Computational Learning Environments for Discrete Mathematics Education (Preproposal)

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We will address the following NSF priority:

"How the evolving capacity of computers and other technologies enables teachers to individualize instruction and students to engage in the process of experimentation, of understanding, of skills acquisition, and of applying content knowledge."

Introduction

The focus of the proposed research and development is the integration of new software technology into discrete mathematics curricula, both at the undergraduate level and in K-12 mathematics teaching. Towards this goal, we propose to bring together a team of researchers in mathematics, mathematics education, and computer science, representing several different types of institution: an NSF Science and Technology Center, a science and engineering university, a liberal arts college, and various secondary schools.

Discrete mathematics pervades many branches of computer science, computer engineering, and operations research, as well as pure and applied mathematics. Furthermore, the visual nature of discrete mathematics enables it to benefit the education process through the integration of computer technology. However, the absence of adequate software systems has prevented the modernization of education in the field of discrete mathematics.

We intend to introduce LINK, a novel system for discrete mathematics, into the curricula of the two educational institutions and into the K-12 Leadership Program at the Science and Technology Center. We will design and disseminate lesson modules concerning fundamental topics in discrete mathematics, and evaluate their effects on curricula at various levels. All relevant materials will be made available free of charge to instructors and students of discrete mathematics all over the country.

Suitability of the Field and the Institutions

Discrete mathematics, which serves as the foundation for computer science, is in a special position to benefit from advances in computer technology. Aside from its many subfields with clear visual interpretations, it is a great "equalizing" field in the sense that many interesting problems can be understood by a child, yet cannot be solved by the top researchers. The fact that so many interesting problems can be stated and understood without much preliminary study makes the field more promising for investigation by non-experts than the more traditional, well-explored classical mathematics.

The field of discrete mathematics also lends itself well to interactive and collaborative forms of learning; these new modes of learning are currently being explored [1] at Rensselaer. The innovations in education at Rensselaer are well recognized: Rensselaer is the recipient of the 1995 Theodore M. Hesburgh Award for Faculty Development to Enhance Undergraduate Teaching (Mark Goldberg is one of the receipients); the Boeing Outstanding Educator Award (1995); and the Pew Leadership Award for the Renewal of Undergraduate Education (1996). The Division of Undergraduate Education of NSF recently awarded grants to an educational initiative [2] of Dartmouth College, Rensselaer Polytechnic Institute, and the University of Pennsylvania. The global environment of change, strong support of the administration, and existing infrastructure for evaluating educational initiatives make Rensselaer an ideal site for the undergraduate initiatives in discrete mathematics and computer science proposed here.

The Center for discrete mathematics and theoretical computer science at Rutgers University sponsors the yearly Leadership Program in which primary and secondary school mathematics teachers travel to New Jersey for an intensive two-week course in combinatorics and graph theory. The success of this program indicates that concepts in discrete mathematics can in fact be incorporated into primary and secondary level mathematics curricula. Below, we will propose a national dissemination strategy for educating teachers in this fashion using modules which combine textual and technological materials. Teachers who have participated in the Leadership Program will provide us with valuable help in evaluating our work.

Elon College is a liberal arts institution in North Carolina with a strong commitment to teaching. The college participates in the statewide Teaching Fellows program, which supports future teachers of all disciplines. Elon is the new home of the *LINK* project described below.

LINK

Although the need for discrete mathematics software is now reflected in the fact that both Maple and Mathematica have graph theory and combinatorics libraries, no system has provided an interface sufficient to allow students to take true advantage of the visual nature of discrete mathematics. Students should be able to edit graphs and their attributes using a mouse-driven interface, program experiments and animations using a command language, and implement algorithms and property tests using object-oriented libraries which spare them most low-level programming chores.

LINK [3] is an NSF project designed to address this need. The system combines the interactive graphics of a drawing and layout program with a computing environment like those of Maple or Mathematica. Furthermore, it is freely available for teaching and research, and is written to be extendible by students and professors. This combination of computing power, interactive graphics, and extensibility distinguishes *LINK* from other software systems for discrete mathematics.

LINK has been used successfully as a presentation and exploration tool in research and education [1, 2]. The system already enables students to develop and control animations of algorithms and to program their own experiments. For example, it is possible, with fewer than 10 lines of command language code, to create and display a series of graphs, assign random weights to their edges, then find, color, and thicken the edges of a minimum spanning tree in each (see Figure 1). Once the graph windows have appeared, the student can manipulate them using both the graphical and command line interfaces. Our proposal for incorporating *LINK* into the educational process on a national scale is sketched in Section .

