general L_i is the set of player i's all decision rules. Power selector's expected payoff is then given by

$$\tilde{u}_P = E[u_P(a_1(g_1), ..., a_m(g_m))] = \hat{F}\mathbf{p}'\mathbf{G}\mathbf{x}, \tag{3}$$

where $\hat{F}$ is the expected value of the inverse of interference.

Definition 1 *A (pure strategy) Bayesian Nash equilibrium for the Bayesian game* $[M, \{A_i\}, \{u_i\}, S]$ *is a profile of decision rules* $(a_1, ..., a_m)$ *that constitutes a Nash-equilibrium of game* $\gamma = [M, \{L_i\}\{\tilde{u}_i(.)\}]$. *That is, for every* $i = 1, ..., m$*: [14]*

$$\tilde{u}_i(a_i(.), a_{-i}(.)) \geq \tilde{u}_i(a'_i(.), a_{-i}(.)) \; \forall \; a' \in L_i. \tag{4}$$

In a Bayesian-Nash-game that corresponds to a mixed strategy Nash-equilibrium, all types of a player have the same payoff function u_i; each player (nature and power selector) will observe only his own type and hence payoff function (c.f. [9]).

Proposition 1 *Given a mixed Nash-strategy equilibrium α (eq. (2)), there exists a Bayesian capacity game with incomplete information against nature, with two players: nature choosing* $\mathbf{p}$ *and the transmit power selector choosing the vector of transmit powers. I.e., A Bayesian Nash Equilibrium in pure strategies is the same as a Nash-equilibrium α (eq. (2)) in mixed strategies.*

Proof 1 *In a mixed strategy equilibrium, nature will randomly choose to play row i with probability p_i. Each type i of the power selector corresponds to a user (i). Thus, in effect, nature chooses the type of the user. The equilibrium mixed strategy of nature corresponding to $(x_1^*, .., x_m^*)$=$\mathbf{x}^*$ is that all types have equal probability. Hence, in a game against nature, a user can be of any type equally likely. Hence, a mixed strategy is individually optimal (in addition to being collectively optimal i.e. Pareto-optimal), since by assumption all users (types) have the same payoff (utility) function $u_i = \alpha = SNR$ (like in [11]).*

In a Bayesian-Nash equilibrium a pure equilibrium strategy gives the player's strategy choice for each realization of his type, assuming each player is rational:

Definition 2 *Player i is rational at s if his action at s maximizes his expected payoff given his information (i.e. his type s_i) [1].*

An event E is a subset of S. If E is an event, define $p(E; s_i)$ as the probability that $s_i's$ theory assigns to $\{s^{-i} \in S^{-i} : (s_i, s^{-i}) \in E\}$. A *conjecture* ϕ_i of i is a probability distribution on $A^{-1} = A_1 \times ... \times A_n$. The theory of i at state s yields a conjecture $\mathbf{\Phi}_i(s)$, called i's conjecture at s, given by [1] $\mathbf{\Phi}(\mathbf{s})(a^{-1}) := p([a^{-1}]; s_i)$. Let $\mathbf{\Phi}$ denote $(\mathbf{\Phi}_1(s), ..., \mathbf{\Phi}_m(s))$. Let $\mathbf{g} = (\mathbf{g}_1(s), ..., \mathbf{g}_m(s))$.

Theorem 1 *(Aumann and Brandenburger [1]) Let there be two players, let g be a game and ϕ a pair of conjectures. Suppose that at some state s, it is mutually known that $\mathbf{g} = g$, that the players are rational, and that $\mathbf{\Phi} = \phi$. Then (ϕ^1, ϕ^2) is a Nash equilibrium of g.*

Proposition 1 is a converse of *Theorem 1* in the case of the $SNR-$model. Given a Nash-equilibrium in mixed strategies, there exists a belief system such that the conditions in *Theorem 1* are fulfilled. The belief system is such that the power selector's types, i.e. users believe that nature will play all rows equally likely, and nature believes that the users choose to be received with the same power.

3.2 Power Control Against Nature with Unknown Type

In a mixed strategy equilibrium of a zero-sum game individual rationality can coincide with collective (group) rationality: a user can be chosen to be of any type equally likely, and hence it can be individually optimal to agree with a Pareto-optimal solution to the problem of finding the optimal transmit powers. This section argues that the equivalence between individual rationality and group rationality is stable even when increasing uncertainty.

