OR Models for Finding Optimal Container Inspection Strategies

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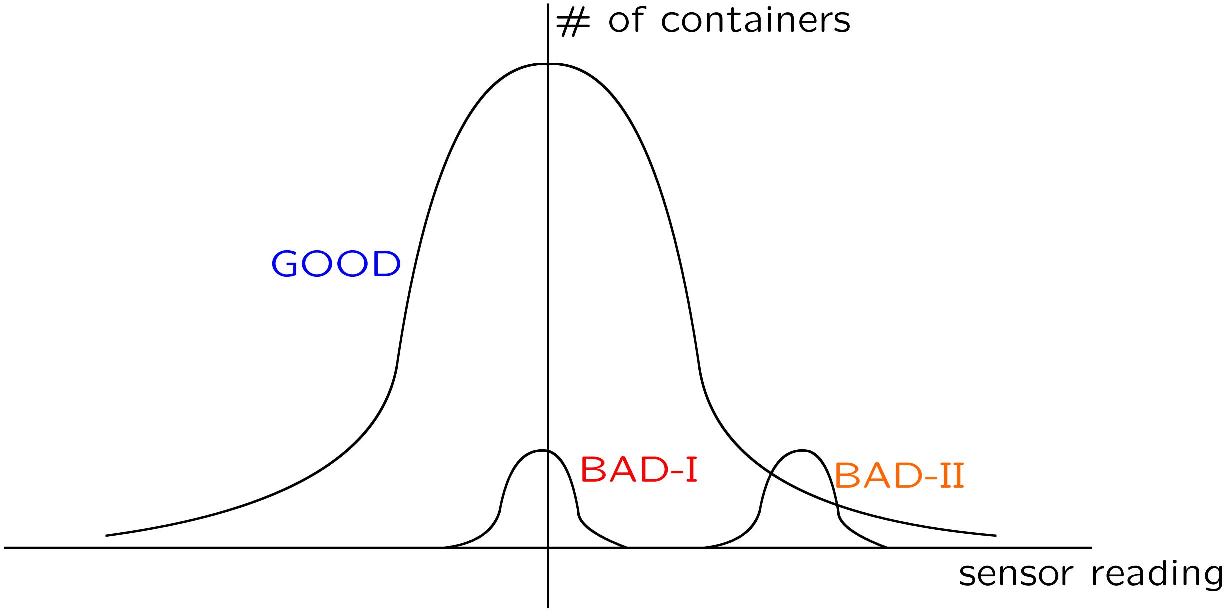
Joint research with L. Fedzhora, P.B. Kantor, and P.D. Stroud

Sensor Model



- For sensor s and container type $\tau \in \mathcal{T}$ the sensor readings (a single real) are normally distributed with *known* mean $\mu_{s\tau}$ and variance $\sigma_{s\tau} = 1$.

[Assumed: $\mathcal{T} = \mathcal{G} \cup \mathcal{B}$, $\mathcal{G} = \{g\}$, $\mathcal{B} = \{b', b''\}$]

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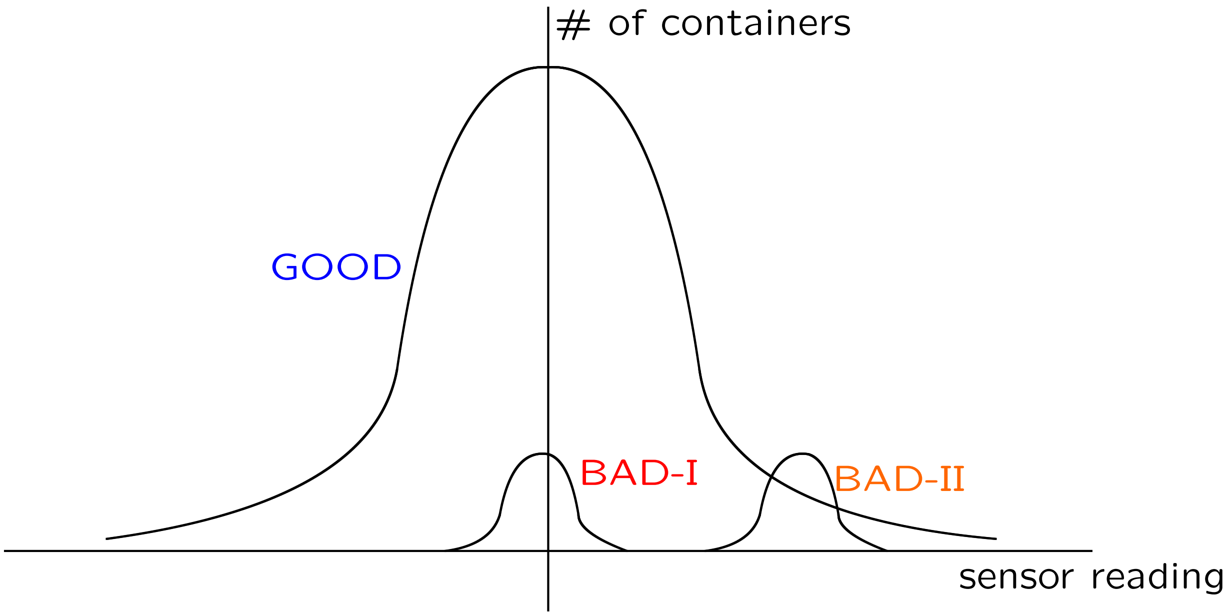