[1] Mark Goldberg, a PI of this proposal, is a co-PI for the NSF supported project *Distributive Collaborative Learning Environment* (grant CA–9634485).

[2] Entitled "Mathematical Sciences and Their Applications Throughout the Curriculum." See SIAM NEWS, volume 29, No. 10, December 1996.

[3] We will discuss *LINK*'s history and competitors in the full proposal.

Proposed Work

Undergraduate Education

To achieve quality and effectiveness of teaching, students must be prompted to actively participate in the learning process. The introduction of *LINK* in discrete mathematics courses will be instrumental in engaging students.

We envision conducting courses in discrete mathematics in a fully interactive fashion, using the studio format. The *lecture method* of instruction will be drastically changed by replacing the current modus operandi, *Theory — Problems & Exercises*, with the method of learning which presents theory through solving a well designed set of problems. To make it possible and productive, we need to employ the web-technology in the preparation of hypertexts that will replace traditional class-notes. The tool we intend to use is *WEBCT (web course tool*) which was developed at the University of British Columbia[4] The focus of every class is problem solving. The students are working in *teams* [5] of 4 to 6 members. Each team has access to a screen (*in the pilot calculus courses given at RPI, each student had a lap-top computer hooked to a network).* Students will start the work by invoking *LINK* and by accessing the corresponding directory prepared by the instructor. For example, the first in-class assignment can be as follows:

`Problem 1-1: Determine all non-isomorphic graphs of order 5.`

Every underlying word is a hyperlink to text with explanations that in turn may refer to other notions, examples, *LINK* programs, or *LINK* features or examples that can help to solve the problem. For instance, if a student clicks on Problem 1-1, a new window might pop up presenting a similar problem and its solution, or hints to a solution.

Use *LINK* to create graphs whose adjacency matrices are

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} \qquad \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

`Are these graphs isomorphic? Randomly permute the matrices using the` *LINK* `permutation-function; construct a new pair of graphs for the new matrices.`

When the problems are simple, as the one above, the teams can be split into smaller parts of two or three members, and joined together for discussions, or more difficult problems. The whole course is a constantly updated network of problems, solutions, facts, definitions, and methods. The flexibility of this approach is one of its main advantages: the students may follow the links with the speed according to their abilities and interest. Therefore, more advanced may explore more problems and learn more than otherwise would be possible; on the other hand, students having difficulties with the material would get more individualized help from the instructor and thus benefit as well. WebCT can keep records of each individual's progress for informal or formal evaluation purposes. This technology-enabled, decentralized approach will more directly engage each student than the traditional approach did, and encourage him or her to explore beyond the minimum requirements. The latter will be ensured by the more traditional tests, homeworks, and quizzes.

[4] We used WEBCT during the Fall of 1997 for teaching an upper level course in Computability; our conclusion was that the tool is very useful for making the course more interactive and the students more engaged in the process.

[5] The team-work is not a prerequisite; in fact, the learning process we are describing is well suited for distant learning and self-learning, although the in-class team work would be most effective.

The function of the teacher will be substantially different in such a setting. The initial preparation for the course may take considerably more time than it does now, but once the materials have been developed, more time can be spent tailoring assignments and helping students. In fact, the main in-class function of the instructor will be to provide individualized help to the students, and to engage them in active explorations and discussions.

Additionally, the instructor will gain experience in using *LINK* and hypertext in order to provide recommendation as how to modernize *LINK*'s functionality and how to change and improve the hyperlinks. The overall goal is the development of an environment which will encourage the students to *think and do* instead of *watch and listen.*

We anticipate that the use of *LINK* and possible other software systems will increase students' interest in discrete mathematics and its applications in computer science, and assist them in solving harder and more elaborate problems. The innovative techniques we intend to create with the introduction of *LINK* will be useful to all students, at all levels of preparation; they will be especially useful for those with disabilities and/or students from disadvantage background.

K-12 Education

Although teachers in the DIMACS's Leadership Program (LP) in discrete mathematics have been enthusiastic about incorporating the topics of the institute into their curricula, one consistent concern that they raise is that, given the visual nature of discrete mathematics, the LP does not sufficiently integrate technological demonstrations and explorations into the learning process.

A primary component of the proposed work will be to determine which discrete mathematics classroom activities can best be carried out in a computer-rich environment. These choices will be made by an education research coordinator under the supervision of LP Project Director Joseph Rosenstein (Professor of Mathematics) and Associate Project Director Valerie DeBellis (Ed.D. in Mathematics Education). The education research coordinator will have had experience with the LP, perhaps as a past participant, and will consult with at least six elementary and secondary school teachers who have been through the program.

