CODES OVER $\mathbb{Z}_{p^k}$ and ASSOCIATION SCHEMES

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ABSTRACT

Association schemes are the appropriate context to obtain all possible p-ary generalizations of binary codes from codes over $\mathbb{Z}_4$ given by the Gray map. The results come within the scope of the lecturer's Chapter 18 *Codes and Association Schemes* in the Handbook of Coding Theory edited by Vera Pless, W. Cary Huffman and Richard A, Brualdi, Elsevier Science Publisher B.V., Amsterdam,1998. Le X be an Abelian group.

A partition $\{X_0 = \{0\}, X_1, \dots, X_n\}$ of X defines an association scheme (X, R) invariant under translation if the relations $R_i = \{(x, y)|x - y \in X_i\},\ i = 0, 1, \dots, n$ are those of an association scheme, as defined by R.C.Bose and T.Shimamoto. Then (X, R) is called an Abelian scheme. It appears that the Gray map σ is an isomorphism from a self-dual Abelian scheme defined on the group $\mathcal{F} = (\mathbb{Z}_4, +)$ to the Hamming scheme $\mathbb{F}_2^2 = H(2, 2)$ consisting of all binary vectors of length 2. This entails that for any subgroup Y of $\mathcal{F}^m$, then $\sigma(Y)$ is an unrestricted but regular code. Another consequence is that the MacWilliams transform of the weight enumerator of Y is the weight enumerator of its dual Y° and of $\sigma(Y^\circ)$ as well. A similar situation can be obtained when $\mathbb{Z}_4$ is replaced with $\mathbb{Z}_{p^k}$.