Now consider extending the game so that nature is allowed to choose any matrix $\mathbf{G}$ (and $\mathbf{F}$) such that the power selector is unable to observe the choice by nature. The question studied in this section is, whether nature can improve its payoff, i.e. decrease α from the above minimax value obtained above (α^*) when both players know the game, i.e which matrices were chosen.

Following [10] define the choice of a particular matrix pair $(\mathbf{G}, \mathbf{F})$ to correspond to a particular "type" nature is playing. Consider first the simple case where nature chooses between appearing in the form of two distinct types: $(\mathbf{G}_1, \mathbf{F}_1), (\mathbf{G}_2, \mathbf{F}_2)$.

For instance, let $\mathbf{G}_1 = \frac{1}{2}\mathbf{I}$ where $\mathbf{I}$ is the identity matrix and let $\mathbf{G}_2 = \mathbf{I}$ (and let $\mathbf{F}_1, \mathbf{F}_2$ denote the corresponding interference matrices). Can nature benefit from privately knowing his type, unrevealed to the power selector?

The minimax solution in [11] suggests that the two types of nature correspond to two exchangeable matrix games; in terms of stochastic utility maximization, both users would agree in exchanging the game with subscripts 1 to get the game $(\mathbf{G}_2, \mathbf{F}_2)$; both games have the same value for the users,

$$\alpha^* = \frac{1}{m-1}. \tag{5}$$

The power selector is thus indifferent between the two games.

Suppose the power selector assumes the two games are equally likely to occur. Would this belief-vector be consistent with a Nash-equilibrium under incomplete information?

The power selector would optimally choose $\mathbf{x} = (\frac{1}{m}, ..., \frac{1}{m})$ in case of both (symmetric) games, to guarantee the same received power $g_i x_i = X$ for all users $i = 1., , m$. An optimal response from nature would be to choose a mixed strategy with $\mathbf{p} = \mathbf{x}$ (uniform distribution). Also, the beliefs of the power selector would constitute "equilibrium beliefs": since both two symmetric games have the same value, nature is indifferent between playing the two types. In fact, with symmetric (w.r.t users) games a pure strategy of nature playing only one type is optimal as well (c.f. exchangeable games).

Figure 1 depicts the extensive form ([14]) for the capacity game; nature N plays first, choosing the game. Second, nature will choose $\mathbf{p}$ for both games, simultaneously with the power selector P choosing transmit powers $\mathbf{x}$ without knowing which game N chose (the dashed line indicates the information set of P, however, N knows the matrix game, assuming perfect recall [14]).

3.3 A Case for no Power Control under Lack of Information

The example in the previous section suggests the following

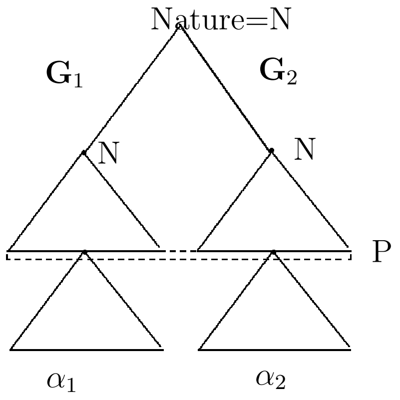

Figure 1: Extensive form capacity game

Proposition 2 *Any transmit power vector* $\mathbf{x}$ *satisfying the condition of being a strictly mixed strategy* $\forall i\ x_i > 0,\ \sum x_i = 1$ *is optimal under a complete lack of information.*

Proof 2 . *Consider the expression for* α^* *under any* $\mathbf{x}$ *satisfying the condition (of strictly mixed strategies) in the proposition. Assume the strategy choice of nature is to play all types (matrix games) with equal probability and to set* $\mathbf{p} = (\frac{1}{m}, ..., \frac{1}{m})$ *in all games. To prove that these strategies constitute a Nash-equilibrium one must show that no player would unilaterally deviate from his strategy.*

