Bull's Eye Prediction Theory for Controlling Large Epidemic Outbreaks

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Agenda

- Predictability - determinism, chaos and randomness
- Review some of the issues in epidemics

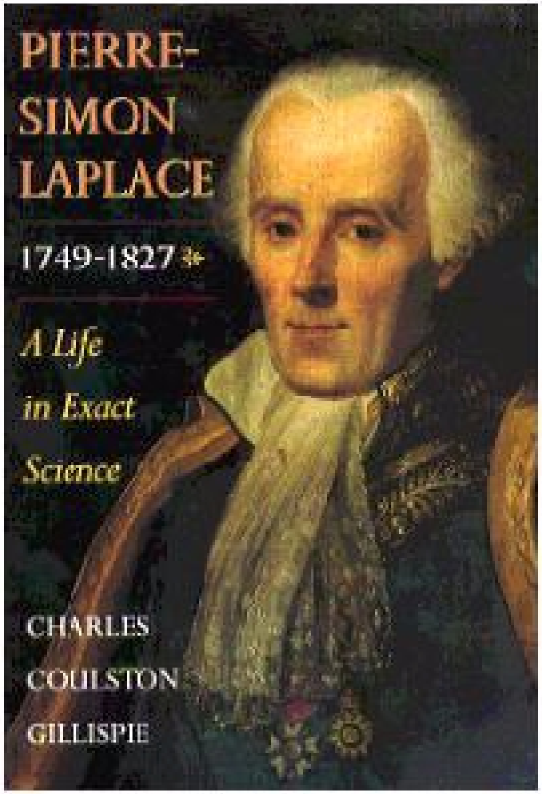
  Outbreaks of childhood diseases-Data
- Modeling with determinism and randomness
- Bull’s eye theory for predicting noise excited outbreaks
- Adaptive vaccine control large outbreaks
- Conclusion

Predictability

Determinism,
Chaos and Randomness

Predictability and Randomness-18-19th Century Thinking

"We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes."-Marquis Pierre Simon de Laplace (Late 18th century)



- **Predictability - deals with foretelling the future given the past**

  Always limited by finite information

  Limited by uncertainty-External environmental perturbations

- **Random processes are unpredictable**

  Ex. Coin toss

- **Deterministic processes are predictable ***

  Ex. Simple pendulum - no friction and forcing

  Exhibits only regular, repeated motion. Planetary motion about the sun.

  * Remember, this is the view of the 18th century

What is CHAOS?

LIFE!

Scientists are the last to know!

J. A. Yorke, 2002

Thinking about Unpredictability

- **Originally discovered by Henri Poincare- Coined the phrase "sensitive dependence to initial conditions"**

If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. but even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon. - in a 1903 essay "Science and Method"

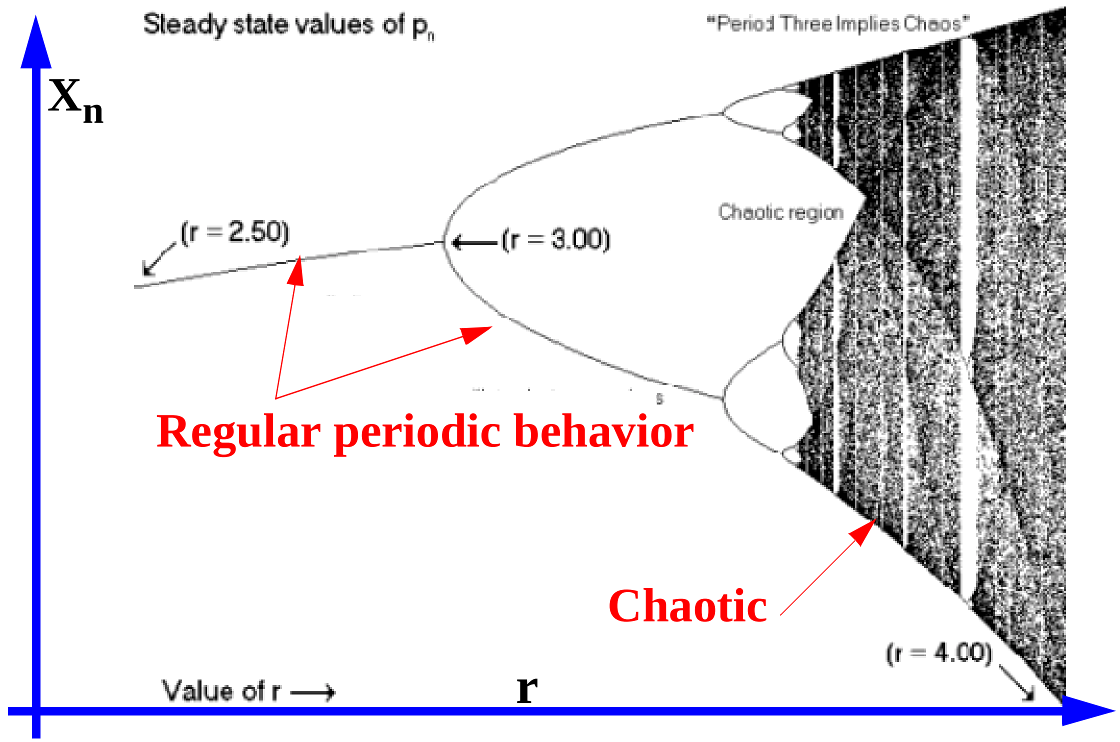
- **Chaos is indeterminism generated by deterministic rules. The scientific usage of the word was first coined by Jim Yorke and Li in their ground breaking paper, "Period Three Implies Chaos (1975)," (Sarkovskii was really first)**

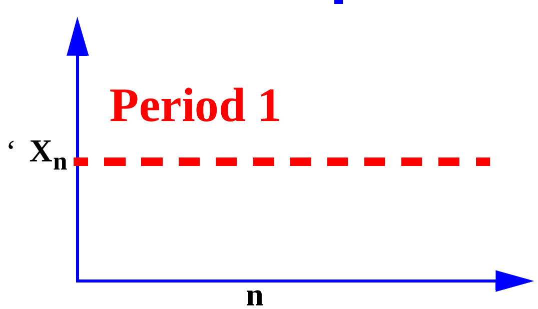
In short, chaos embodies three important principles:

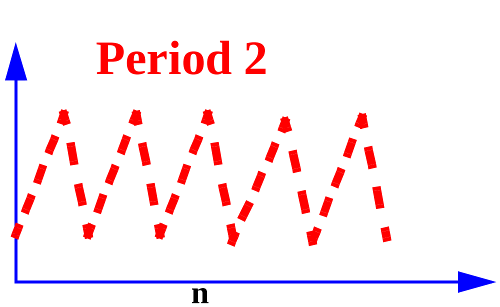
- **extreme sensitivity to initial conditions**
- **cause and effect are not correlated over long times**
- **nonlinearity**

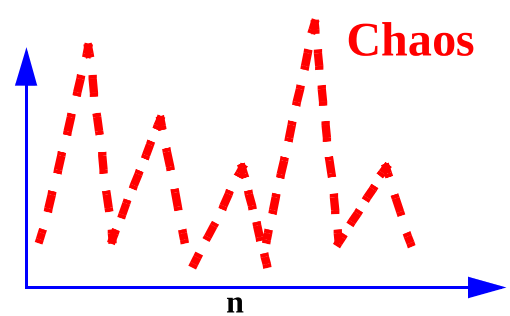
Demonstration

- **The Deterministic Rule:** $x_{n+1} = rx_n(1 - x_n)$









Review some issues in epidemics

Epidemics - Why study them

Why are we interested in childhood diseases?

