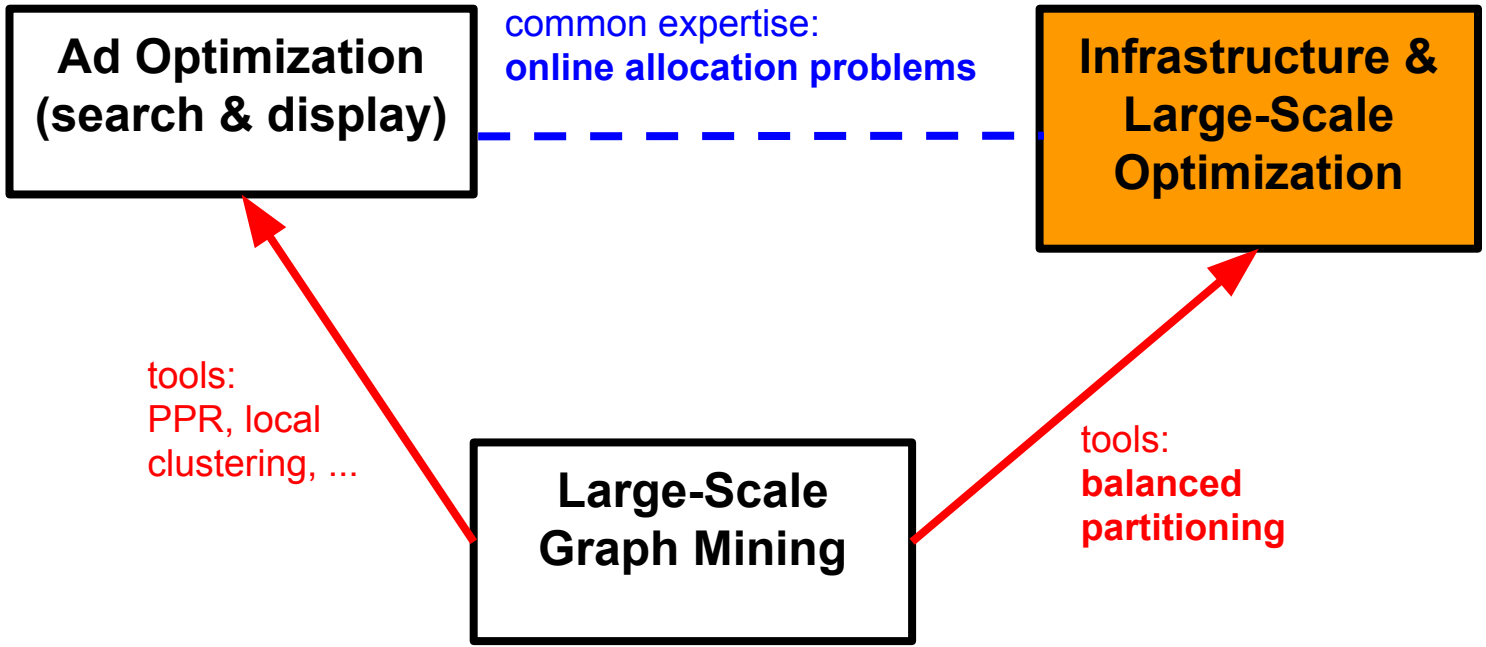
Google

Optimization for Search via Consistent Hashing & Balanced Partitioning

Vahab Mirrokni

NYC Algorithms Research, Google Research

NYC Algorithms overview

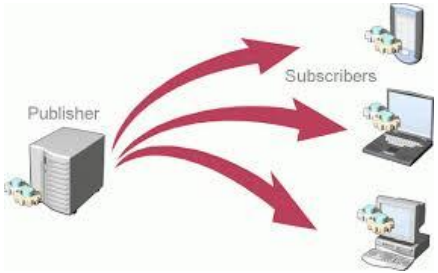


Outline: Three Stories

- Consistent Hashing for Bounded Loads
- Application of Balanced Partitioning to Web search
  - Main idea: cluster query stream to improve caching
  - Balanced Graph Partitioning: Algorithms and Empirical Evaluation
- Online Robust Allocation
  - Simultaneous Adversarial and Stochastic Optimization
  - Mixed Stochastic and Adversarial Models

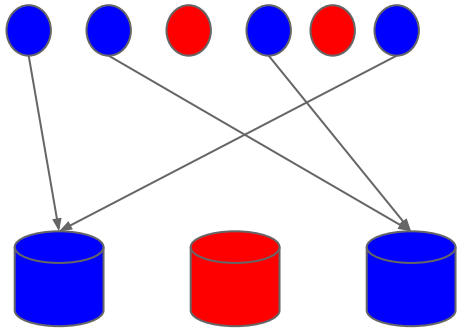
Consistent Hashing with Bounded Loads for Dynamic Bins

- Vahab Mirrokni (Google NYC)
- Mikkel Thorup (Visitor / U. Coppenhagen)
- Morteza Zadimoghaddam (Google NYC)



Problem: Consistent Hashing for Dynamic Bins

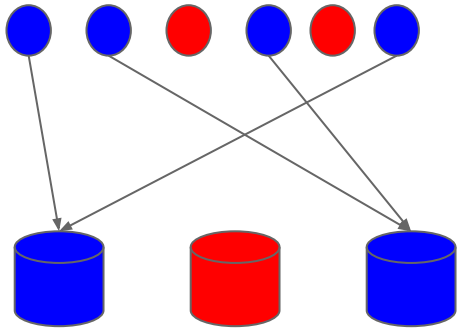
- Hash balls into bins
- Both balls and **bins are dynamic**
- Main Objectives:
  - Uniformity: **Hard capacities**
  - Consistency: Minimize movements
- Remarks:
  - Update time is not the main concern
  - We need a memoryless system based on state (balls/bins)



Active balls and bins are marked with blue.

Previous Approaches

- Consistency Hashing/Chord (Dynamic): Hash balls and bins into a circle, and put each ball in the next bin on the circle.
- Power of two choices (Static): Try two random bins & send to the smaller one.



Active balls and bins are marked with blue.

Related Work

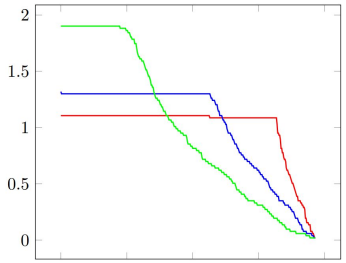
	Max Load	Avg Relocation
Chord [Stoica, Morris, Karger, Kaashoek, Balakrishnan 2001] Consistent Hashing [Karger, Lehman, Leighton, Panigrahy, Levine, Lewin 1997]	density × log(n)/loglog(n)	O(density)
Totally Random Hash Function	density × log(n)/loglog(n)	O(density)
Balanced Allocations [Azar, Broder, Karlin, Upfal 1999] Cuckoo Hashing [Pagh, Rodler 2001]	density × loglog(n)	O(density)
Linear Probing with tight capacity	density	Large in simulations - Cycle length in a random permutation Ω(n)?
Our approach: Linear Probing with (1+ε) extra multiplicative capacity	**density × (1+ε)**	**O(density/ε^2)**

density is the average load, i.e.number of balls divided by number of bins

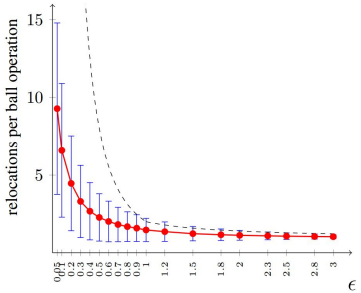
Results: Provable performance guarantees

