

Double Trouble: Attack of Two or More Separate Viral Species

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...and my friends

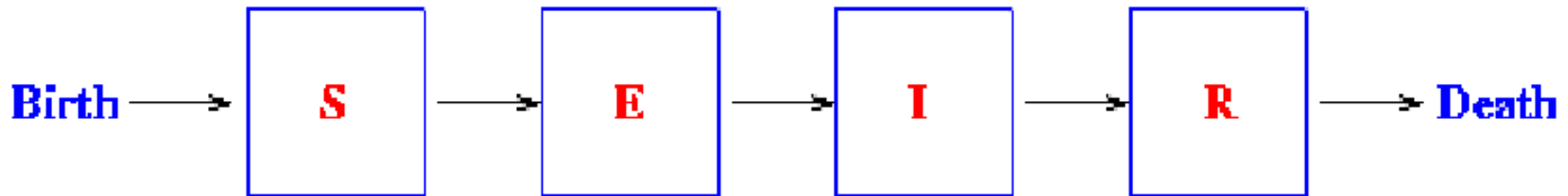
- Ira Schwartz, NRL
- Lora Billings, Montclair State U., NJ.

Two *Only SLIGHTLY* Pertinent Anecdotes

- My Comcast connection – the accidental “bolt virus.”
- USNA, China, the internet, recruiting and network services.

A “Typical” Disease: The SEIR Model

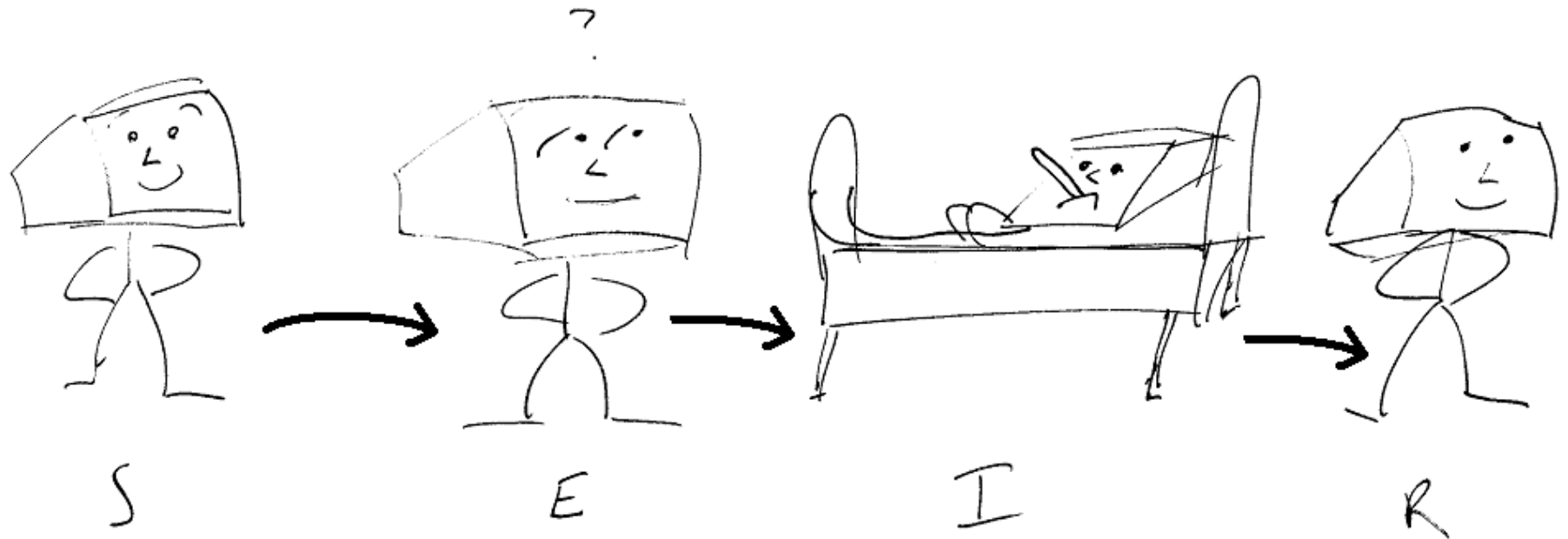
- assigned variables to the four main groups of a population
 - S=susceptibles
 - E=exposed
 - I=infectives
 - R=recovered
- normalize the variables so that $S+E+I+R=1$



- assumed that the population is large and well mixed

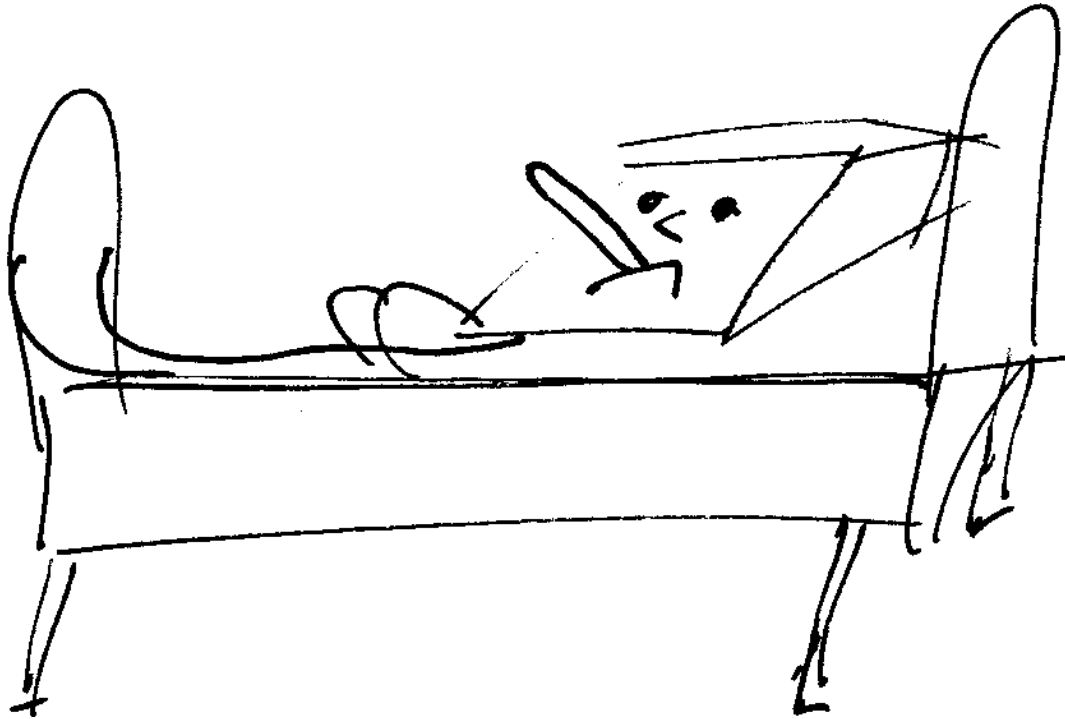
Thanks Lora!

