A Generalization of the Terwilliger Algebra of a Commutative Association Scheme

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Given a commutative association scheme Γ, Terwilliger considered the $\mathbf{C}$-algebra generated by the associated Bose Mesner algebra M and dual Bose Mesner algebra M^*. This algebra is now known as the Terwilliger algebra and is usually denoted by T. Terwilliger showed that each vanishing intersection number and Krein parameter of Γ gives rise to a relation on certain generators of T. The existence of these relations makes T useful in the study of commutative association schemes with many vanishing intersection numbers and Krein parameters. In particular, T is useful in the study of P- and Q-polynomial association schemes, which play an important role in coding theory.

Terwilliger's relations determine much of the structure of T, though not all of it in general. To illuminate the role these relations play, we consider a certain generalization $\mathcal{T}$ of T. To go from T to $\mathcal{T}$, we replace M and M^* with a pair of dual character algebras C and C^*. We define $\mathcal{T}$ by generators and relations; intuitively $\mathcal{T}$ is the associative $\mathbf{C}$-algebra with identity generated by C and C^* subject to the analogues of Terwilliger's relations. $\mathcal{T}$ is infinite dimensional and noncommutative in general. We construct an irreducible $\mathcal{T}$-module which we call the primary module; the dimension of this module is the same as that of C and C^*. We find two bases of the primary module; one diagonalizes C and the other diagonalizes C^*. We compute the action of the generators of $\mathcal{T}$ on these bases. We compute the central primitive idempotent of $\mathcal{T}$ associated with the primary module in terms of the generators of $\mathcal{T}$. We then use this central primitive idempotent to show that $\mathcal{T}$ is a direct sum of two sided ideals $\mathcal{T}_0$ and $\mathcal{T}_1$ with $\mathcal{T}_0$ isomorphic to a full matrix algebra.