A Proposal for Valuing Information and Instrumental Goods

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Questions of Information Value

- Websites IQPort.com and FatBrain.com allow anyone to post information. What's the value?
- What is the value to Dow Chemical of a new process for PVC? What is the value of Wal-Mart's process knowledge of "cross-docking?"
- How do you account for the 90% of Microsoft's market capitalization that isn't tangible?

How do you value anything?

A Boggs Bill

Valuing Known Tangibles

Valuing Known Tangibles

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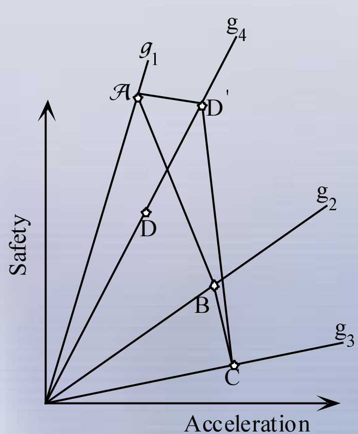
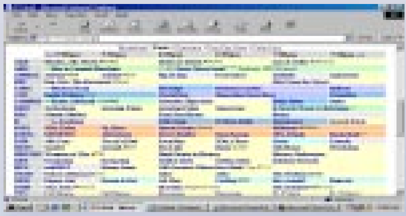
Example: How a marketing economist might value a Honda Accord



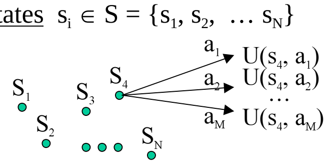
V6 Engine	$5000
Airbags	1000
High MPG	4000
Climate Control	800
Reliable (Cons Rpt)	2000
Stereo	450
Comfortable Interior	3300
Security System	1200
Handling	750
Storage Space	120
Power Steering	600
Power Locks & Mirrors	300
Total	$19,520

Information as Facts



Classic Bayesian Decision Theory

States $s_i \in S = \{s_1, s_2, \dots s_N\}$



Actions $a_j \in A = \{a_1, a_2, \dots a_M\}$

Utilities $U(s_i, a_j)$ has range $|S| \times |A|$

Prior Beliefs $\sigma = \{\sigma_1, \sigma_2, \dots \sigma_N\}$
with $\sigma_i < 1$ and $\Sigma_s \sigma_i = 1$

Signals $y_j \in Y = \{y_1, y_2, \dots y_L\}$

Information $\pi = p(y_l | s_j) =$ range $|S|x|Y|$
$\{\{p(y_l | s_1), p(y_2 | s_1), \dots p(y_L | s_1)\},$
$\{p(y_l | s_2), p(y_2 | s_2), \dots p(y_L | s_2)\},$
$\{p(y_l | s_N), p(y_2 | s_N), \dots p(y_L | s_N)\},\}$

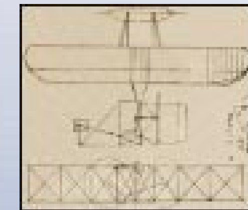
Information is a signal on the state of nature that changes beliefs.

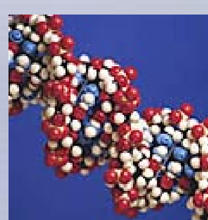
Its *value* is the utility of the best decision with information net of best decision without information:

$\Sigma_S \Sigma_Y U(s_j, a(s_j)) p(s_j | y_k) p(y_k)$ -
$\Sigma_S U(s_j, a(s_j)) \sigma(s_j)$

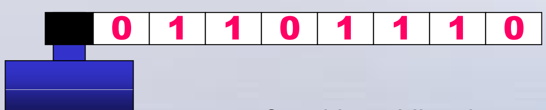
More informative signals are more precise $\{\{1, 0, \dots 0\},$
$\{0, 1, \dots 0\},$
$\{0, 0, \dots 1\}$

Information as Instructions...





