Immunization and Epidemic Outbreaks in Complex Heterogeneous Networks

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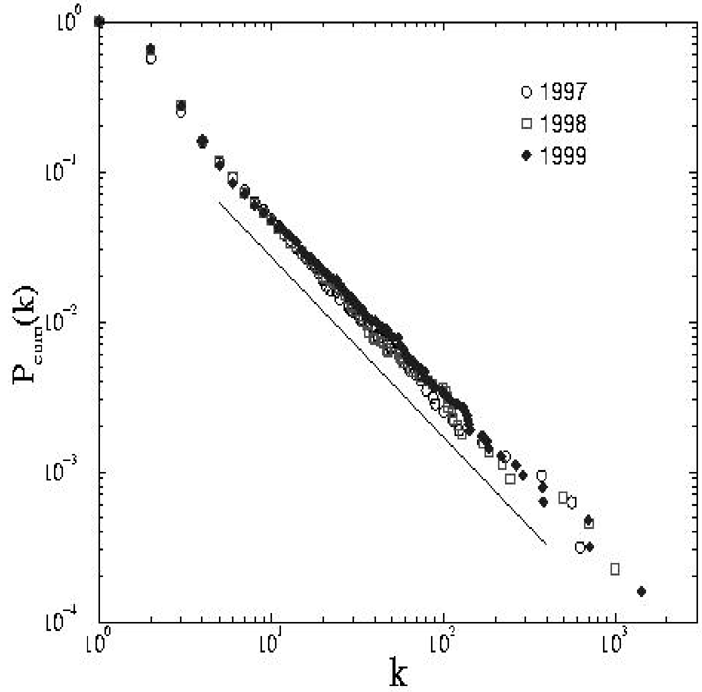
Main properties:

- **complex network**
- **preferential attachment**
- **local clustering**

Modeling of scale-free networks by Barabasi et al. (1999)



<u>Connectivity distribution</u>

- $P(k)$ = **probability that a node has k links**

$$P(k) \sim k^{-\gamma} \qquad (2 < \gamma \leq 3)$$

- $\langle k \rangle$ = **const**
- $\langle k^2 \rangle \to \infty$

Scale-free properties ⟷ Diverging fluctuations

Examples of Scale-Free Networks in Nature

- **The Internet and the World-Wide-Web**
- **Protein networks**
- **Metabolic networks**
- **Social networks**
- **Food-webs and ecological networks**

SIS Model

- The topology of the network has a great influence in the overall behavior of epidemic spreading.
- The connectivity fluctuations of the network strongly enhance the infection’s incidence.
- Scale-free networks are very weak in face of infections.
- The epidemic threshold is vanishing in the thermodynamic limit (infinite system size limit).
- The SIS model on SF networks qualitatively fits the observation from real data of computer virus spreading.

SIR MODEL ON SF NETWORKS

What are the differences between SIS and SIR models ?

- SIR does not have a stationary state while the SIS has one.
- The order parameters of the two models are different.
- The basic equations describing the model are different.
- Critical exponents are different.
- Activity profiles and time evolution are different.

May & Lloyd PRE (2001)
Moreno, Pastor-Satorras & Vespignani EPJB (2002)

In order to address the effects of contact heterogeneity in epidemic spreading, let us consider the SIR model defined on a network with general connectivity distribution $P(k)$ and a finite average connectivity $\langle k \rangle = \Sigma_k \, k P(k)$.

$$
\begin{aligned}
\frac{d\rho_k(t)}{dt} &= -\rho_k(t) + \lambda k S_k(t)\Theta(t), \\
\frac{dS_k(t)}{dt} &= -\lambda k S_k(t)\Theta(t), \\
\frac{dR_k(t)}{dt} &= \rho_k(t).
\end{aligned}
$$

- **A link is more likely connected to a node with high connectivity**
- **The probability that a link points to a node with *s* links is proportional to *sP(s)*.**