SEIR

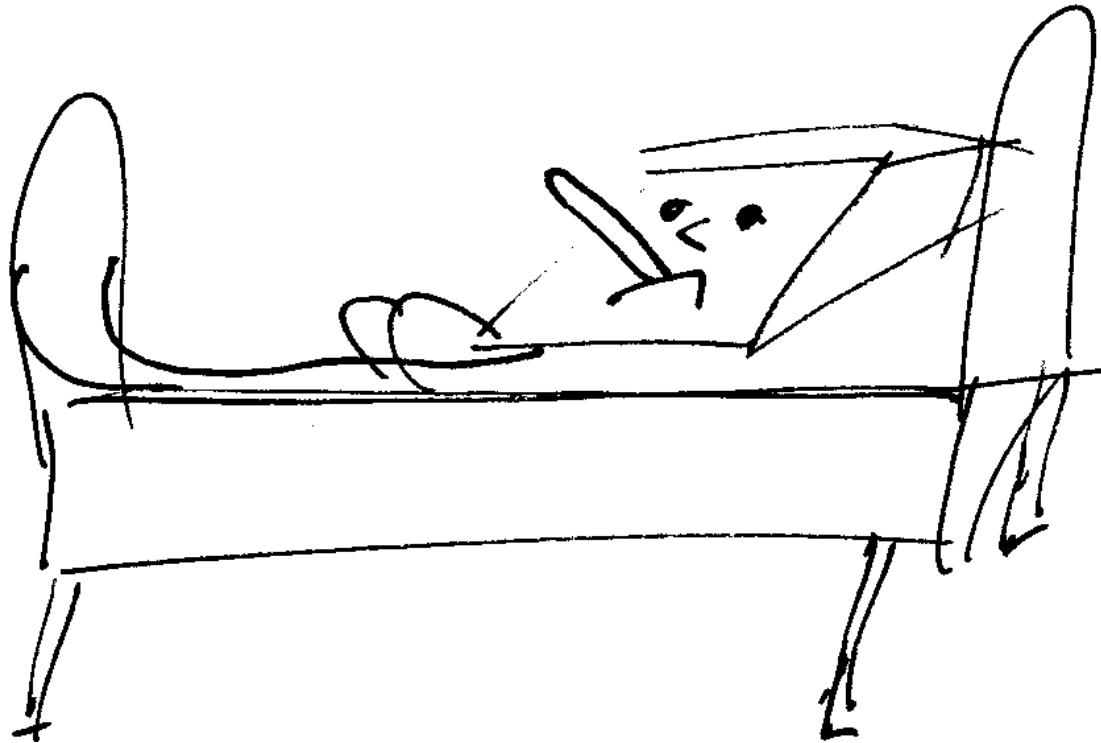


Original.

But Sometimes
You (Your Computer) Gets Sick



And then Sick Again,
With Something Else



Double Trouble

Assumptions of SEIR

- Markov – Memoryless
- Homogeneous – averaged
- One illness – recovery rate is fast relative to infection rates of any new diseases.

History and K-W

- One of the earliest and simplest virus models is K-W model.
- J. O. Kephart and S. R. White (K-W) 1991, "Directed-Graph Epidemiological Models of Computer Viruses," in Proc. Comp. Soc. Symp. on Research in Security and Privacy, pp. 343--359, 1991.
- Directed graph: each node in one of several states for state of "infection," rules for changing the state of node, and each link between nodes represents a potential route of infection.
- Nodes are homogeneous and react similarly.
- The network is isotropic and nodes see the same statistical view as all the other nodes.

A Suggestion of a sort of Model Subgraph.

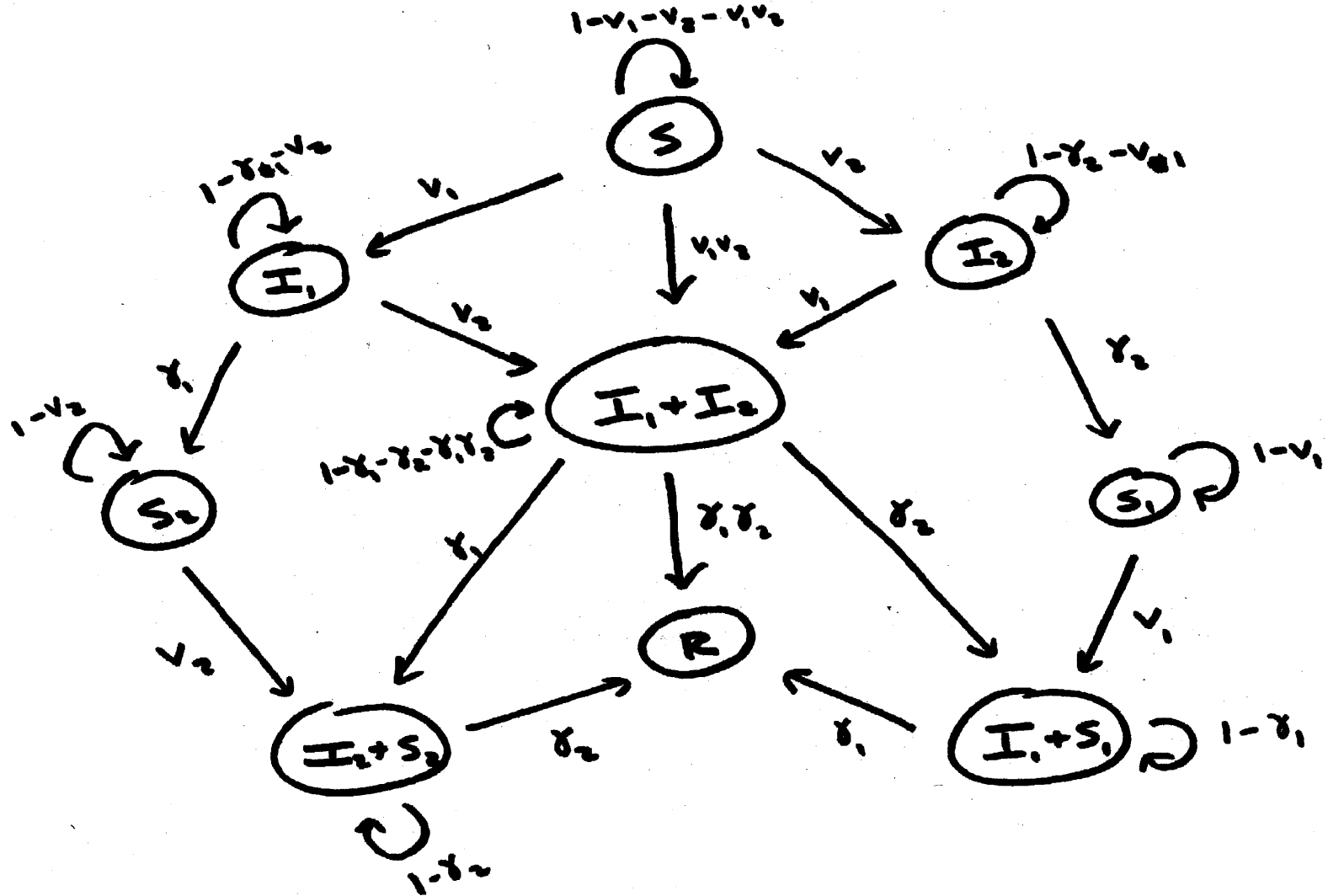
- **Scales of modeling:**

1. Homogeneous. S,E,I,R represent population (percentages).
2. City Clumps.
3. Full simulation of accurate network topologies.

- **Two main areas of study of network**

- Topology of network
 - Connectivity and percolation? Components and Giant Components?
- Dynamics on the network 1, 2, or 3.
 - Evolution of individuals health on Networks:
 - Discrete Time or Continuous time.
 - Epidemic Diffusion.
 - Bifurcation analysis
 - Noise.