Interpreting Turing Machine Instructions



Consider adding the numbers 2 and 3

State Action Description

In State	Symbol on Tape 0	1
S_1	**0 S_1 R**	**0 S_2 R**
S_2	**1 S_3 R**	**1 S_2 R**
S_3	**Halt**	**Halt**

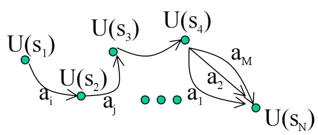
This machine simply moves right along the tape combining the two strings of ones into one contiguous string:

0	**1**	**1**	**1**	**1**	**1**	**0**	**0**

Tuples represent the instructions for moving from state to state

Instructional Model of Information

<u>States</u> $s_i \in S = \{s_0, s_1, s_2, \dots s_N\}$



<u>Actions</u> $a_j \in A = \{a_1, a_2, \dots a_M\}$

<u>Utilities</u> $U(s_i)$ and <u>Costs</u> $C(a_j)$

<u>Prior Beliefs</u> $\sigma = \{\sigma_1, \sigma_2, \dots \sigma_N\}$ with $\sigma_1 < 1$ and $\Sigma_s \sigma_i = 1$

<u>Instruction</u> $y_1 = \{\{s_1, a_1, s_2,\}$, and $\pi(y_1) = p(s_2 | s_1, a_1)$. Instructions can be chained as in $\omega=\{\{s_1, a_1, s_2\}, \{s_2, a_2, s_3\}, \dots\{s_i, a_j, s_k\}\}$

Information describes how to move between states.

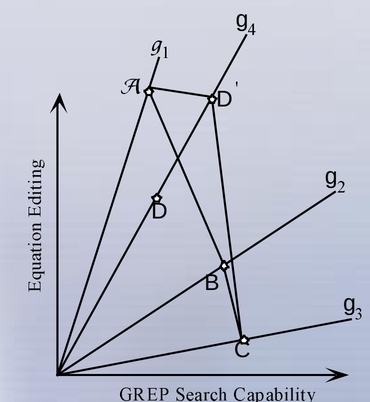
Its *value* is the option on net utility from start to end states:

$$\Sigma_S \Sigma_Y [U(s_j)-C(a(s_j))]p(s_j|y_k)p(y_k) - \Sigma_S [U(s_j)-C(a(s_j))]\sigma(s_j)$$

Easily includes the Bayesian model: To report on states, use instructions with null actions $\{s_i, \varnothing, s_i, \pi_j\}$

But following instructions can *alter* states $s_0 \rightarrow s_1 \rightarrow s_2$

Example: How one might value an information good.



Equation Editing	$60
GREP Search	45
Colleague Compatible	95
WYSIWYG	110
Supports Biblio	35
Supports Tables	25
Supports Inline Graphics	25
<u>On Line Help</u>	<u>60</u>
Total	$455

Proposition: The value of a series of information options v, arriving at rate λ, with interest rate r is $v\lambda\,(1\text{-}e^{-rt})/r$.

$$V(t+\Delta t) = \lambda \Delta t v + e^{-rt} V(t)$$

$$V(t+\Delta t) - V(t) = \lambda \Delta t v + (e^{-r\Delta t} - 1) V(t)$$

$$V(t+\Delta t) - V(t) = \lambda \Delta t v - r \Delta t V(t) + o(-r\Delta t) V(t)$$

$$V'(t) = \lambda v - r V(t)$$

$$\left[V'(t) + r V(t)\right] e^{rt} = \lambda v e^{rt}$$

$$V(t) e^{rt} = \frac{\lambda v}{r} e^{rt} + C_0$$

$$V(0) = 0 \Rightarrow C_0 = -\frac{\lambda v}{r}$$

$$\Rightarrow V(t) = \frac{\lambda v}{r}(1 - e^{rt}) \quad \blacksquare$$

Proposition: The Instrumental model contains the Bayesian model or $M=\{S', A', U', \pi', \alpha\} \supseteq B=\{S, A, U, \pi, \alpha\}$

Intuition:

- By allowing null actions, $\varnothing \in A$, a person can gather information on the current state without changing it.

- Probabilities $\pi \subseteq \pi'$ are only those probabilities associated with states, not actions.

- And, utilities on (*state* x *action*) pairs in the Bayesian model are mapped onto the increment to utilities in the Instrumental model $U(s_i,a_j) = V(s_i) - V(s_0) - C(a_j)$ with $V(s_0) \equiv 0$.

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Corollary: The Bayesian model does not contain the Instrumental model, $M \not\subset B$, as the information capacity is less.