For those interested in nonlinear dynamics:

Records of incidence exhibit periodic or more complex behavior

In many situations, it's clear that the observe dynamics reflect the interaction between nonlinearities and stochastic effects

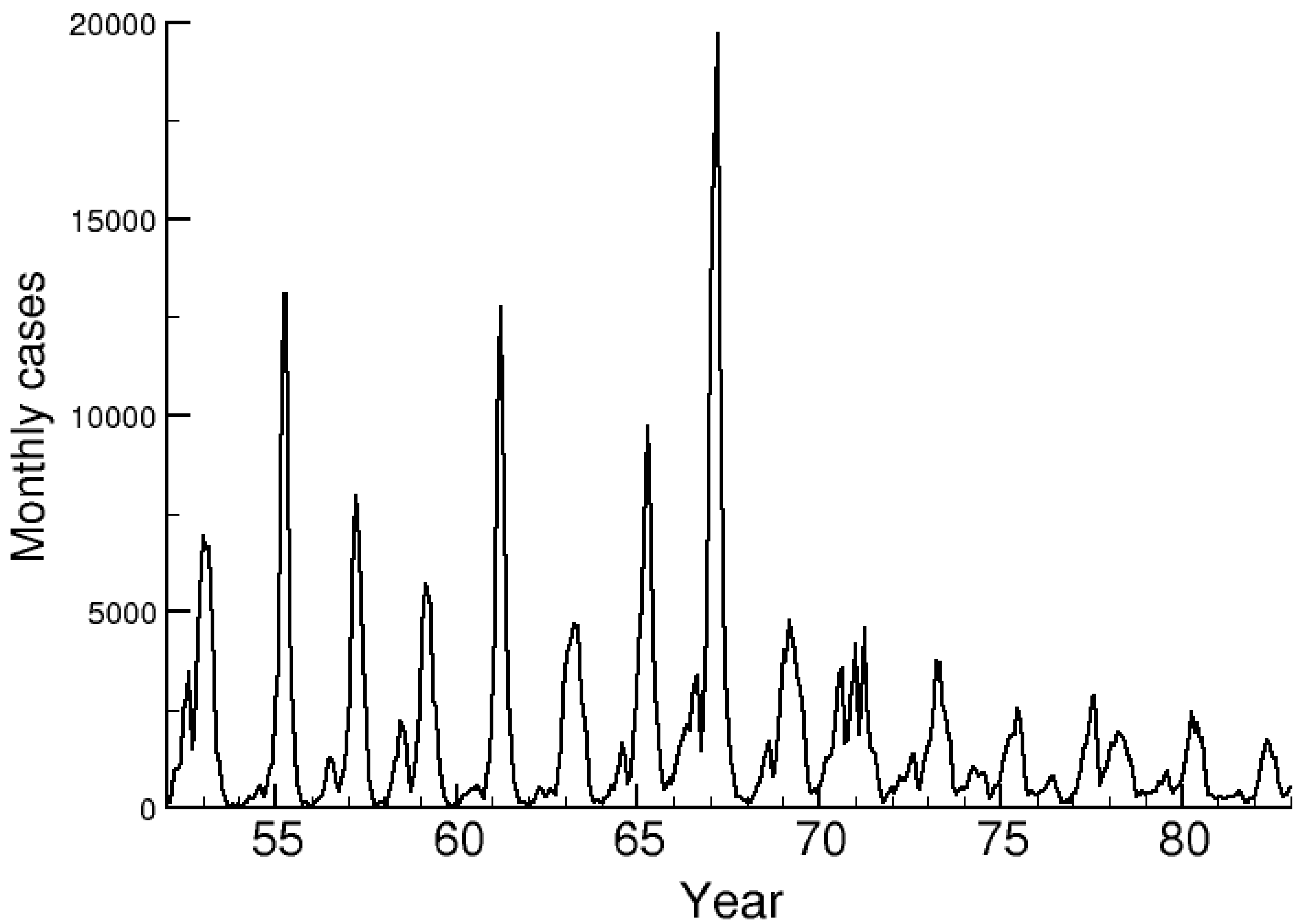
For modelers:

Excellent data is available
can ask detailed questions about spatial and temporal dynamics and can test and improve models

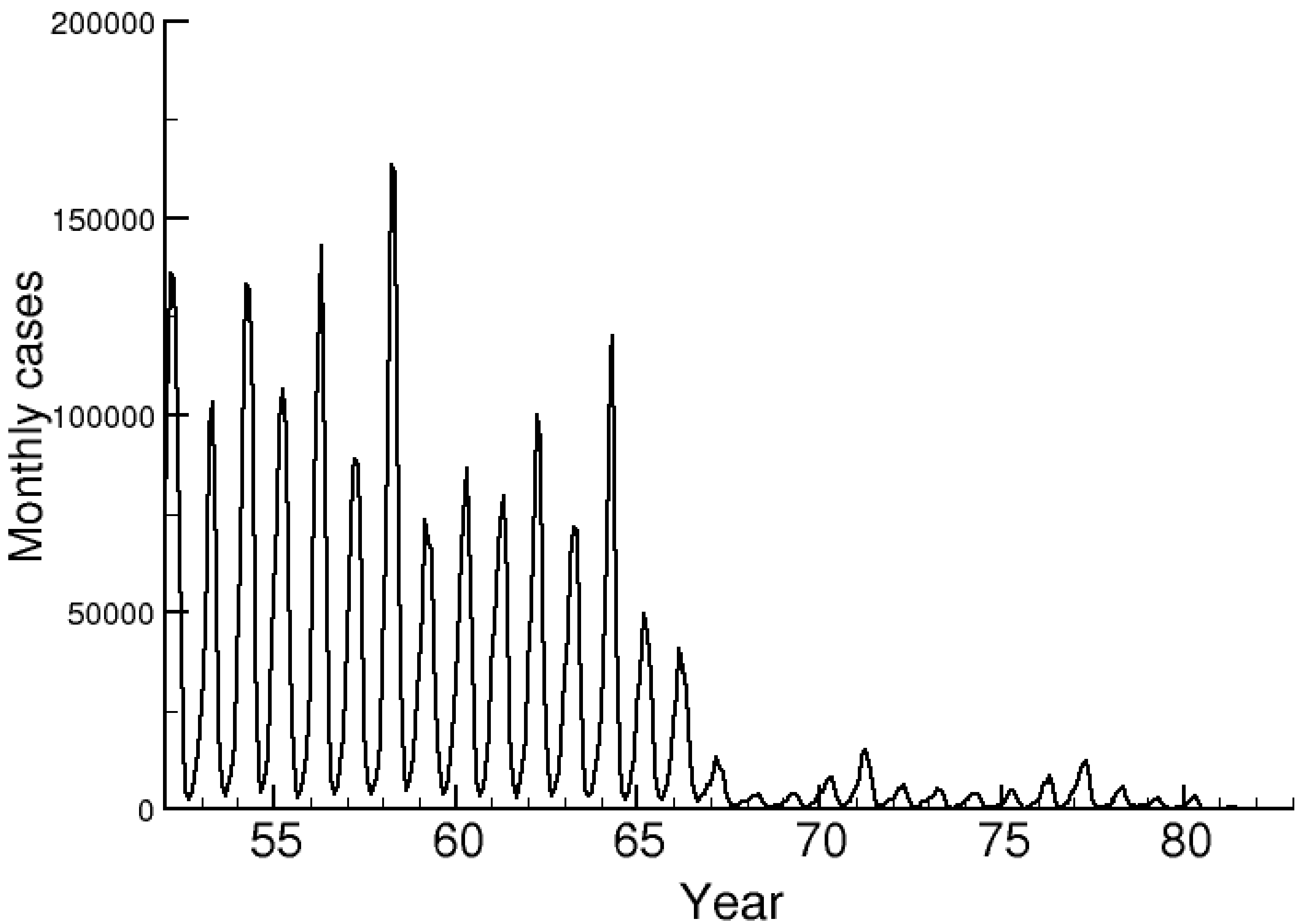
Data also covers the introduction of mass vaccination

