On Graph Packing Theorems by Sauer and Spencer

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Graphs $G_1, G_2, \ldots, G_k$ (on n vertices each) *pack*, if there exists an edge disjoint placement of all these graphs into the complete graph K_n. A number of basic problems in graph theory can be naturally stated as packing problems. Examples are: Turán problems, equitable coloring, problems on Hamiltonian cycles. Directions of studying extremal problems on packing of graphs were outlined almost 30 years ago in fundamental papers by Bollobás and Eldridge and Sauer and Spencer. We review refinements of the three main theorems by Sauer and Spencer. Our main result is about their theorem which states that if $|V(G_1)| = |V(G_2)| = n$ and $|E(G_1)||E(G_2)| < \binom{n}{2}$, then G_1 and G_2 pack. The restriction on the product of sizes is sharp, but we show that for every $\epsilon > 0$ for sufficiently large n, if $|E(G_1)||E(G_2)| \leq (1-\epsilon)n^2$, then G_1 and G_2 *do not pack* if and only if the pair (G_1, G_2) belongs to one of three well defined families. The talk is based on joint results with H. Kaul and G. Yu.