Halting viruses in scale-free networks

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Phys. Rev. E, **65**,055103(R)

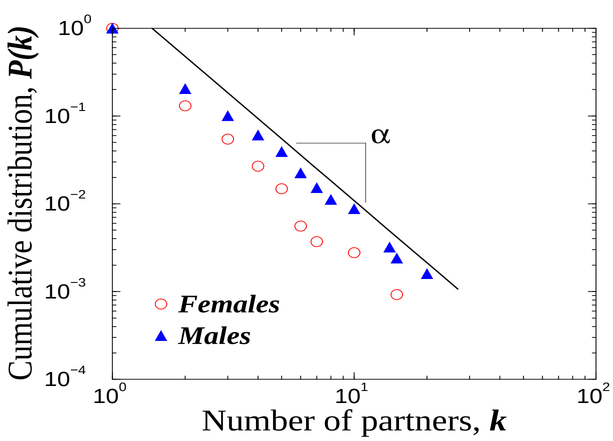
Where do viruses spread?

Biological viruses:

Sex-web

Nodes: people

Links: sexual relationships



4781 Swedes;
age 18-74.

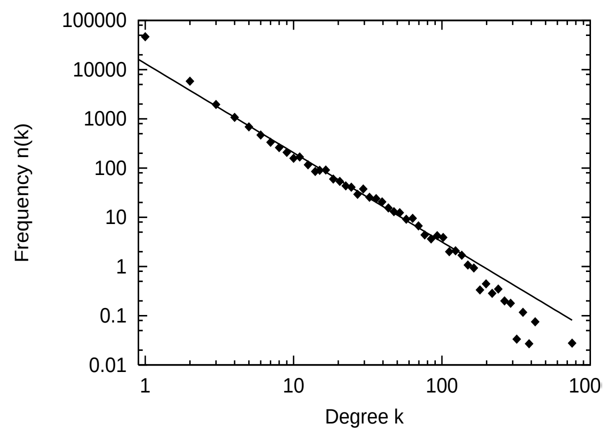
Liljeros et al.
Nature 2001

Computer viruses:

E-mail network

Nodes: e-mail addresses

Links: e-mails



Kiel University: period:
112 days; nodes: 59912.

Bornholdt et al.
cond-mat/0201476.

Modeling the spreading of viruses on complex networks

The SIS model:

- **nodes**: individuals can be in two states: "healthy" or "infected"

- **links**: connections between the individuals along which the infection can spread.

- **dynamics:** In each time step a healthy node is infected with probability ν if it is connected to at least one infected node. At the same time an infected node is cured with probability δ, defining a spreading rate $\lambda \equiv \frac{\nu}{\delta}$ for the virus.

Epidemic threshold (λ_c)

For a regular lattice or a random network studies indicate that:

- if the spreading rate exceeds a critical threshold the virus will persist
- if the spreading rate is under the critical threshold the virus will die out

Surprising results:

The epidemic threshold, $\lambda_c = 0$ for scale-free network!

The origin of the zero threshold is the infinite variance of the power law distribution:
$\lambda_c = \frac{<k>}{<k^2>} = 0$ because $< k^2 > \to \infty$

Reducing the spreading rate won't eradicate the epidemic.

R. Pastor-Satorras and A. Vespignani, Phys. Rev. Lett. **86**, 3200 (2001);

Can we eradicate a virus in a scale-free network?

Stopping epidemics on scale-free networks

Solutions:

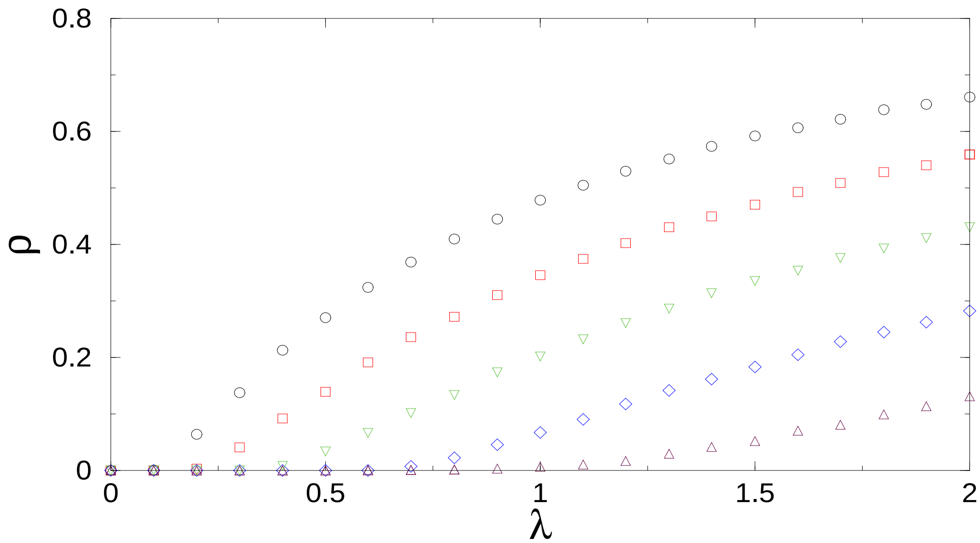
- curing simultaniously all infected nodes
  Problem: **number of available cures is often limited**

