Large Scale LP Models for Finding Optimal Container Inspection Strategies

E. Boros
Rutgers University

Joint research with L. Fedzhora, P.B. Kantor (Rutgers University) and K. Saeger and P. Stroud (LANL)

Main Questions

- How to evaluate the utility of new sensor technology?
- How to design/expand most efficiently inspection stations/capacities?
- How to use optimally existing sensor technologies to inspect arriving containers to detect illegal radioactive content?

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- Maximize **detection rate**.
- Minimize **average inspection cost**.
- Minimize **false positives**.
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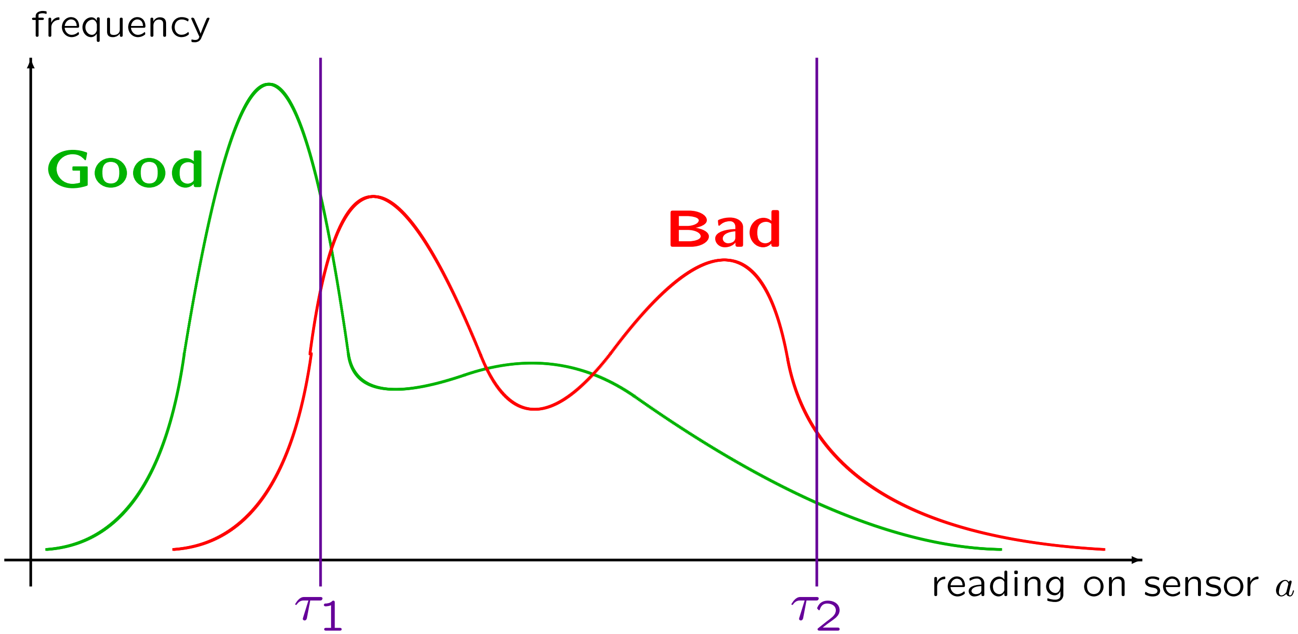
Background: Stroud and Saeger (2003,2004)

Modeling Difficulties and Assumptions

- It is hard to get data for sensor readings (in particular for dangerous containers).

Standard Sensor Model



For every choice of thresholds $\tau_1 \leq \tau_2$ we can compute the proportion (expected fraction) of containers which have their sensor readings in the interval $[\tau_1, \tau_2]$:

- $\rho(a, \tau_1, \tau_2)$ for **Good** containers, and
- $\xi(a, \tau_1, \tau_2)$ for **Bad** containers.

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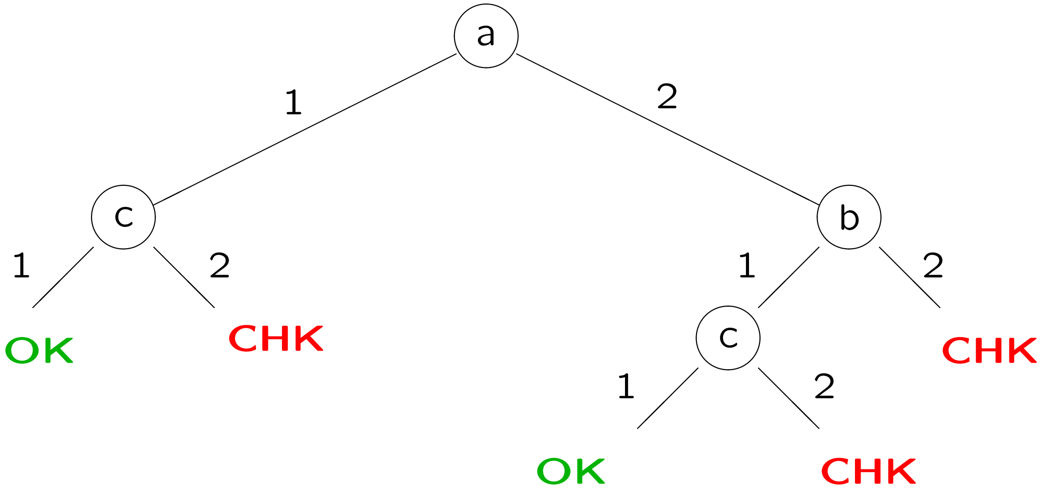
- It is hard to get reliable estimate of cost of a false negative.

  Handle separately **inspection cost** and **detection rate**.

- We did not see (yet) one **Bad** container.

  Compute **inspection costs**, **false positives**, and **container delays** only for **Good** containers.

What is an Inspection Policy?