One Possible Two Virus Life-Cycle



Individual's health subgraph: **Double Trouble Graph**

Definitions:

State Definitions:

S -susceptible to everything

I₁ -Infection 1 contracted

I₂ -Infection 2 contracted

S₁ -Cured of 2, but still 1 – susceptible

S₂ -Cured of 1, but still 2 – susceptible

R – Recovered from everything

Assume: For our discrete time model

Rate = Probability

$\Rightarrow \sum \text{Probability from any vertex} = 1$

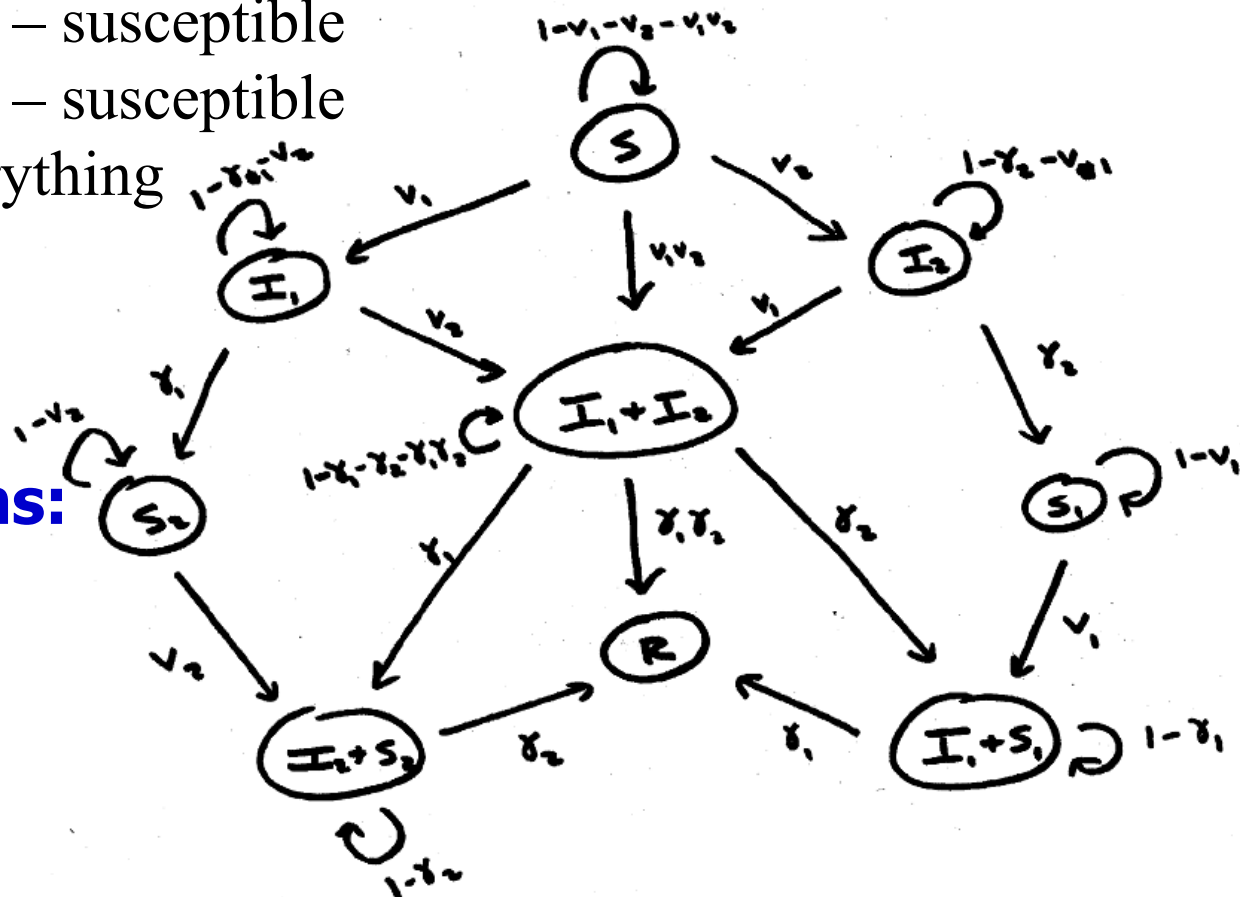
Transition Definitions:

v₁ – I₁ contraction rate

v₂ – I₂ contraction rate

γ₁ – I₁ cure rate

γ₂ – I₂ cure rate



Previous Model is Homogeneous

- Represents Either 1 members health cycle or many averaged members lifecycle
- Put this individual's health subgraph on a large (random) acquaintance graph.
- Latency in Double Trouble.
- Does I1 make you more susceptible, weakened to I2?
- Why Double Trouble
 - Deliberate delayed two part attack, for slow ubiquitous attack.
 - Accidental. Particularly in Biology.

Spatial Structure: Topology of Networks.

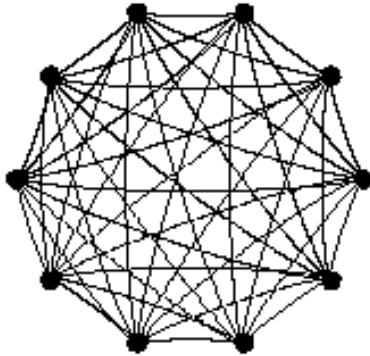
- Distinction between:
 - Topology of underlying *network*, say *routers*.
 - Topology of activity of network due to say, email, which is *social network of users*.
 - A graph is **$G=(E,V)$** , where vertices is collection of IP addresses, and edges is ordered pairs of vertices (links).

Taxonomy of Popular Random Graph Communities: Global Topology

Which Best Describes the World-Wide Web?