- curing the hubs (all nodes with degree $k > k_0$)
  It works: $\lambda_c = \frac{<k>}{<k^2>} \neq 0$
  Problem: **We cannot effectively identify the hubs!**

- **probability of curing a node is proportional to k^{α};**
  $-\alpha = 0$ for random immunization;
  $-\alpha = \infty$ for a policy treating all hubs with degree larger than k_0.

Numerical Results

The probability of curing a node is proportional to k^{α}



Prevalence, ρ, measured as the fraction of infected nodes in function of the effective spreading rate λ for $\alpha = 0$(O), 0.25($\square$), 0.50($\bigtriangledown$), 0.75($\diamond$) and 1($\bigtriangleup$), as predicted by Monte-Carlo simulations using the SIS model on a scale-free network with N=10,000 nodes.

For which value of α will appear a finite epidemic threshold?

Mean-field equation:

$$\partial_t \rho_k(t) = -\delta_0 k^\alpha \rho_k(t) + \nu(1-\rho_k(t))k\theta(\lambda).$$

- **first term** in rhs the probability that an infected node is cured:
  - $\delta_0 k^\alpha$ is the probability that a node with k links will be selected for a cure
  - $\rho_k(t)$ density of infected nodes
- **second term** in the rhs is the probability that a healthy node with k links is infected
  - ν is the infection rate
  - $[1-\rho_k(t)]$ the number of healthy nodes with k links
  - $\theta(\lambda)$ is the probability that a given link points to an infected node.

The probability $\theta(\lambda)$ is proportional to $kP(k)$:

$$\theta(\lambda) = \sum_k \frac{kP(k)}{\sum_s sP(s)} \rho_k.$$

Imposing the stationary condition $\partial_t \rho_k(t) = 0$ the average density of the infected nodes can be written as:

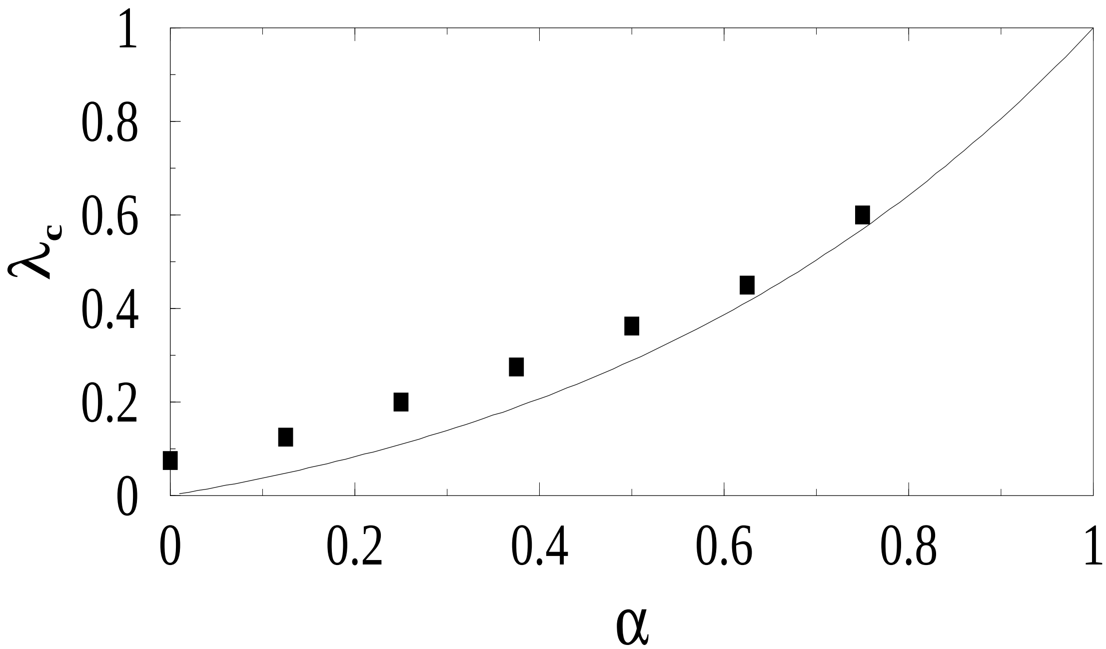
$$\rho(\lambda) = 2m^2 \lambda \theta(\lambda) \int_m^\infty \frac{dk}{k^3(k^{\alpha-1} + \lambda\theta(\lambda))}$$

