Population heterogeneity

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- What are the effects of homogeneity?

The implications of homogeneity

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- What conclusions can we reach if we assume that people are effectively the same for the purposes of transmitting a particular disease?

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- A disease can invade a population $\iff \mathcal{R}_0 > 1$.

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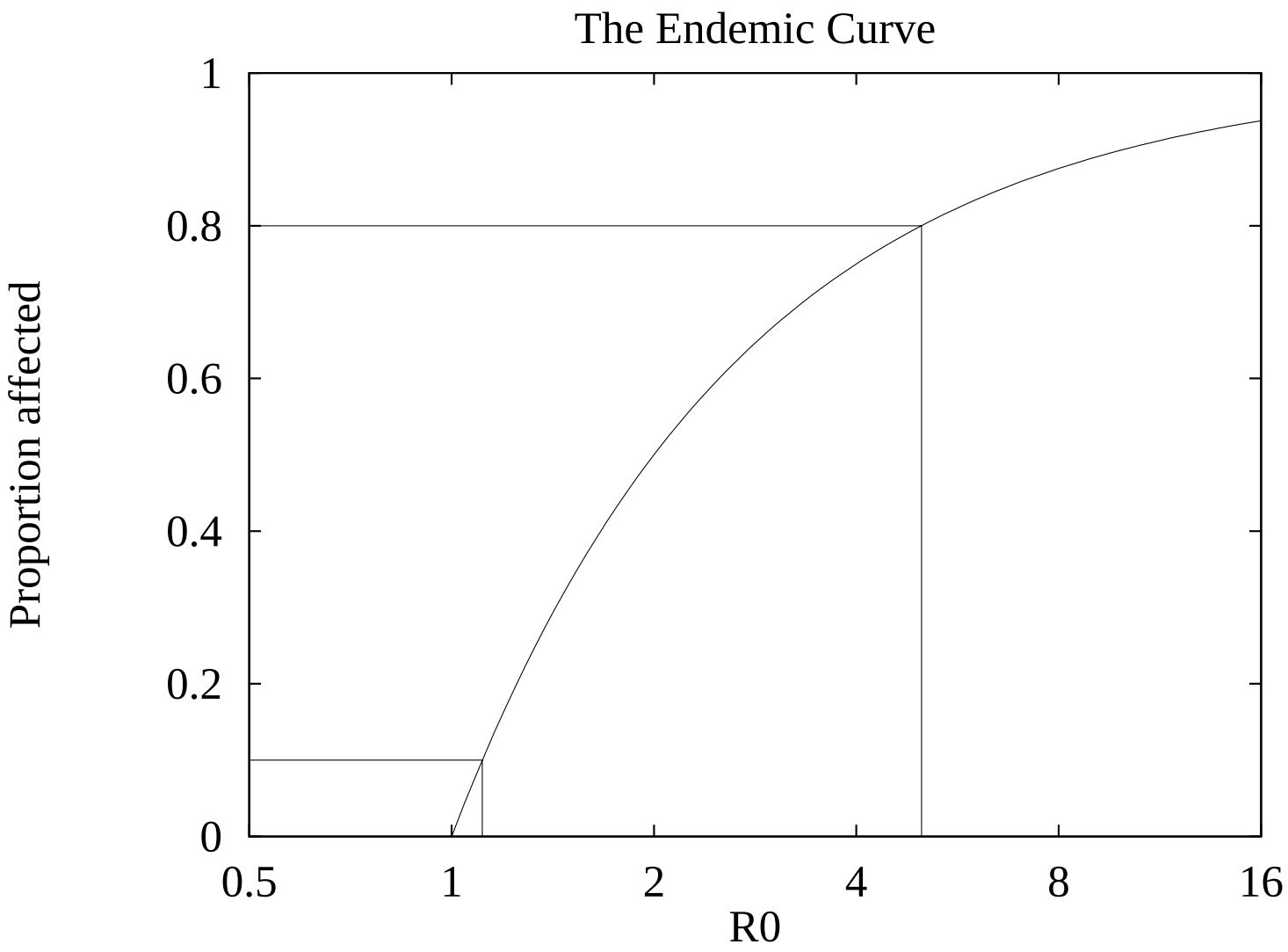
- Proportion 'affected' is $V = 1 - S/N = 1 - 1/R_0.$