- For sensor s and container type $\tau \in \mathcal{T}$ the sensor readings (a single real) are normally distributed with *known* mean $\mu_{s\mathcal{T}}$ and variance $\sigma_{s\mathcal{T}} = 1$.

  [Assumed: $\mathcal{T} = \mathcal{G} \cup \mathcal{B}$, $\mathcal{G} = \{g\}$, $\mathcal{B} = \{b', b''\}$]

- Containers' a priori distribution: $\mathbf{P}_\tau$, $\tau \in \mathcal{T}$ (where $\sum_{\tau \in \mathcal{T}} \mathbf{P}_\tau = 1$).

  [unknown; assumed to be $\mathbf{P}_g \approx 1$, $\mathbf{P}_{b'} = \mathbf{P}_{b''} \approx 0$]

Inspection Model

A *decision tree* (**BDT**), in which each node corresponds to one of the sensors having its own thresholds, and with **two terminal states**:

- **CHK**: manually inspect it (each at $\mathbf{Q}_\tau$ cost for $\tau \in \mathcal{T}$);
  [$\mathbf{Q}_g = \mathbf{Q}_{b'} = \mathbf{Q}_{b''} =$ \$600]

- **OK**: let it go in peace (each at $\mathbf{R}_\tau$ additional expected cost for $\tau \in \mathcal{T}$);
  [$\mathbf{R}_g = 0$, $\mathbf{R}_{b'} = \mathbf{R}_{b''} =? \gg 0$]

Inspection Model

A *decision tree* (**DT**), in which each node corresponds to one of the sensors having its own thresholds, and with **two terminal states**:

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Nodes corresponding to the same sensor share a common capacity (with a fixed cost capacity extension model).

Average Cost of Inspection

$$
\begin{aligned}
F(\mathbf{D},\mathbf{t}) &= \frac{K_f}{\mathbf{q}} + \sum_{\tau\in\mathcal{G}} \mathbf{P}_\tau \left(C_\tau + \delta_\tau \mathbf{Q}_\tau\right) + \sum_{\tau\in\mathcal{B}} \mathbf{P}_\tau \left(C_\tau + (1-\delta_\tau)\mathbf{R}_\tau\right) \\
&\approx \left[\frac{K_f}{\mathbf{q}} + \sum_{\tau\in\mathcal{G}} C_\tau + \sum_{\tau\in\mathcal{G}} \delta_\tau \mathbf{Q}_\tau\right] + \sum_{\tau\in\mathcal{B}} (1-\delta_\tau)\mathbf{P}_\tau\mathbf{R}_\tau
\end{aligned}
$$

- $\mathbf{D}$ is a DT,
- $\mathbf{t}$ is the vector of chosen thresholds,
- $\mathbf{q}$ is the number of inspected containers,
- $K_f = K_f(\mathbf{D},\mathbf{t},\mathbf{P}) \approx K_f(\mathbf{D},\mathbf{t})$ is the fixed cost of creating the necessary capacity to handle the container flow for this instance,
- $C_\tau = C_\tau(\mathbf{D},\mathbf{t})$ is the average cost of inspecting a container of type $\tau \in \mathcal{T}$, and
- $\delta_\tau = \delta_\tau(\mathbf{D},\mathbf{t})$ is the fraction of containers of type $\tau \in \mathcal{T}$ checked manually by the policy $(\mathbf{D},\mathbf{t})$.

Optimization Problem

Minimize $F(\mathbf{D}, \mathbf{t})$

over **all** DT-s $\mathbf{D}$, and **all possible** threshold selections $\mathbf{t}$.

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over **all** DT-s $\mathbf{D}$, and **all possible** threshold selections $\mathbf{t}$.

- The number of inspected containers $\mathbf{q}$ must be large (long time period) in order to amortize the fixed cost K_f in a realistic way!

- The expected costs of false negatives, $\mathbf{P}_\tau \mathbf{R}_\tau$, $\tau \in \mathcal{B}$, must be estimated carefully!