The probability of pointing to an infected node is

$$\Theta(\lambda) = \sum_k \frac{kP(k)\rho_k}{\sum_s sP(s)}.$$

ρ_k themselves are a function of $\Theta(\lambda)$

Self-consistent equation

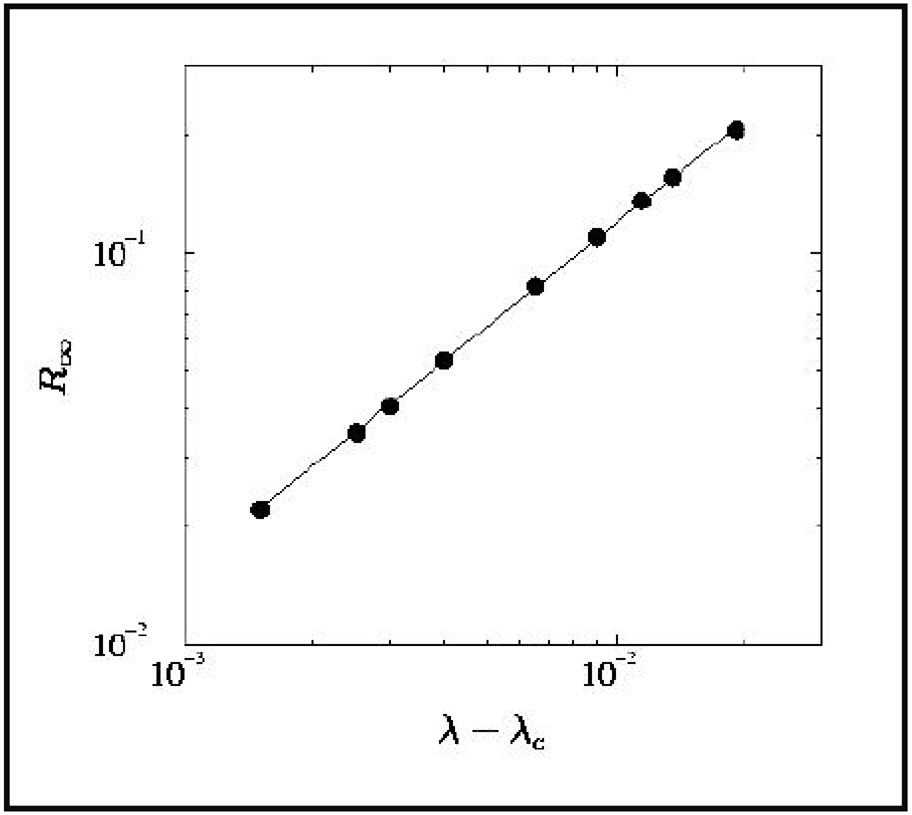
Finally, the equation for the order parameter is