Method: Linear Probing with (1+ε) extra multiplicative capacity

- Uniformity: max load is (1+ε) × average load
- Relocations is at most:
  - $O(1/\varepsilon^2)$ per ball operation for $\varepsilon < 1$
  - $1 + O(\log(1+\varepsilon)/\varepsilon^2)$ per ball operation for $\varepsilon > 1$ (theoretical)
  - The bounds for bin operation is multiplied by density = #balls / #bins
- For ε > 1, the extra relocation term disappears in the limit



Load Distributions for different values of ϵ



Take-home point 1

- You want to achieve desirable load balancing with consistency in dynamic environments? Then use:

  Linear probing with (1+ε) extra multiplicative capacity

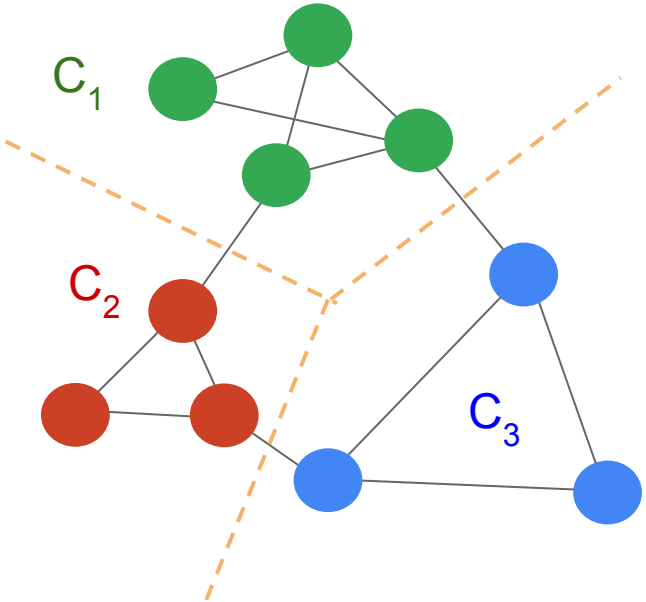
- Good theoretical and empirical properties for:
  - Load Balancing: Deals with hard capacities
  - # of Movements: Bounded by a constant **($O(\text{density}/\varepsilon^2)$)**

Application of Balanced Partitioning to Web search

- Eng Team: Bartek Wydrowski, Ray Yang, Richard Zhuang, Aaron Schild (PhD intern, Berkeley)
- Research Team: Aaron Archer, Kevin Aydin, Hossein Bateni, Vahab Mirrokni

Balanced graph partitioning

- Given graph G=(V,E) with:
  - node weights w_v
  - edge costs c_e
  - # clusters k
  - imbalance tolerance $\epsilon>0$



- Goal: partition V into sets $P=\{C_1,...,C_k\}$ s.t.
  - node weight balanced across clusters, up to $(1+\epsilon)$ factor
  - minimize total cost of edges cut

Some observations in Web search backend

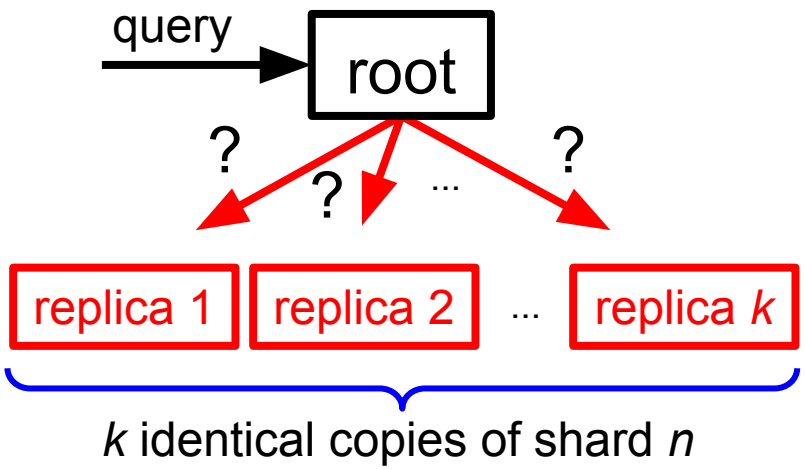
- Caching is very important for efficient Web search.
- Query stream more uniform → caching more efficient.
- A lot of machines are involved.

Idea:

Try to make query stream more uniform at each cache.

Routing Web search queries

- Machine layout: *R* roots, sharing *L* leaves
- The corpus is doc-sharded.
- Each leaf serves 1 shard.
  - Root forwards query to 1 replica in each shard, combines leaf results.

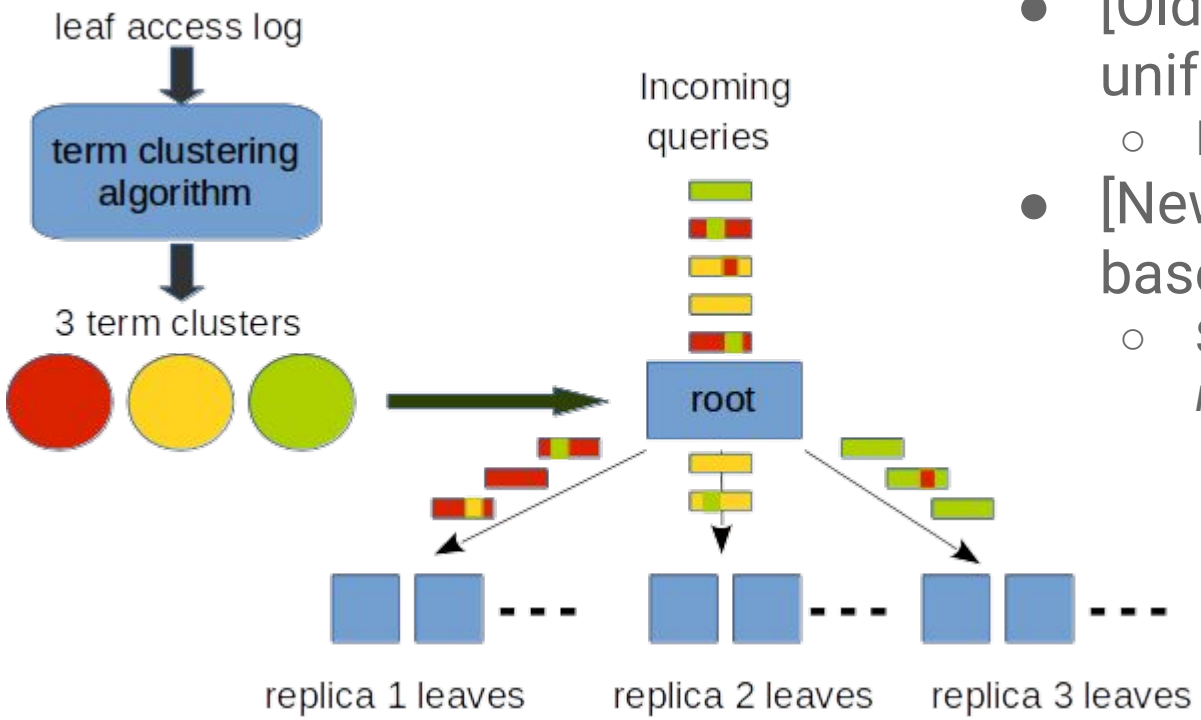


Q: For each shard, which replica to pick?