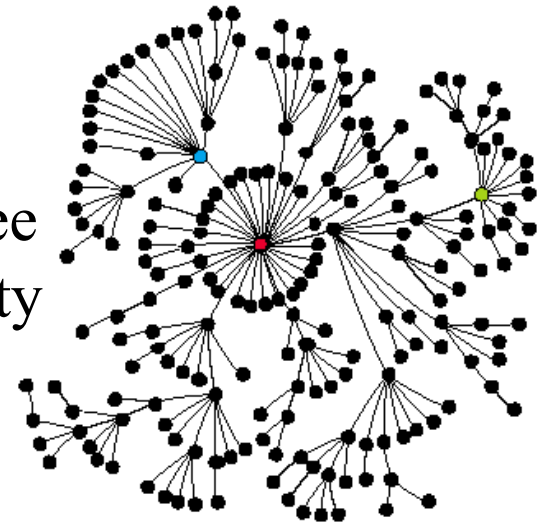
All-All

- $E=10, N=10$



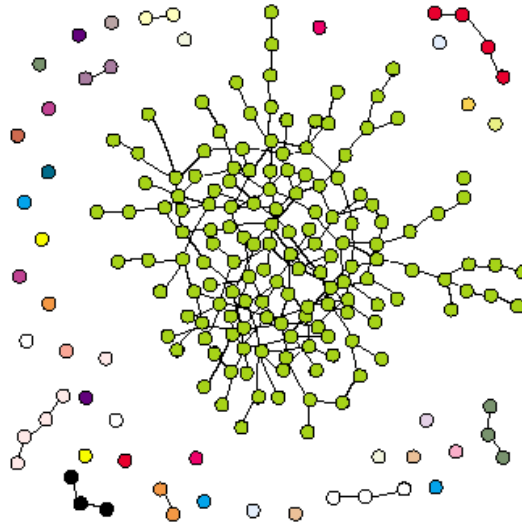
Scale-Free

- $E=199, N=200$
- Red $\rightarrow k=33$ degree
- $\phi(k)$ - probability



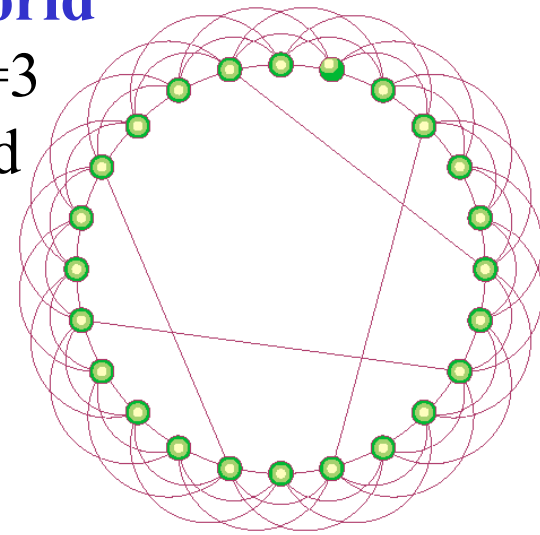
Random

- $E=193, N=200$
- Giant Component
- Expect if $E > N/2$
- No Central Hub



Small-World

- $N=24, k=3$
- $\phi(k)$ add

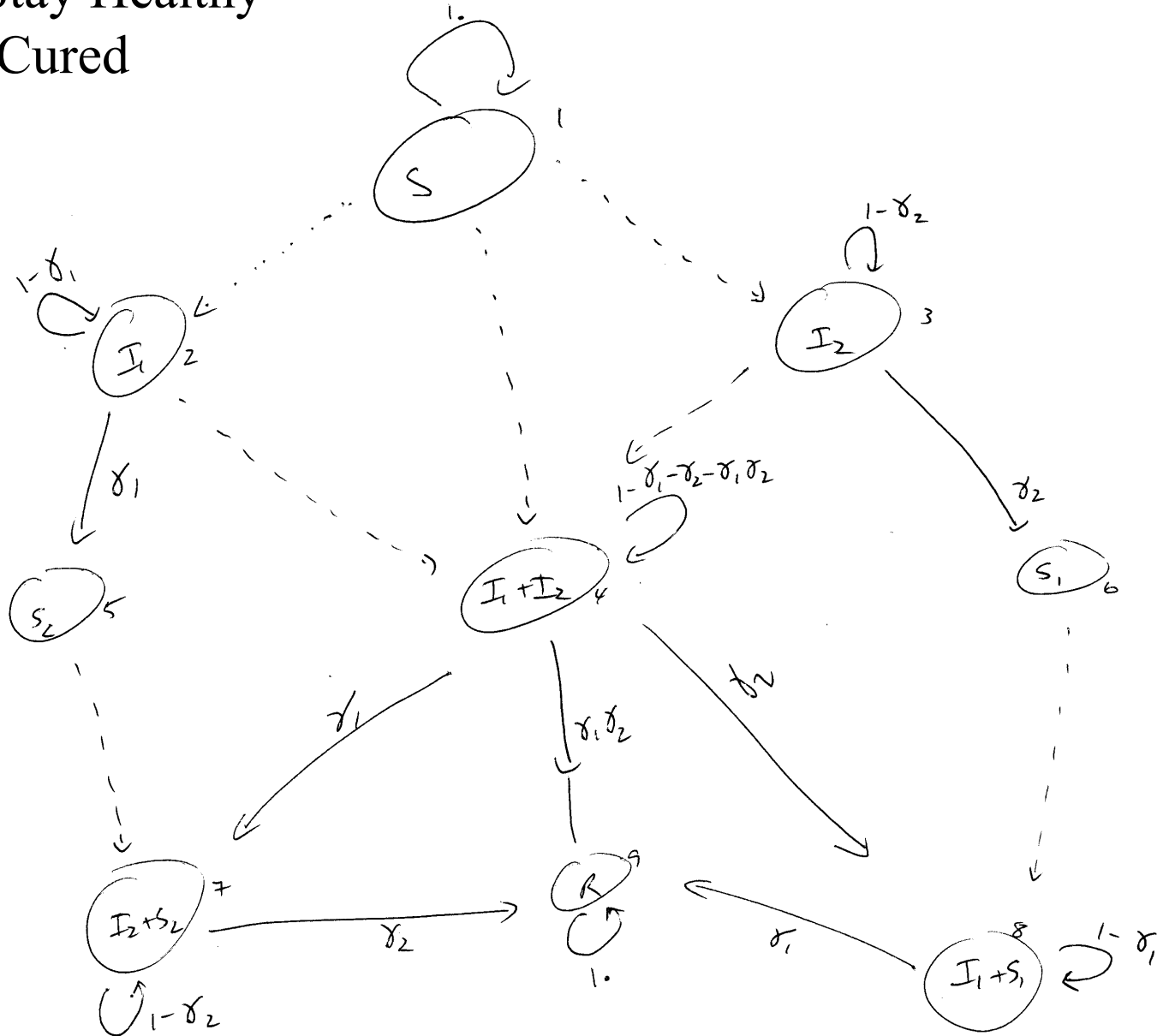


In Theory and Practice: Random Graph of Double Trouble “Islands.”

- Graph g generated by transfer matrix S , $g_S: Z_2^9 \dashrightarrow Z_2^9$
- Transfer Matrix G for global graph g ,
- Full Transfer Matrix: $GS = G \otimes S$
- Generates Global Dynamics: $g_{GS}: Z_2^{9n} \dashrightarrow Z_2^{9n}$
- BUT! Easier in Practice as a composite Markov operator:
 - $\text{Process}(\text{states}) = \text{Process3}(\text{Process2}(\text{Process1}(\text{states})))$
- And this one is nonlinear: Ceiling, or like a CA.

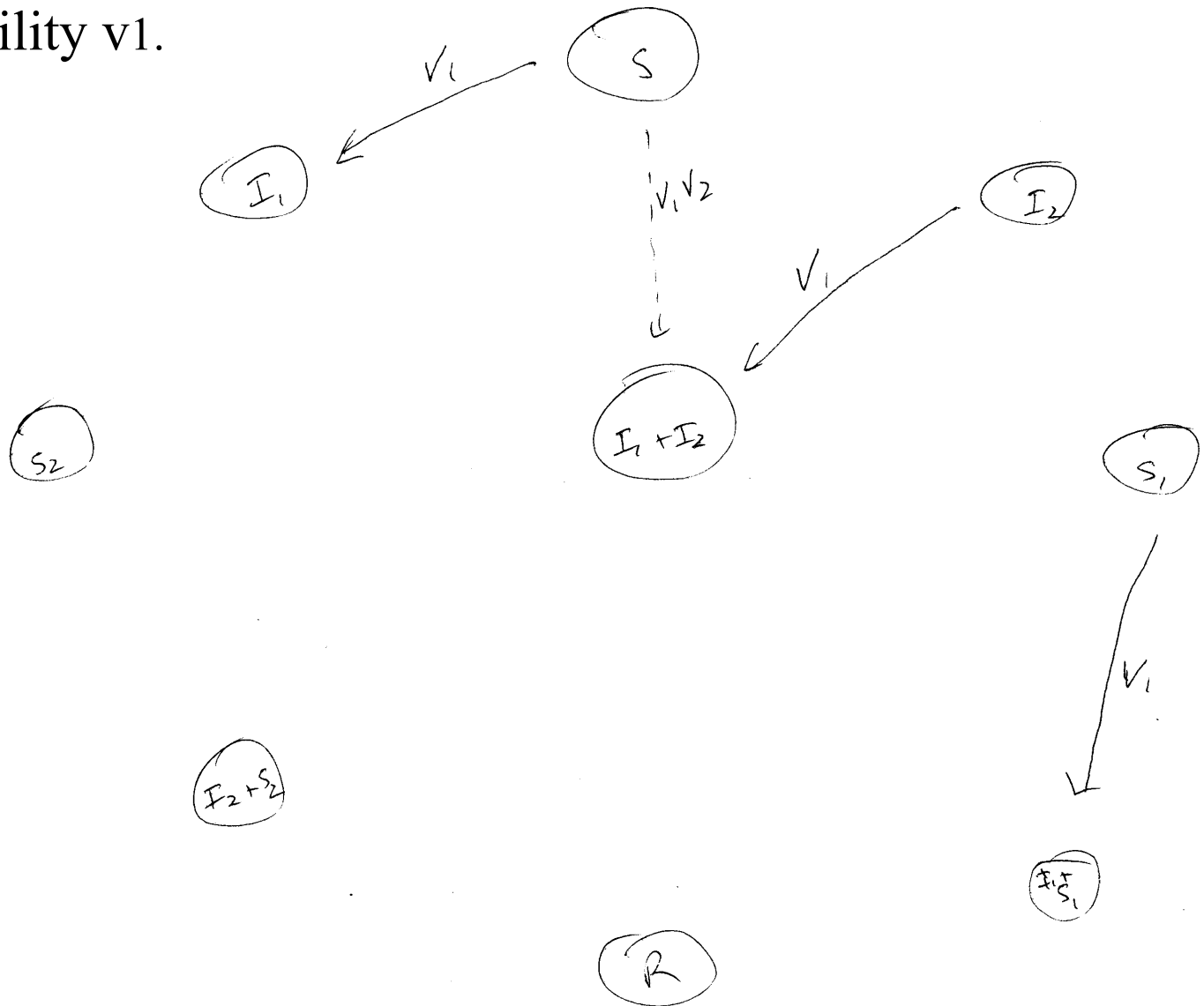
Process 1:

- The Healthy Stay Healthy
- The Sick Get Cured



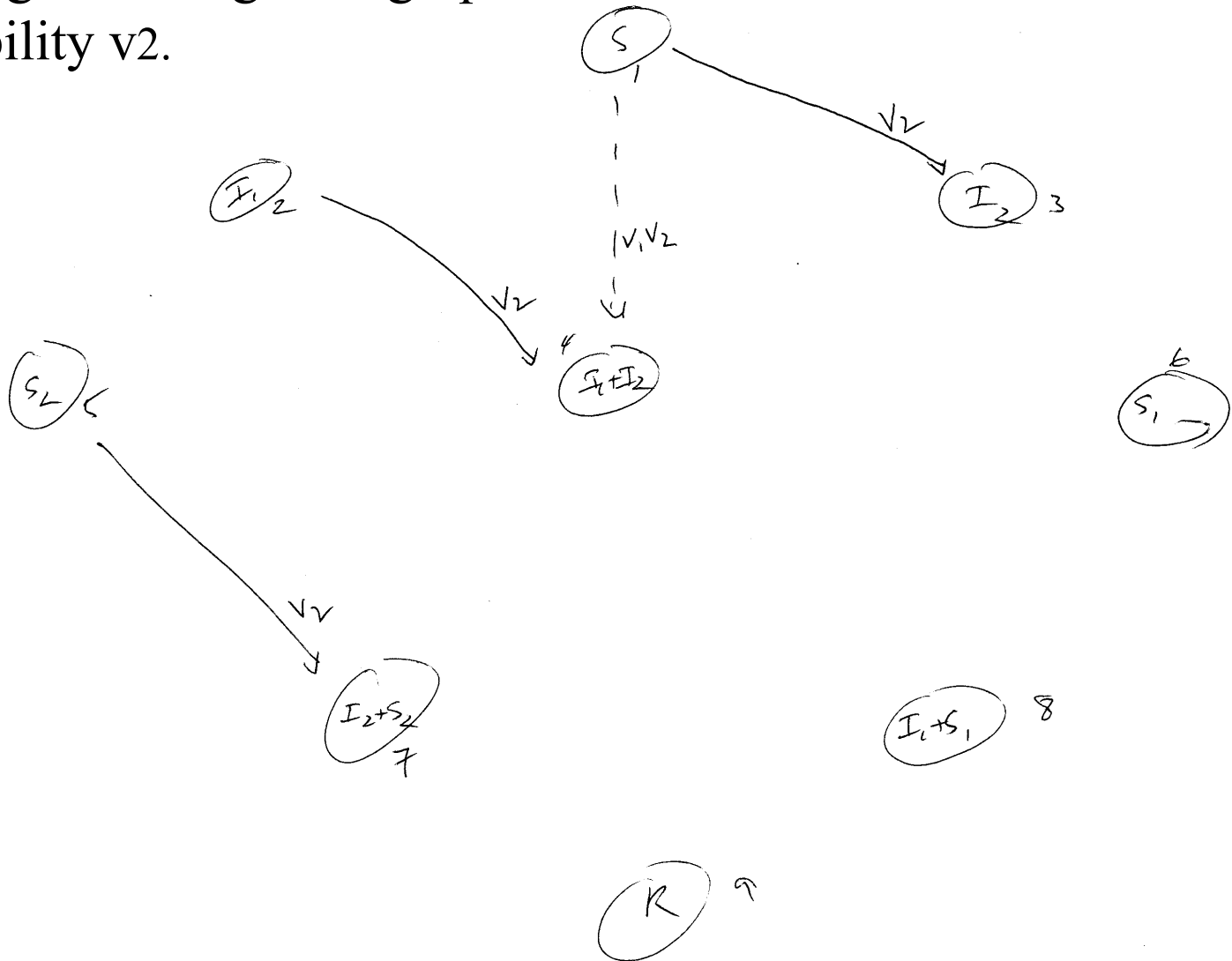
Process 2:

- Those infected with virus-1 (I1's) all neighbors in global graph G with probability v_1 .



