ABSTRACT

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On the Complexity of Tensor Formulae

We consider matrix formulae on base of sum, product, and tensor product. In particular, we consider tensor formulae over the Boolean semi-ring B=(0,1,AND,OR) and over the two-element field $\mathbf{F}_2 = (0, 1, +, x)$.

The TENSOR FORMULA EVALUATION PROBLEM over a certain semi-ring S (denoted TEP_S**) is the set of tensor formulae that over S evaluate to 1. We prove: (1)** TEP_B **and** TEP_{F_2}**, respectively, are complete for NP and PP, respectively, under polynomial time reductions. (2) When restricted to tensor formulae containing only small sub-formulae, these problems are complete for LOGCFL and Parity-LOGCFL under log-space reductions, respectively. Restriction to formulae without tensor products leads to problems that are complete for NL and Parity-L, respectively.**

We demonstrate the strength of these characterizations by giving a unified proof for the inclusions

- **NP/poly** $\subseteq Parity - P/poly,$
- **LOGCFL/poly** $\subseteq Parity - LOGCFL/poly, and$
- **NL/poly** $\subseteq Parity - L/poly,$

that were first proved using Valiant-Vazirani's Lemma and the Isolating Lemma of Vazirani, Vazirani, and Mulmuley. Our proof instead relies on randomized polynomial testing. In case of the latter two inclusions our proof uses less random bits than earlier constructions.