Core Groups, Cooperative Behavior and Peer Pressure: the dynamics of fanaticism by ultra-ideologically driven individuals

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Introduction

The concept of core group was introduced in epidemiology by Hethcote and Yorke (1984) in the context of gonorrhea dynamics. It has played a fundamental role in the study of disease dynamics and in the development of health policy ever since. In 1995, Hadeler and Castillo-Chavez introduced a dynamic concept of Core Group. In their model, the size of the core group was allowed to vary. The rate of recruitment from the non-core into the core depended heavily on the state of infection within the core, that is, it was driven by the local (within core) disease dynamics. The model has been recently extended by Huang and Castillo-Chavez (2002) to populations structured by age. These models, supported subcritical bifurcations a troublesome epidemiological behavior. This early work represents the starting point for the development of a *simple* model for the dynamics of fanatic behavior in ultra-ideologically driven populations. From the formulation of a very *simple* model, it becomes evident that the resulting dynamics (subcritical bifurcations) are typical of peer-driven structures and, in this sense, the results may be general and troubling.

In classical epidemiology, the concept of basic reproductive number, R_0, plays a dominant role. R_0 gives the number of secondary "conversions" generated by a "typical" extreme individual in a population of susceptible non-converts. The classical result says that $R_0 > 1$ is a necessary and sufficient (local condition) for the spread of extremism, that is, for the population of individuals with extreme behaviors to grow and become established.

In this lecture, a *simple* model with compartments for core and non-core subpopulations is introduced. The dynamics of individuals with extreme behaviors (I-class) are analyzed within this *simple* framework. Two types of R_0s are found: R_d ("demographic" R_0) and R_0 ("epidemiological" R_0). It is shown that the establishment of a core-group subpopulation is not possible when $R_d \leq 1$. Furthermore, it is also shown that multiple extreme populations, within the core subpopulation, may become established when $R_d \geq 1$ and $R_0 < 1$.

The results are quite troublesome because they point out to the tremendous impact that "invading" small subpopulations of individuals with extreme behavior may have on the establishment of extremist groups when the conditions are not "ideal" ($R_d > 1$ and $R_0 \leq 1$).

Some possible scenarios will be discussed as well as potential strategies for reducing the impact of individuals with extreme behaviors.

Some of the specifics follow (work in progress):

The model

$$\frac{dG}{dt} = \Lambda - \beta_1 G \frac{C}{T}, \tag{1}$$

$$\frac{dS}{dt} = \beta_1 G \frac{C}{T} - \beta_2 S \frac{E+F}{C} - \gamma_S S, \tag{2}$$

$$\frac{dE}{dt} = \beta_2 S \frac{E+F}{C} - \beta_3 E \frac{F}{C} - \gamma_E E, \tag{3}$$

$$\frac{dF}{dt} = \beta_3 E \frac{F}{C} - \gamma_F F \tag{4}$$

$T = G + S + E + F$ denotes the total population size (core and non-core);
$C = S + E + F$: denotes the number of individuals in the core group;
G: number of individuals who are not members of the core group but who may join it (source of recruits);
S: Susceptibles in core group;
E: Semi-fanatic members of the core group;
F: Fanatics members of core group;

Summary of partial results

1. If $R_d = \frac{\beta_1}{\gamma_S} \leq 1$ then the core group cannot become established, that is, $\lim_{t\to\infty} C = 0$.
   Hence, it is assumed that
$$R_d = \frac{\beta_1}{\gamma_S} > 1.$$

2. The equilibrium $EQ_2 = \{\frac{\Lambda}{\beta_1-\gamma_S}, \frac{\Lambda}{\gamma_S}, 0, 0\}$ always exists and it is stable whenever $R_0 = \frac{\beta_2}{\gamma_E} \leq 1$.

3. There is a third threshold,
$$R_1 \equiv \frac{\beta_3}{\gamma_F}$$
   If $R_1 = \frac{\beta_3}{\gamma_F} \leq 1$ then the superplane $F = 0$ is a global attractor, that is, $\lim_{t\to\infty} F = 0$. In this case the model can be reduced to a three-dimensional model

4. Whenever $R_d > 1$, $R_0 > 1$ and $\beta_1\beta_2 > \gamma_E(\gamma_S + \beta_2 - \gamma_E)$ (which is identical to $\frac{R_d R_0 - 1}{R_0 - 1}\frac{\gamma_S}{\gamma_E} > 1$) then the equilibrium $EQ_3 = \{G_3^*, S_3^*, E_3^*, 0\}$ appears, that is, becomes positive. Letting $n = \frac{\gamma_E}{\beta_2 - \gamma_E}$ allows for the rewriting of EQ_3 as:
$$\begin{aligned}
E_3^* &= \frac{\Lambda}{\gamma_E}\left(\frac{\beta_2 - \gamma_E}{\gamma_S + \beta_2 - \gamma_E}\right), \\
S_3^* &= nE_3^*, \\
G_3^* &= \frac{(1+n)\Lambda E_3^*}{(1+n)\beta_1 E_3^* - \Lambda}.
\end{aligned}$$
   It can be shown that EQ_3 is stable whenever $\frac{R_1(R_2-1)}{R_2} < 1$ and unstable otherwise.

5. If $R_1 = \frac{\beta_3}{\gamma_F} > 1$ and $\frac{R_0(R_1-1)}{R_1} > 1$ there are two I-positive equilibria. This happens when $R_0 < 1$, that is, the system undergoes a subcritical bifurcation.

References

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