The endemic curve



The effects of heterogeneity

Beyond the homogeneous model

- Heterogeneity among hosts

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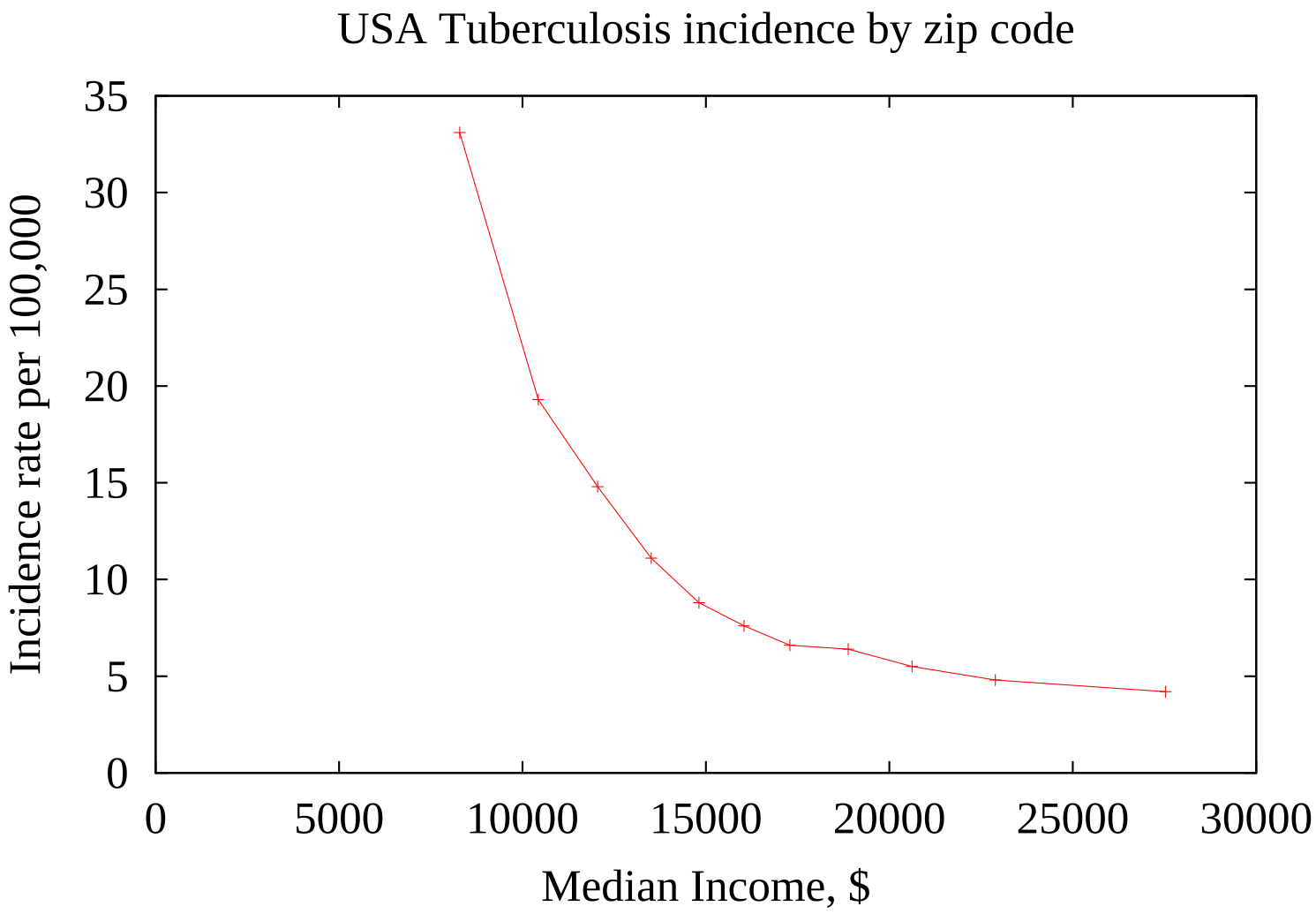
  - Why do we assume this is multiplicative?

Heterogeneity in TB



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- *Many others*

Parametric heterogeneity

Imagine an aspect space indexed by $a \in \mathcal{A}$. Each a is associated with a value for $\sigma(a)$, $\tau(a)$, $c(a)$ and $D(a)$. We can model heterogeneity explicitly, using boxes, or distributions, or seek approximations based on the moments of the parameters: e.g., means, variances and covariances across the aspect space.

$$\bar{c} = \int_{\mathcal{A}} c(a)N(a)da$$

$$\sigma_c^2 = \int_{\mathcal{A}} (c(a) - (\bar{c}))^2 N(a)da$$

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  - The endemic equilibrium is where a distribution of cases regenerates itself for the next generation.