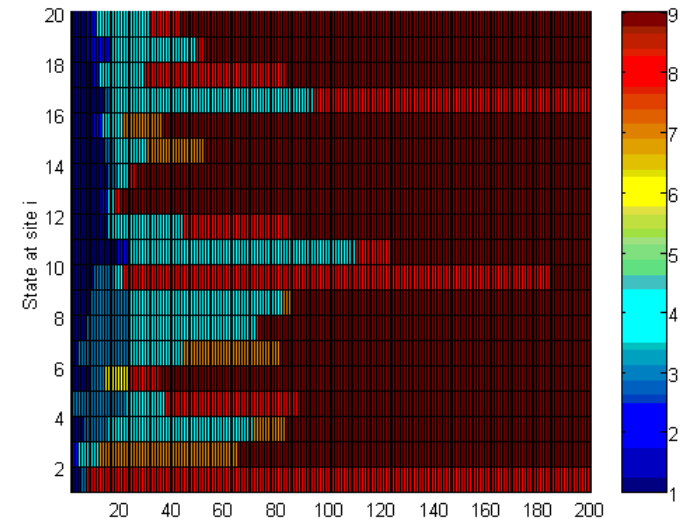
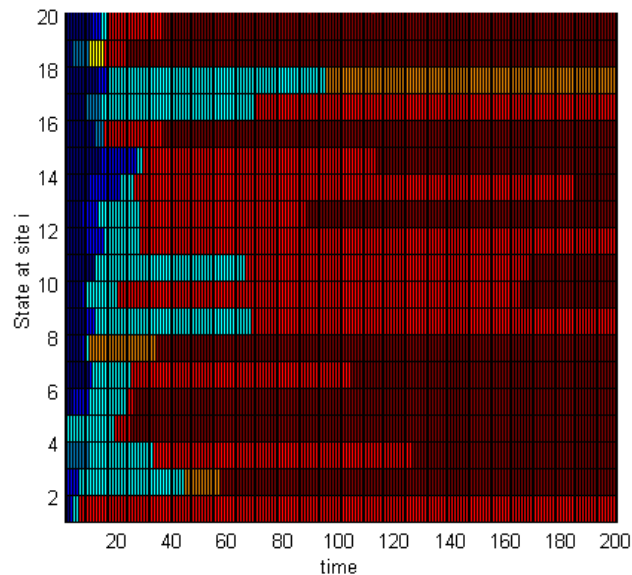
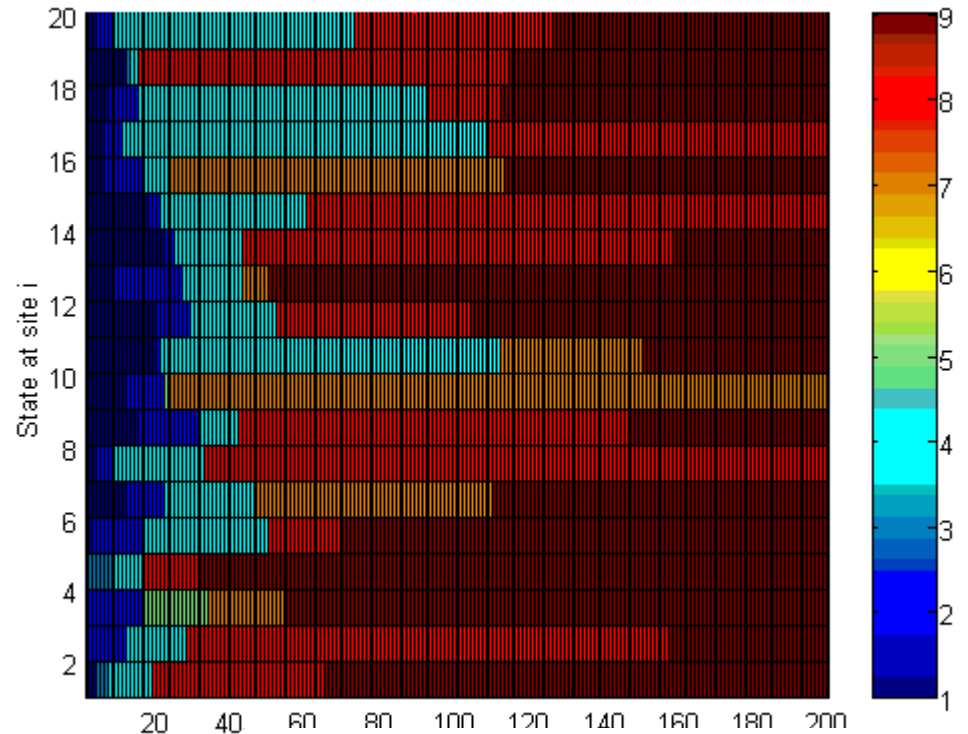
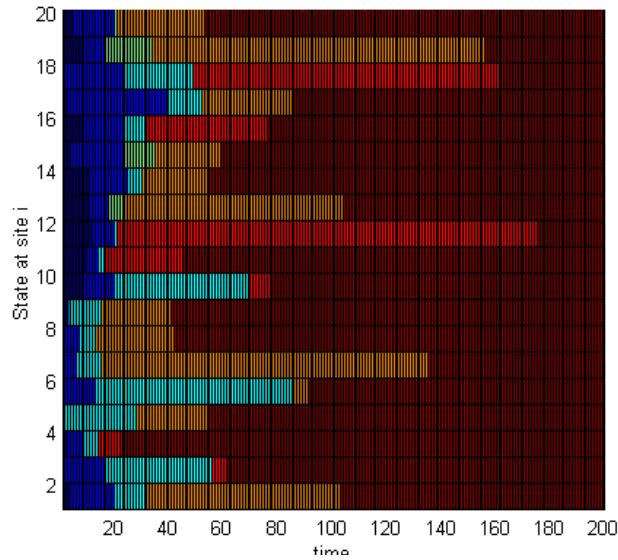
Process 3:

- Those infected with virus-2 (I_2 's) all neighbors in global graph G with probability v_2 .