Simplifying Assumptions

$$\boxed{\mathbf{R}_g = 0,\ \mathbf{P}_g \approx 1,\ \text{and}\ \mathbf{P}_{b'}\mathbf{R}_{b'} = \mathbf{P}_{b''}\mathbf{R}_{b''} = \mathbf{K}^*}$$

yielding

$$F(\mathbf{D}, \mathbf{t}) = C(\mathbf{D}, \mathbf{t}) + \mathbf{K}^* (1 - \Delta(\mathbf{D}, \mathbf{t}))$$

where

- $C(\mathbf{D}, \mathbf{t}) = \frac{K_f}{\mathbf{q}} + C_g$ is the average inspection cost of **good** containers

- $\Delta(\mathbf{D}, \mathbf{t}) = \frac{1}{2}(\delta_{b'} + \delta_{b''})$ is the so called **detection rate** of policy $(\mathbf{D}, \mathbf{t})$

Surrogate Problem I

$$\textbf{Minimize} \quad C(\mathbf{D}, \mathbf{t}) = \frac{K_f(\mathbf{D}, \mathbf{t})}{\mathbf{q}} + C_g(\mathbf{D}, \mathbf{t})$$

subject to

$$\Delta(\mathbf{D}, \mathbf{t}) = \frac{1}{2}\left(\delta_{b'}(\mathbf{D}, \mathbf{t}) + \delta_{b''}(\mathbf{D}, \mathbf{t})\right) \geq 0.85$$

over **all** DT-s $\mathbf{D}$, and **all possible** threshold selections $\mathbf{t}$.

[Stroud and Saeger, 2004; one threshold/sensor]

Surrogate Problem II

$$\textbf{Maximize}\quad \Delta(\mathbf{D},\mathbf{t}) = \frac{1}{2}\left(\delta_{b'}(\mathbf{D},\mathbf{t}) + \delta_{b''}(\mathbf{D},\mathbf{t})\right)$$

subject to

$$C(\mathbf{D},\mathbf{t}) = \frac{K_f(\mathbf{D},\mathbf{t})}{\mathbf{q}} + C_g(\mathbf{D},\mathbf{t}) \leq Budget$$

over **all** DT-s $\mathbf{D}$, and **all possible** threshold selections $\mathbf{t}$.

Comments

All these problems can be solved by

- complete enumeration of **all** DT-s $\mathbf{D}$,

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- and by solving a difficult nonlinear optimization problem to find the best thresholds $\mathbf{t}$ for each DT $\mathbf{D}$

- **finds only pure strategies, which are rarely the optimal ones!**

Method does not scale well – growth rate even in the 1-1 threshold case is

$$\mathbf{\Omega}\Big(\mathbf{n!}\ \cdot\ \mathbf{M(n)}\Big)$$

where n is the number of sensors, and

$$\mathbf{M(n)}\ \approx\ \mathbf{2}^{\frac{\mathbf{2^n}}{\sqrt{n}}}.$$

Comments cont'd

n	$M(n)$	$n!$
0	2	1
1	3	1
2	6	2
3	20	6
4	168	24
5	7581	120
6	7828354	720
7	2414682040998	5040
8	56130437228687557907788	40320

Dedekind asked first to determine the number M(n) of distinct monotone functions of n variables in 1897. There is still no concise closed-form expression for M(n). As of 1999 only the above values were known.

Proposed Approach

All these problems can be formulated as (large scale) mixed integer linear programming problems (network type, with only a *few* integer variables):

- fix a permutation of the sensors and consider only inspection strategies which agree with this permutation, and
- fix a grid of possible threshold values.