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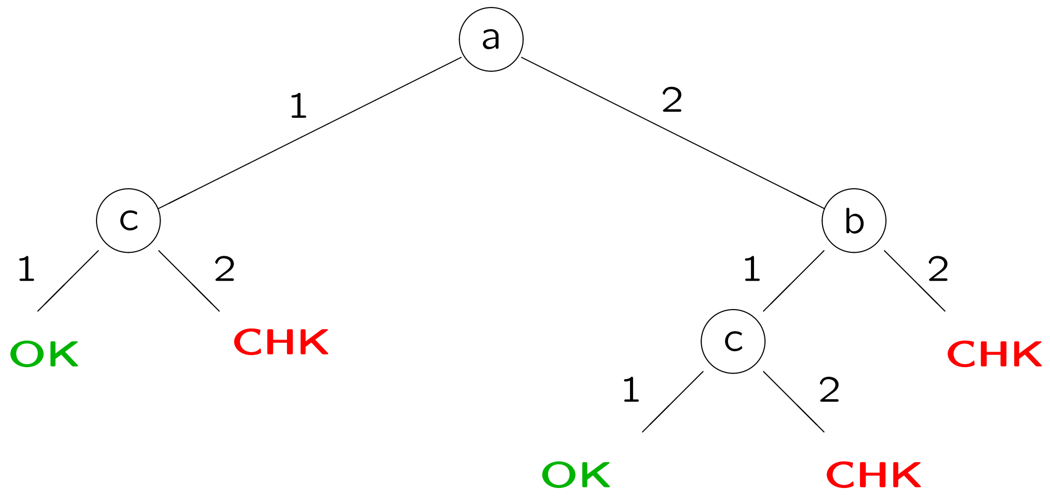

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- More than $\mathbf{s}!\,\mathbf{2}^{\left(\mathbf{2}^{\mathbf{s}}/\sqrt{s}\right)}$ binary decision trees with $\mathbf{s}$ sensors!

  Enumeration is possible for $\mathbf{s} \leq 4$ (Stroud and Saeger, 2003).

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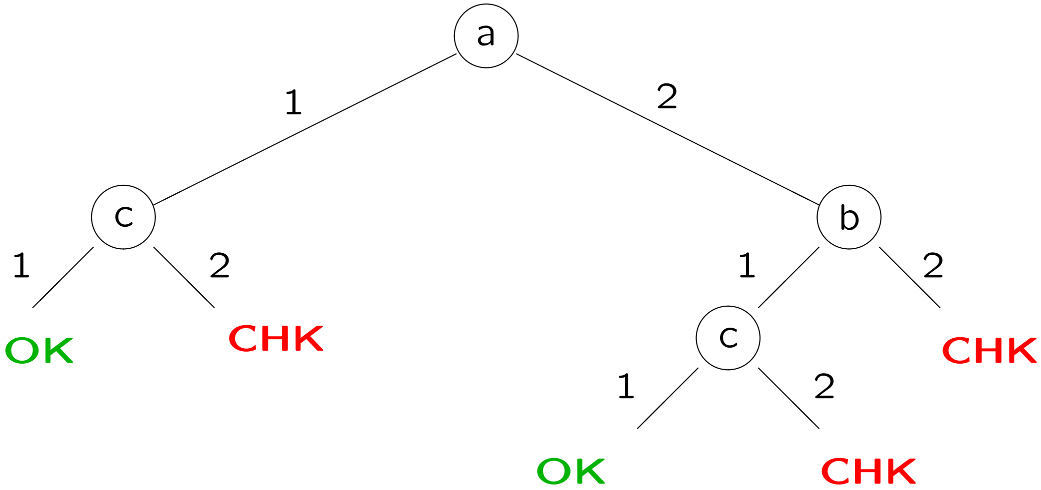

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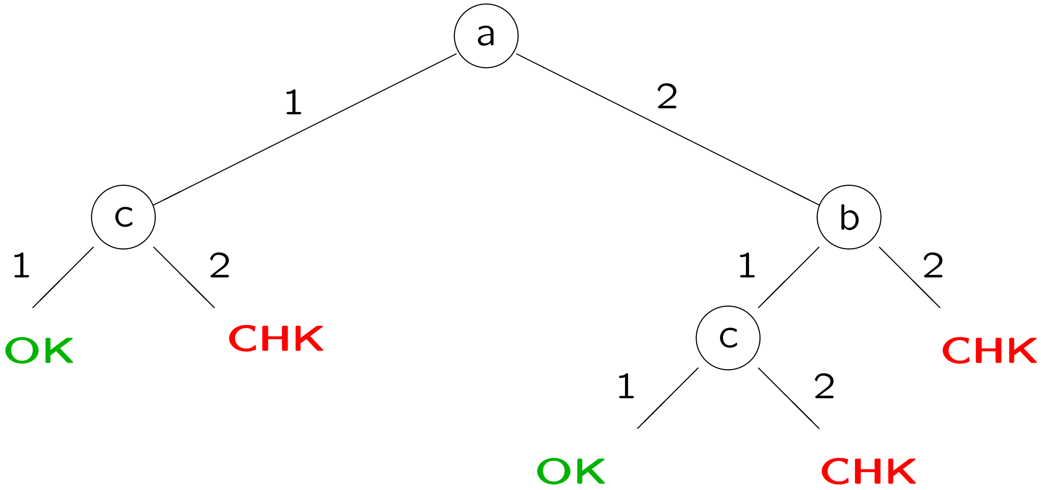

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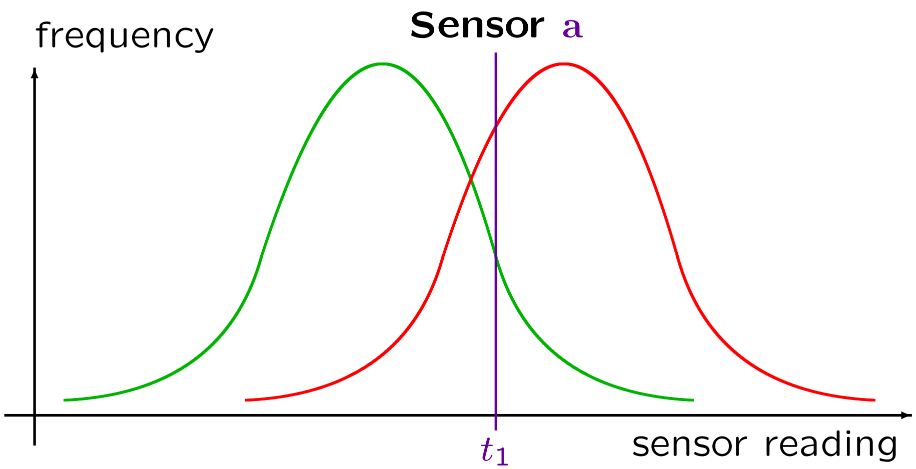
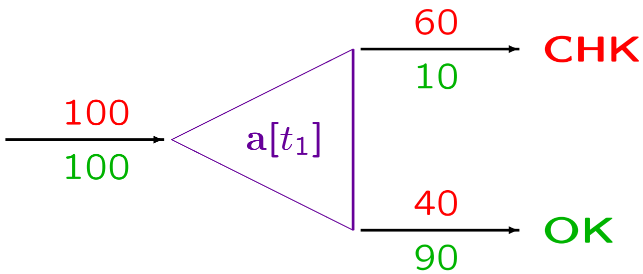
- Why binary? Why not more than one threshold for some/all sensors?