Fixing $\mathbf{x}$*, nature obtains from a representative matrix game a value of*

$$\alpha = \frac{\frac{1}{m}\sum_i^m x_i g_i}{\frac{1}{m}\sum_j^m \sum_{j \neq i} x_j g_j}$$
$$= \frac{1}{m-1}.$$

Since this value coincides with α^**, the power selector prefers to play the mixed strategy* $\mathbf{x}$*; the informational disadvantage does not reduce the expected payoff. But by playing any mixed strategy, the power selector can force nature to choose a uniform (entropy maximizing) distribution. Only by choosing the uniform distribution, nature can guarantee the upper bound* α^* *for the value of the game the value of which it tries to minimize. This exemplifies the use of mixed strategies to overcome an informationally disadvantageous position.*

Proposition 2 also follows from *Theorem 1*; in particular, it is the conjectures that constitute the Nash equilibrium: a randomized strategy $\mathbf{x}$ is optimal under the conjecture that nature could have chosen any game. Randomization is rationalized by the conjectures. In Harsanyi's view of mixtures, the mixing represents the uncertainty, the conjecture about the choice of the other player. Unlike the traditional view, this approach to mixed strategies is based on a theory about the positive incentives of the players to play mixed strategies. However, like in the traditional view, the aim of mixing remains to make the other player indifferent between his strategies: nature is indifferent between the various types, when conjecturing that the power selector has this conjecture; the conjectures are mutually consistent and correct. Thus, α^* defines a Nash equilibrium that is also a *weak perfect Bayesian equilibrium*:

Definition 3 *A set of strategies constitutes a weak perfect Bayesian equilibrium (PBE) if two conditions are satisfied*

- *First, strategies must be sequentially rational given beliefs.*
- *Second, beliefs must be consistent with strategies: the beliefs are derived from the strategies using Bayes rule.*

The definition of sequential rationality in the weak Bayesian equilibrium (see [14]) only requires beliefs to be consistent with equilibrium strategies; there are no restrictions on beliefs at information sets not reached with positive probability when playing equilibrium strategies. Strengthenings of the concept PBE (see [14]) improve this shortcoming.

By [11], the minimax equilibrium value of α is unique. *Proposition 2* demonstrates that the equilibrium strategies need not be unique. Under incomplete information, the average user attains the same signal-to-noise ratio as that for all users under complete information. In so far as the welfare of the average user is a good indicator of the reliability of the system, the mixed strategy equilibrium remains optimal under incomplete information. This result is in line with Harsanyi's prediction that under "sufficiently" small perturbations in the payoff functions of the players (here matrices $\mathbf{G}$, $\mathbf{F}$), mixed strategy equilibria remain optimal:

Proposition 3 *The minimax equilibrium α^* is optimal under small random perturbations across the elements of the link gain and interference matrices.*

Proof 3 *By [10], "the players may make no deliberate efforts to use their pure strategies with the probability weights prescribed by their mixed equilibrium strategies. Nevertheless, as a result of random fluctuations in their payoffs, they will in fact use their pure strategies with the prescribed probabilities". By* Proposition 2 *this result holds for minimax matrix games with two nonnegative payoff matrices* $\mathbf{G}$ *and* $\mathbf{F}$ *with the requirement that the equilibrium strategy vector is the same for both games (c.f. equation (2)).*

In conclusion, under lack of information, it can be optimal for the transmit power controller to play any mixed strategy, preferably one with minimal sum of powers. This strategy gives information capacity $I = ln(1 + \frac{1}{m-1})$. A topic for future research would be to allow the players to consider the matrix games $\mathbf{G}$ and $\mathbf{F}$ as separate and allow the players to choose separate strategies accordingly. Also, allowing for repeated games with incomplete information is likely to change equilibria [2].

4 Conclusion

This paper has studied optimum power control of a multimedia network, addressing the issue of optimal transmit power control under incomplete information. A solution based on mixed strategies to the optimal transmit power control under incomplete information is presented. A game-theoretical rationale for no power control is given in the case of lack of information.

A minimax solution to power control is that all users agree to get the same received power; thus, the dsitributed solution is Pareto-optimal. The analysis in this paper is based on the minimax game approach to power control. Applying more advanced techniques based on stochastic programming constitutes a topic for future research.

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