[Old answer] Uniformly at random.

[New answer] This talk.

Design



- [Old] Root selects leaf uniformly at random.
  - Leaf caches look ~same.
- [New] Terms in query vote based on clustering.
  - Specializes cache in replica *r* to terms in cluster *r*.

Example diagram with k=3 replicas.

Algorithm

Offline:

Leaf logs → term-query graph.

Cluster terms into k buckets, using balanced graph partitioning.

Store term-bucket affinity mapping.

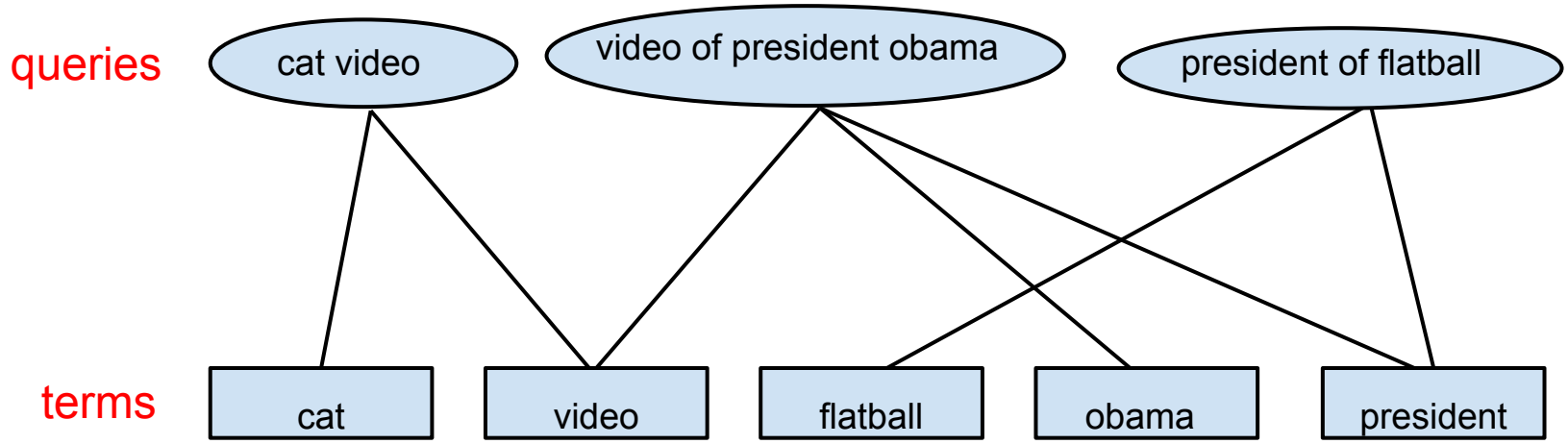
Online:

Root loads term-bucket affinities into memory at startup.

Terms in query hold weighted vote to select replica *r*.

Send query to replica *r* for each doc shard.

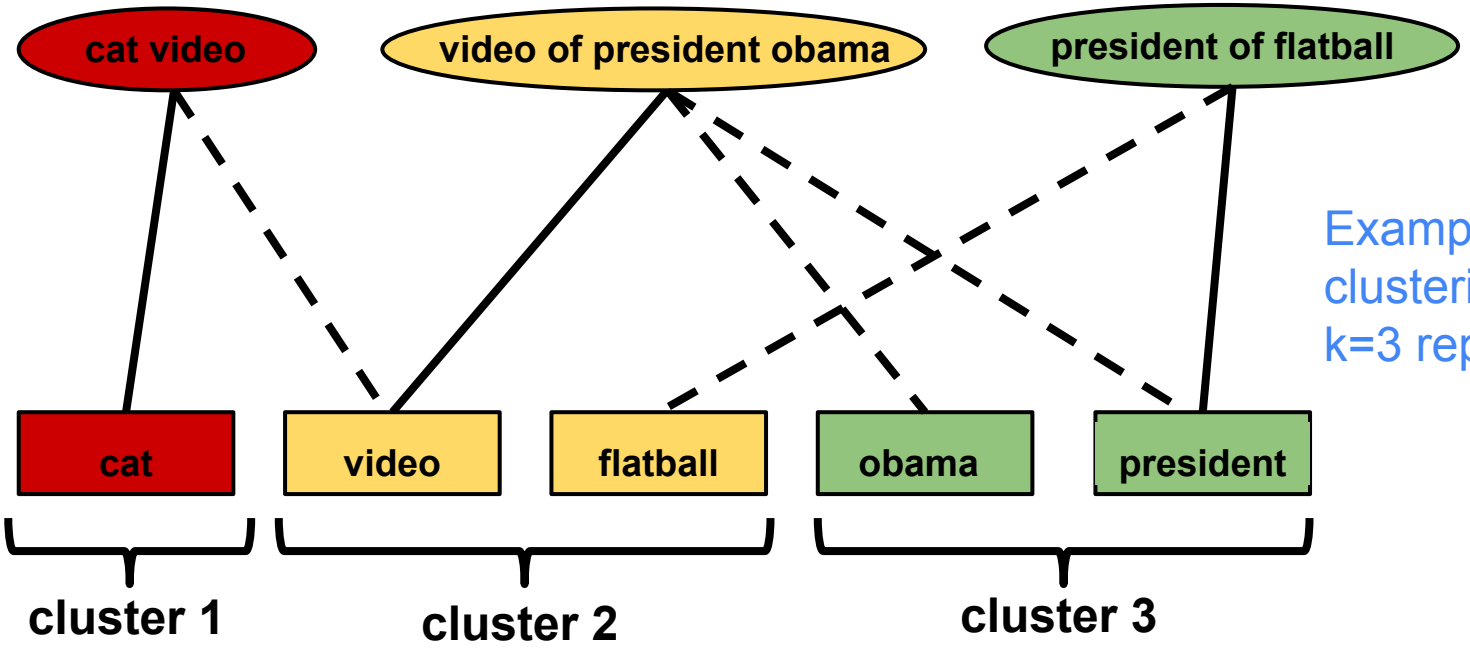
Clustering objectives



Balanced: Aim for roughly equal working set size in each cluster.

Small cut size: cut {term, query} edge ↔ query assigned to different cluster than term, so probable cache miss.