The biological system is fairly simple

Measles: Total Numbers in UK

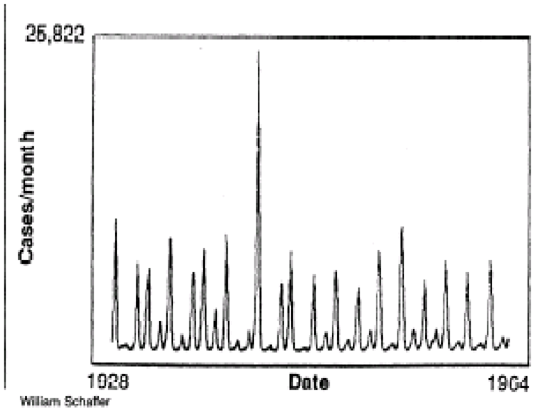


Measles: US Total Cases



Some Amusing Questions from Data

- Given outbreaks that are:
  Annual in peaks
  Appear random in amplitude



A mix of order and disorder: *Measles in New York City, 1928 to 1964.*

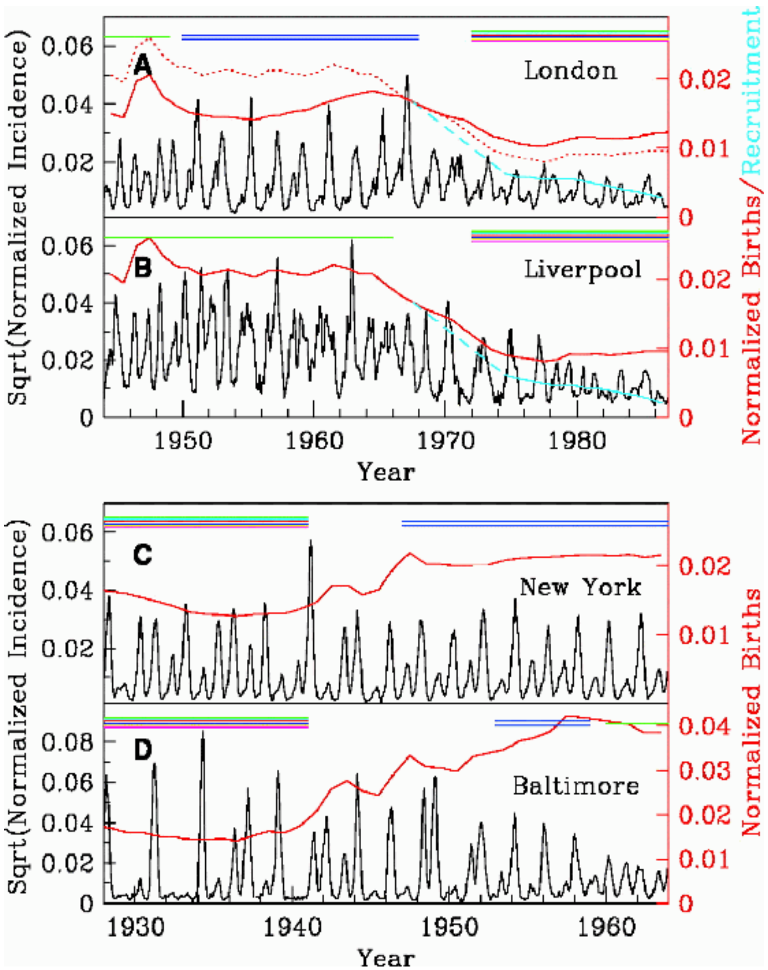
RESEARCH NEWS 25

Science, 1989

- Can one:
  understand large outbreaks
  predict large outbreaks
  reduce the amplitude through vaccine introduction

Modeling Outbreaks with randomness

Measles Data - Social Parameter Fluctuations

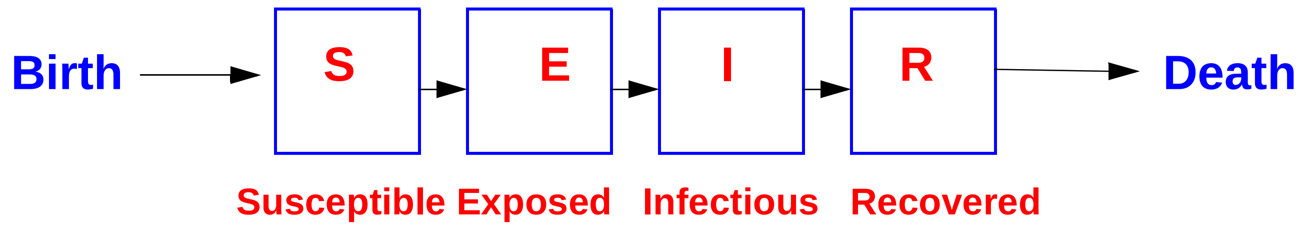


Science, 2001

Annual peaks-
Schools opening and closing

Noise in population-
Random births
Migration

The "Simplest" Epidemic Model



Equations of Motion

$$S' = \mu - \beta(t)IS - \mu S$$
$$E' = \beta(t)IS - \alpha E - \mu E$$
$$I' = \alpha E - \gamma I - \mu I$$
$$R' = \gamma I - \mu R$$
$$\beta(t) = \beta_0(1 + \delta\cos 2\pi t)$$

Parameters

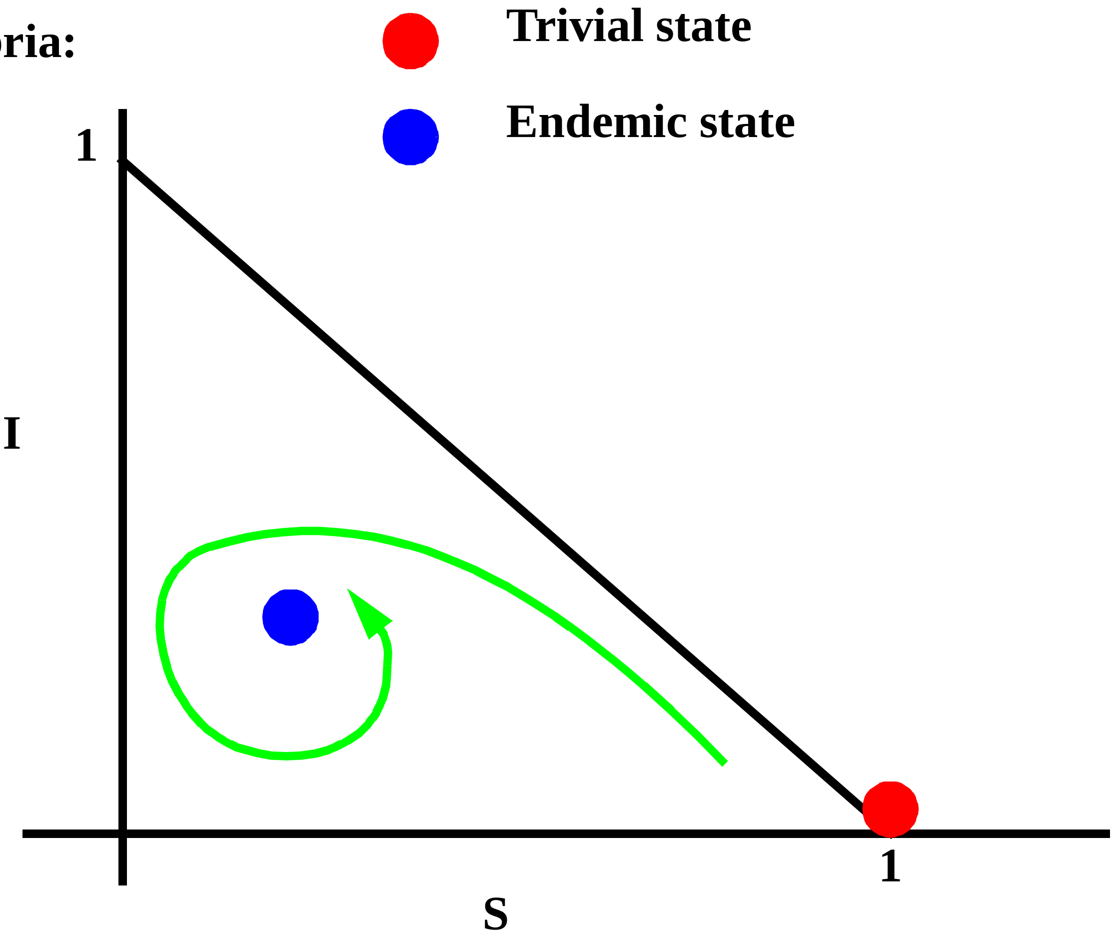
α^{-1}, mean latent period in the exposed class
γ^{-1}, mean infectious period in the infected class

