ABSTRACT

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Synthesis of Least-Depth Circuits for Aperiodic Computation

We describe an algebraic approach along the lines of Krohn-Rhodes theory to the asynchronous synthesis of finite-state systems, and explain asynchronous emulation for finite-state automata. This viewpoint is applied to yield cascade-parallel decomposition of aperiodic finite-state automata (in terms of asynchronous emulation) and hence of those recognizing star-free languages (or, equivalently, counter-free languages). [Aperiodic finite automata are those which cannot emulate a modulo n counter for any $n > 1$. By Schützenberger's Theorem, the class of such 'non-counting' automata consists of exactly those that recognize the star-free languages, i.e. those languages specifiable by regular expressions (allowing complement) without invoking the star operation.]

We show the least depth of feedback-free circuits needed to synthesize any aperiodic automaton, and as a consequence, to recognize each star-free language is computable. Although no corresponding result is known for finite automata in general, this implies that the natural complexity hierarchy on aperiodic finite automata is decidable. Moreover, the hierarchy is shown to induce a strict, non-collapsing filtration of aperiodic finite-state automata, and we present a constructive technique to (asynchronously) synthesize any aperiodic finite-state computation in least depth circuits consisting of cascaded layers of flip-flops.