Clustering solution

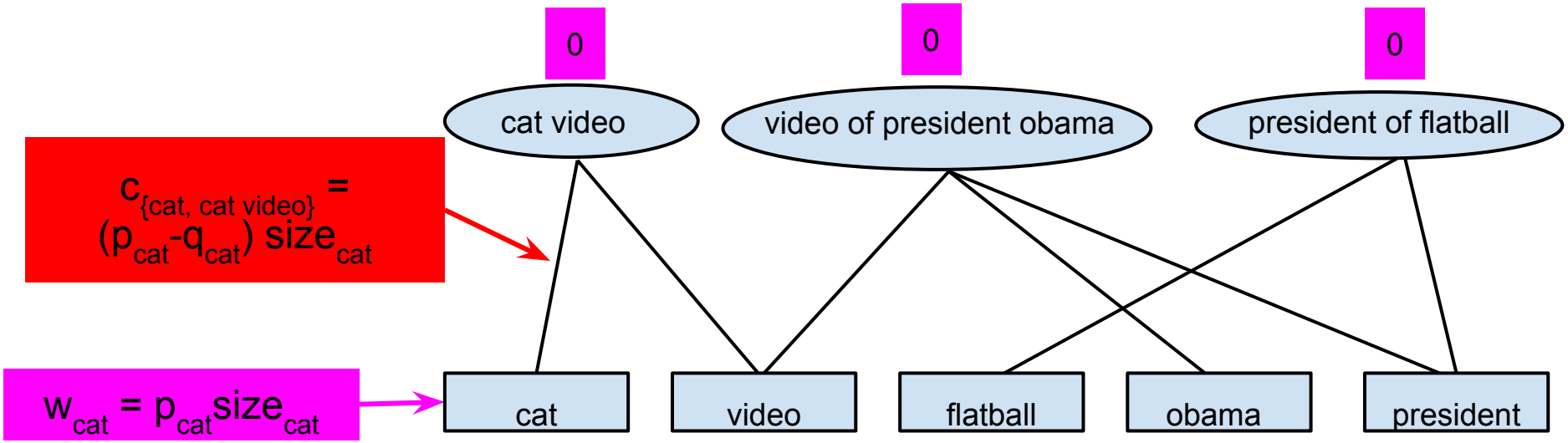


Example clustering with k=3 replicas.

cut edges: – – – – – query routed to non-preferred replica for that term, so less likely to be in cache

Input to balanced partitioner

- p_t = Pr[term t in cache in ***preferred*** replica]
- q_t = Pr[term t in cache in any ***non-preferred*** replica]
- $size_t$ = size of t's data in memory pages = cost of cache miss



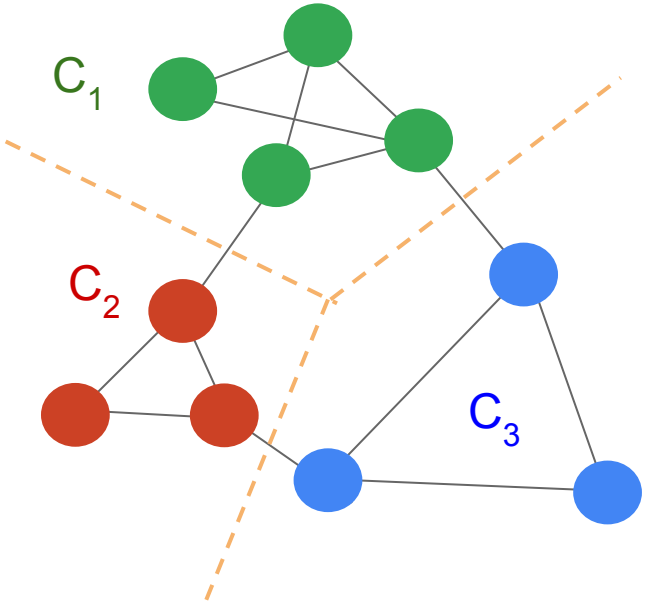
Balanced Partitioning via Linear Embedding

Kevin Aydin, Hossein Bateni, Vahab Mirrokni, WSDM 2015

Paper Here

Balanced graph partitioning

- Given graph G=(V,E) with:
  - node weights w_v
  - edge costs c_e
  - # clusters k
  - imbalance tolerance $\epsilon>0$



- Goal: partition V into sets $P=\{C_1,...,C_k\}$ s.t.
  - node weight balanced across clusters, up to $(1+\epsilon)$ factor
  - minimize total cost of edges cut

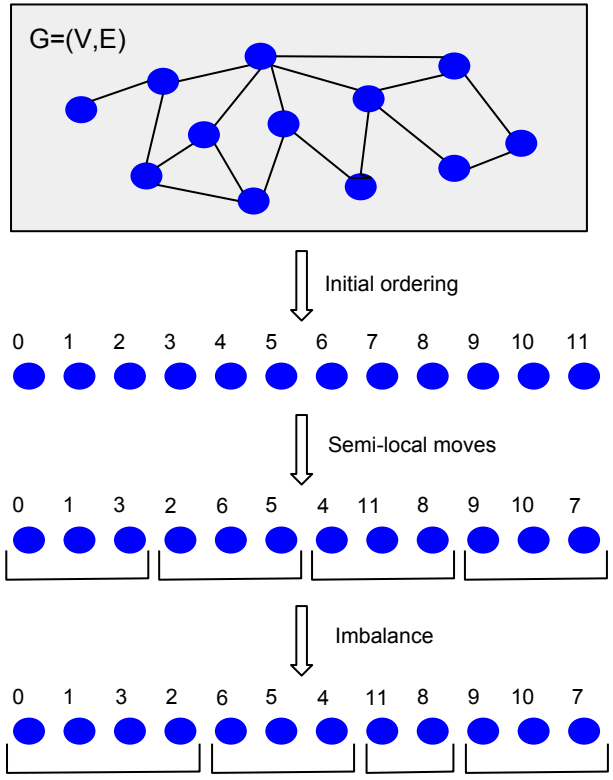
We need scalable, *distributed* algorithms

- O(1)-apx. NP-hard, so rely on principled heuristics.
- Example run of our tool:
  - 100M nodes, 2B edges
  - <1 hour on 1000 machines
- Uses affinity clustering as a subroutine.
- Affinity scalability:
  - 10B nodes, 9.5T edges
  - 20 min on 10K machines

Linear embedding: outline of algorithm

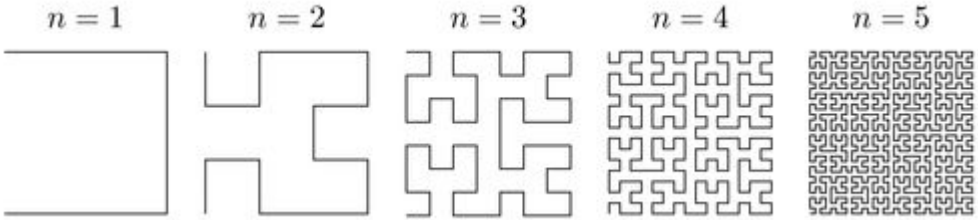
Three-stage algorithm:

1. Reasonable initial ordering
    - hierarchical clustering
2. Semi-local moves
    - improve by swapping pairs
3. Introduce imbalance
    - dynamic programming
    - min-cut

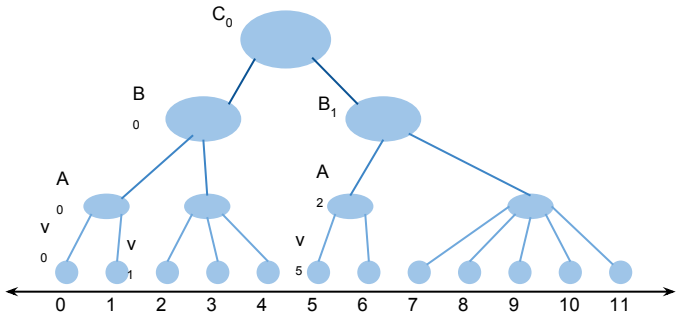


Step 1: initial embedding

- Space-filling curves (geo graphs)



