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"Two Functional Equations Arising in Information Theory and the Analysis of Algorithms"

We shall talk about two functional equations arising in the analysis of algorithms and information theory, namely, in the performance evaluation of the generalized digital search trees (i.e., generalized Lempel-Ziv compression scheme) and the asymmetric leader election algorithm. These functional equations are Poisson version of the following general recurrence that encompasses both algorithms

$$x_{n+b} = a_n + cp^n x_n + u \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} (x_k + x_{n-k})$$

with some initial condition, where u and c are constants, a_n is a given sequence, and $b \geq 1$ is a parameter. This recurrence was extensively investigated either for $b = 1$ or $c = 0$. We present asymptotic expansions of x_n up to $O(1)$ term. Interestingly enough, for both algorithms there appear a constant that must be evaluated numerically from the original recurrence. Analytic techniques of (precise) analysis of algorithms are used to establish these conclusions. In particular, we use analytic poissonization/depoissonization, Mellin transform and singularity analysis.

The results presented in this talk were obtained jointly with S. Janson, G. Louchard and J. Tang.