- Why a single decision tree? Why not a mixed strategy?

Small Example





Detection rate	$\boldsymbol{\Delta} =$	0.60
False Positives	$\mathbf{FP} =$	0.10
Average Cost	$\mathbf{C} =$	$C_a + 0.1 C_{CHK}$

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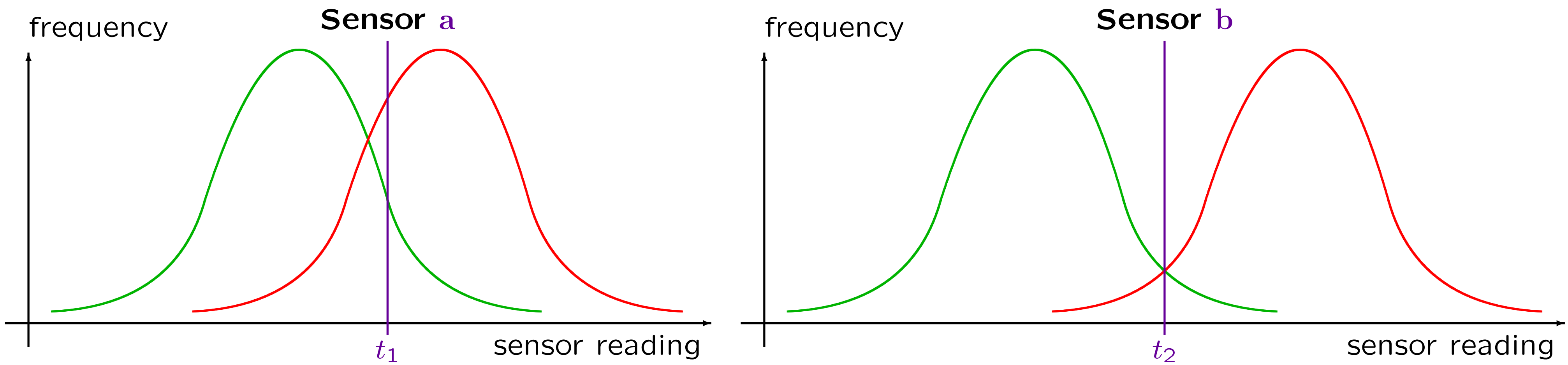

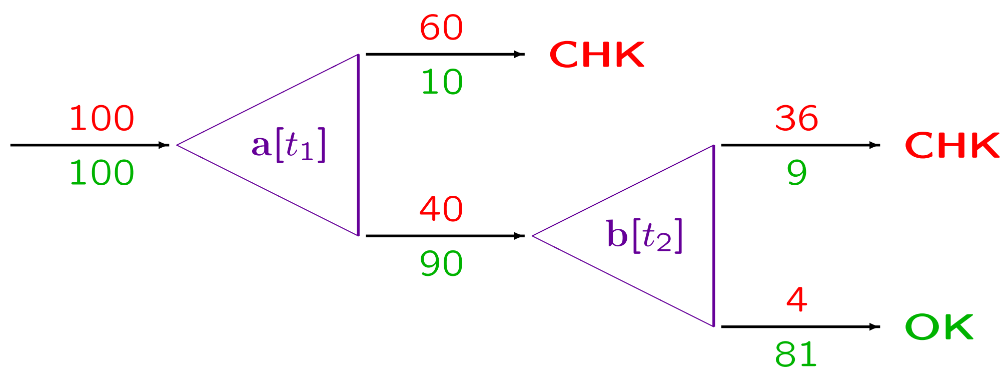

Detection rate	$\mathbf{\Delta} =$	0.60	0.96
False Positives	$\mathbf{FP} =$	0.10	0.19
Average Cost	$\mathbf{C} =$	$C_a + 0.1 C_{CHK}$	$C_a + 0.9 C_b + 0.19 C_{CHK}$

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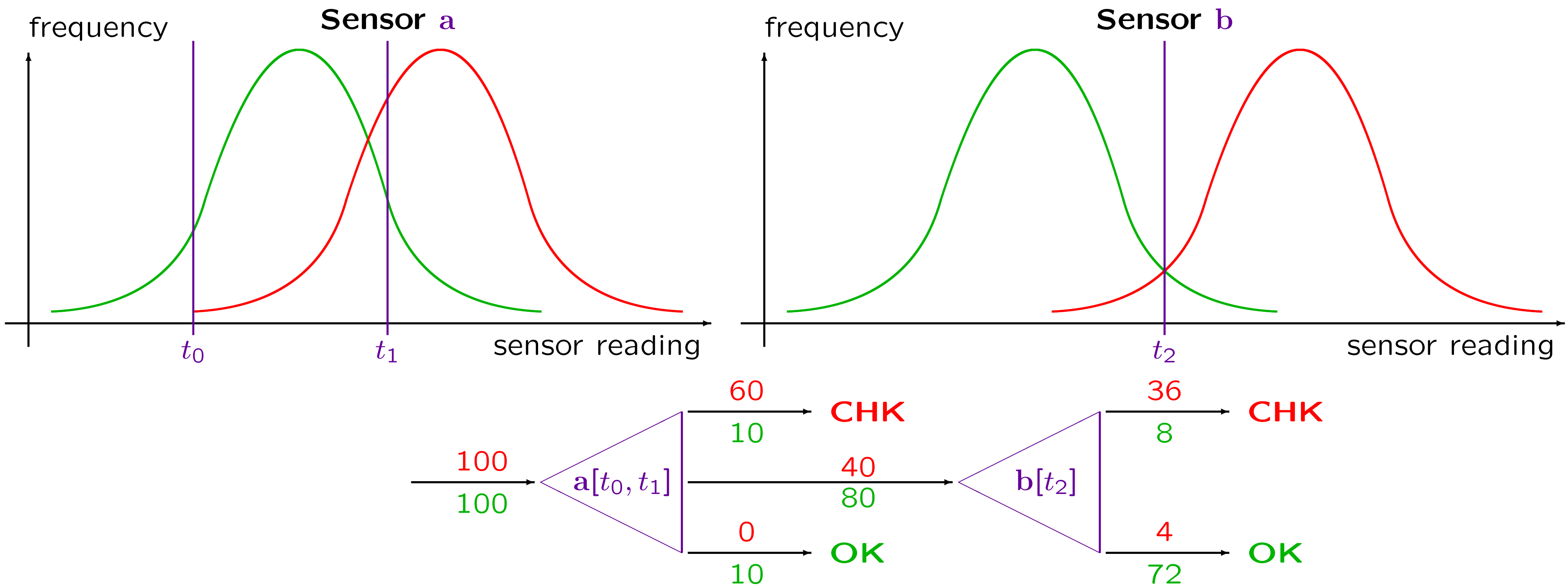