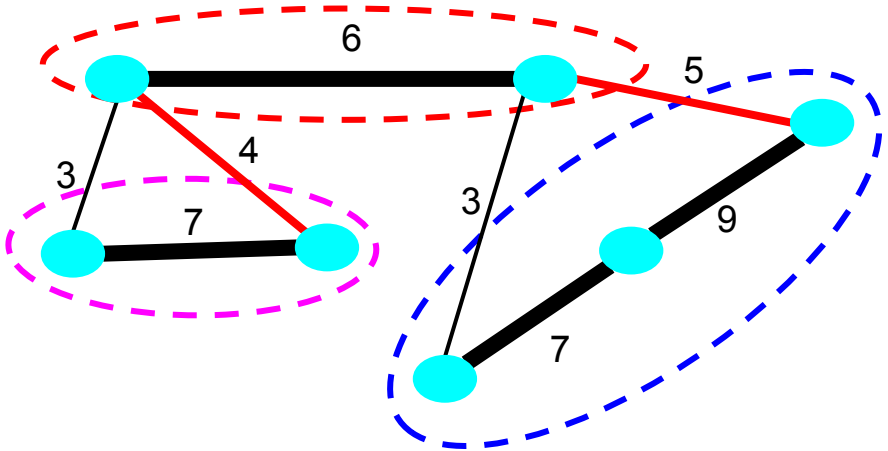
- Hierarchical clustering (general graphs)



Affinity hierarchical clustering

- Keep heaviest edge incident to each node.
- Contract connected components.
- Scalable parallel version of Boruvka's algorithm for MST.

iterate



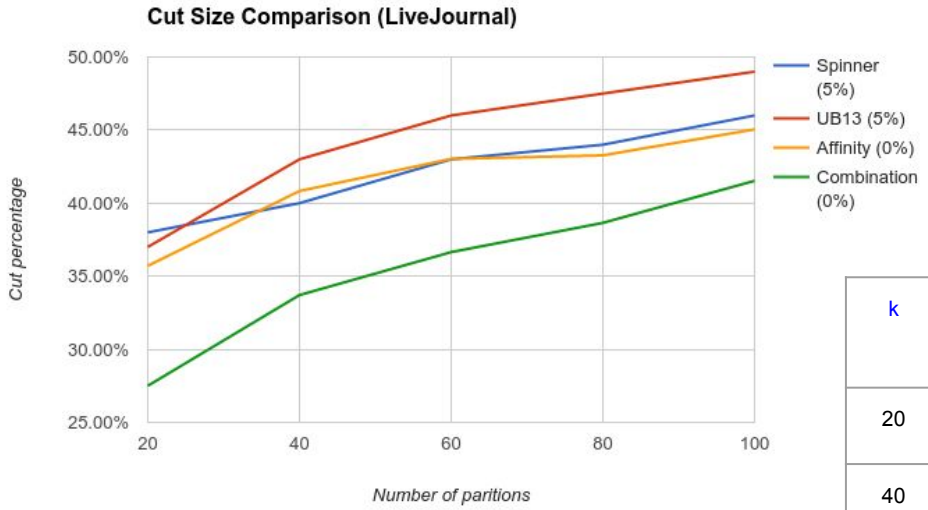
Datasets

- Social graphs
  - **Twitter**: 41M nodes, 1.2B edges (source: [KLPM'10])
  - **LiveJournal**: 4.8M nodes, 42.9M edges (source: SNAP)
  - **Friendster**: 65.6M nodes, 1.8B edges (source: SNAP)
- Geo graphs
  - **World** graph: 500M+ nodes, 1B+ edges (source: internal)
  - **Country** graphs (filtered versions of **World** graph)

Related work

- FENNEL [Tsourakakis et al., WSDM’14]
  - Microsoft Research
  - Streaming algorithm
- UB13 [Ugander & Backstorm, WSDM’13]
  - Facebook
  - Balanced label propagation
- Spinner [Martella et al., arXiv'14]
- METIS (in-memory) [Karypis et al. '95-'15]

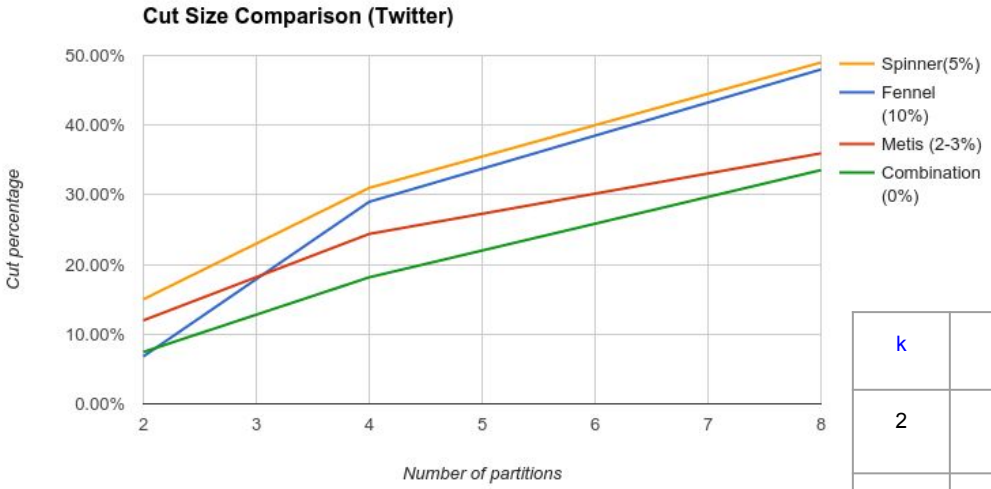
Comparison to previous work: LiveJournal graph



Cut size as a percentage of total edge weight in graph. (x%) denotes imbalance.

k	Spinner (5%)	UB13 (**5%**)	Affinity (0%)	Combination (**0%**)
20	38%	37%	35.71%	**27.5%**
40	40%	43%	40.83%	**33.71%**
60	43%	46%	43.03%	**36.65%**
80	44%	47.5%	43.27%	**38.65%**
100	46%	49%	45.05%	**41.53%**

Comparison to previous work: Twitter graph



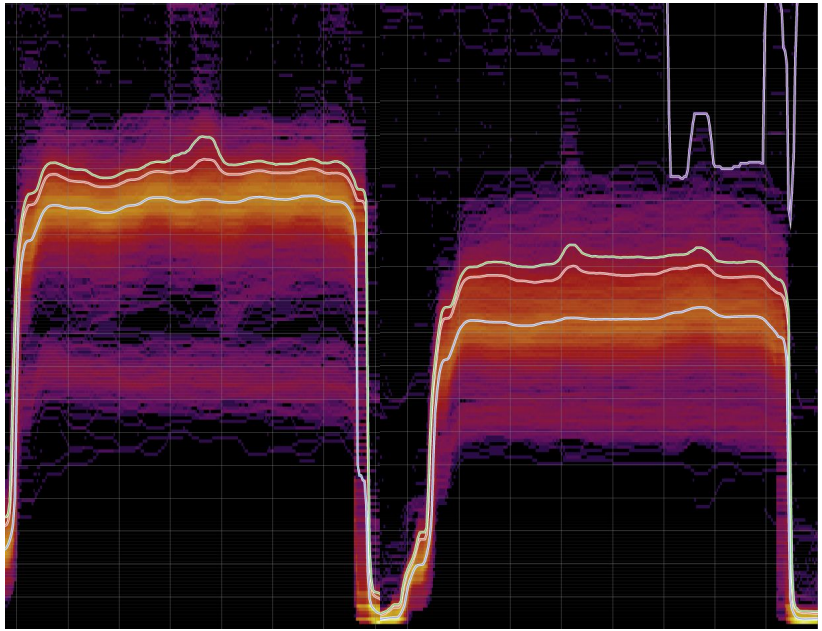
Cut size as a percentage of total edge weight in graph. (x%) denotes imbalance.

k	Spinner (5%)	Fennel (10%)	Metis (2-3%)	Combination (0%)
2	15%	**6.8%**	11.98%	7.43%
4	31%	29%	24.39%	**18.16%**
8	49%	48%	35.96%	**33.55%**

Main result of 2nd part

25% fewer cache misses!