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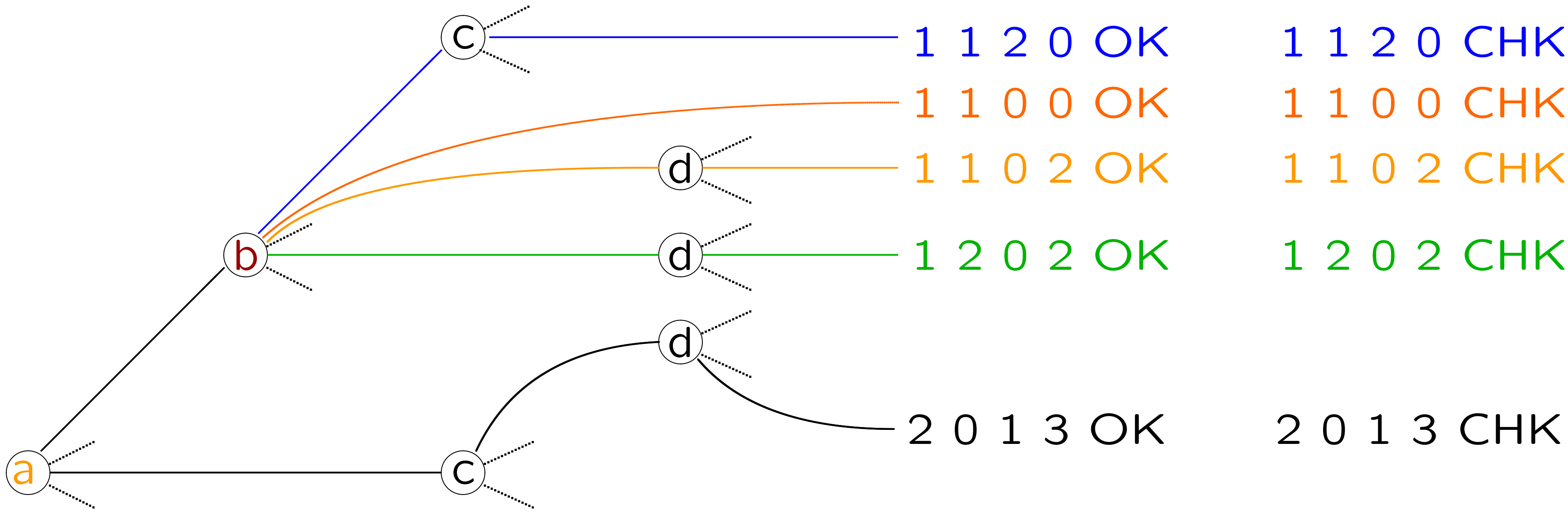
- **can consider mixed strategies, as well!**

This method scales a bit better – growth rate is only (!?)

$$\mathbf{\Omega\,(n!)}$$

where n is the number of sensors (in current practice $n \approx 4-5$).

Network Model



Flow values: $y_\pi \geq 0$ for $\pi \in \mathbf{F} \;=\; X_a \times X_b \times X_c \times X_d \times \{\text{OK,CHK}\}$

where $X_s = \{0, 1, ..., r_s\}$ (r_s ranges for sensor s).

No skipping first: $X_a = \{1, 2, ..., r_a\}$.

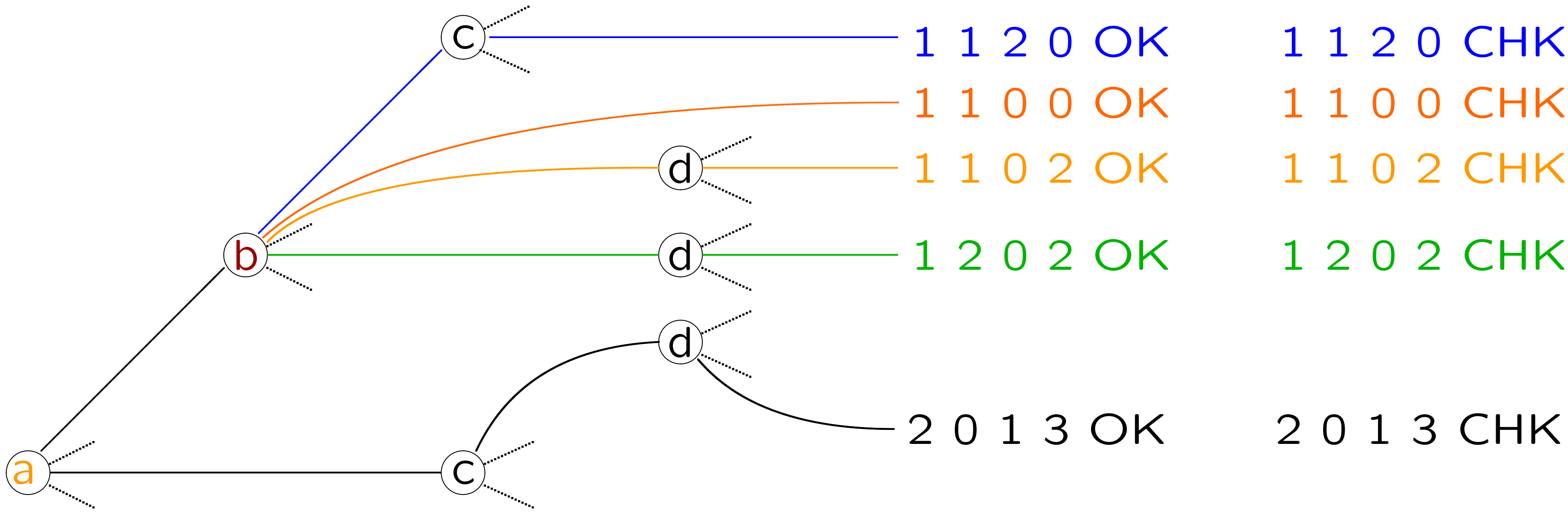
Throughputs:

$$Good(\pi) \;=\; \rho(a, \pi_1)\rho(b, \pi_2)\rho(c, \pi_3)\rho(d, \pi_4)$$

$$Bad(\pi) \;=\; \xi(a, \pi_1)\xi(b, \pi_2)\xi(c, \pi_3)\xi(d, \pi_4)$$

where $\rho(s, 0) = \xi(s, 0) = 1$ for all sensors s

Network Model



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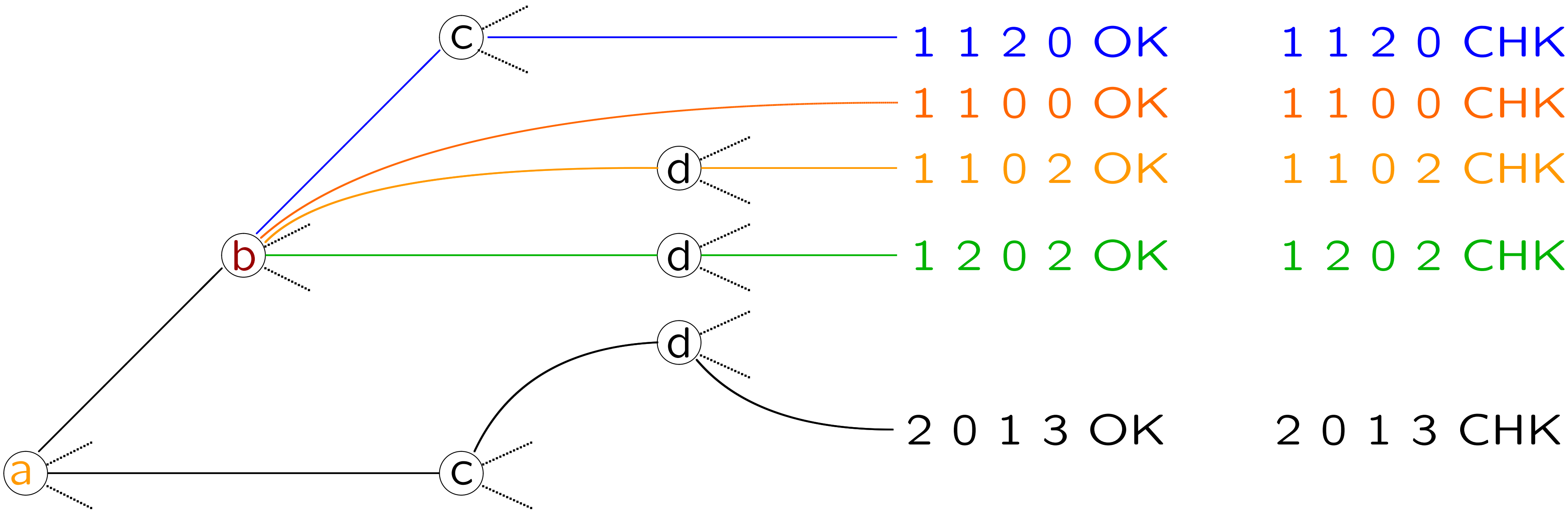
Initialization: $\sum_{\pi \in \mathbf{F}} y_\pi \;=\; 1$ (we normalized by $\mathbf{q}$)