The education coordinator will work closely with a Rutgers graduate student, who will design and implement the proposed technology using *LINK*. Jonathan Berry will also be a consultant in this process.

As modules become available for use, selected past or current participants in the LP will be asked to assess the effectiveness of the new activities and suggest improvements. When a core set of activities have become generally accepted, they will be included in a national dissemination of a "book" entitled, "Discrete Mathematics in the Classroom: A Guide for Teachers" that is being developed separately by Joseph G. Rosenstein (in collaboration with program participants).

LINK

Although it is usable by capable computer scientists at the moment, *LINK* must be improved to be truly helpful in accomplishing the goals set above. Several fundamental improvements must be made:

- The incorporation of a world wide web interface to a large database of discrete mathematical knowledge, which students can access at any time during their experiments.
- The extension and streamlining of the interface to make the system more intuitive, usable, and easily customized.

- The extension of the algorithms library.

LINK was designed using freely available, high-level tools, meaning that it is possible for undergraduate students under competent direction to extend the system. The proposed systems work will be supervised by Jonathan Berry, the *LINK* project leader. The work will be broken down into tasks for Elon undergraduates and RPI and Rutgers graduate students. Also, since *LINK* is a free software system, help in certain fundamentals might come from the internet. The project supports an active web site and mailing list, and the former currently experiences roughly 500 hits per month.

Project Coordination

The project will be centered at DIMACS and coordinated by Joseph Rosenstein, who will direct the K-12 work. Mark Goldberg will be responsible for the undergraduate initiative, and Jonathan Berry will direct the systems work. All three centers of activity will need to stay coordinated, and Steven Mahaney will provide guidance and ensure a high level of communication between the mathematics and computer science project centers.

Budget

The following are rough estimates of costs for each year of the three year project. These have not been institutionally endorsed.

$10,000	Rosenstein, 1 month, Rutgers
$15,000	DeBellis (20%), Rutgers
$10,000	Mahaney, 1 month, Rutgers
$25,000	Designer/Implementor for K-12, Rutgers
$25,000	Education Research Coordinator, Rutgers
$7,000	Technical support for the above
$18,000	Teacher Support: 6 teachers
$5,000	DIMACS undergraduate component
$12,000	Computing fee at DIMACS Center, Rutgers (3 people $4000)
$18,000	Berry (summer support and released time), Elon College
$6,000	Undergraduate assistants, Elon
$6,000	Software and Hardware expenses, Elon
$15,000	Goldberg, 1.5 months summer support, RPI
$25,000	Graduate RA, RPI
$5,000	Travel and publication
$20,000	Secretarial support for the project: 50% time

TOTAL DIRECT COSTS (including fringe): $222,000, INDIRECT COSTS: $130,000
TOTAL $352,000

Biographies

- Joseph Rosenstein
- Jonathan Berry

Education	**Ph.D. in Computer Science**, Rensselaer Polytechnic Institute, 1995. **M.S. in Computer Science**, Rensselaer Polytechnic Institute, 1989. **B.S. in Computer Science and Economics**, The American University, 1987

Teaching Experience

Assistant Professor of Computing Sciences, Elon College, 1996-present.
Visiting Assistant Professor of Computer Science, DePauw University, spring 1995.
Instructor, Rensselaer Polytechnic Institute, 1993-1994.
Teaching assistant, Rensselaer Polytechnic Institute, 1990 and 1992.

Selected Publications

1. J. Berry and M. Goldberg. Path optimization and near-greedy analysis for graph partitioning: Am empirical study. In *Proceedings of the Sixth Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 223–232, 1995.
2. J. Berry. Improving discrete mathematics and algorithms curricula with LINK. In *SIGCSE/SIGCUE Conference on Integrating Technology into Computer Science Education*, pages 14–20, 1997.
3. J. Berry. LINK: A software system for experimentation with graphs and hypergraphs. *SIAM Activity Group on Discrete Mathematics Newsletter*, 7(2), 1997.
4. J. Berry and M. Goldberg. Path optimization for graph partitioning problems. *Discrete Applied Mathematics, Special Issue on Combinatorial Methods for VLSI Design.*, to appear.
5. J. Berry, N. Dean, M. Goldberg, G. Shannon, and S. Skiena. Graph drawing and manipulation with LINK. *Springer Verlag's Lecture Notes in Computer Science*, to appear.

