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This equation should be regarded as our best current estimate of the location of the cross-over point.

(atend)bpbpbp=0bp-0

Figure 4: Empirical values for the number of clauses required for cross-over as a function of the number of variables.

4 Conclusion

Our experimental results show that the hardest satisfiability problems seem to be those that are *critically constrained* — *i.e.*, those that are neither so under-constrained that they have many solutions nor so over-constrained that the search tree is small. This confirms the results of [Cheeseman et al. 91, Mitchell et al. 92]. For randomly generated problems, these critically constrained problems seem to be found in a narrow band near the cross-over point. We have determined that the number of clauses required for cross-over seems to be a *linear* function of the number of variables. Our best empirical estimate of this function is $clauses = 4.24vars + 6.21$. Finally, the complexity of our satisfiability algorithm seems to be on average roughly linear below the cross-over point (though even here there are some problems that are as hard as those at the cross-over point). At the cross-over point the complexity seems to be about $2^{n/17}$ (where n is the number of variables). Above the cross-over point we conjecture that the growth rate will be of form $2^{n/k}$ where k is less than 17 and decreases as one gets further from the cross-over point.

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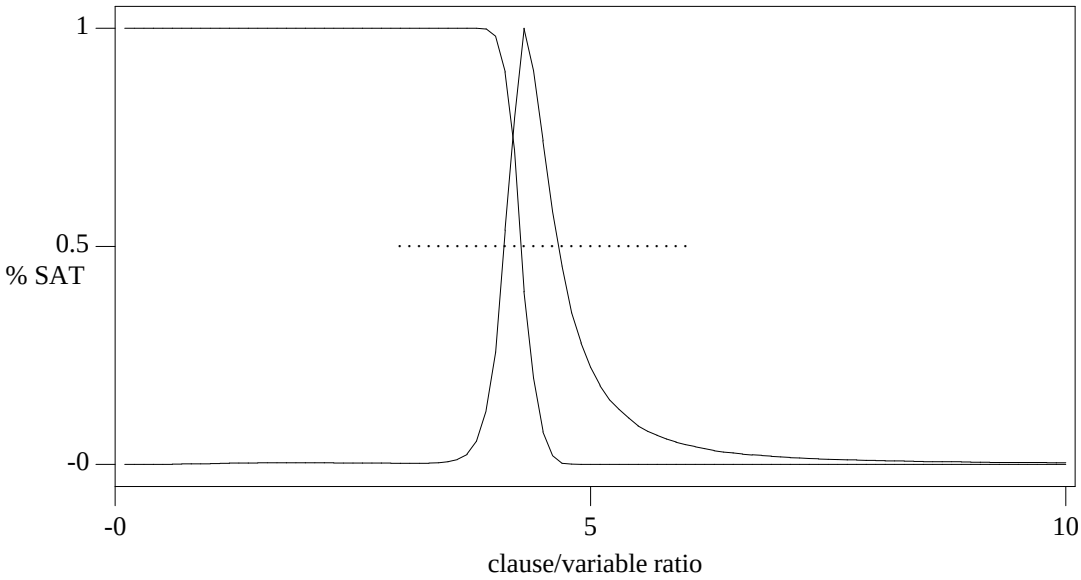
1
0.5
% SAT
-0
-0
5
10
clause/variable ratio