ABSTRACT

Speaker: Bit-Shun Tam

Title: On Matrices with Cyclic Structure

Abstract: A square matrix A is said to be m-cyclic if A is permutationally similar to a matrix in the superdiagonal $(k_1, \cdots, k_m)$-block form for some m-tuples $(k_1, \cdots, k_m)$of positive integers, or equivalently, if the digraph of A is cyclically m-partite. In the setting of a general square complex matrix A, the logical relations between several necessary conditions for A to be m-cyclic are examined. Some of these conditions are known to be equivalent if A is an irreducible nonnegative matrix whose index of imprimitivity is m. The concept of a cycle (which extends the concept of a circuit) and the characterization of diagonal similarity between matrices in terms of matrix cycle products due to Saunders and Schneider play a key role in our development. In the second half of this talk, we consider the connection between the m-cyclicity of a square complex matrix and that of its associated collection of elementary Jordan blocks. In particular, for an m-cyclic collection $\mathcal{U}$ it is found that the problem of determining all m-tuples $(k_1, \cdots, k_m)$ of positive integers for which there exists an m-cyclic matrix A in the superdiagonal $(k_1, \cdots, k_m)$-block form such that the collection of elementary Jordan blocks associated with A is $\mathcal{U}$ is equivalent to determine all row sum vectors of $(0, 1)$-matrices that have a prescribed column sum vector and each of whose column vectors has cyclically consecutive equal components. As a by-product we also obtain an equivalent condition on m given square complex matrices $B_1, \cdots, B_m$ so that there exist complex rectangular matrices $A_1, \ldots, A_m$ that satisfy $B_j = A_j \cdots A_m A_1 \cdots A_{j-1}$ for $j = 1, \ldots, m$, thus completing the work of Flanders for the case $m = 2$.