Translates to greater QPS throughput for the same hardware.



Baseline Experiment

Take-home point 2

- Fundamental optimization models + good logs data + scalable algorithms → big improvements in data center operations.
- When splitting query stream to distribute load, might as well cluster it to improve caching.
  - Idea is generally applicable; nothing special about Web search!

Google

Online (Robust) Ad Allocation

Why (not) to rely on data

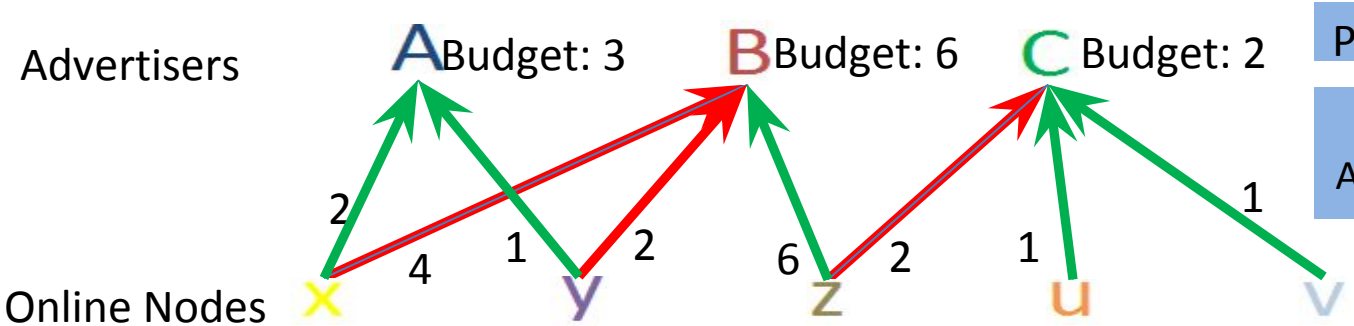
Online Ad Allocation: Budgeted Allocation

- Budgeted Fixed Nodes (Advertisers), Online Nodes (Users), and Weighted Edges between them

Goal: Assign online nodes to Advertisers maximizing revenue respecting budgets

Revenue(Greedy) = 4 + 2 + 2 = 8

Revenue(Optimum) = 2 + 1 + 6 + 1 + 1 = 11



Performance Ratio = 8/11

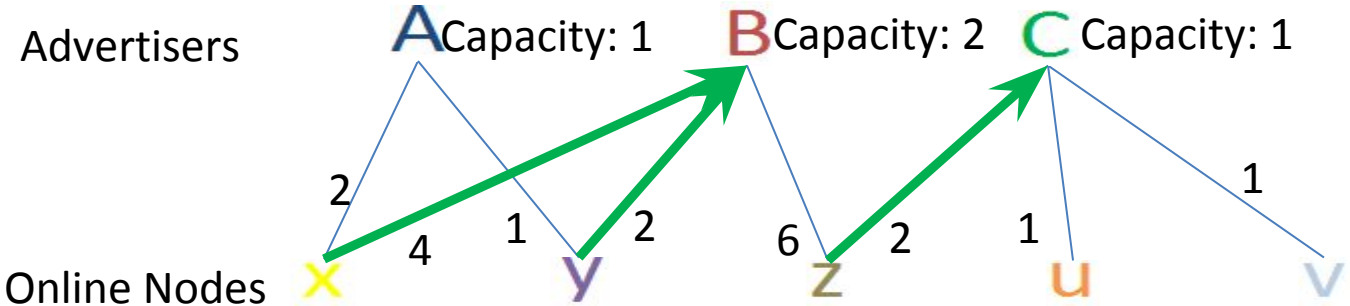
Depends on Instance and Arrival Order of Online Nodes

Online Weighted Matching (Display Ads)

Fixed Nodes (Advertisers) with Capacities, Online Nodes (Users), and weighted edges between them

Goal: Assign online nodes to Advertisers maximizing weight of the allocation respecting capacities

Performance of Green Allocation: Cardinality 3, Weight 8



Arrival Models and Competitive Ratio

Algorithm Alg is α-competitive if:

Worst Case/ Adversarial

$$\frac{\text{Revenue(Alg)}}{\text{Revenue(OPT)}} > \alpha$$

should hold for all instances and arrival orders

Stochastic

$$\frac{\text{E[Revenue(Alg)]}}{\text{Revenue(OPT)}} > \alpha$$

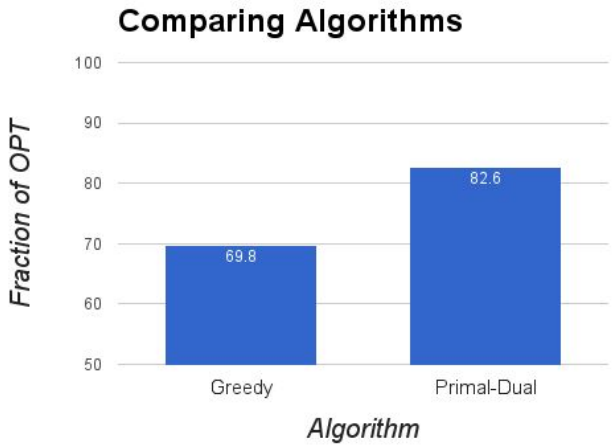
should hold for all instances

We take expected values over all instances

Online Ad Allocation: Adversarial order

Theorem [MSVV'05, FKMMP'09]: In worst-case, Primal-dual Algorithm is (1-1/e)-competitive.

