New Approximation Results for the Maximum Scatter TSP

(Abstract)

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We consider the following *maximum scatter traveling salesperson problem* (TSP): Given an edge-weighted complete graph $\mathcal{G} = (S, E)$, find a Hamiltonian path or cycle such that the length of a shortest edge is *maximized*. In other words, the goal is to have each point be far away (most "scattered") from the points that are visited just before or just after it in the path/cycle. This is also referred to as the *max-min 1-neighbor TSP*. More generally, in the *max-min m-neighbor TSP* problem, the goal is to maximize the minimum distance between any point and all of its "m-neighbors" along the path/cycle. An *m-neighbor* of p is a point that is at most m points away from p in the path/cycle. The problem, then, is to find a path/cycle, $(p_1, \dots, p_n)$, on the n points of S in order to maximize $\min_{j \in \{i-m, \dots, i-1, i+1, \dots, i+m\}} d(p_i, p_j)$, where $d(p_i, p_j)$ denotes the length of edge (p_i, p_j) (*not* the length along the path/cycle linking p_i and p_j). The indices in the minimization should be modified appropriately to account for wrap-around effects in cycles, or endpoint effects in paths.

Motivation As described in [2], this problem arises in some manufacturing processes where it is important to have substantial separation (distance) between consecutive (or *nearly* consecutive) operations on a workpiece [6]. When fastening sheets of metal together, to prevent nonuniform deformation of the sheet metal plates, it is important to sequence the riveting process so that the distance between one rivet and the next one is large. In operations that involve heating the workpiece, it is important not just to have each point well separated from its immediate predecessor and successor, but also from its m-neighbors, in order to allow for cooling time in the vicinity of each operation [2, 6].

The maximum scatter TSP also arises in some medical imaging applications, as described in [2]. When imaging physiological functions using a Dynamic Spatial Reconstructor (DSR), the radiation sources are placed along the top half of a circular ring, with sensors placed directly opposite, in the bottom half of the ring. The "firing sequence" determines the order in which the sources and their partnered sensors are activated, usually in a periodic pattern [5]. The sensors collect intensity data of the energy that passes through the patient, who is placed in the center of the ring. When source i is activated, some amount of scattering occurs, so it is important not to activate sensors of nearby sources (e.g., $i+1, i-1, i+2, \dots$) soon after i is activated [5]. This motivates the study of the cycle version of the maximum scatter TSP [2].

Previous Work Previously Arkin, Chiang, Mitchell, Skiena, and Yang [2] gave the first algorithmic study of the problem. It was shown that the maximum scatter TSP is, in general, NP-complete and that no constant-factor approximation algorithm exists unless P=NP. In the case that distances obey the triangle inequality, they proposed factor-2 (best factor) approximation algorithms for the

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max-min 1-neighbor TSP, for both the path and cycle versions. The methods also extended to give a factor-2 (best factor) approximation for the max-min 2-neighbor TSP, for the cycle version and some cases ($n \neq 3k+1$) of the path version of the problem. However, these extensions are highly impractical—the underlying *Regularity Lemma* [1,4] needed is valid only for $n > Q$ where $\log^* Q \approx 2^{20}$ [1,2]! They also gave practical algorithms for the max-min 2-neighbor TSP with the triangle inequality, for both the path (factor 32) and cycle (factor 64) versions. These methods extended to yield an approximation factor of $4 \cdot 8^{\lceil m/2 \rceil}$ for the path version of the max-min m-neighbor TSP, for any *constant* $m > 2$, when n is a multiple of $m+1$. We remark that although not explicitly stated in [2], this max-min m-neighbor TSP algorithm can be easily extended to the case of $n = (m+1)k + m$ in the path version, with the same approximation factor. For the max-min 1-neighbor TSP with Euclidean distances, they gave linear-time (optimal) exact algorithms for the case of points on a line or on a circle, for both path and cycle versions [2]. Later, Fekete [3] proved that the Euclidean maximum scatter TSP is NP-hard in three or more dimensions.

Our Results In this paper, we give $O(n^{2.5})$-time approximation algorithms for the max-min 2-neighbor TSP with the triangle inequality, which do not use the *Regularity Lemma* and are practical. We achieve an approximation factor of 12 for the path version, and a factor of 18 for the cycle version of the problem, improving the previous best factors of 32 and 64, respectively, of the algorithms of [2] that do not use the *Regularity Lemma*. As mentioned above, the factor-2 approximation algorithms of [2] do not work for the case of $n = 3k+1$ in the path version and are valid only when n is much larger than the estimated number of atoms in the entire universe(!), and thus should not be taken into account for any practical purposes. We remark that the case of $n = 3k+1$ in the path version, as also observed in [2], is surprisingly difficult to handle; we give two novel algorithms to deal with this case. Both algorithms, in addition to improving the approximation factor given in [2], are much simpler than the method of [2], by exploiting some novel and elegant structural properties of the point set S under the distance metrics, and may be of independent interest.

Moreover, we obtain an $O(mn^{2.5})$-time algorithm that achieves a *constant* approximation factor of 16 for the path version of the max-min m-neighbor TSP with the triangle inequality when n is a multiple of $m+1$ or when $n = (m+1)k + m$, where $m \in (2, n)$ is *part of the input.* This significantly improves the previous best approximation factor of $4 \cdot 8^{\lceil m/2 \rceil}$ [2], from exponential in m to a small constant. Our algorithm utilizes novel, combined matching and swapping techniques such that the approximation factor stays the same (a fixed constant) throughout the $\Theta(m)$ phases of the construction process, and may be interesting by its own.

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