μ, birth rate‘
$\beta(t)$, annual fluctuating contact rate
δ is the control parameter
Noise is added to the (birth) population

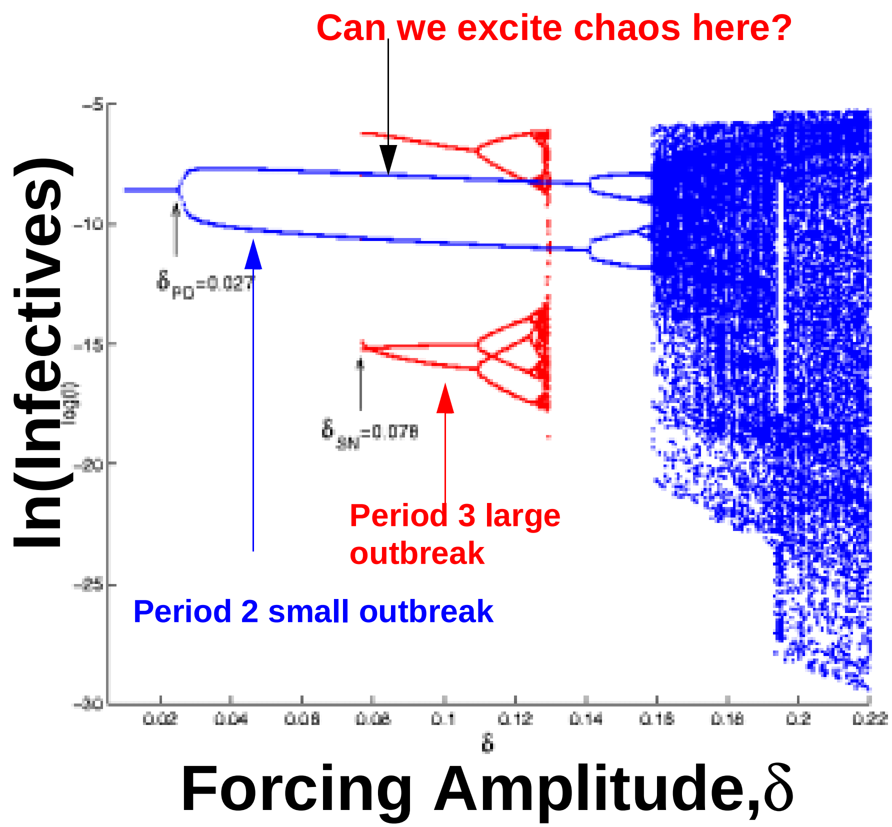
Normalization of population: $S + E + I + R = 1$