- Mark Goldberg
- Stephen Mahaney

References

[1] J. Berry. Improving discrete mathematics and algorithms curricula with LINK. In *SIGCSE/SIGCUE Conference on Integrating Technology into Computer Science Education*, pages 14–20, 1997.

[2] J. Berry, N. Dean, M. Goldberg, G. Shannon, and S. Skiena. Graph drawing and manipulation with LINK. Springer Verlag's Lecture Notes in Computer Science, to appear, 1997.

Figures

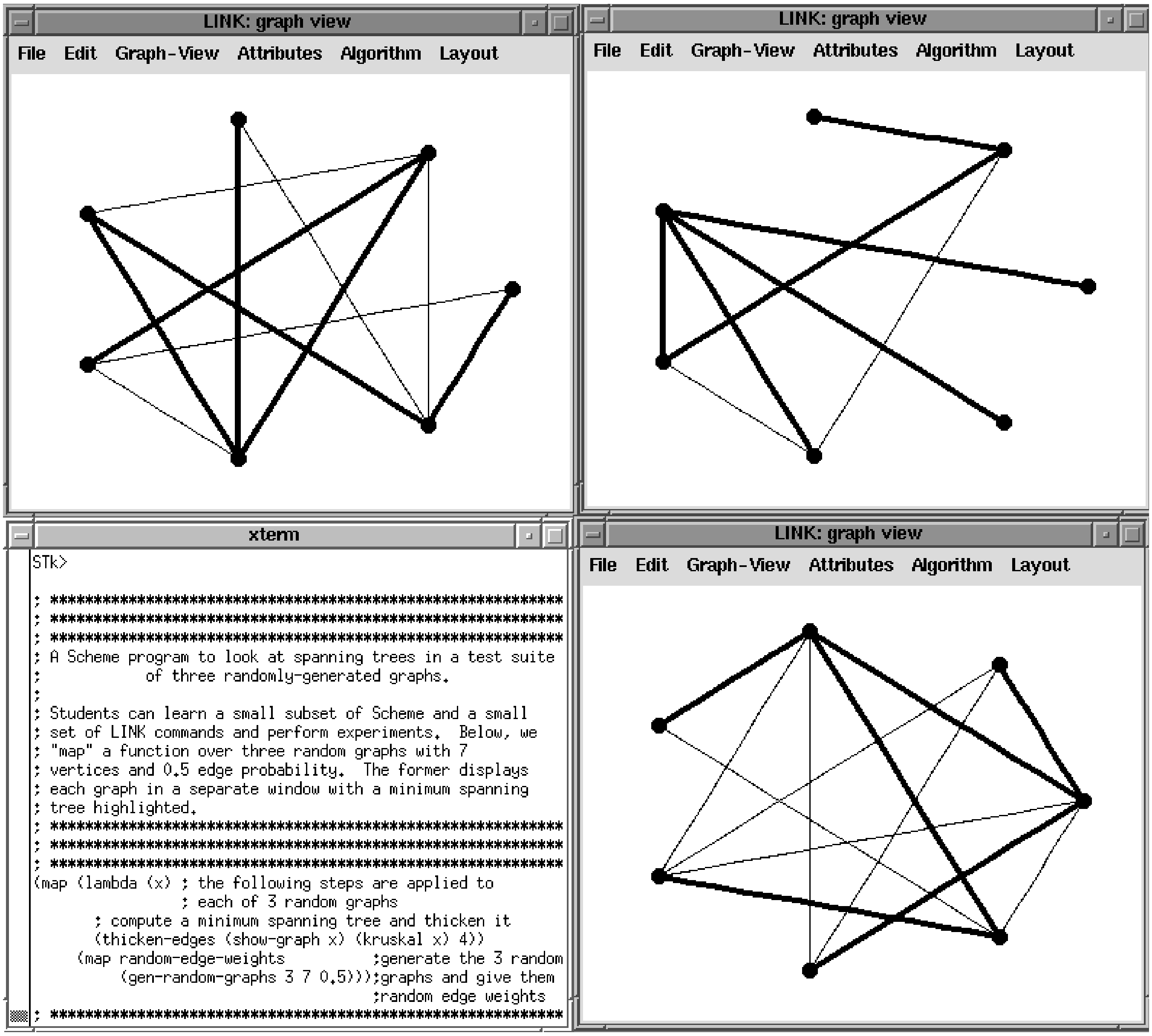

Figure 1: An example Scheme program to visualize spanning trees in random graphs