Heterogeneity and $\mathcal{R}_0$

Proportional mixing

$$\mathcal{R}_0 = \frac{\langle c^2 \sigma \tau D \rangle_N}{\langle c \rangle_N}$$

- Heterogeneity in susceptibility, transmissibility or duration alone has no effect on $\mathcal{R}_0$ and thus on whether a disease persists in a population.

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- Correlations between other parameters can have a similar effect (e.g., if those who are more susceptible are on average also more able to transmit).

Heterogeneity and $\mathcal{R}_0$

Assortative mixing

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- Assortative mixing combined with any kind of heterogeneity will increase $\mathcal{R}_0$.

Heterogeneity and equilibrium

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- In a heterogeneous population, the infected population and the susceptible population generally both become less risky on average as the disease spreads!
  - Makes equilibrium disease level lower for a given risk
  - Conversely, risk may be higher than you think

Heterogeneity and equilibrium

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- $\mathcal{R}_0 = \langle c\sigma \rangle_N \frac{\langle c\tau D \rangle_{\hat{I}}}{\bar{c}}; \quad 1 = \langle c\sigma \rangle_{\tilde{S}} \frac{\langle c\tau D \rangle_{\tilde{I}}}{\bar{c}} \frac{S}{N}$

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  - Effect on equilibrium level of disease is mixed.

Homogeneous population

- Population heterogeneity

The implications of homogeneity

The effects of heterogeneity

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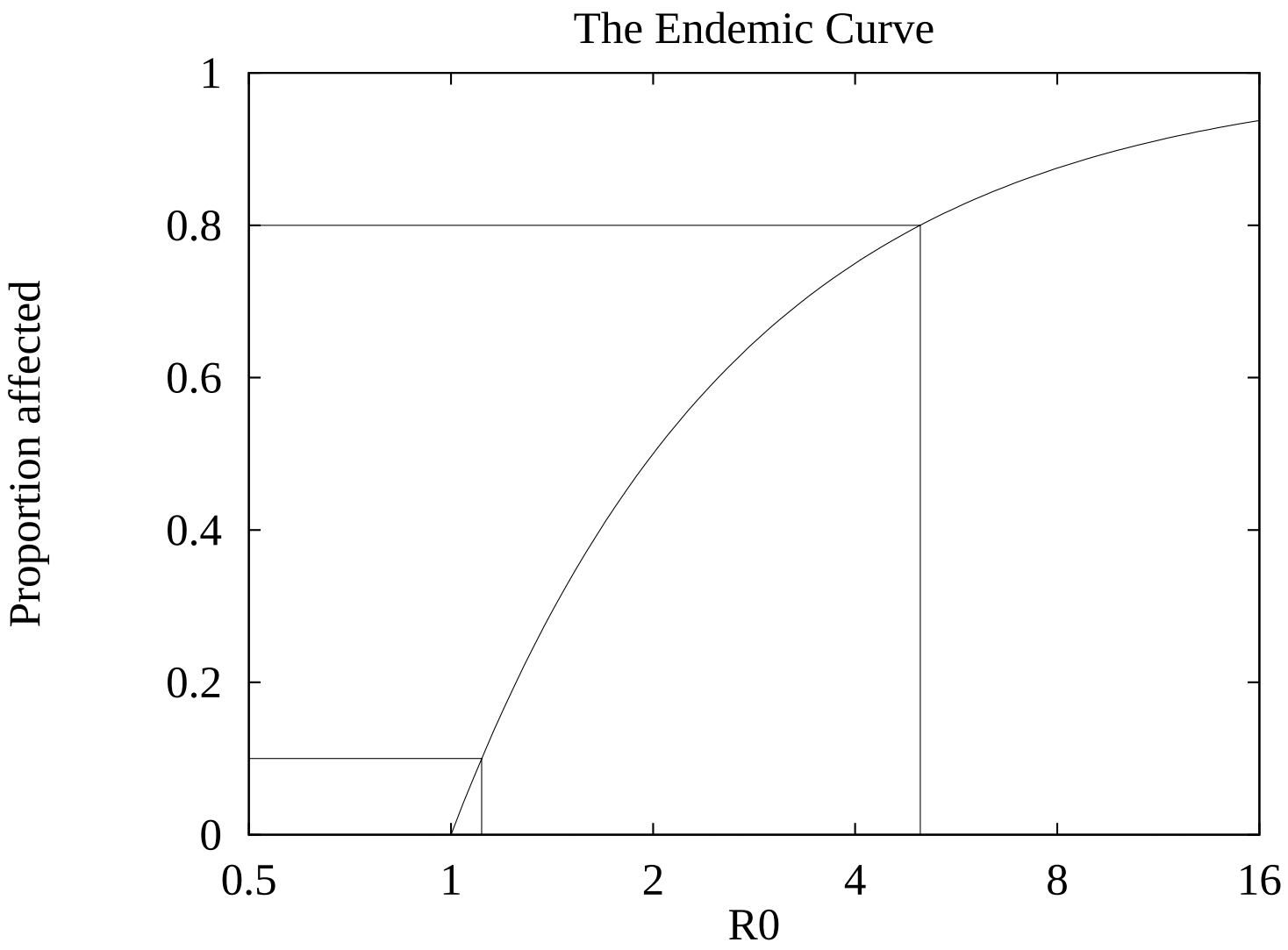
- Next-generation framework
- Heterogeneity and $\mathcal{R}_0$
- Heterogeneity and equilibrium