Why Annual Fluctuations are Important

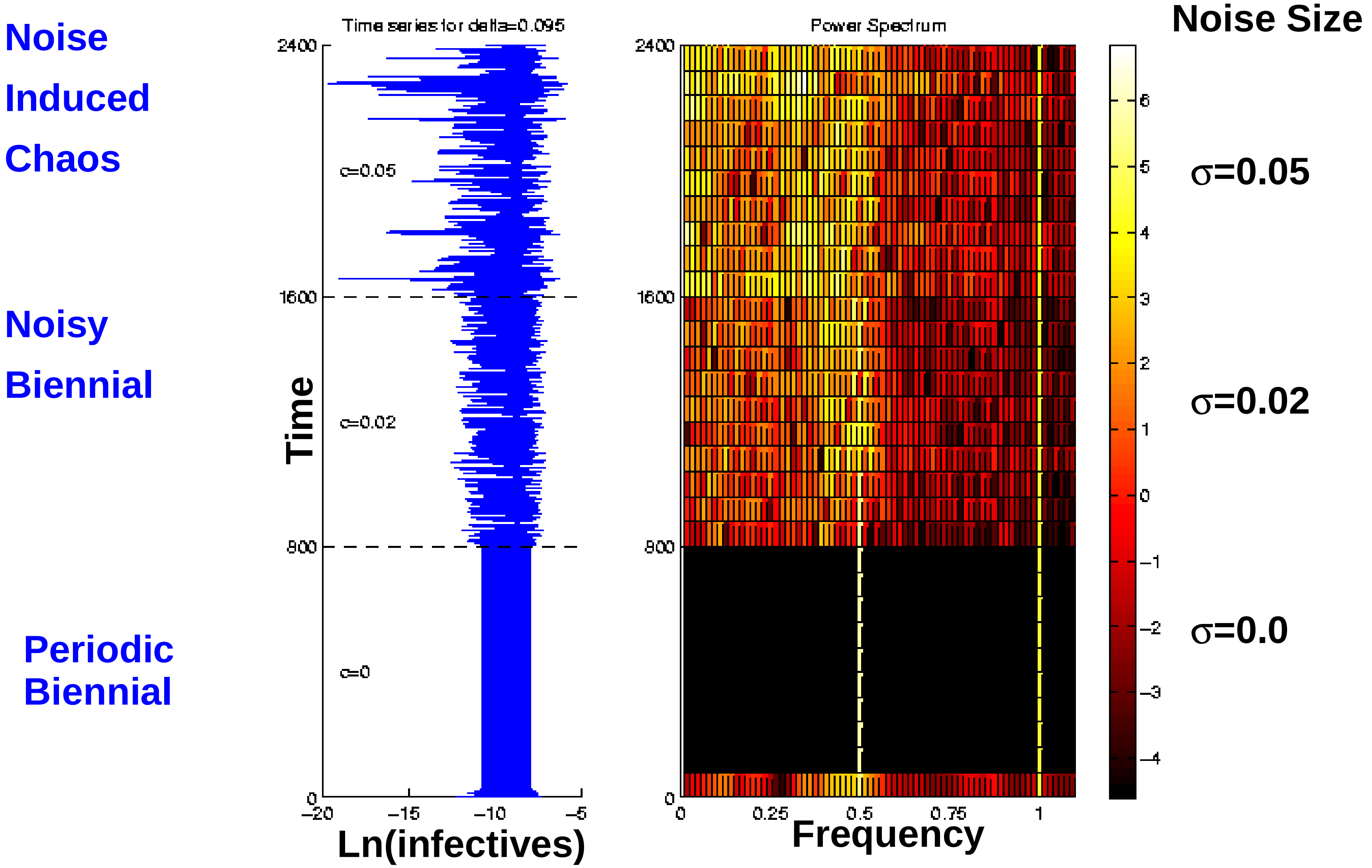
- $\delta = 0$
- **Two equilibria:**



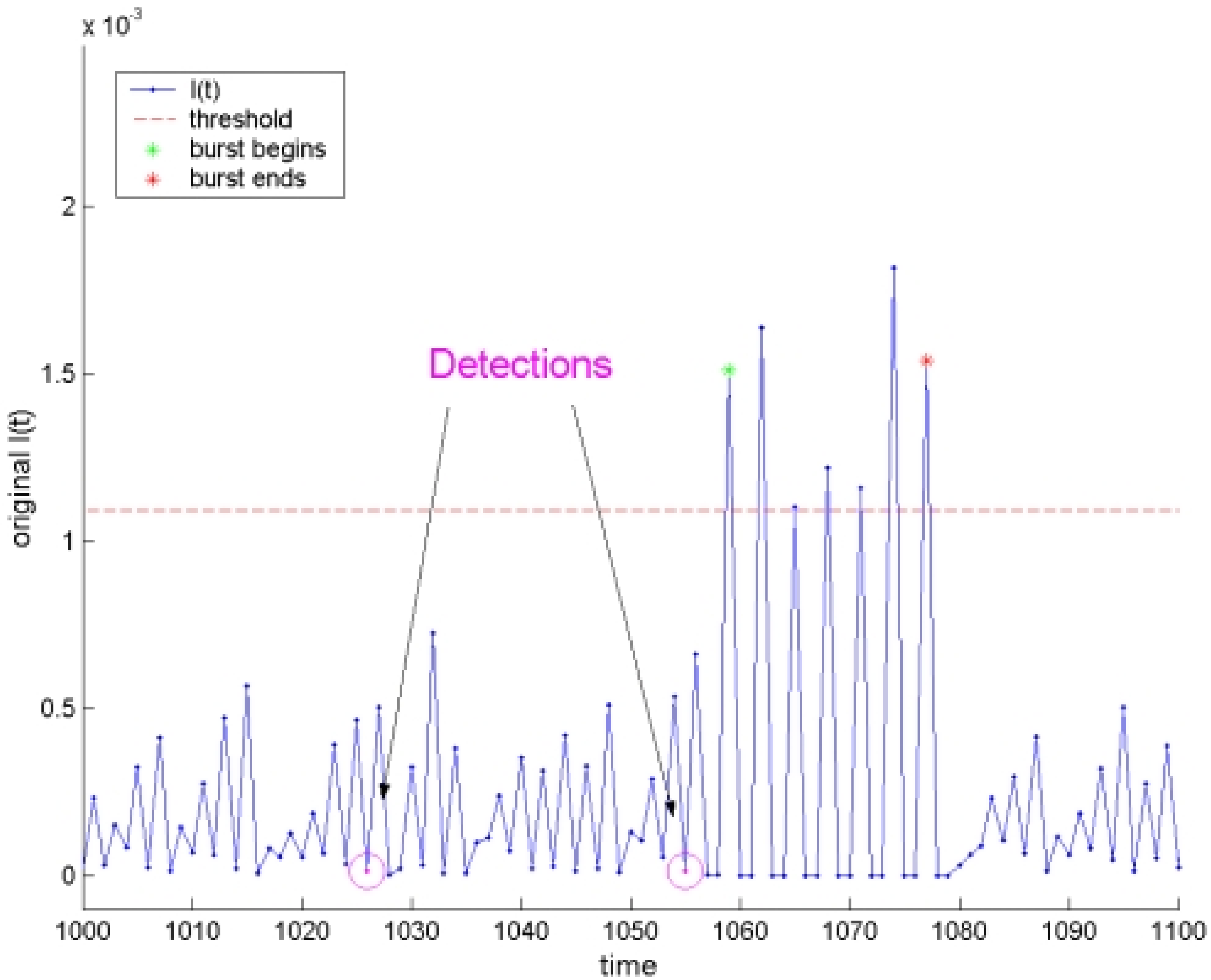
Bifurcation Diagram for Realistic Parameters



Added Noise Creates New Dynamics
Power Spectral Analysis



Time Series Example of Noise-Induced Outbreaks



Defining and Measuring Sensitive Dependence

Deterministic

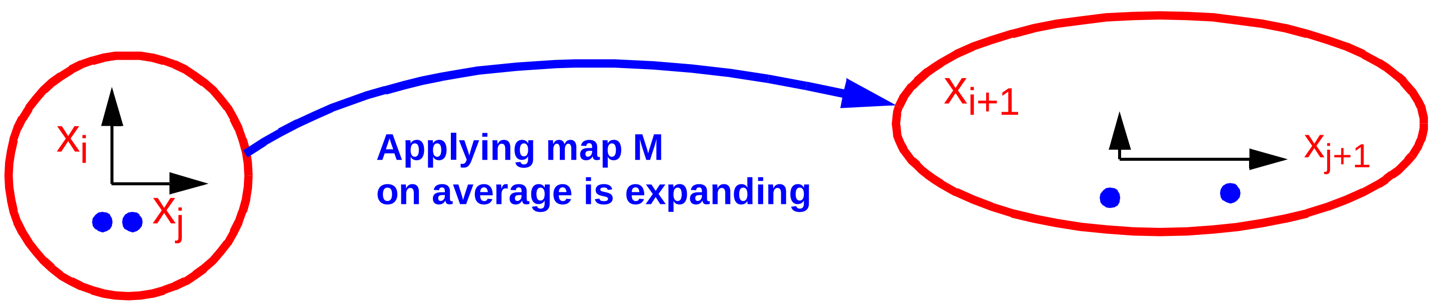
At least one long time averaged expanding direction (positive Lyapunov exponent)

Noise Induced Chaos

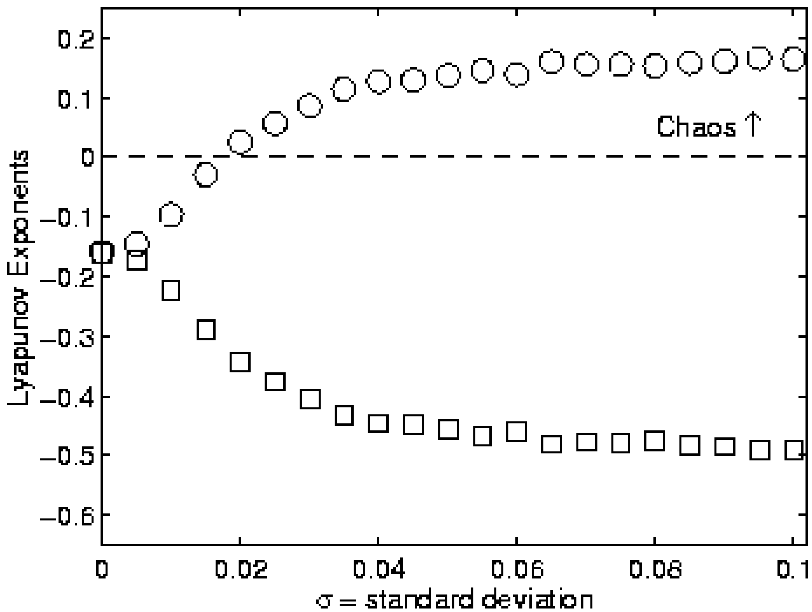
At least one "short term" time averaged positive expanding direction