Proof:

Use Shannon's entropy function $H(S) = \Sigma p(s_i)\log(1/p(s_i))$ to measure channel capacity.

Maximal uncertainty in B is given by $p(s_i) = 1/N$.

$$H_B \equiv \sum_{i=1}^{N} p(s_i)\log\left(\frac{1}{p(s_i)}\right) = \sum_{i=1}^{N} \frac{1}{N}\log(N) = \log(N)$$

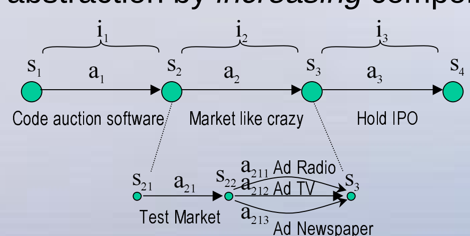
Maximal uncertainty in M is given by $p(s_0, a_j, s_i) = 1/|A|$.

$$H_M \equiv \sum_{i=1}^{N}\sum_{j=1}^{A} p(s_0, a_j, s_i)\log\left(\frac{1}{p(s_0, a_j, s_i)}\right) = \sum_{i=1}^{N}\sum_{j=1}^{A} \frac{1}{A}\log(A) = N\log(A)$$

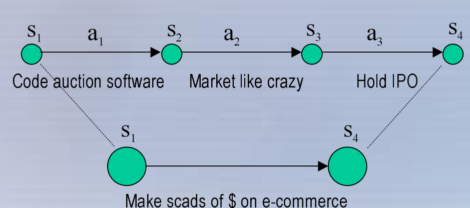
$H_M > H_B$ for all values of $A \geq 2$ and $N \geq 1$. ■

Treating information as instructions allows a possible model of abstraction...

Reduce abstraction by *increasing* component detail



Increase abstraction by *reducing* component detail



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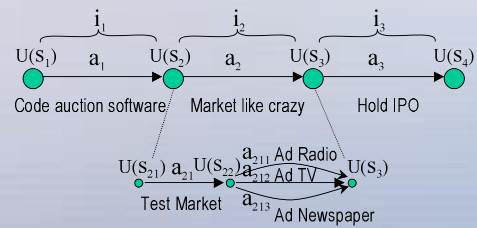
This formulation also allows a possible model of context...

What is the value of instruction $i_2=(s_2,a_2,s_3)$ in the context of i_1, and i_3? Or, i_{211} in the context of i_{212} and i_{213}?



Let a knowledge K base be a collection of instructions $\{i_1, i_2, \ldots i_n\}$. Then the value of resource i_i can be measured according to a standard resource allocation formula

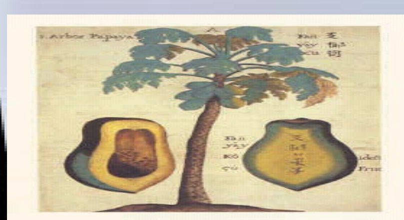
$$V(i\,|\,K) = \sum_{k \subseteq K} p(k\,|\,K)\left[V(k) - V(k \setminus \{i\})\right]$$

where $p(k\,|\,K) = \dfrac{(|k|-1)!(|K|-|k|)!}{|K|!}$

The Shapley Formula

- Treats all resources symmetrically.
- Awards nothing to non-contributing resources.
- Is Pareto efficient
- Sets the expected value of the sum = the sum of the expected values.
- And leads to a Walrasian Equilibrium!

Examples of Dual Information



Questions of Information Value

- For websites IQPort.com and FatBrain.com, resolve the “inspection paradox” by providing samples or limited functionality. Use *hedonics* to value features sets.
- Value Dow’s process for PVC and Wal-Mart’s method of “cross-docking” by valuing the *option* on the result, estimating the frequency of use, volatility, and discount rate.
- To value a firm’s collected intangibles, value the *portfolio* of options provided by it’s information (data *and* process knowledge).

Conclusions

- Information can be modeled as instructions, not just signals on the states of nature.
- The framework has precedent in ideas from Austin, Turing, and von Neumann.
- Bayesian models are captured within the same proposed framework.
- Combined with *options* pricing and *hedonics,* this allows us to value algorithms, blueprints, gene sequences, production know-how as well as data.
- This framework permits models of abstraction and context at the micro level that lead to reasonable behavior at the macro level.