The epidemic threshold depends on α as:

$$\lambda_c = \alpha m^{\alpha - 1},$$

where $m = \frac{<k>}{2}$.

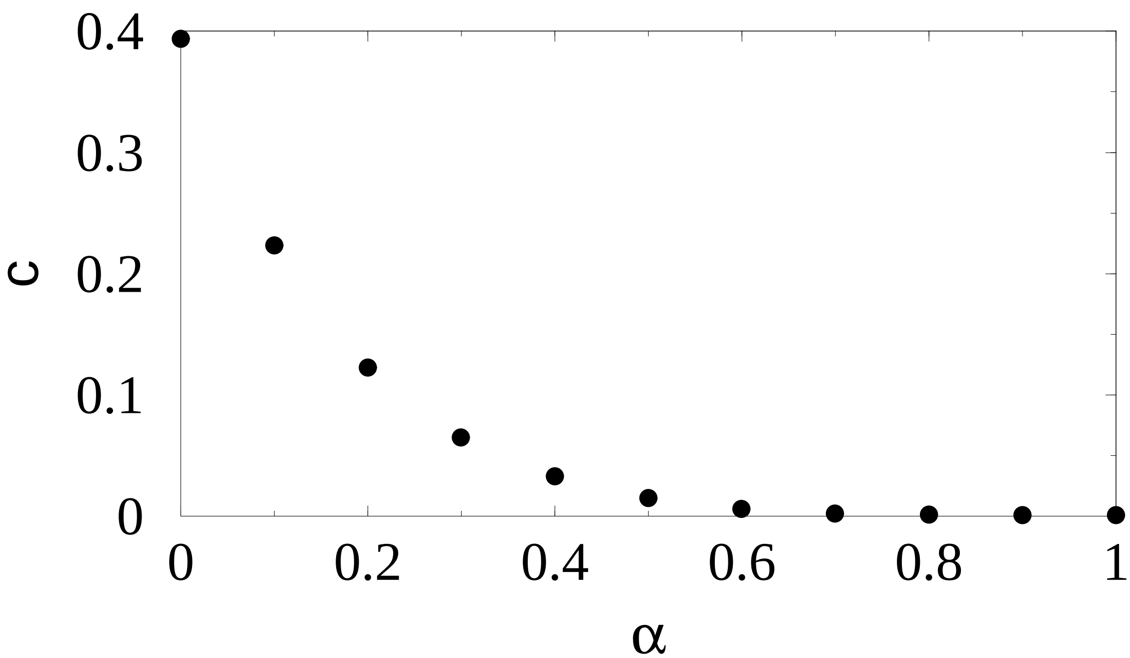
The dependence of the epidemic threshold on α:



Dependence of the epidemic threshold λ_c on α as predicted by our calculations (continuous line) based on the continuum approach, and by the numerical simulations based on the SIS model (boxes).

For *any* non-zero α the epidemic threshold is finite

How cost-effective is a biased policy?



Number of cures, c, administered in an unit time per node for different values of α.

The more successful is the policy in curing the hubs the fewer cures are required

Conclusions:

- **policies biased towards the hubs restore the epidemic threshold**

- **the epidemic threshold is finite for any** $\alpha \neq 0$ **value**

- **biased treatment policy is less expensive than random immunization**

(Phys. Rev. E, **65**,055103(R))

Note: Similar results for permanent immunization and $\alpha = 1$ were obtained by R. Pastor-Satorras and A. Vespignani (Phys. Rev. E **65**, 036104 (2002)).