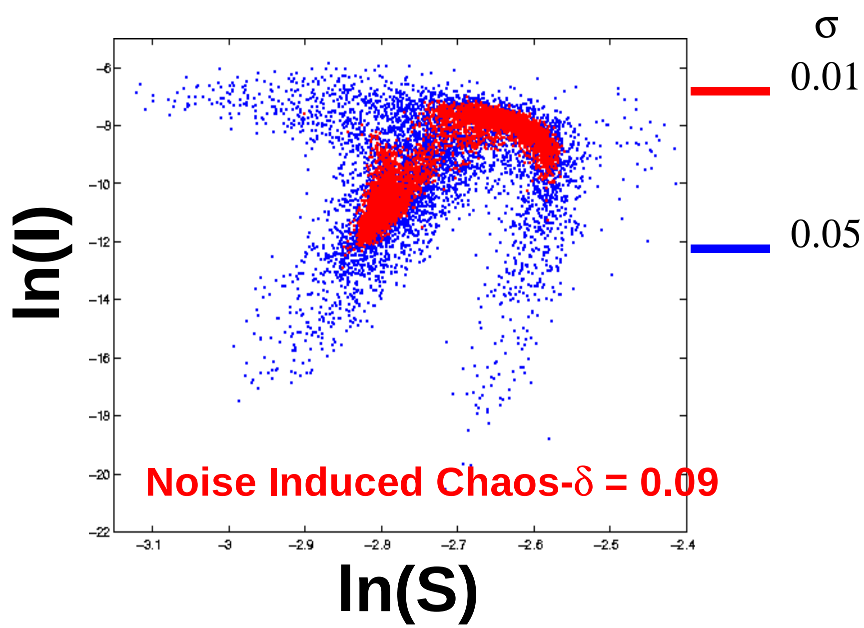
OR

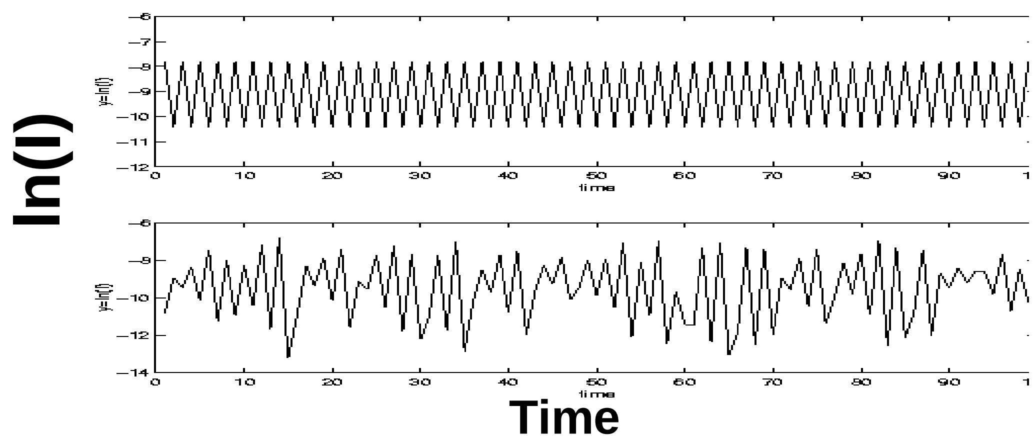
At least one spatially averaged positive expanding direction