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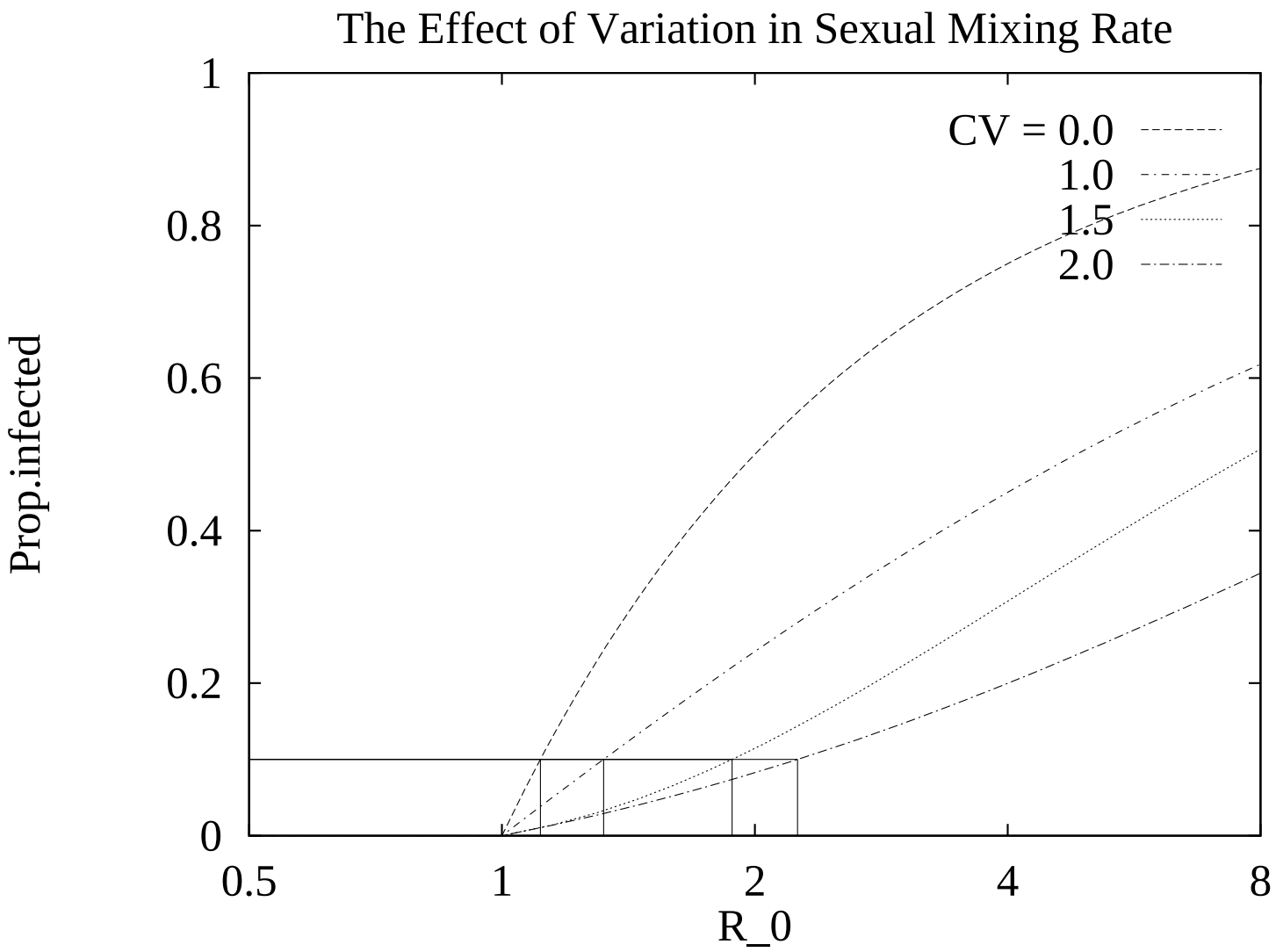