Greedy is (1/2)-competitive



$$\max \sum_{i,a} b_{ia} x_{ia}$$

$$\sum_{a} x_{ia} \leq 1 \qquad (\forall i)$$

$$\sum_{i} b_{ia} x_{ia} \leq B(a) \qquad (\forall a)$$

$$x_{ia} \geq 0 \qquad (\forall, i, a)$$

Primal-dual Algorithm [FKMMP'09, FKHMS'11]

Primal

$$\max \quad \sum_{i,a} w_{ia} x_{ia}$$

$$\sum_a x_{ia} \leq 1 \quad (\forall\ i)$$

$$\sum_i x_{ia} \leq n(a) \quad (\forall\ a)$$

$$x_{ia} \geq 0 \quad (\forall\ i, a)$$

Dual

$$\min \sum_a n(a)\beta_a + \sum_i z_i$$

$$\beta_a + z_i \geq w_{ia} \quad (\forall i, a)$$

$$\beta_a, z_i \geq 0 \quad (\forall i, a)$$

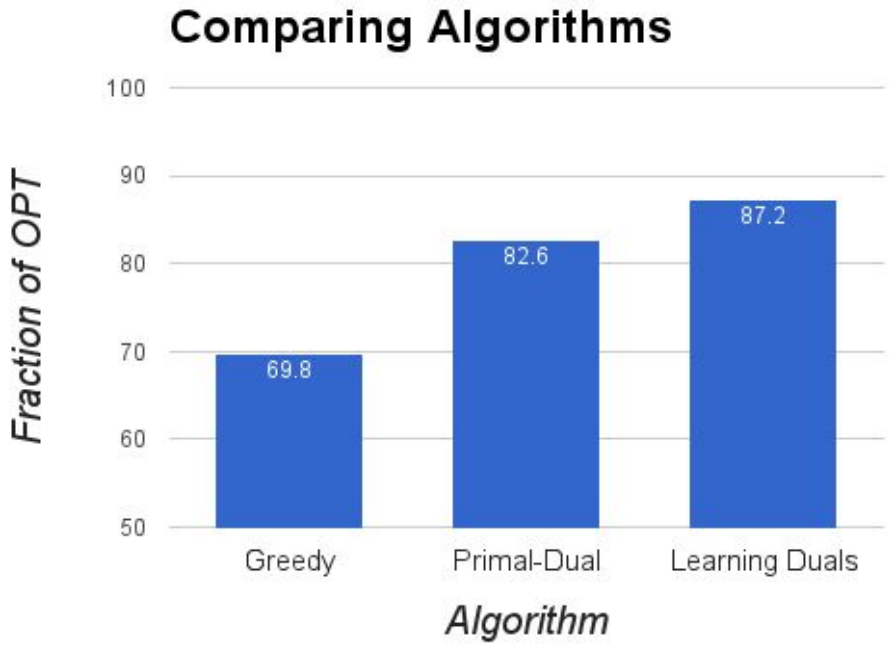
- Primal solution shows the allocation
- Maintain Dual Variable β_a for each advertiser a
- Assign *i* to advertiser *a*, maximizing: $w_{ia} - \beta_a$
- Update β_a online after each allocation (function of capacity constraint). Initialize at 0.

Stochastic Model:
Learn Optimal Parameters from Forecast / Distribution

Theorem [DH'09, FHKMS'11]:
In stochastic model, dual-based

algorithm is a (1-ε)-competitive.

Assumptions are invalid!
Data is an imperfect guide.

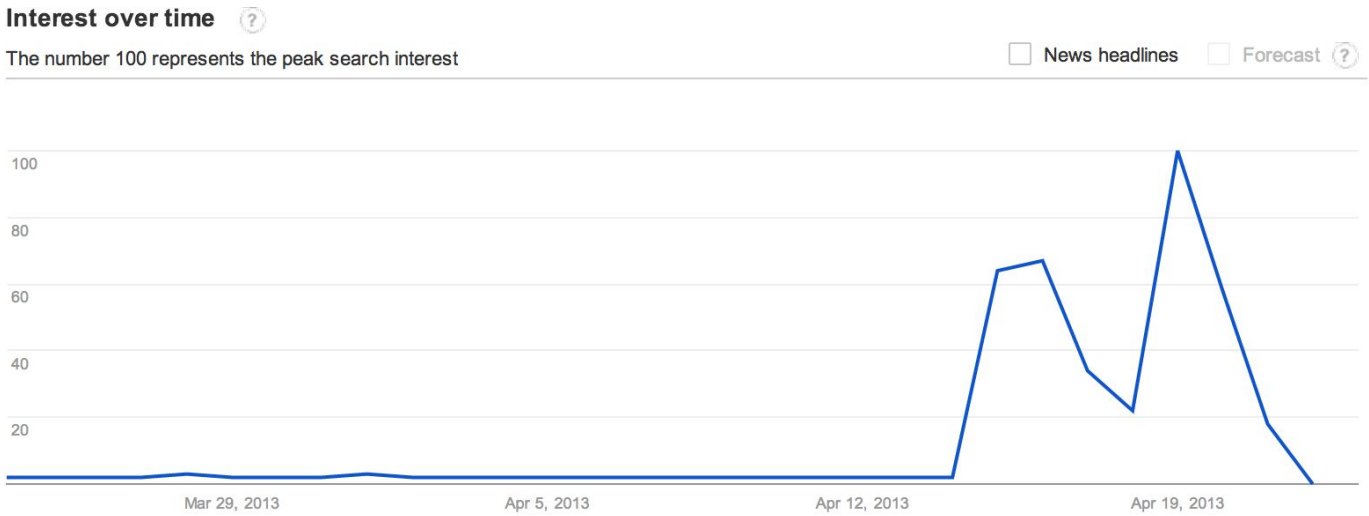


Reality is Far from Predictions: Traffic Spikes

Breaking news

One-off events

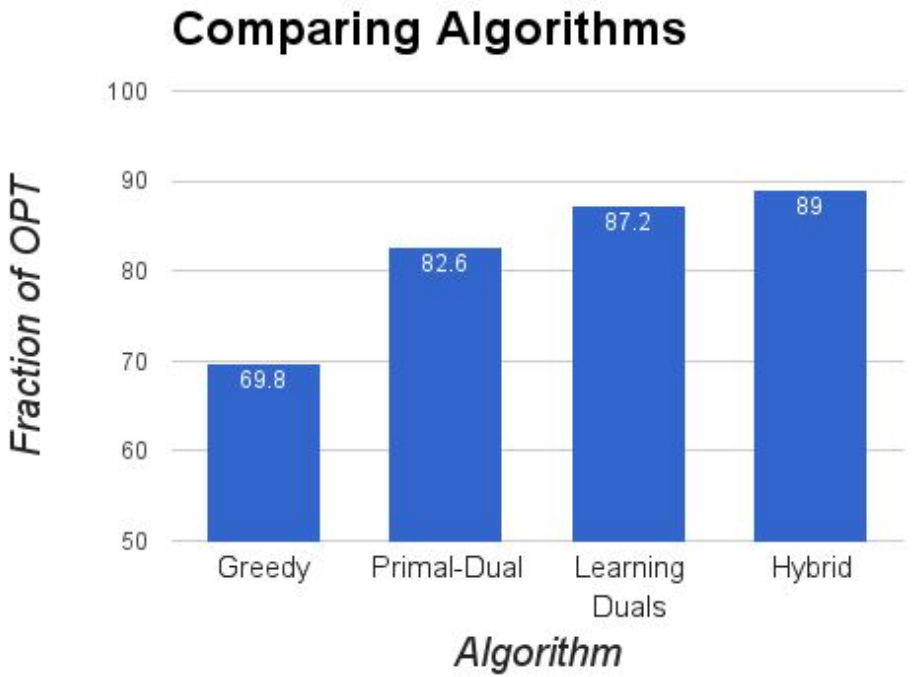
Exciting sporting events



Use Forecasts, but Don’t Trust Them Completely

Hybrid algorithm:
Learn Duals, blend them with adversarial duals

Theorem: ???



Goal: A Theory of Partially Accurate Forecasts

Simultaneous Adversarial & Stochastic Approximation

- Adversarial Setting: too pessimistic, real world data has non-adversarial structure
- Stochastic Input: too optimistic, violated by *traffic spikes*

Practice: mixture of these two settings is used
Goal: theoretically model these mixture settings
One way to model: simultaneous competitive algorithms

Goal: Design robust algorithms that achieve the best competitive ratios in each case, i.e. robust against traffic spikes and performs better in stochastic case.