Detection rate	$\mathbf{\Delta} =$	0.60	0.96	0.96
False Positives	$\mathbf{FP} =$	0.10	0.19	0.18
Average Cost	$\mathbf{C} =$	$C_a + 0.1 C_{CHK}$	$C_a + 0.9 C_b + 0.19 C_{CHK}$	$C_a + 0.8 C_b + 0.18 C_{CHK}$

Large Scale MILP Model

Set of sensors: $\mathcal{S} = \{a, b, c, d\}$

Number of containers: $\mathbf{q}$ (over a longer period; only for capacity planning)

Inspection strategy: decision tree $\mathbf{D}$ and threshold selection $\mathbf{t}$

Decision variables: $x_{(\mathbf{D},\mathbf{t})}$ = fraction of containers inspected by $(\mathbf{D}, \mathbf{t})$

MILP model:

$$\begin{array}{rcll}
\sum_{(\mathbf{D},\mathbf{t})} \Delta(\mathbf{D},\mathbf{t}) x_{(\mathbf{D},\mathbf{t})} & \to & \text{max} & \\
C_0 + \sum_{(\mathbf{D},\mathbf{t})} C(\mathbf{D},\mathbf{t}) x_{(\mathbf{D},\mathbf{t})} & \leq & Budget & \\
\sum_{(\mathbf{D},\mathbf{t})} FP(\mathbf{D},\mathbf{t}) x_{(\mathbf{D},\mathbf{t})} & \leq & Limit & \\
\mathbf{q} \sum_{(\mathbf{D},\mathbf{t})} \mathbf{s}(\mathbf{D},\mathbf{t}) x_{(\mathbf{D},\mathbf{t})} & \leq & K(\mathbf{s}) & \forall \mathbf{s} \in \mathcal{S} \\
\sum_{(\mathbf{D},\mathbf{t})} x_{(\mathbf{D},\mathbf{t})} & = & 1 & \\
x_{(\mathbf{D},\mathbf{t})} & \geq & 0 & \forall (\mathbf{D},\mathbf{t})
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For a fixed order of the sensors and a fixed grid of possible threshold values we can describe efficiently a polyhedron in a higher dimensional space, the vertices of which are exactly the inspection strategies $(\mathbf{D},\mathbf{t})$ in which sensors follow the fixed order and $\mathbf{t}$ belongs to the fixed grid.

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Efficient column generation is possible!