Lyapunov Exponent Computations





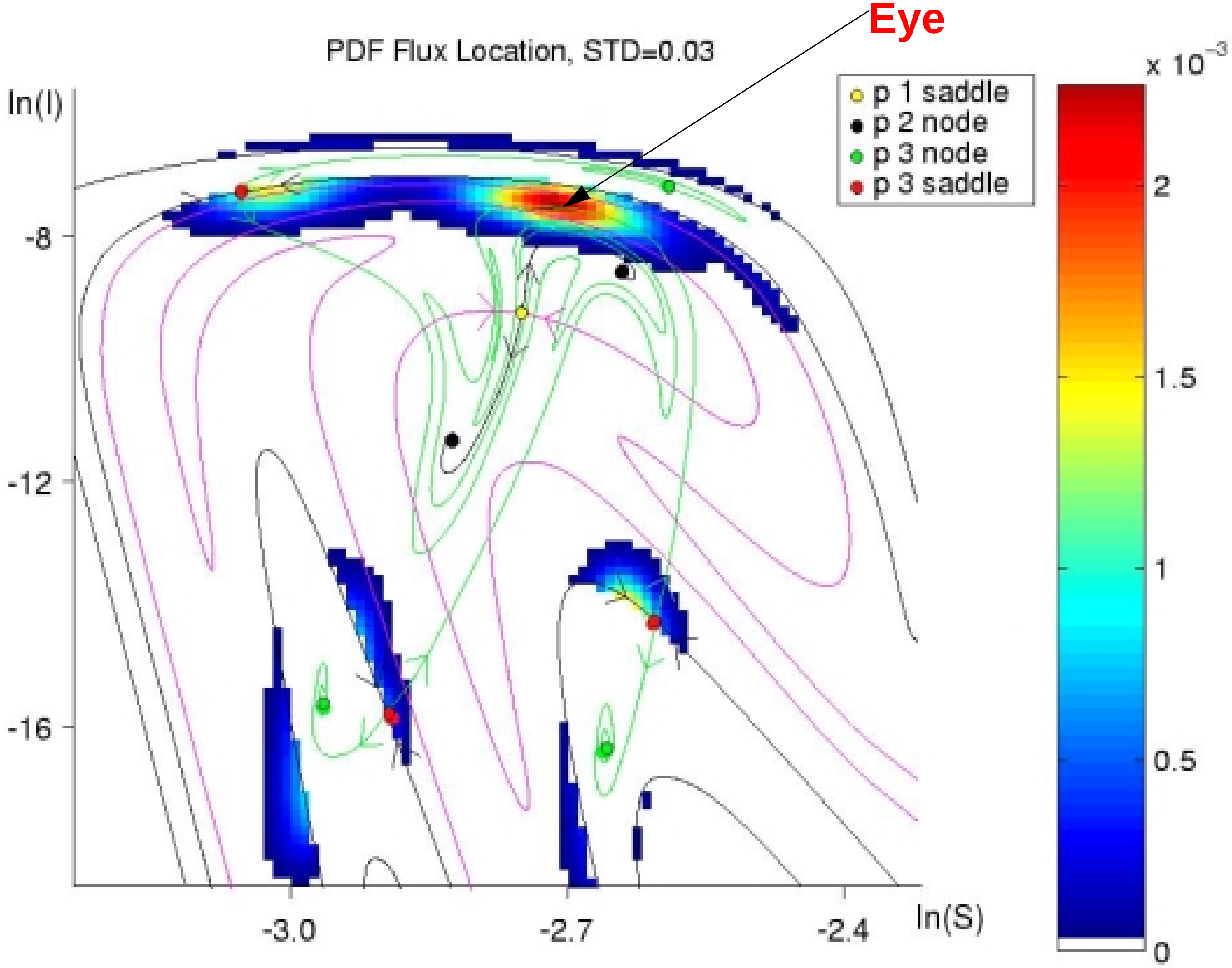


N.B. Lyapunov Exponents may yield false results for stochastic systems

Bull’s eye theory for predicting noise excited outbreaks

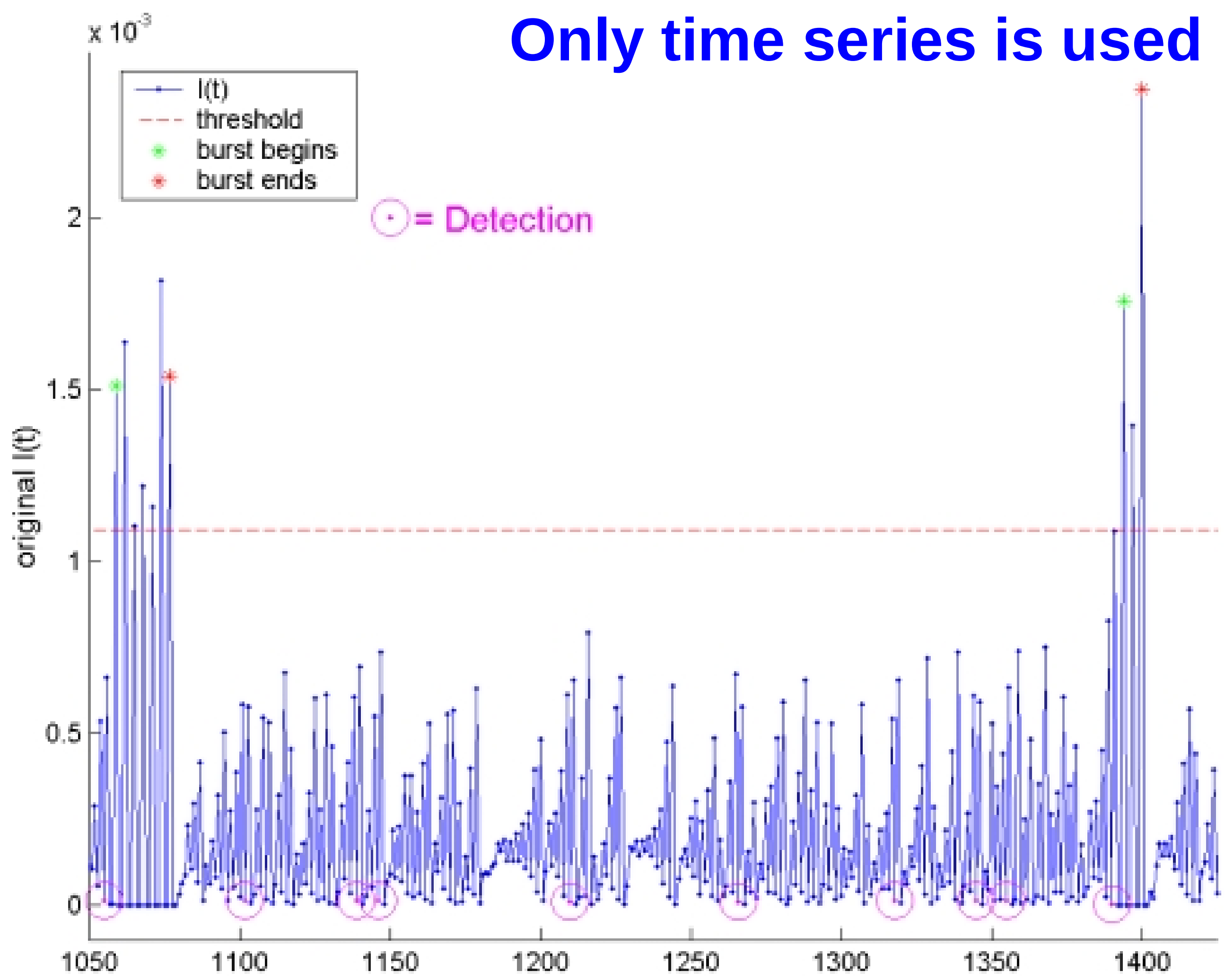
See Lora’s Talk for PDF Details

Stochastic Chaotic Transport-Bull's Eye Prediction

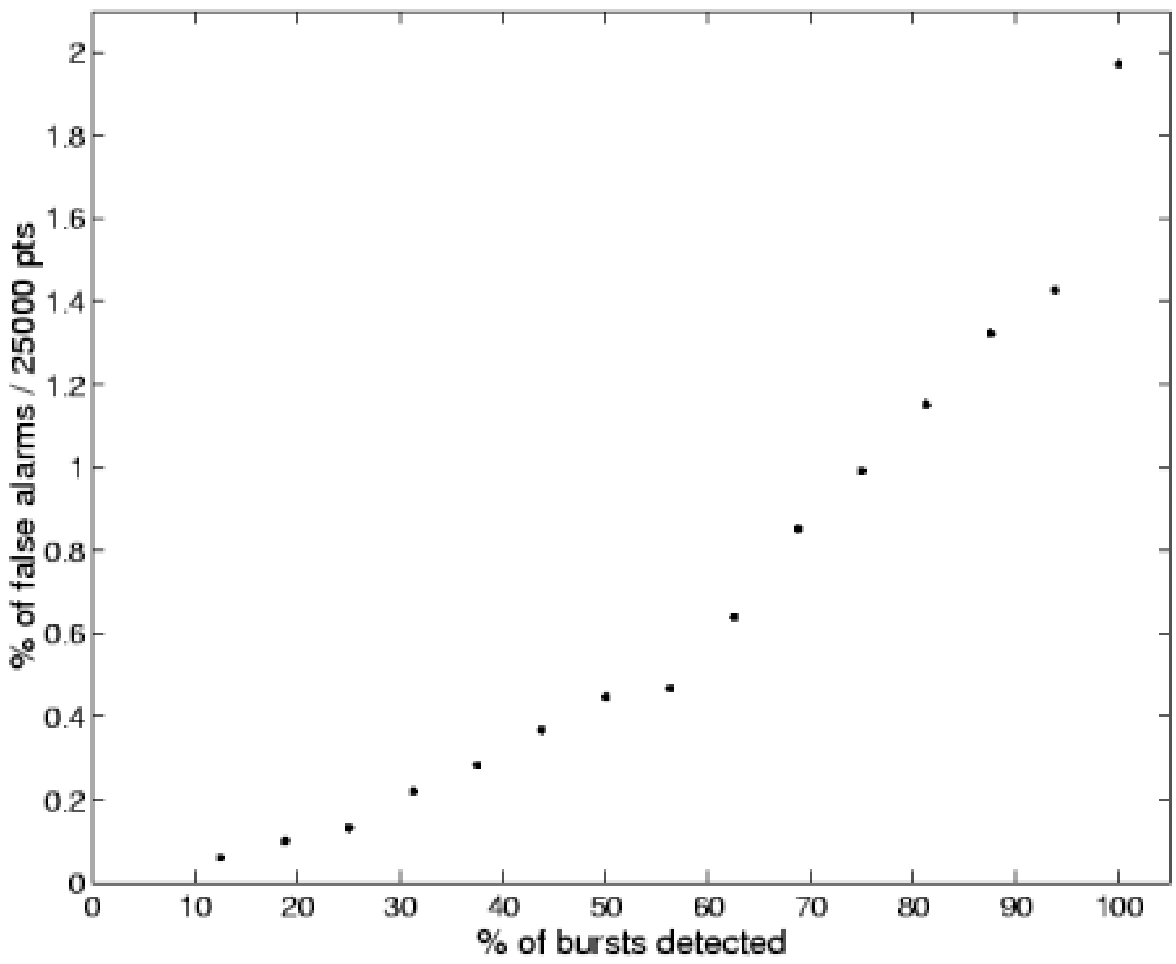


Eye Region- Probability that a large outbreak occurs after a small outbreak

Predicting Large Outbreaks Using the Bull's Eye

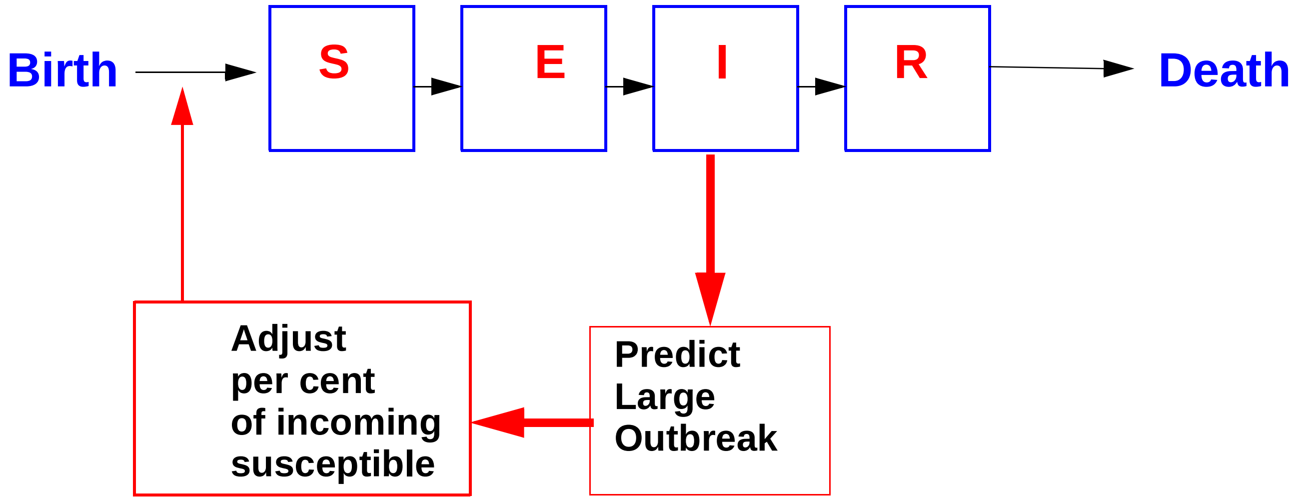


Measuring Success of Prediction



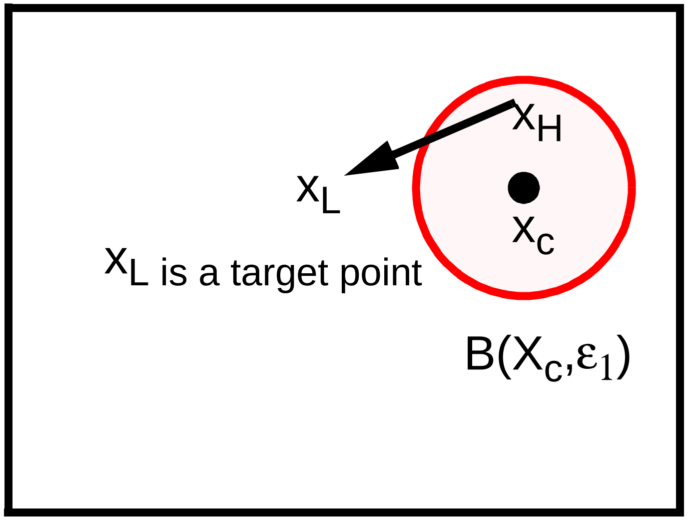
Adaptive vaccine control large outbreaks