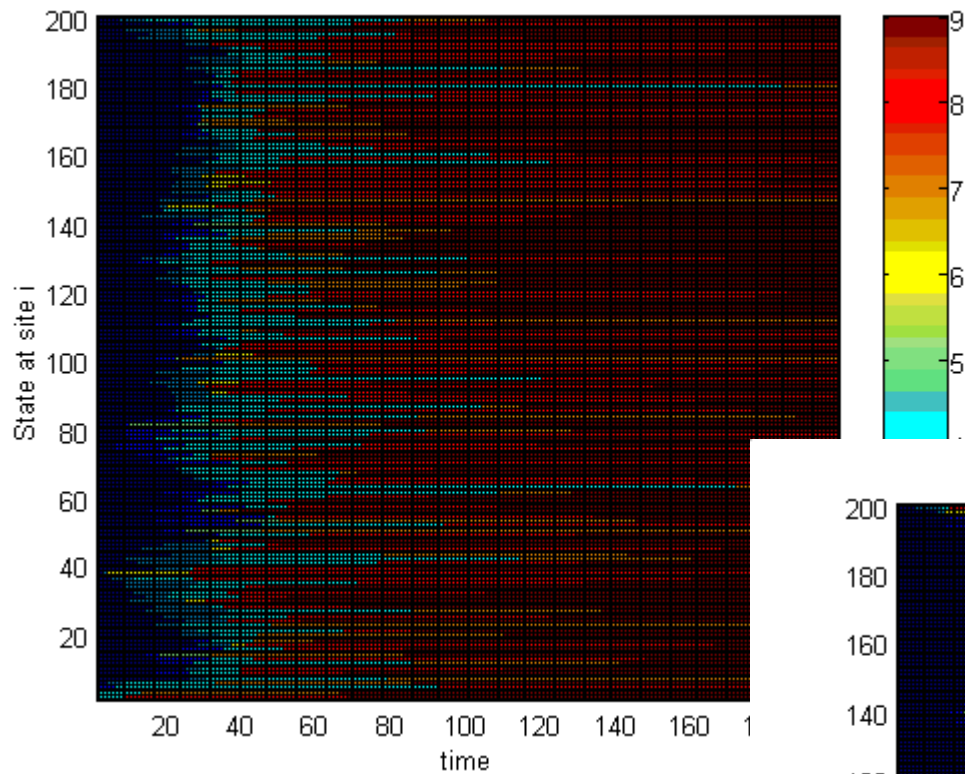


Some Simulations of Double Trouble in a Small World

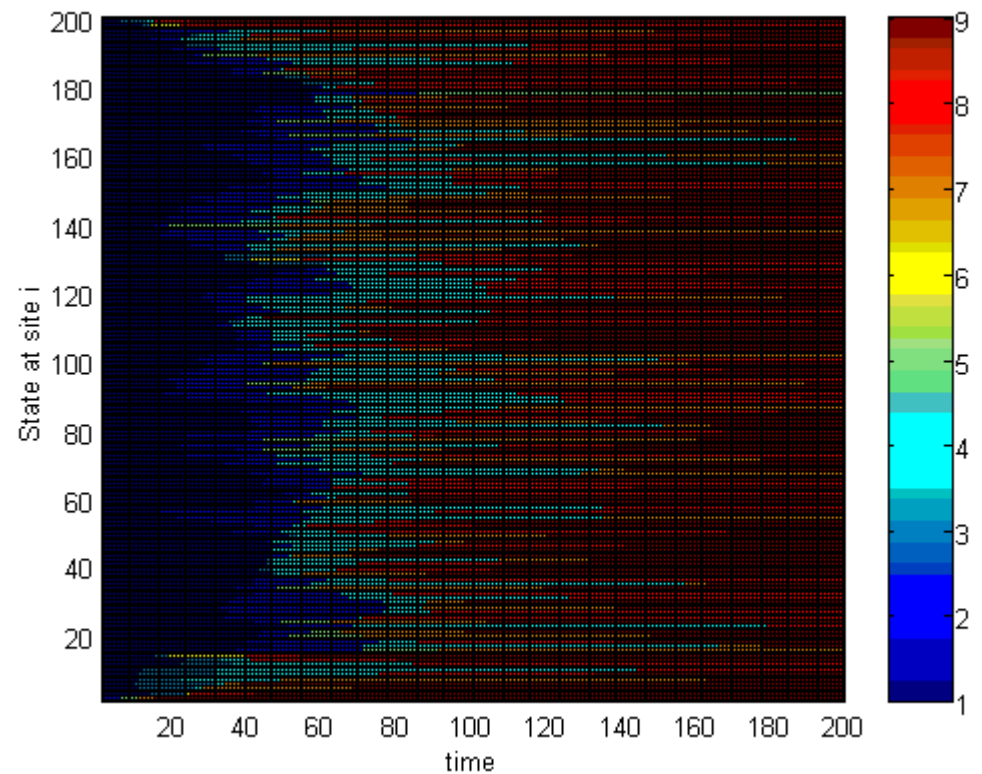
$N=20$, $k=3$, $\phi=0.075$



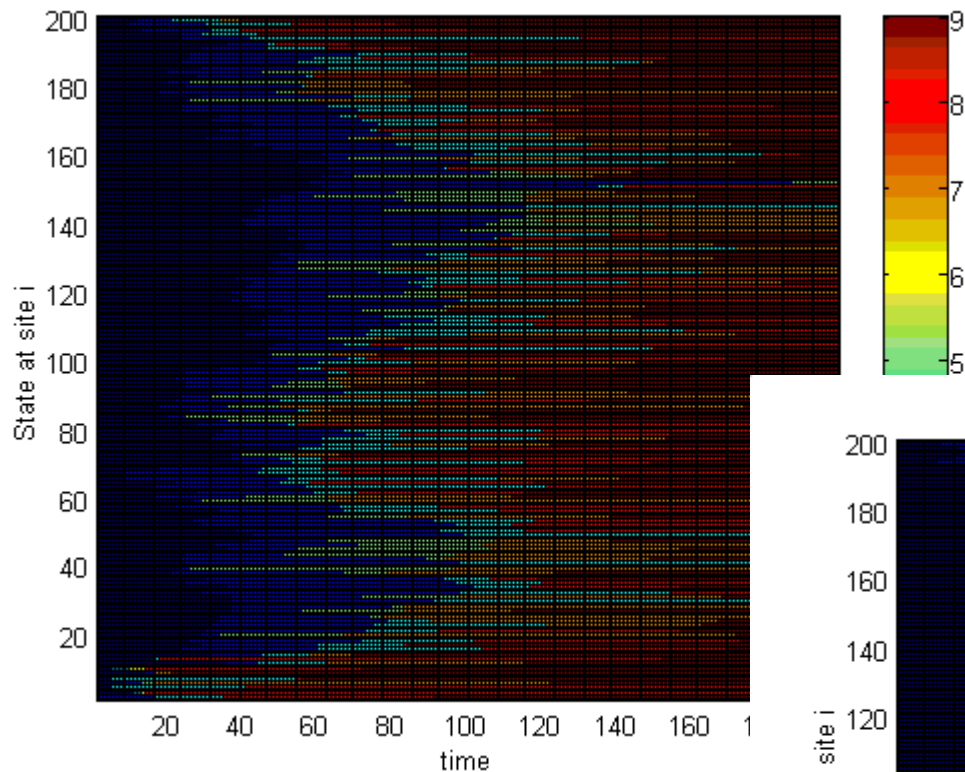
More Sims – Bigger Small World



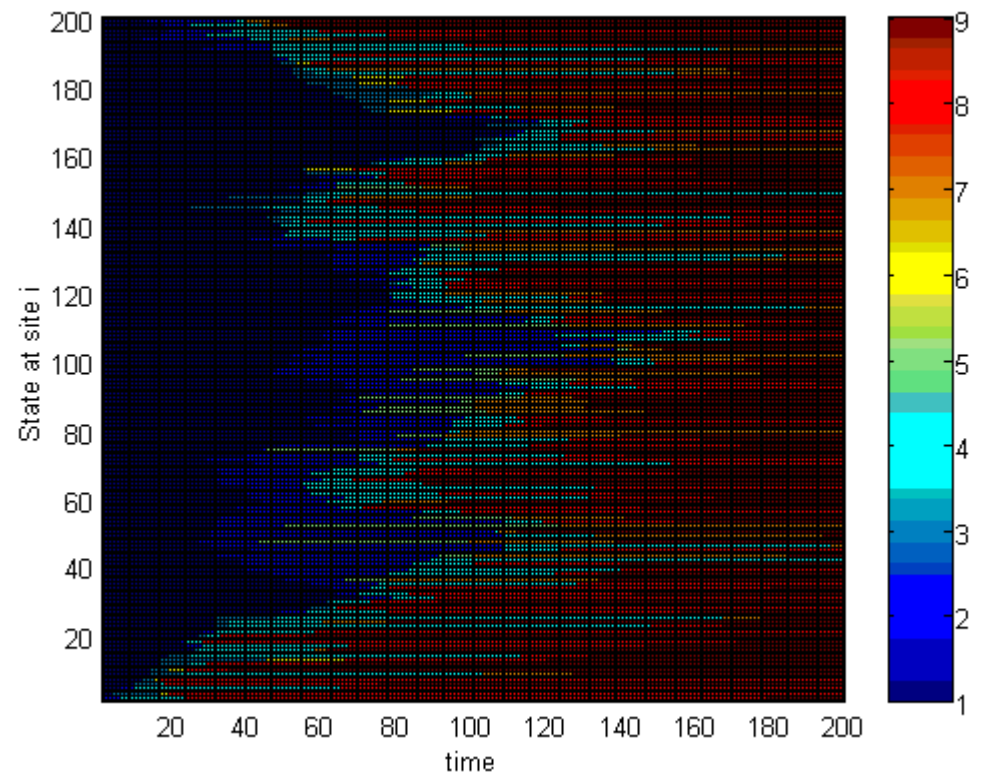
$N=20, k=3, \phi=0.075$



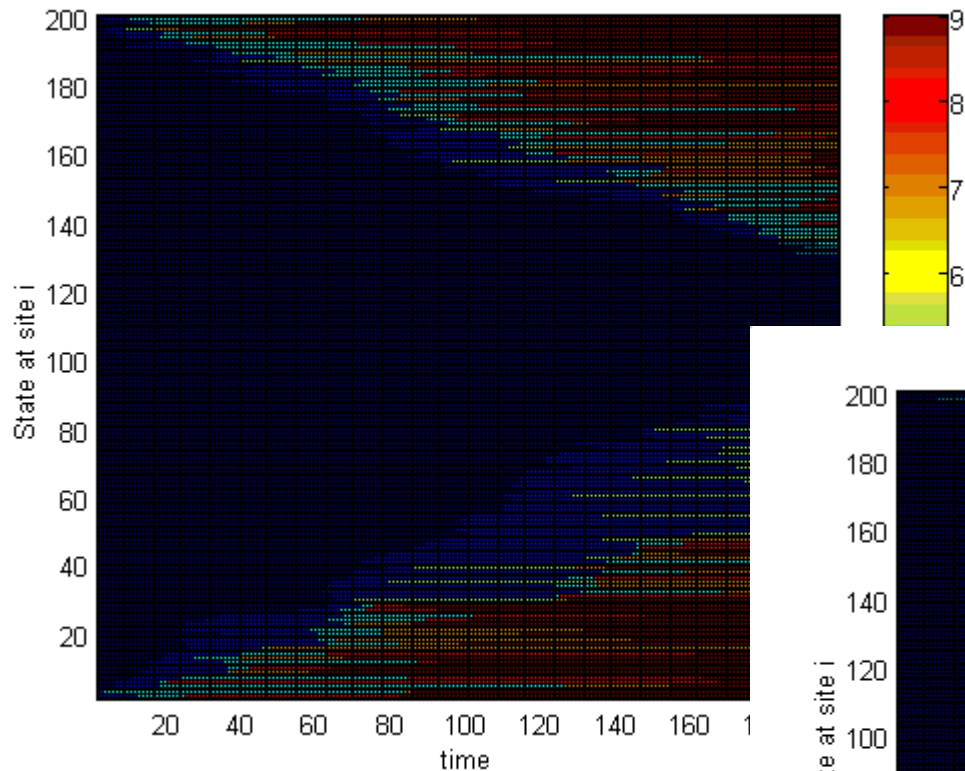
Small World With Fewer Long Range Connections