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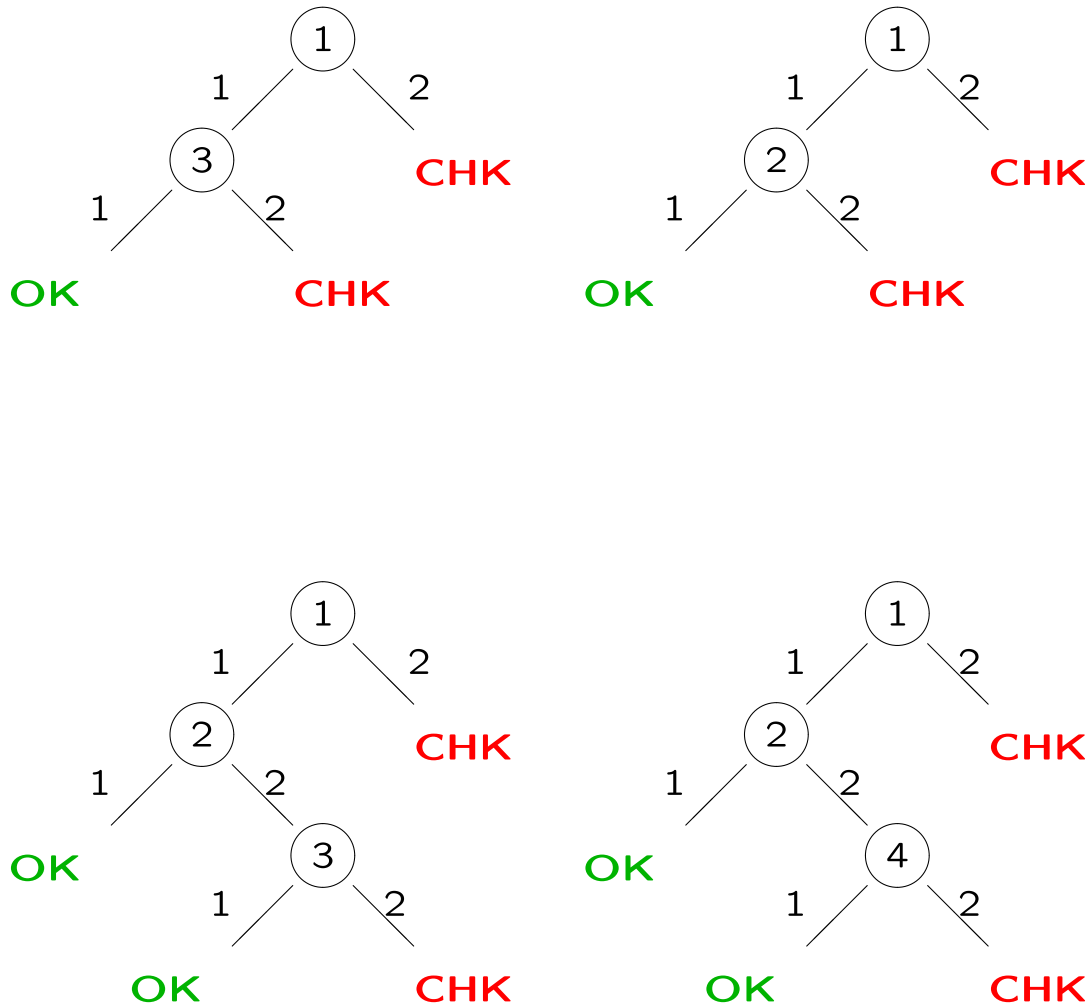
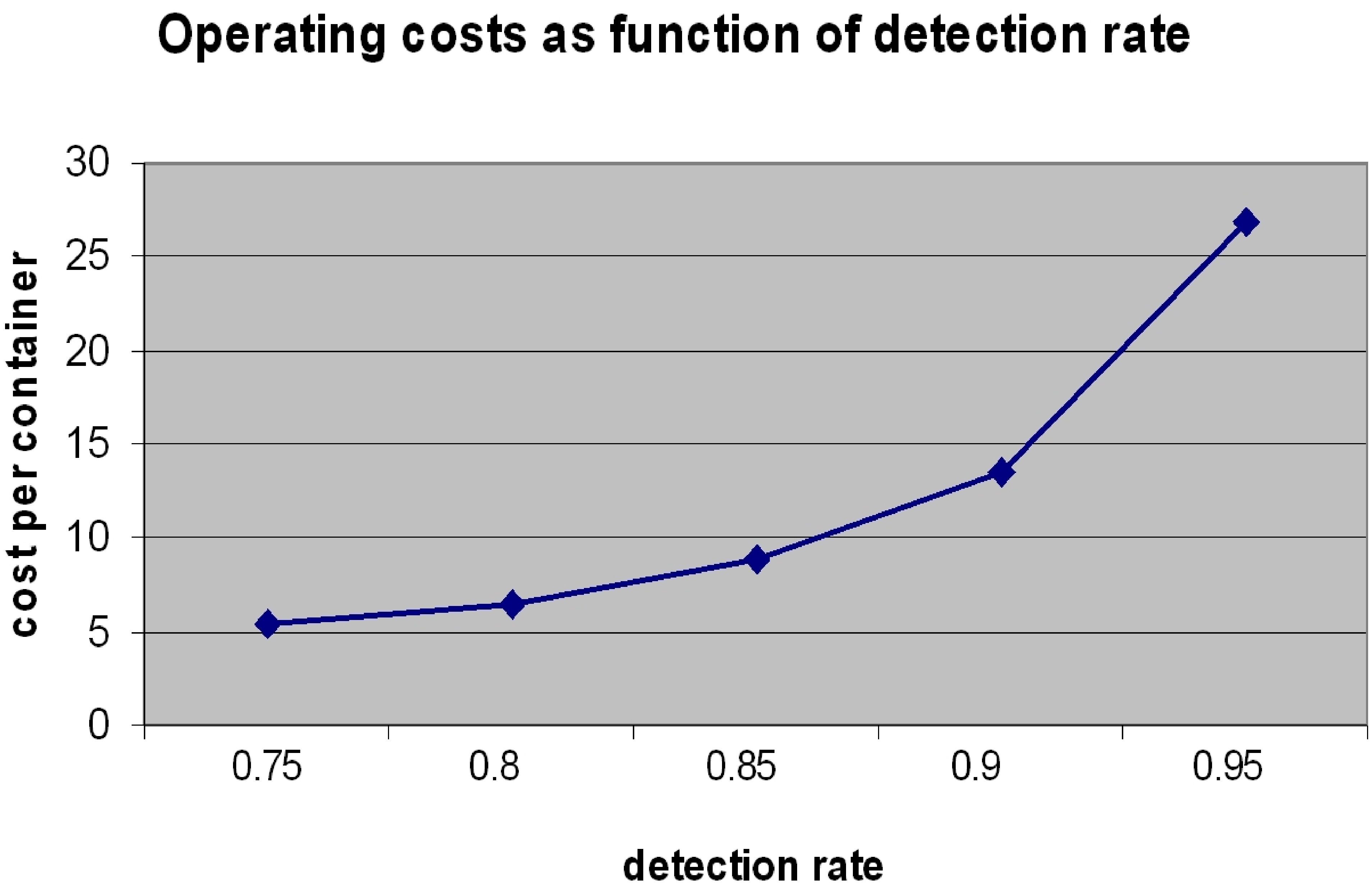
LP-s can be solved faster than the time needed to write them down!