Vaccine Implementation Modeling



Parameter Perturbations - A Targetting Approach

Suppose $x_H \in B(x_c, \varepsilon)$ **,and** $\iota \notin B(x_c, \varepsilon)$



Consider the map:

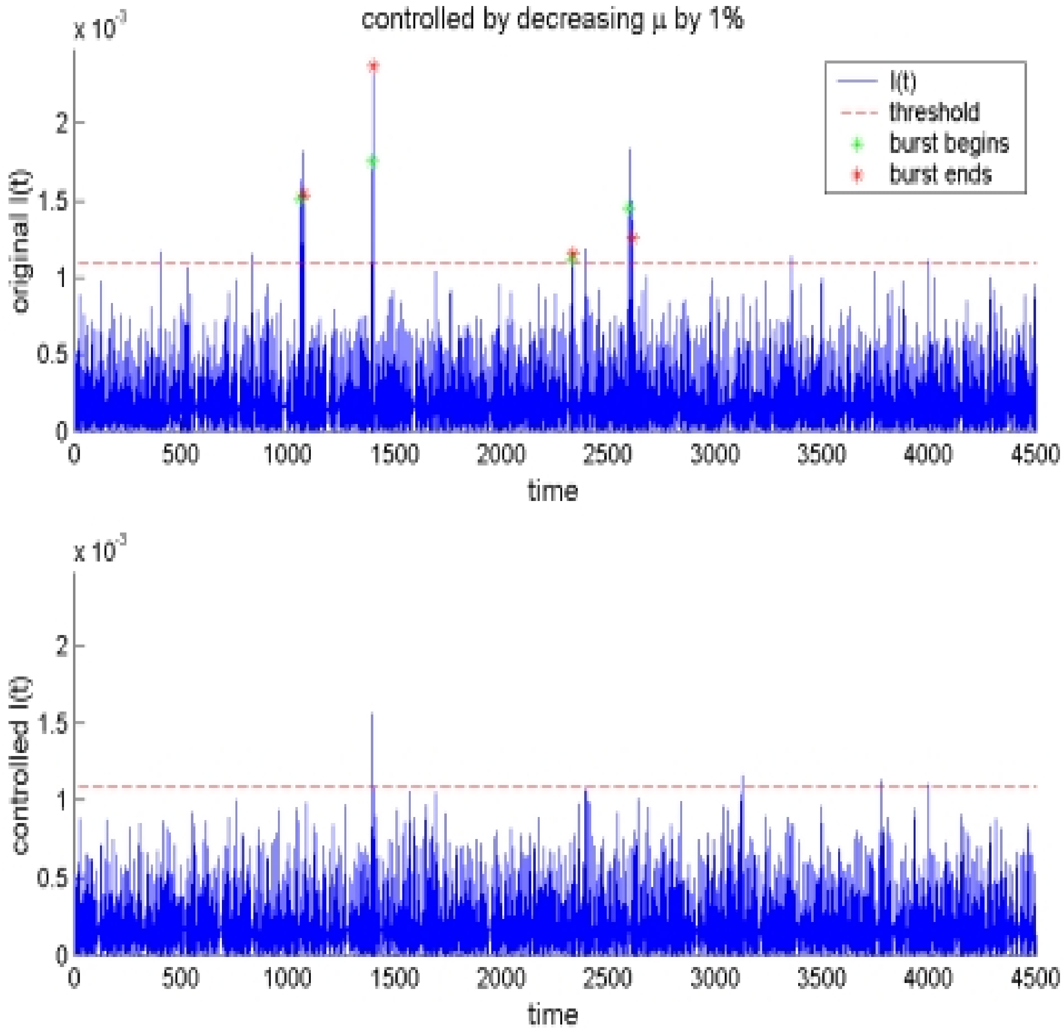
$$\begin{aligned} X_L &= T(X_H, \nu + h) \\ &= T(X_C + \alpha y, \nu + h) \end{aligned}$$

, where α **is a scalar**

Taylor expanding, and solving for h:

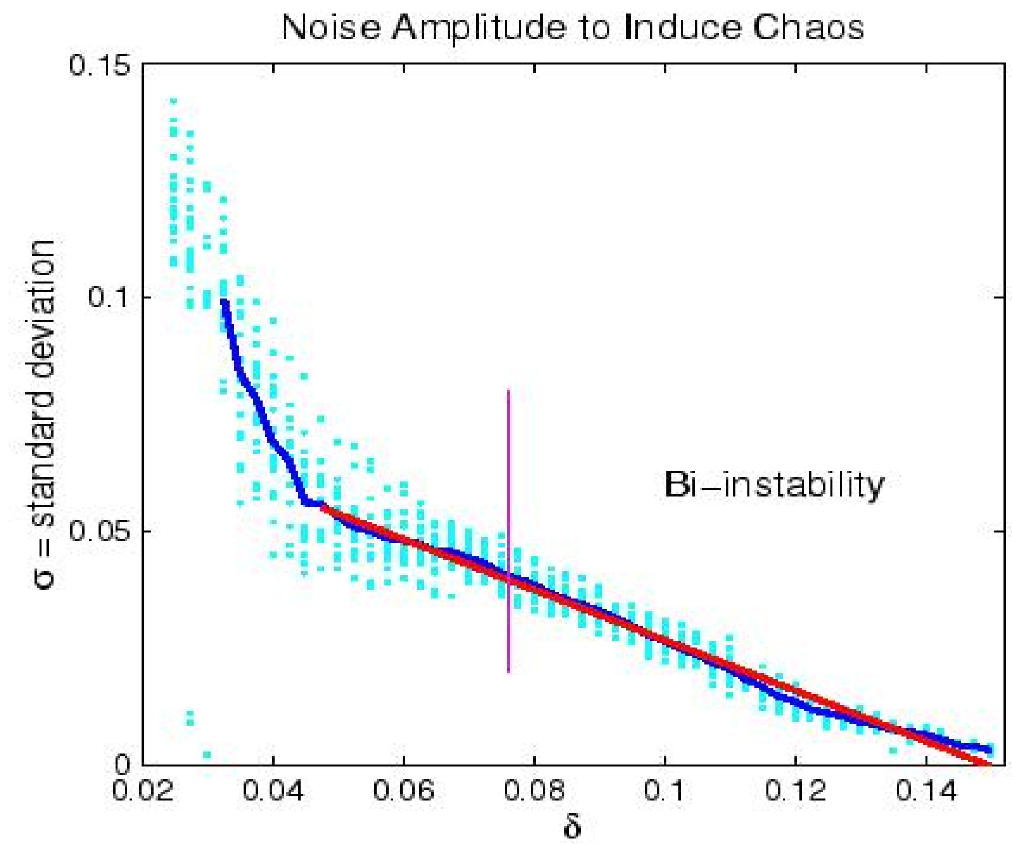
$$h = \frac{(X_L - \alpha D_x T(X_C, \delta) y)^T D_\delta T(X_C, \delta)}{\left|D_\delta T(X_C, \delta)\right|^2}$$

Prediction and Control of Large Outbreaks



Conclusion

- You can have noise generate chaos like behavior - some structure
- Mixing of small and large amplitude outbreaks
- Predict large outbreaks
- Control large outbreaks with minimal intervention



Future Work

- **Coupled populations**

  Large populations coupled to small populations

  Effective birth rate drivers - Modulated plus noise
- **Novel stochastic controls of virus**

  Eliminate large outbreaks

  Control of stochastic density
- **Population gene modeling**

  Malaria

  Aids

  Influenza
- **Social impact factors**

  Core groups

  Computer virus modeling