Network Model



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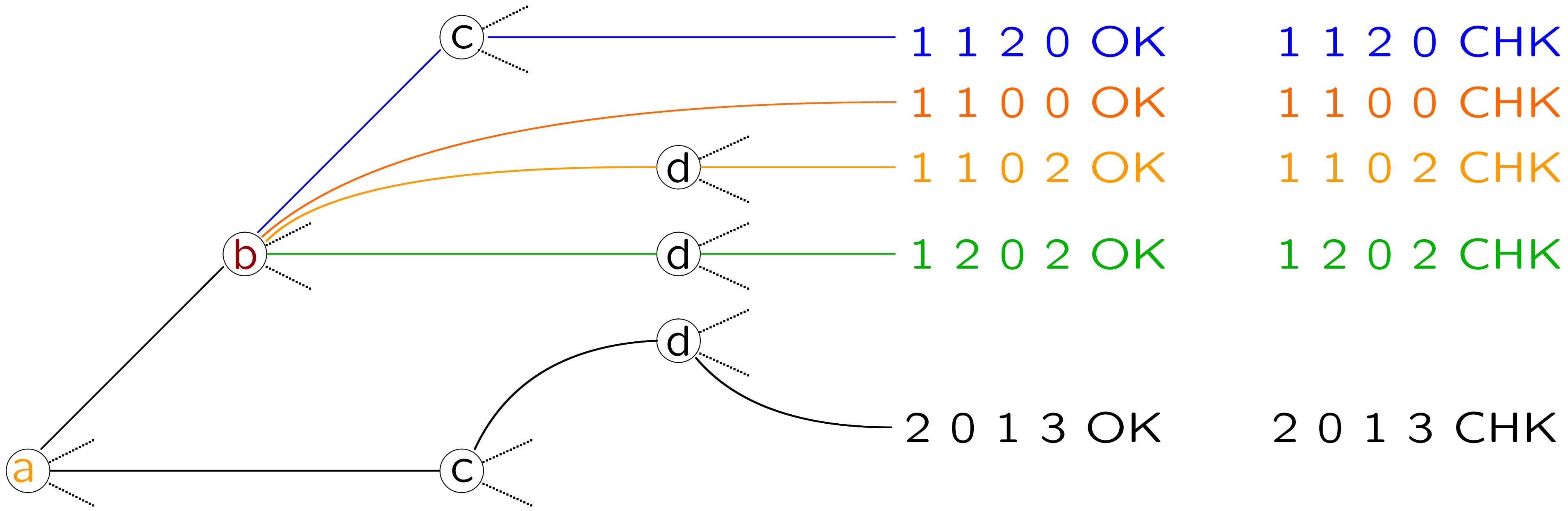
Flow conservation:

$$\begin{aligned} y_{11*} &= \rho(b,1)\,(y_{11*} + y_{12*} + \ldots) \\ y_{12*} &= \rho(b,2)\,(y_{11*} + y_{12*} + \ldots) \\ &\vdots \end{aligned}$$

where e.g., $y_{11*} = \sum_{\gamma \in X_c} \sum_{\delta \in X_d} \sum_{\varepsilon \in \{\text{OK,CHK}\}} y_{11\gamma\delta\varepsilon}$.

Network Model



Flow values: $y_\pi \geq 0$ for $\pi \in \mathbf{F} \;=\; X_a \times X_b \times X_c \times X_d \times \{\text{OK,CHK}\}$

where $X_s = \{0, 1, ..., r_s\}$ (r_s ranges for sensor s).

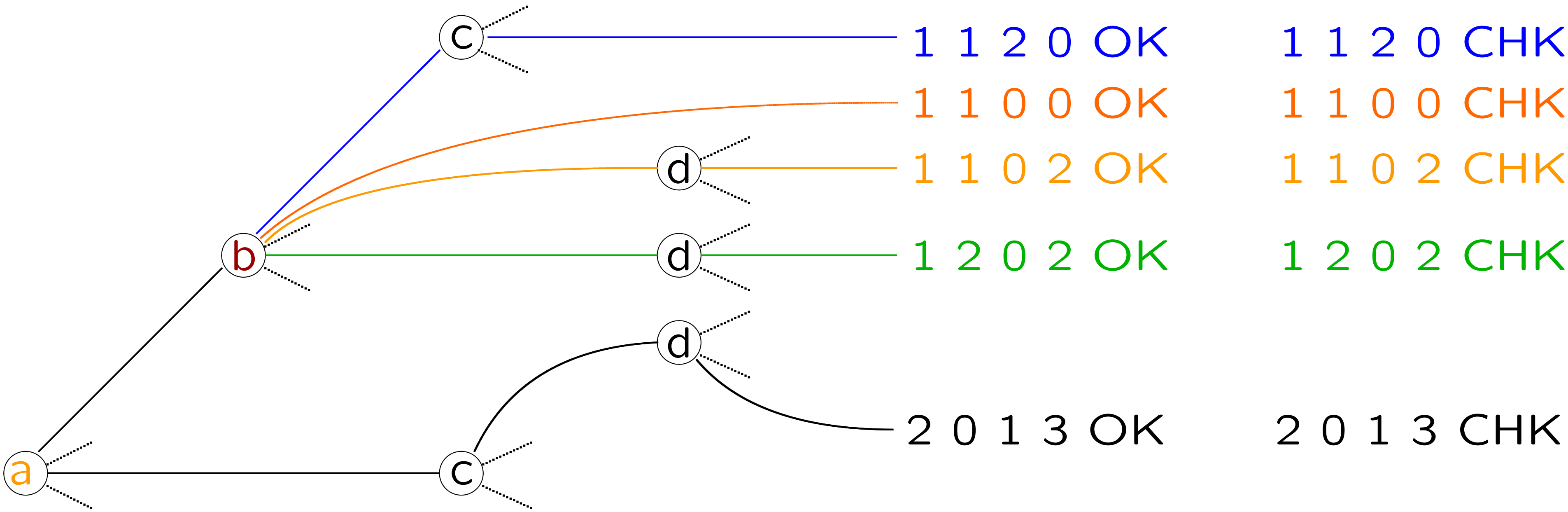
Capacity constraints:

$$\sum_{\alpha \in X_a \setminus \{0\}} y_{\alpha *} \;\leq\; K(a)/\mathbf{q}$$

$$\sum_{\alpha \in X_a} \sum_{\beta \in X_b \setminus \{0\}} y_{\alpha\beta *} \;\leq\; K(b)/\mathbf{q}$$

$$\vdots$$

Network Model



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where $X_s = \{0, 1, ..., r_s\}$ (r_s ranges for sensor s).