Simultaneous Adversarial & Stochastic Approximation

[Mirrokni Oveis Gharan Zadimoghaddam, SODA 12]

	Algorithm	Hardness
un-weighted	(1-1/e , 1-ε) [KVV] , Ours	(1-1/e , 1) [KVV]
weighted	Balance gets (1-1/e , 0.76) [MSVV], Ours	($4\varepsilon^{1/2}$, 1-ε) (1-1/e , 0.976) Ours

Simultaneous Adversarial & Stochastic Approximation

For **unweighted instances**: Assigning each online node to the least congested advertiser achieves (1-1/e, 1-ε)-approximation in adversarial and stochastic settings with large budgets.

1-ε competitive ratio for stochastic input → at most $4\varepsilon^{1/2}$ competitive ratio for adversarial input

1-1/e competitive ratio for adversarial input → at most 97.6% competitive ratio for stochastic input

Our Result for weighted instances:
We show that primal-dual algorithm achieves (1-1/e, 0.76)-approximation of with large budgets.

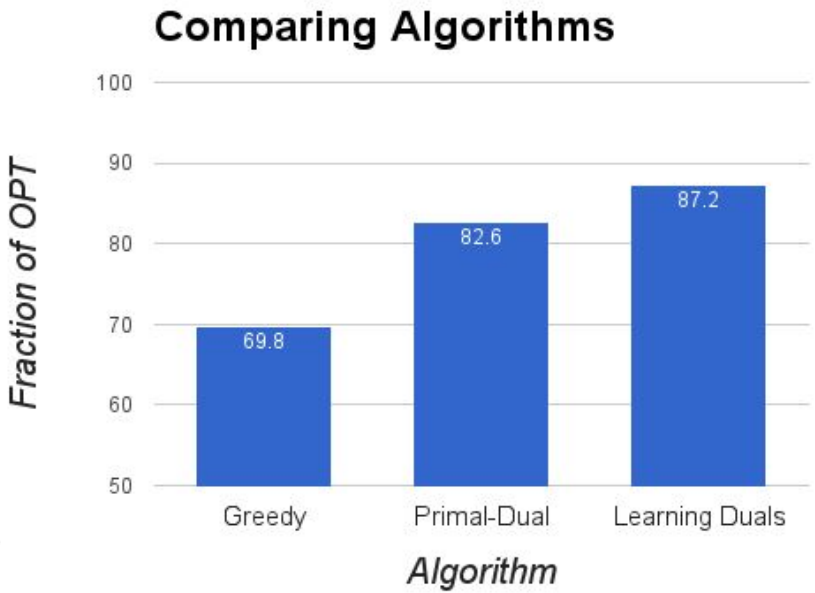
Neither Adversarial nor Stochastic

Reality is not bimodal!

Every day, forecasts are a ‘little’ inaccurate.

Small, but non-random (adversarial?) deviations from forecast.

Design algorithms with performance that degrades gracefully with forecast accuracy?



Modeling Traffic Spikes

Algorithm knows forecast: f items from distribution D (with finite support)

1. At each time step, adversary can either:
   a. Create an arbitrary item
   b. Draw an item from D
2. After f items have been drawn from D, adversary can terminate input.

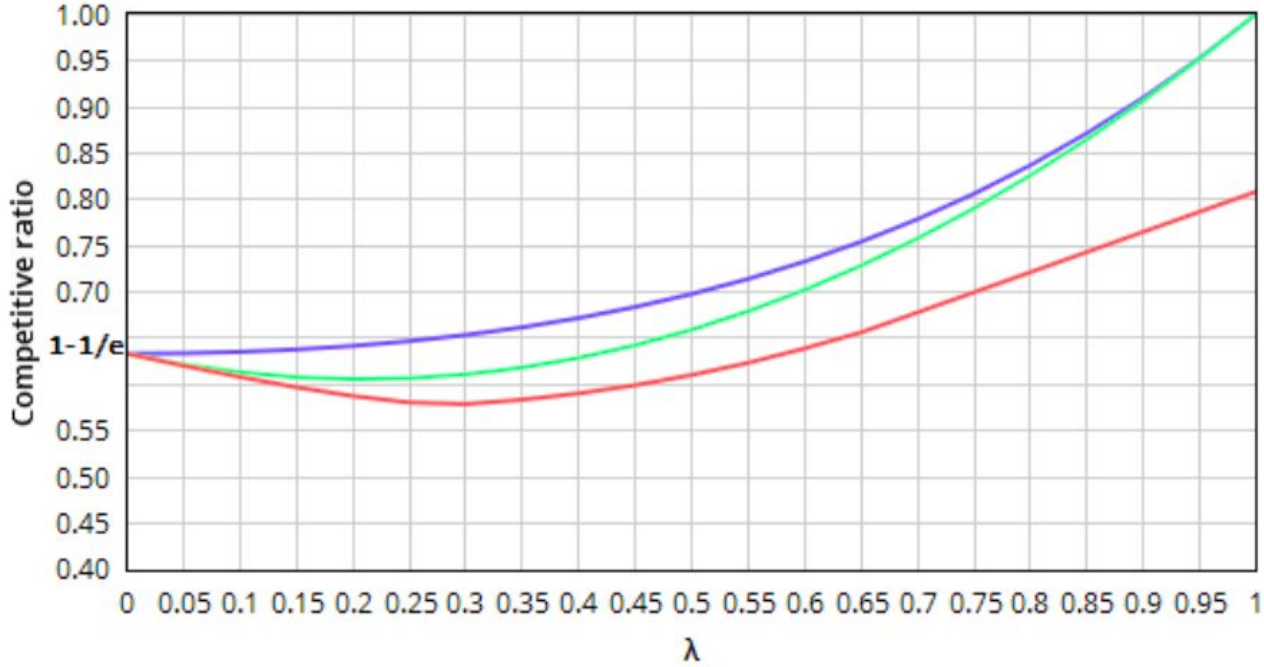
Measure forecast accuracy by parameter λ: How much noise did adversary add?

λ = OPT(Forecast) / OPT(Forecast $\cup$ Adversarial Items)

[Esfandiari, Korula, Mirrokni, EC'15]

Allocating with Traffic Spikes

Allocate items according to forecast, ‘reserving’ budget for forecast items. When algorithm detects adversarial items, use worst-case algorithm to assign, using remaining budgets.



Take-home Points 3: Robust Ad Allocation

- Good algorithms for online allocation in adversarial settings
- Good algorithms in stochastic settings
- Hybrid settings? Need better models for partially accurate forecasts!
  - Simultaneous Adversarial and Stochastic Approximation → SODA 2012 paper
  - Mixed Adversarial and Stochastic Models → EC 2015 paper

Conclusion: Examples of “Algorithms in the Field”

“Examples of “Algorithms in the Field of Infrastructure Optimization”

1. Dynamic load balancing with bounded hashing: Enhanced Linear Probing
2. Balanced partitioning to improve caching in web search and beyond
3. Online Optimization: Hybrid adversarial and stochastic models.
    - Simultaneous Adversarial and Stochastic Approximation → SODA 2012 paper
    - Mixed Adversarial and Stochastic Models → EC 2015 paper

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