$$\rho = \sum_k P(k)\rho_k$$

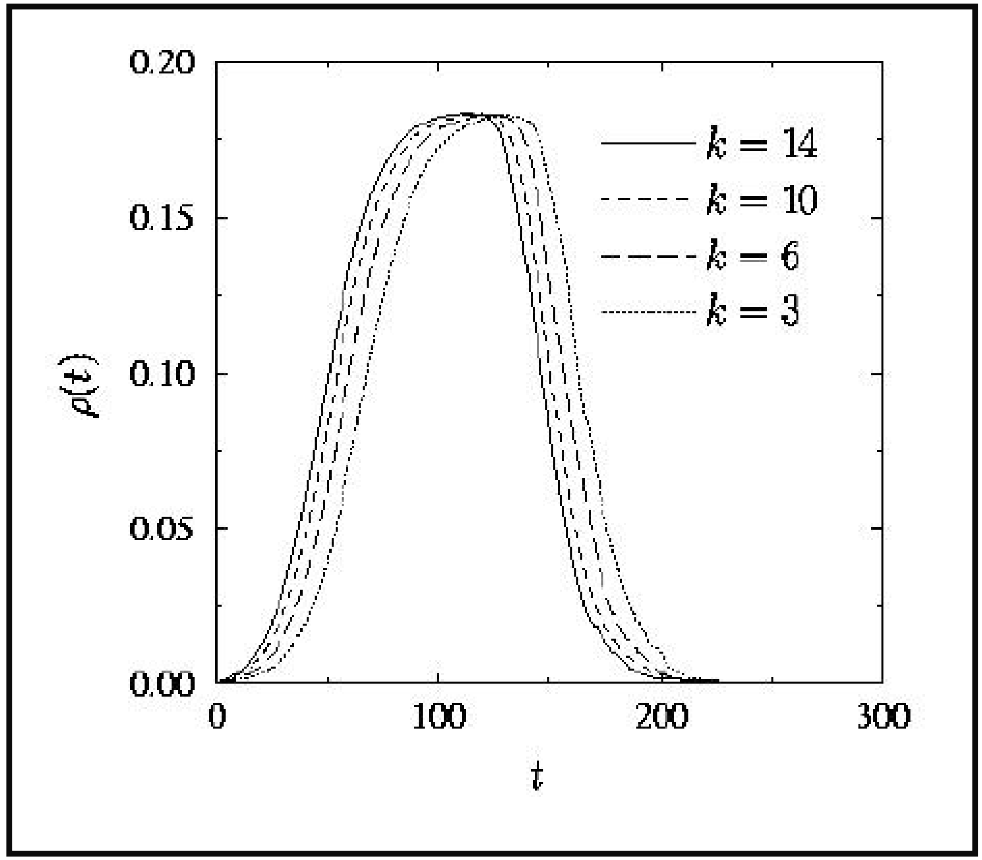
$$\frac{1}{\langle k \rangle} \sum_k k P(k)(\lambda k) = \lambda \frac{\langle k^2 \rangle}{\langle k \rangle} > 1.$$

$$\lambda_c = \frac{\langle k \rangle}{\langle k^2 \rangle}$$

below which the epidemic incidence is null, and above which it attains a finite value. That is, the threshold is inversely proportional to the connectivity fluctuations $\langle k^2 \rangle$.