Operating Cost: $\sum_{\pi \in \mathbf{F}} C(\pi) y_\pi$ $\leq Budget$ or *minimize*

Detection Rate: $\sum_{\substack{\pi \in \mathbf{F} \\ \pi_5 = \text{CHK}}} \frac{Bad(\pi)}{Good(\pi)} y_\pi$ *maximize* or ≥ 0.85

False Positives: $\sum_{\substack{\pi \in \mathbf{F} \\ \pi_5 = \text{CHK}}} y_\pi$ minimize or ≤ 0.05

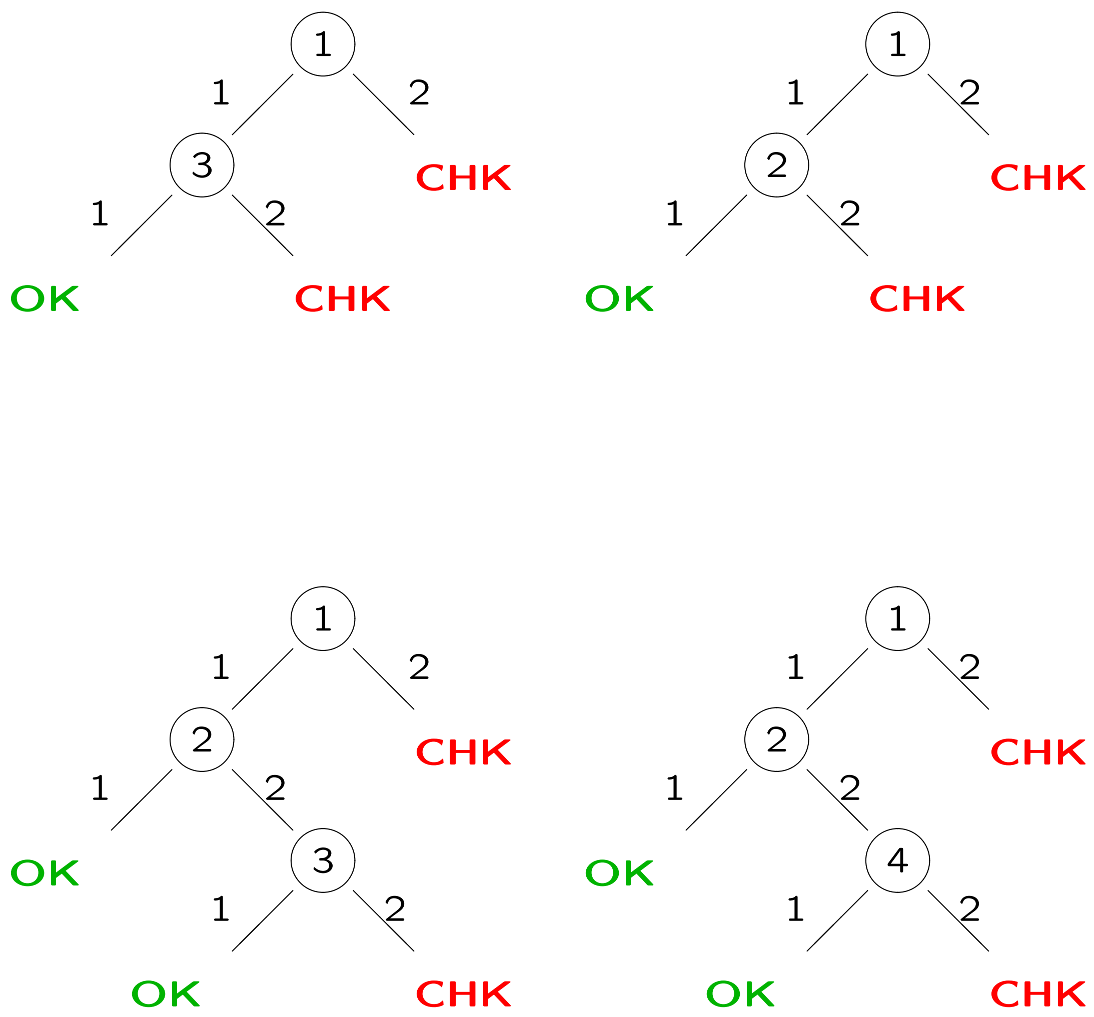
Typical output

Result for $n = 4$ sensors with $1 - 1$ thresholds T=[3; 1; 2; 3]:

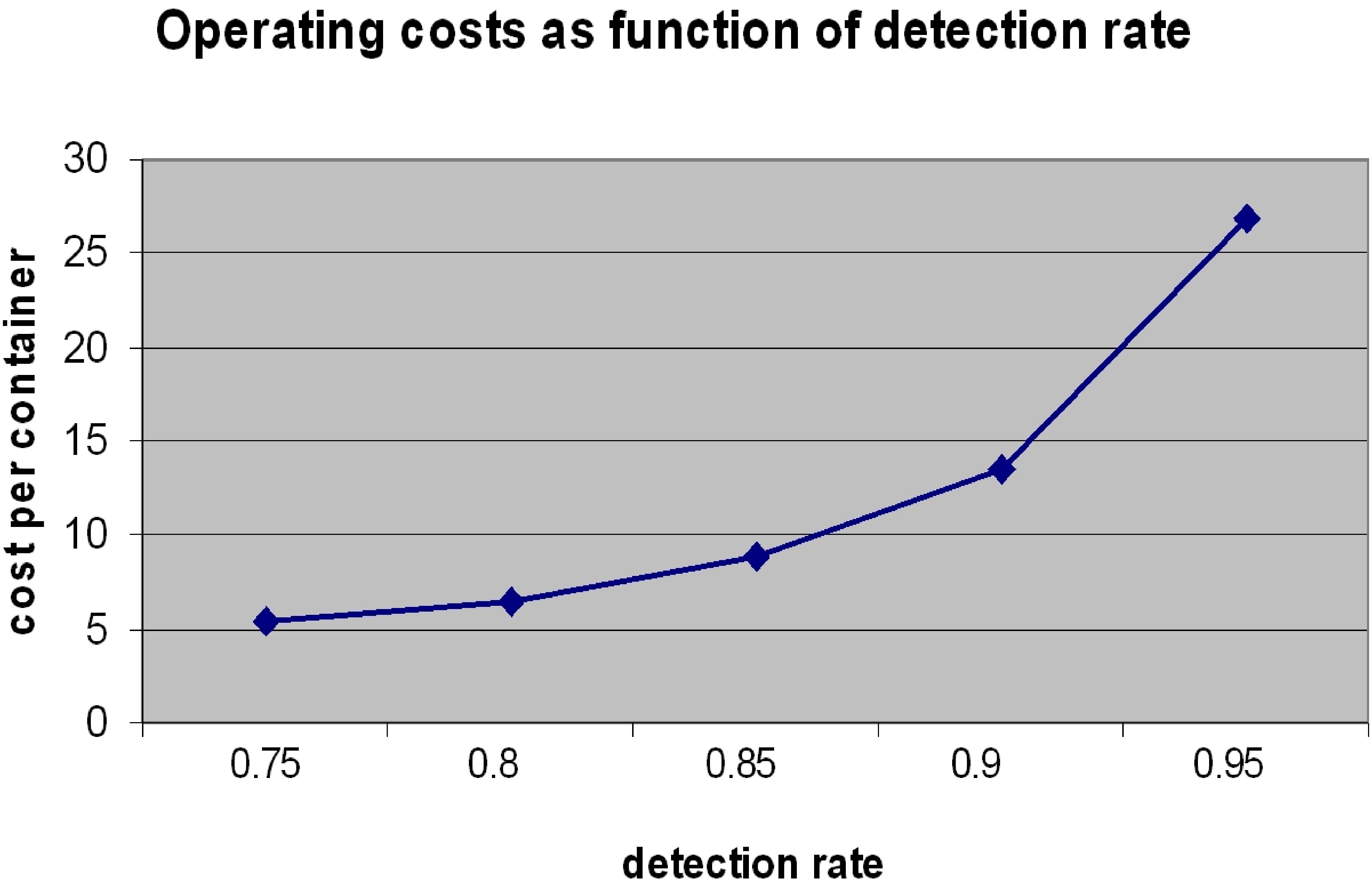
1	2	3	4	decision
1	0	1	0	ok
1	0	2	0	chk
1	1	0	0	ok
1	2	0	0	chk
1	2	0	1	ok
1	2	0	2	chk
1	2	1	0	ok
1	2	2	0	chk
2	0	0	0	chk

Detection rate	0.778474
False positives	0.031250
Operating cost	$10.42

Typical output



Sample Results with 4 Sensors and 5 Thresholds for Each



Sample Results at fixed Detection Rate