Fault-Tolerant Geometric Spanners

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Abstract

Let $G = (V, E)$ be a Euclidean graph on a set of n points and let $t > 1$. Graph G is called a *t-spanner for V* if for every pair of vertices $v, u \in V$ there is a vu-path in G of length upper bounded by $t \cdot |vu|$. For any non-negative integer k, graph G is called a *k-vertex fault-tolerant t-spanner for V* if for any $V' \subseteq V$ of size at most k and for any pair of points $v, u \in V \setminus V'$, the graph $G \setminus V'$ contains a vu-path of length at most $t \cdot |vu|$.

The problem of constructing efficient vertex fault-tolerant t-spanners in Euclidean spaces $\mathbb{R}^d$ has been proposed recently by Levcopoulos et. al. (STOC'98). In that paper three algorithms are given that construct (i) in $\mathcal{O}(n \log n) + n\, 2^{\mathcal{O}(k)}$ time a k-vertex fault-tolerant t-spanner whose maximum degree is $2^{\mathcal{O}(k)}$ and whose total cost is at most $2^{\mathcal{O}(k)}$ times the cost of a minimum spanning tree for V, (ii) in $\mathcal{O}(n \log n + k^2 n)$ time such a spanner with $\mathcal{O}(k^2 n)$ edges, and (iii) in $\mathcal{O}(n\, k \log n)$ time such a spanner with $\mathcal{O}(n\, k \log n)$ edges. Later, Lukovszki (WADS'99) constructed a k-vertex fault-tolerant t-spanner with the optimum number of edges $\mathcal{O}(n\, k)$; the running time of this algorithm is $\mathcal{O}(n \log^{d-1} n + n\, k \log\log n)$. Lukovszki gave also a construction that achieves the maximum degree of $\mathcal{O}(k^2)$ and considered spanner constructions that allow Steiner points.

In this paper we present a new construction of vertex fault-tolerant spanners in Euclidean spaces that achieves optimal bounds for the maximum degree and the total weight. We present a greedy algorithm that for any $t > 1$, any non-negative integer k, and any d, constructs a k-vertex fault tolerant t-spanner in which every vertex is of degree $\mathcal{O}(k)$ and whose total cost is $\mathcal{O}(k^2)$ times the cost of minimum spanning tree. One can show that the bounds for both the degree and the cost are optimal.

Our next contribution is an efficient algorithm for constructing fault tolerant spanners. We present a new, sufficient condition for a graph to be a k-vertex fault tolerant t-spanner. Using this condition, we design a $\mathcal{O}(n\, k \log^d n)$-time algorithm that finds a k-vertex fault tolerant t-spanner in which every vertex has degree $\mathcal{O}(k)$ and whose total cost is $\mathcal{O}(k^2 \cdot \log n)$ times the cost of a minimum spanning tree for V.

(In all the bounds above the constant implicit in the $\mathcal{O}$ notation depend on t and d.)