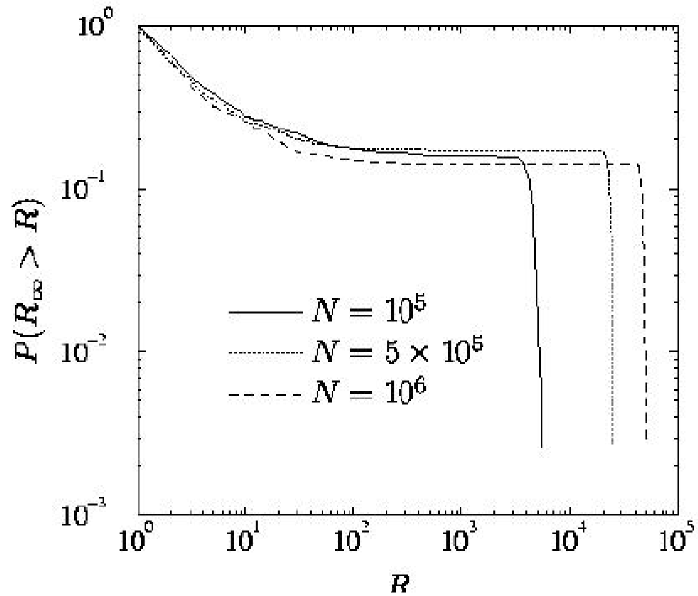


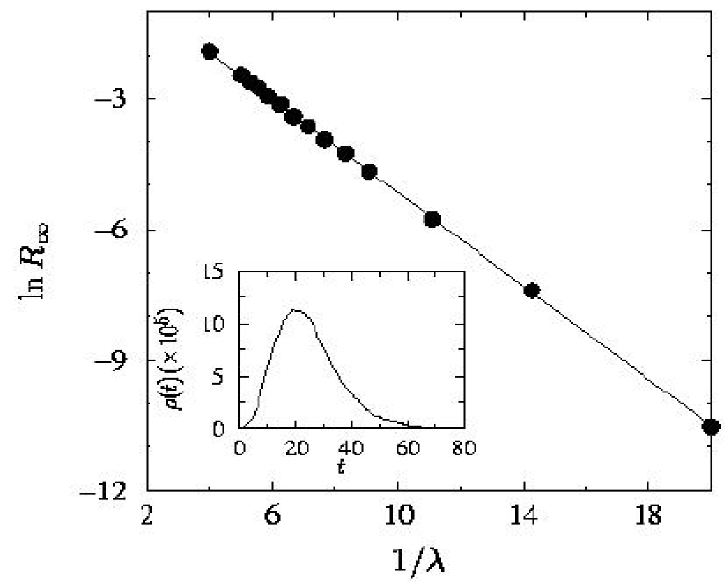
WS Model

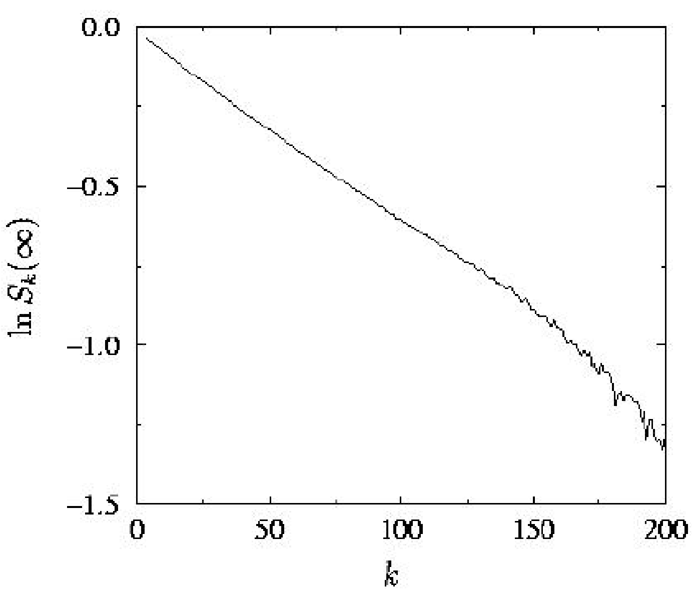


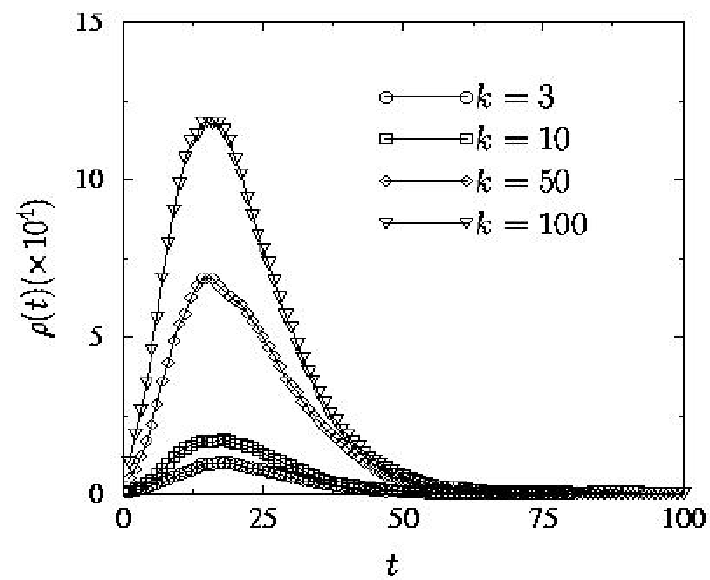
The total incidence does not depend on the connectivity of the initial seed.

Scale-free Networks.









The total incidence depends on the connectivity of the initial seed.

