Unsolved Problems in Visibility

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There are a number of unsolved problems in combinatorial geometry concerning visibility that are accessible to be investigated (and perhaps solved!) by high-school students. Many of these problems arose in the area known as discrete and computational geometry. Here are three examples:

1. *Illuminating Disjoint Triangles.* How many exterior lights are needed to illuminate the boundaries of n disjoint triangles in the plane? It has been conjectured that $n+1$ lights always suffice; for example, one triangle needs two lights. But no one has been able to prove this. The best bound known is $(5n+1)/4n$ [Tót01]. There is a gap between the conjectured n-factor 1 and the known factor 1.25, a gap large enough that improvement should be possible. This problem may be awaiting just the right idea.

2. *Illuminating Convex Bodies.* How many exterior lights are needed to illuminate and penetrate the boundary of one convex polyhedron in 3D? The notion of "illuminate" used here is more demanding than in the previous problem. In 2D, the answer is known: 3 lights suffice, except 4 are needed to illuminate a parallelogram in this sense. In 3D, it has been conjectured for 50 years [MS99] that, similarly, 7 lights suffice, except that 8 are needed for a parallelopiped.

3. *Enchanted Mirror Forrest.* Does a match lit in a finite forest of disjoint circular mirror trees necessarily send light to infinity? In other words, can one trap light rays from a point source with such mirrors? I worked on this problem with an undergraduate at Smith. We wrote a paper [OP01] proving that circular mirrors can "narrow" light from a point source to an arbitrarily small angle. However, we could not trap all rays, and we conjecture that it is impossible to do so.

This talk will present a number of unsolved visibility problems and conjectures, some old and some new, at a level that all can understand, and perhaps initiate efforts at resolving them.

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References

[MS99] H. Martini and V. Soltan. Combinatorial problems on the illumination of convex bodies. *Aequationes Matheamticae*, 57:121–152, 1999.

[OP01] Joseph O'Rourke and Octavia Petrovici. Narrowing light rays with mirrors. In *Proc. 13th Canad. Conf. Comput. Geom.*, pages 137–140, August 2001.

[Tót01] Csaba D. Tóth. Guarding disjoint triangles and claws in the plane. In *Abstracts 17th European Workshop Comput. Geom.*, pages 137–139. Freie Universität Berlin, 2001.