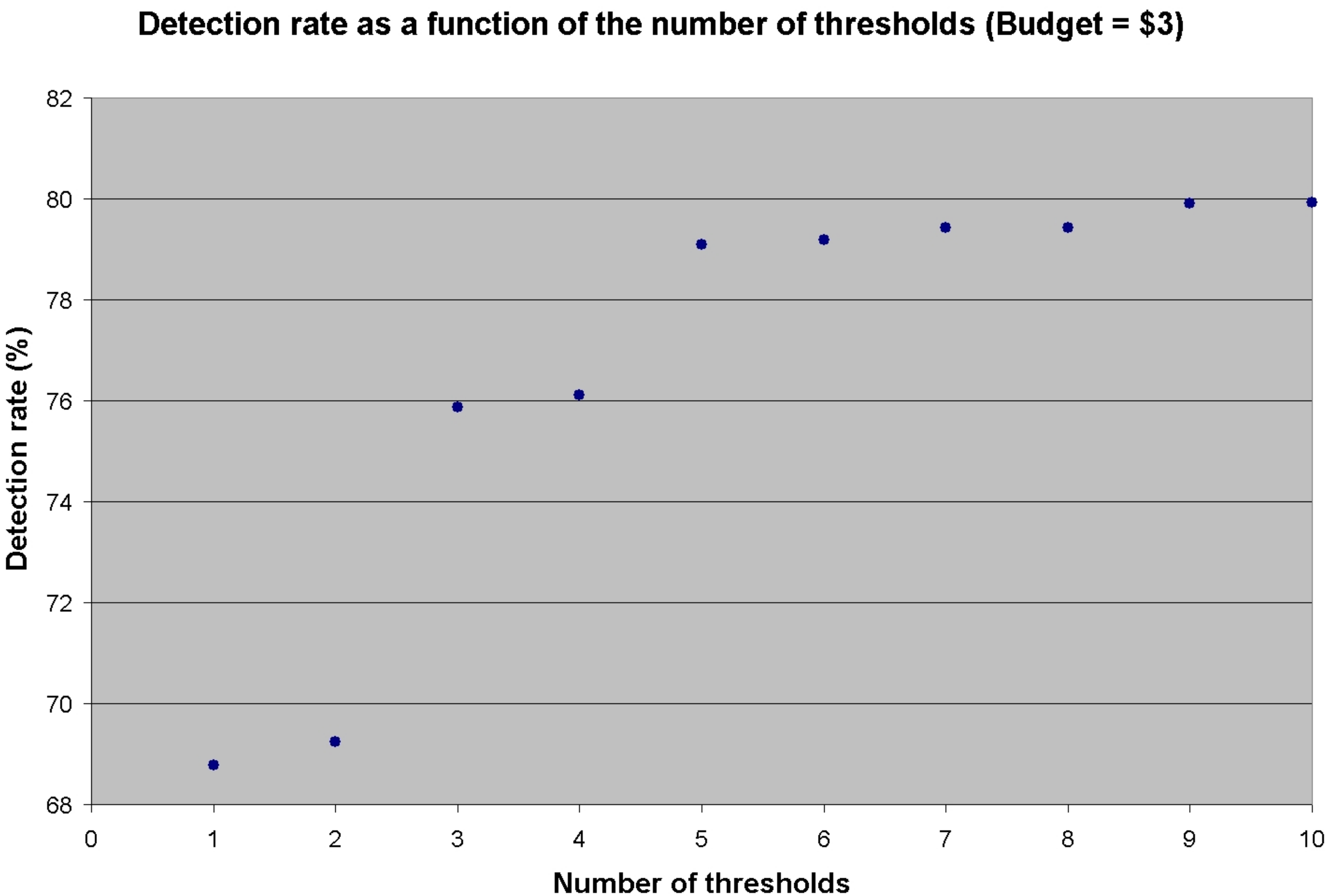
Typical output is a mixed strategy



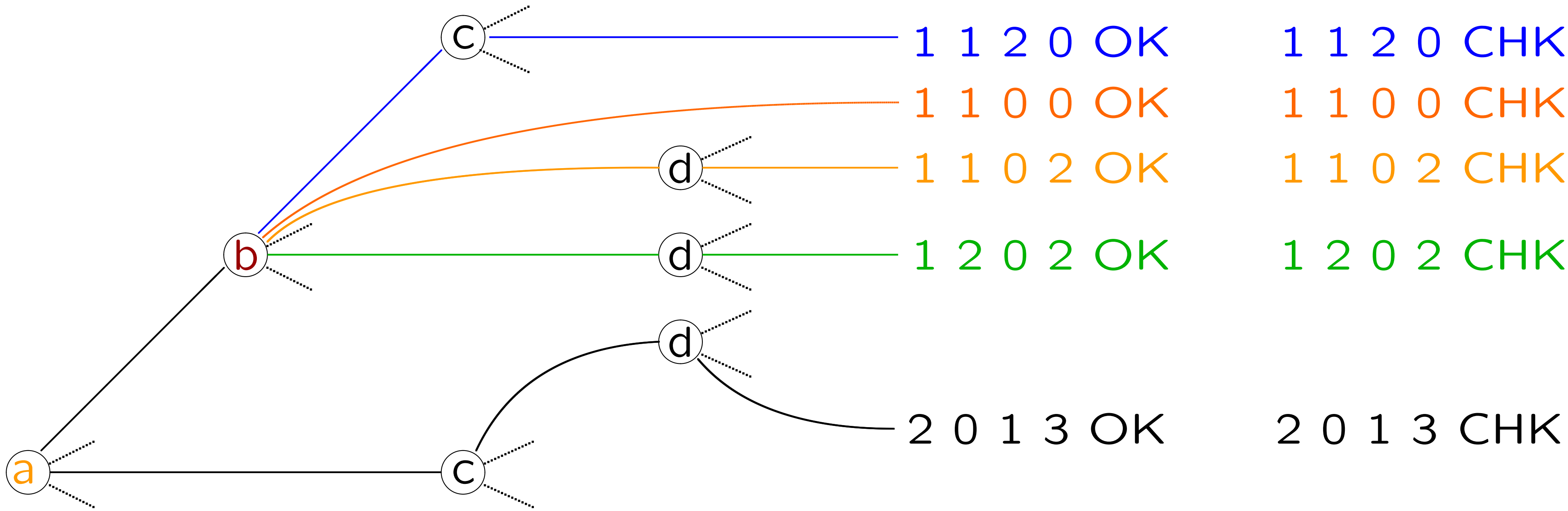
Sample Results with 4 Sensors and up to 5 Thresholds for Each



Sample Results at a Fixed Budget of $3



Network Model



Flow values: $y_\pi \geq 0$ for $\pi \in \mathbf{F} \; = \; X_a \times X_b \times X_c \times X_d \times \{\text{OK,CHK}\}$

where $X_s = \{0, 1, ..., r_s\}$ (r_s ranges $s \in \{a, b, c, d\}$).