SIS or SIR

Depends on the users:

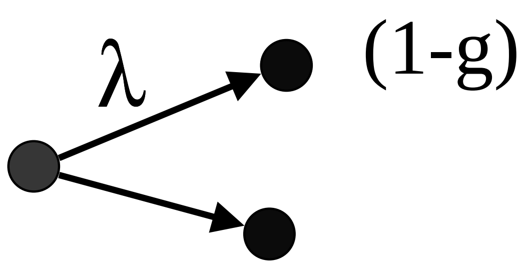
- It seems that SIR is more appropriated since once the antivirus is available I class reduces to R class.
- Point 1 does not depend only on the capacity of the users to buy the antivirus but also on whether or not they are willing to solve the problem. (or whether or not they are aware of the antivirus). Here SIS applies.

$$\Downarrow$$

SIS with variable spreading rate λ, i.e., $\lambda(t)$?

Immunization

- **Random immunization:**
  g = density of immune nodes



$$\boxed{\lambda \rightarrow \lambda\,(1-\mathbf{g})}$$

Epidemic dies if $\lambda\,(1-\mathbf{g}) \leq \lambda_c$

Regular or random networks

$$\rho_g = \rho_0 (g_c - g)/g_c$$

Immunization threshold

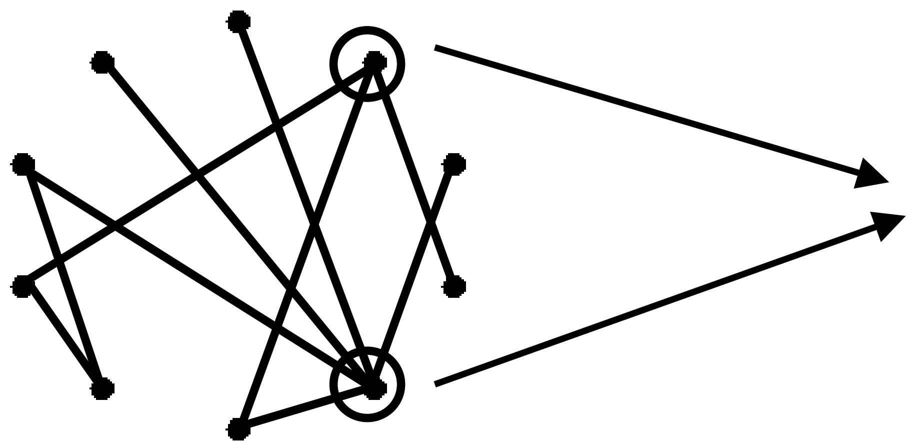
$$g_c = (\lambda - \lambda_c)/\lambda$$

Scale-free networks $\Rightarrow$ $\boxed{\rho_g \sim \exp(-C/(1-g))}$

$$\boxed{\textbf{Immunization threshold } g_c = 1}$$

- **<u>Random immunization is totally ineffective</u>**
- **<u>Different immunization specifically devised for highly heterogeneous systems</u>**