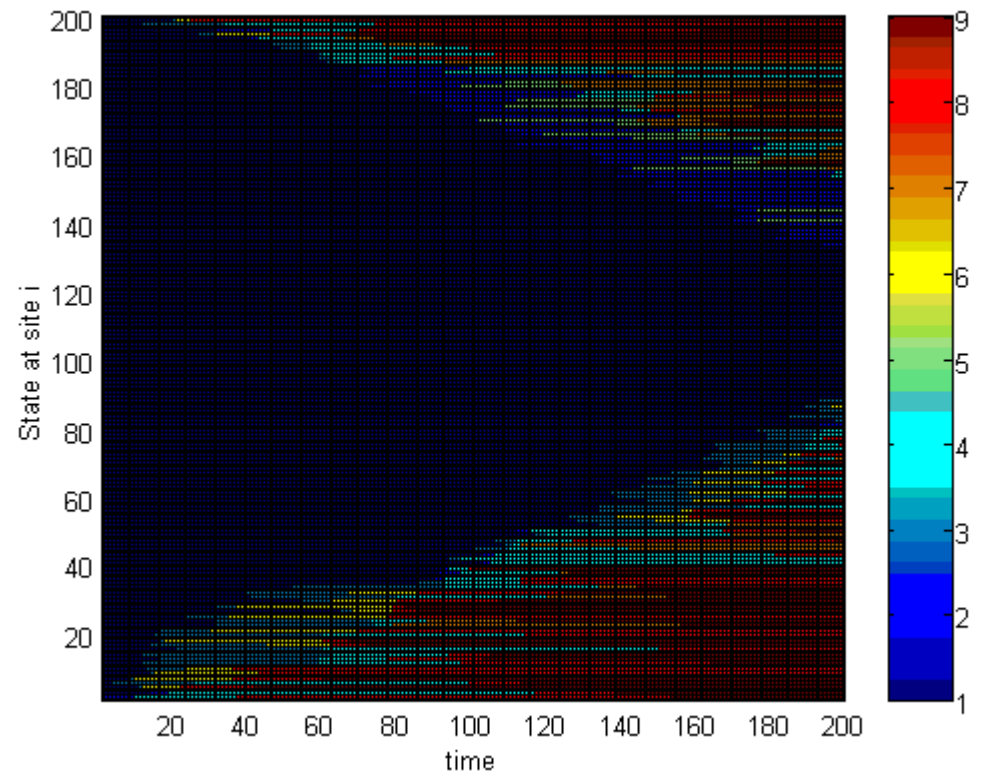
$$N=20, k=3, \phi=0.0075$$



Small World With **EVEN** Fewer Long Range Connections

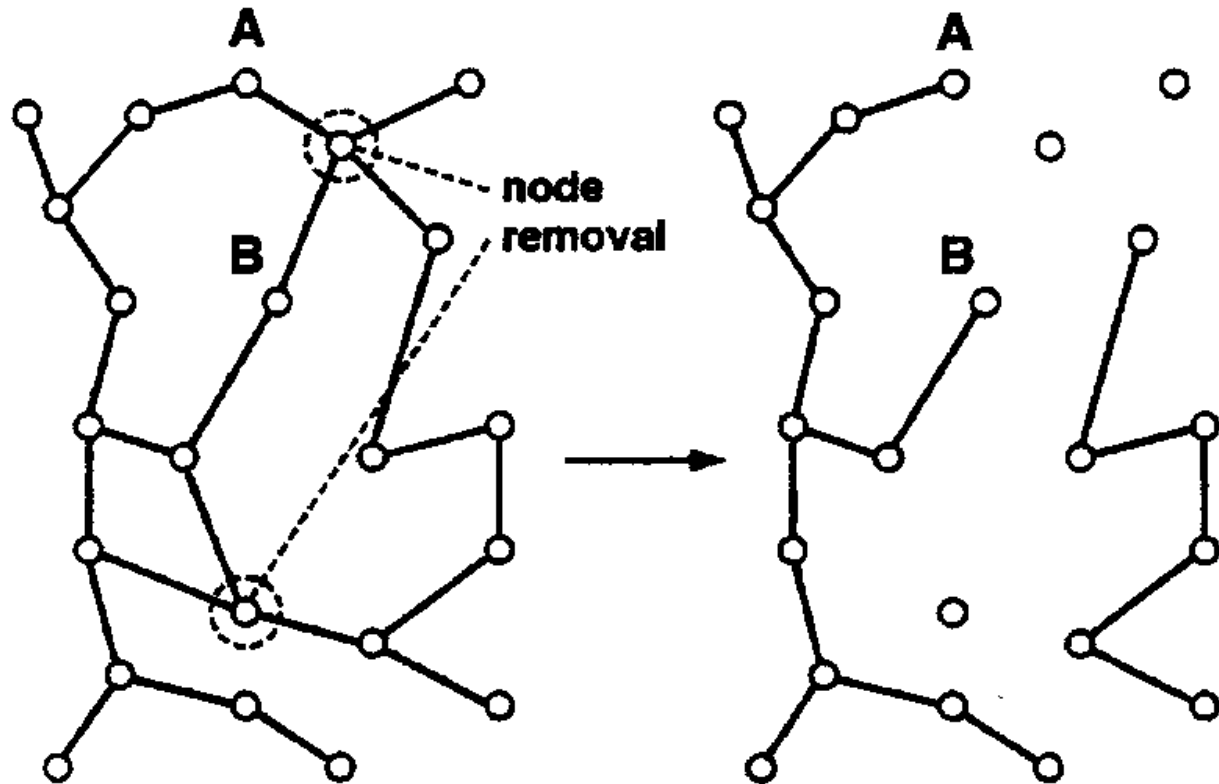


$$N=20, k=3, \phi=0.00075$$



Topology Questions

- Network Robustness
- Inverse Percolation and Topology – Avoiding this – Redundancy
- How does this play into Double Trouble?



- All-all becomes,
- $\text{Dist}(A,B)=2 \rightarrow 6$
 - 5 isolated clusters

Thinking about...

- Nonlinear couplings?
- What topology or network evolution model?
- How realistic is this model when used to study real world virus spread?
- How many "infection" states need to be modeled? Is a random directed graph a realistic model?
- Compare percolation graphs, small-world graphs, and other random graph models. What is the connectivity of the graphs?
- ***Parameter Estimation:*** What are the viral transmission and cure rates?
- Are assumptions of homogeneity and isotropy too restrictive?
- **Control!**

Before Control:

What do the bad guys end-up doing?

- Adversely adjust parameters.
- Knock out nodes-IP's
- Knock out links-vertices
 - Inverse percolation and network consequence.

Control Strategies for Scales.

- **Global in Mind.**
 - **Full Models.**
 - Kill/close Hubs? This with large k .
 - Inverse Percolation.
 - **Averaged models.**
 - Control bifurcation/adjust an accessible parameter.
 - Vaccination.
- **Local/Selfish in Mind.**
 - Local virus scanner
 - Turn off my laptop.
- **Pinpoint and Bulls-eye, lobe dynamics and partial barriers to transport.**

Conclusion

- Just a start.
- With Double Trouble Subgraph, Parameters, Weights and Topologies to vary.
- **Control.**

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