Throughputs:

$$Good(\pi) \quad = \quad \rho(a, \pi_1)\rho(b, \pi_2)\rho(c, \pi_3)\rho(d, \pi_4)$$

$$Bad(\pi) \quad = \quad \xi(a, \pi_1)\xi(b, \pi_2)\xi(c, \pi_3)\xi(d, \pi_4)$$

where $\rho(s, 0) = \xi(s, 0) = 1$ for all sensors s

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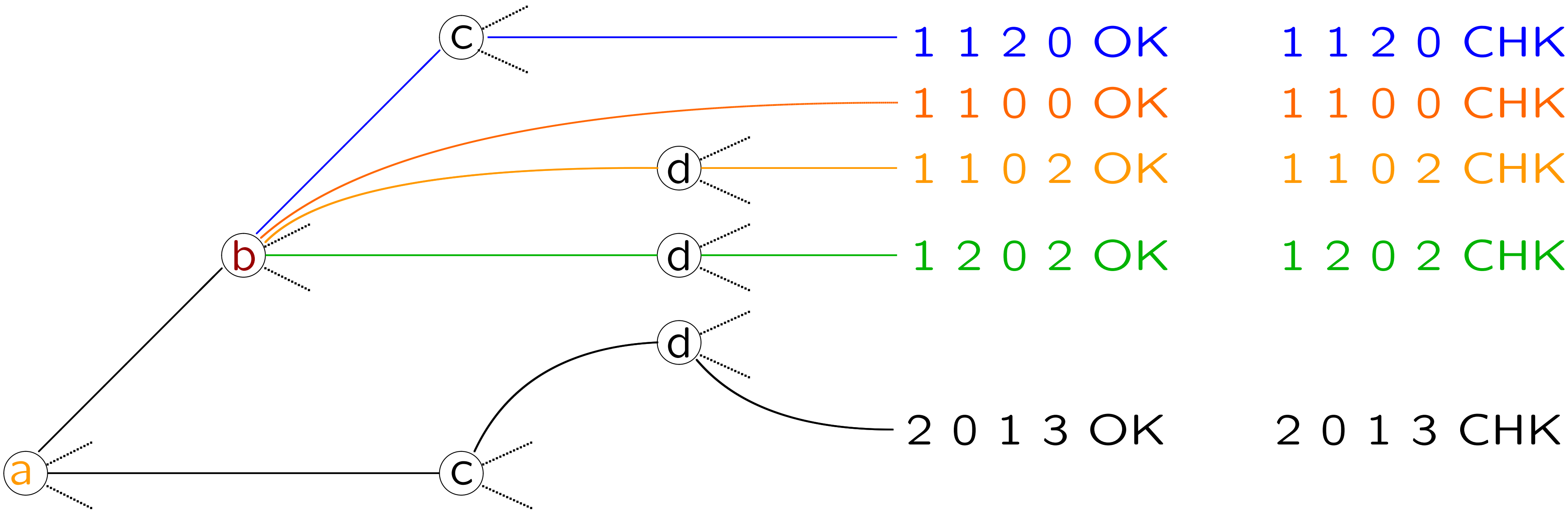

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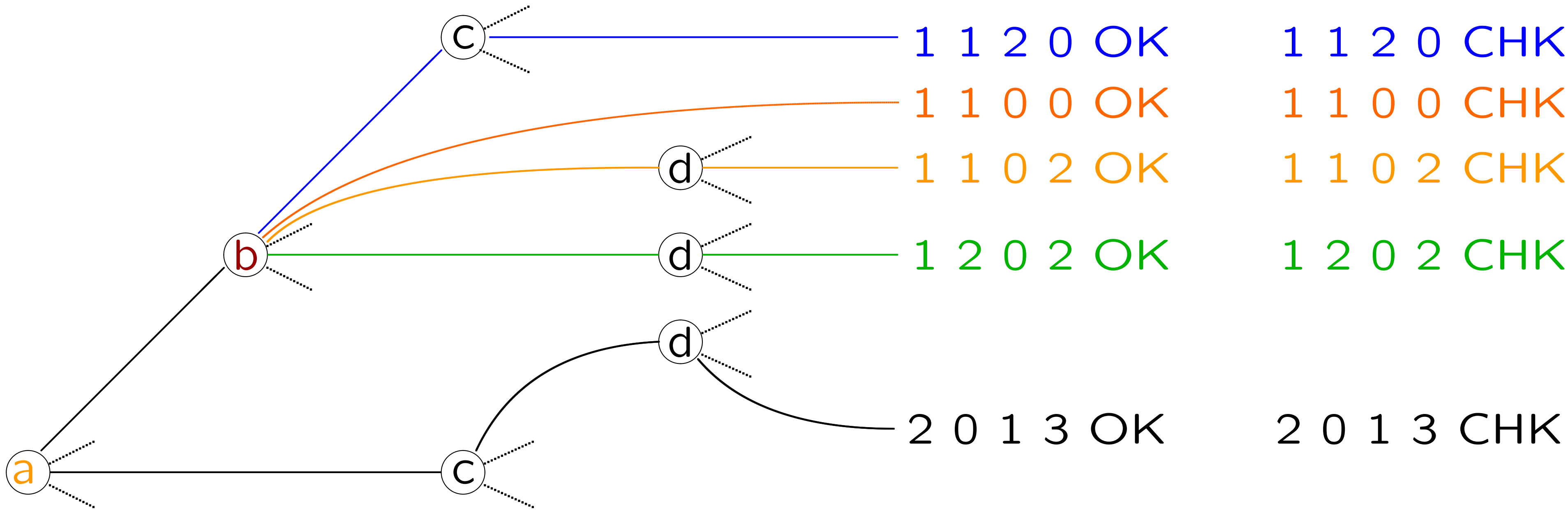
Initialization: $$\sum_{\pi \in \mathbf{F}} y_\pi \;=\; 1$$ (we normalized by $\mathbf{q}$)

No skipping first: $X_a = \{1, 2, ..., r_a\}$.