Targeted immunization strategies

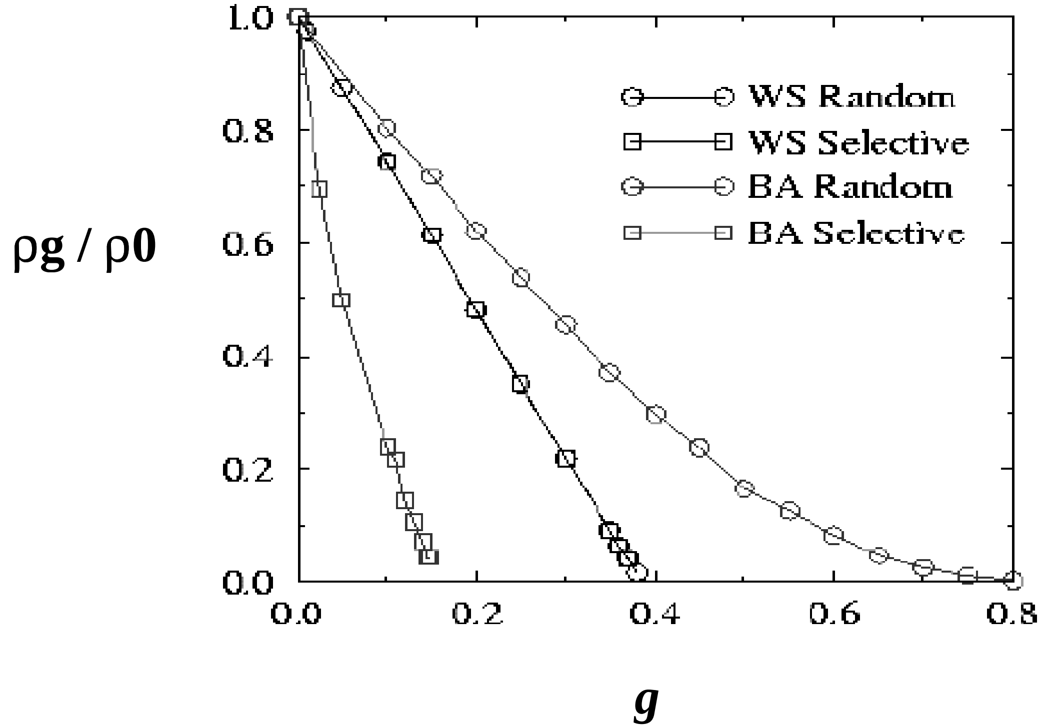


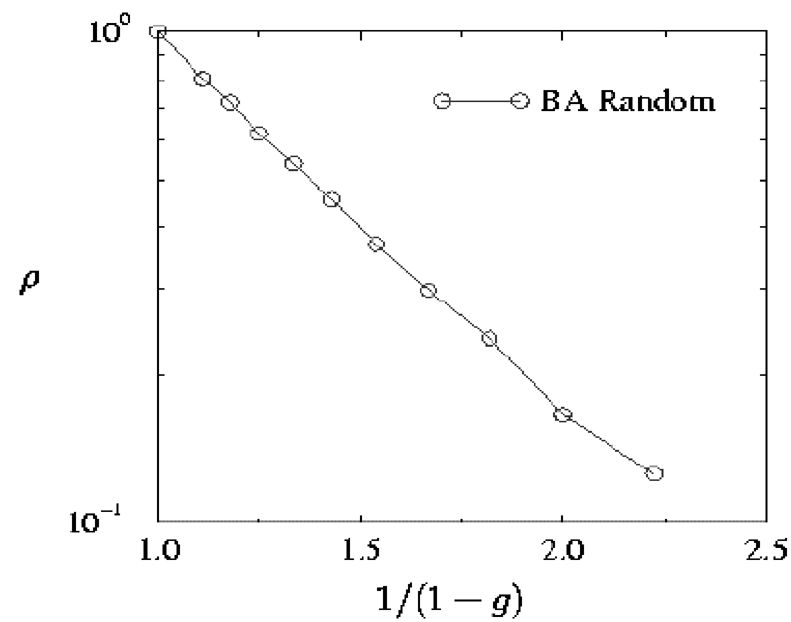
Progressive immunization of crucial nodes

Epidemic threshold is reintroduced

$$\boxed{g_c = \exp(-2/m\lambda)}$$

Numerical Simulations on Scale-free and Random Networks





[Pastor Satorras &Vespignani, PRE 65, 036104 (2002)]

[Dezso & Barabasi cond-mat/0107420]

MAIN RESULTS FOR S-F NETWORKS

- **Absence of an epidemic/immunization threshold.**

- **The total incidence depends on the connectivity of the initial seed.**

- **Progressive random immunization is totally ineffective.**

- **Targeted immunization strategies are more effective since the epidemic threshold is reintroduced.**

References :

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- **A. Vazquez, R.Pastor Satorras and A.Vespignani, cond-mat/0112400 (2001)**