Network Model



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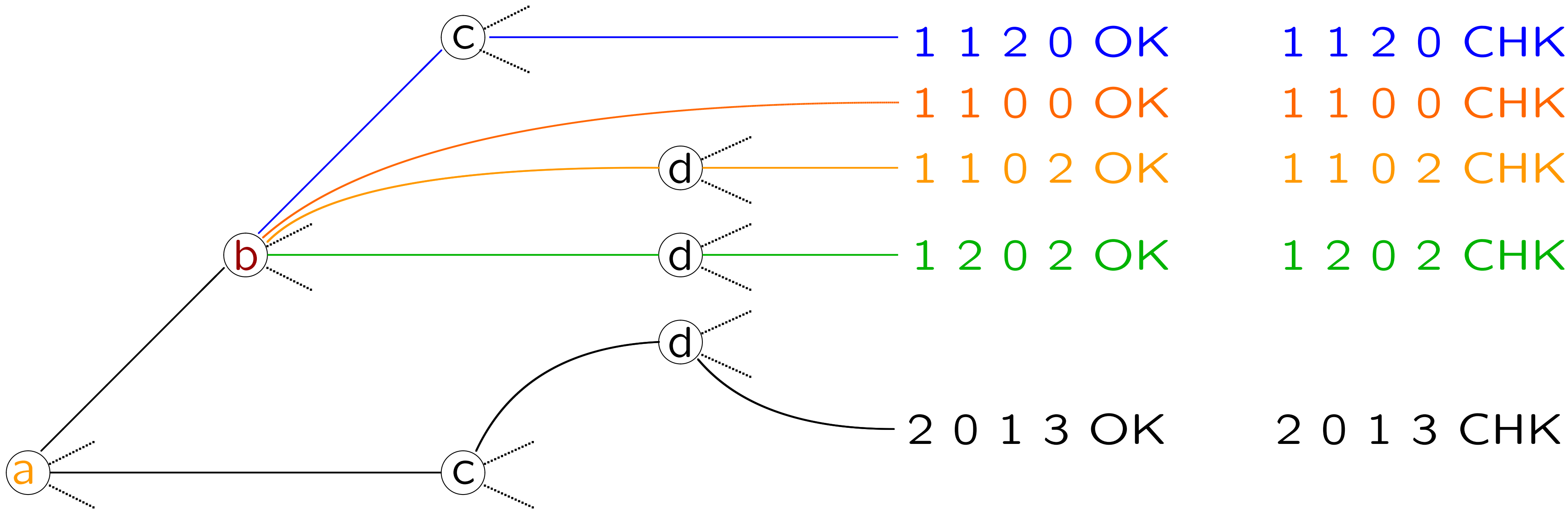
Flow conservation:

$$\begin{aligned} y_{11*} &= \rho(b,1)\,(y_{11*} + y_{12*} + ...) \\ y_{12*} &= \rho(b,2)\,(y_{11*} + y_{12*} + ...) \\ &\vdots \end{aligned}$$

where e.g., $y_{11*} = \sum_{\gamma \in X_c} \sum_{\delta \in X_d} \sum_{\varepsilon \in \{\text{OK,CHK}\}} y_{11\gamma\delta\varepsilon}$.

Network Model



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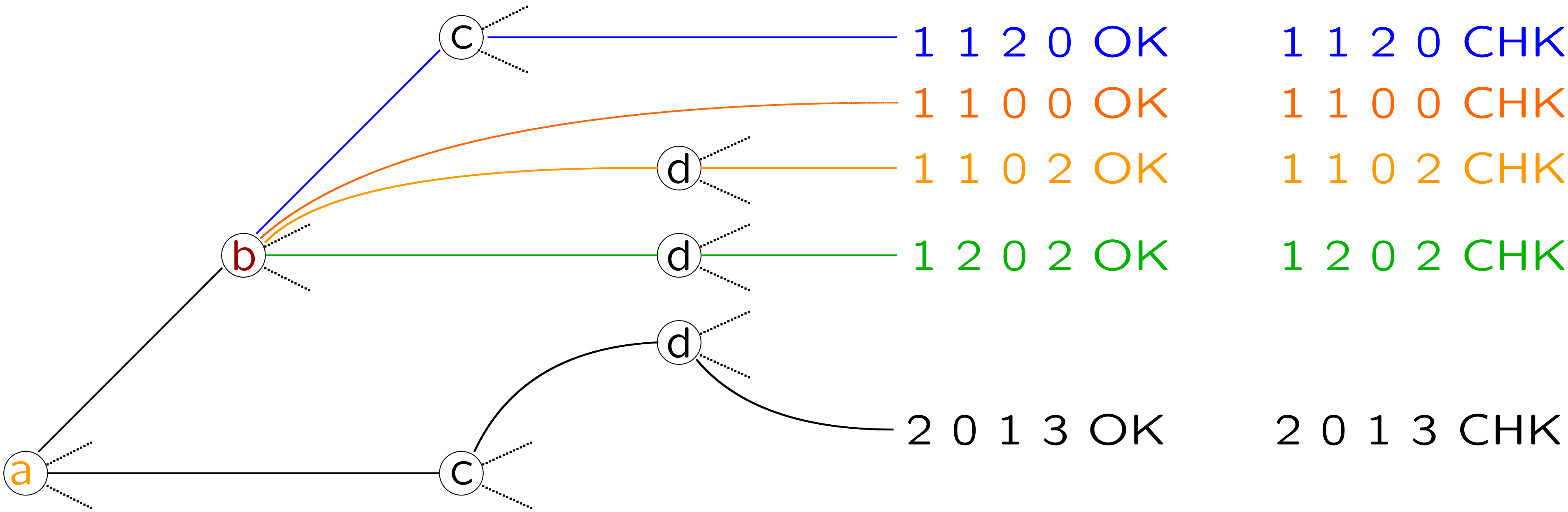
Capacity constraints:

$$\sum_{\alpha \in X_a \setminus \{0\}} y_{\alpha *} \;\leq\; K(a)/\mathbf{q}$$

$$\sum_{\alpha \in X_a} \sum_{\beta \in X_b \setminus \{0\}} y_{\alpha\beta *} \;\leq\; K(b)/\mathbf{q}$$

$$\vdots$$

Network Model



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Operating Cost: $\sum_{\pi \in \mathbf{F}} C(\pi) y_\pi$ $\quad \leq Budget$ or *minimize*

Detection Rate: $\sum_{\substack{\pi \in \mathbf{F} \\ \pi_5 = \text{CHK}}} \frac{Bad(\pi)}{Good(\pi)} y_\pi$ $\quad$ *maximize* or ≥ 0.85

False Positives: $\sum_{\substack{\pi \in \mathbf{F} \\ \pi_5 = \text{CHK}}} y_\pi$ $\quad$ minimize or ≤ 0.05

Typical output

Result for $n = 4$ sensors with $1 - 1$ thresholds T=[3; 1; 2; 3]:

1	2	3	4	decision
1	0	1	0	ok
1	0	2	0	chk
1	1	0	0	ok
1	2	0	0	chk
1	2	0	1	ok
1	2	0	2	chk
1	2	1	0	ok
1	2	2	0	chk
2	0	0	0	chk

Detection rate	0.778474
False positives	0.031250
Operating cost	$10.42