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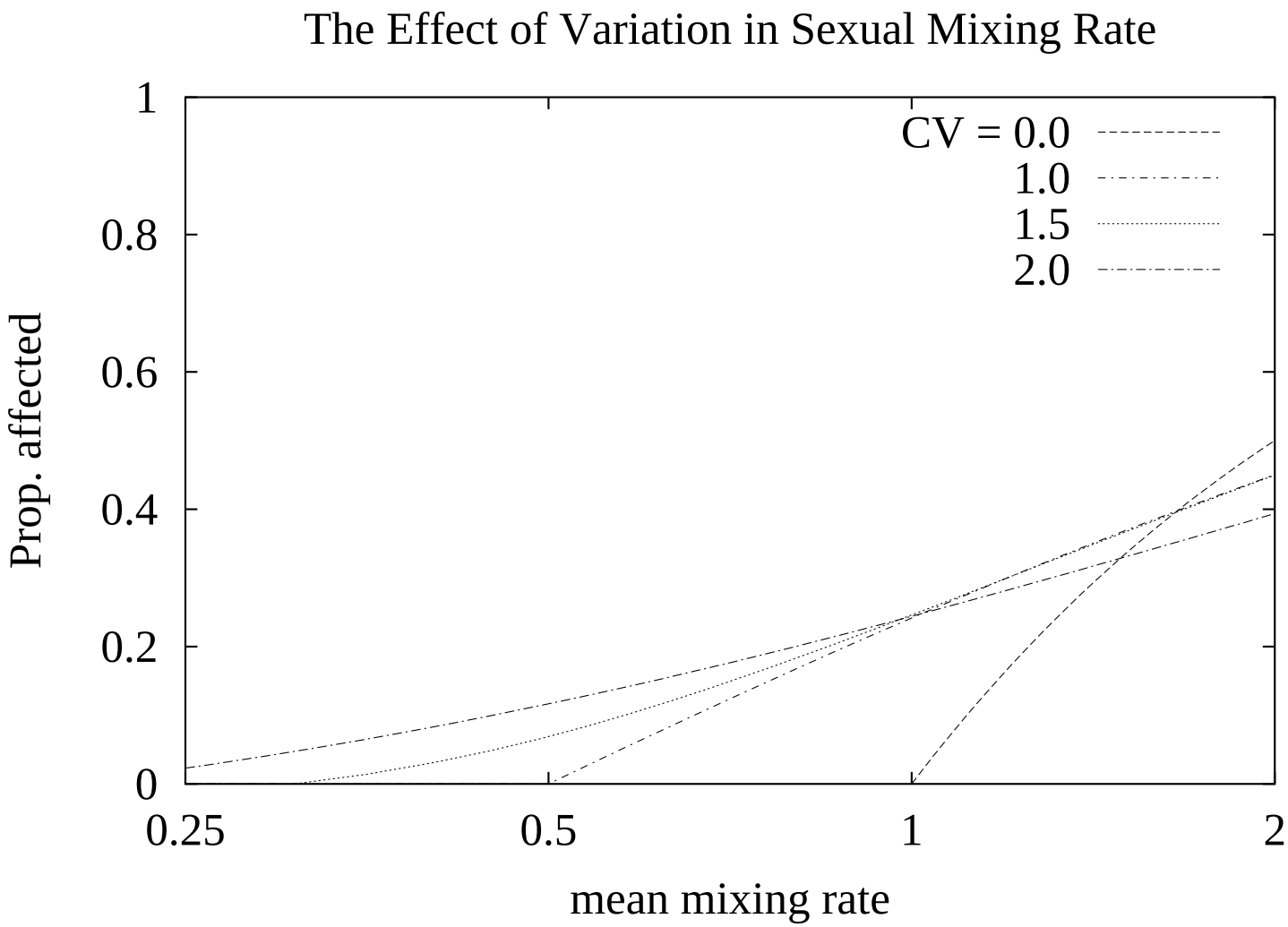
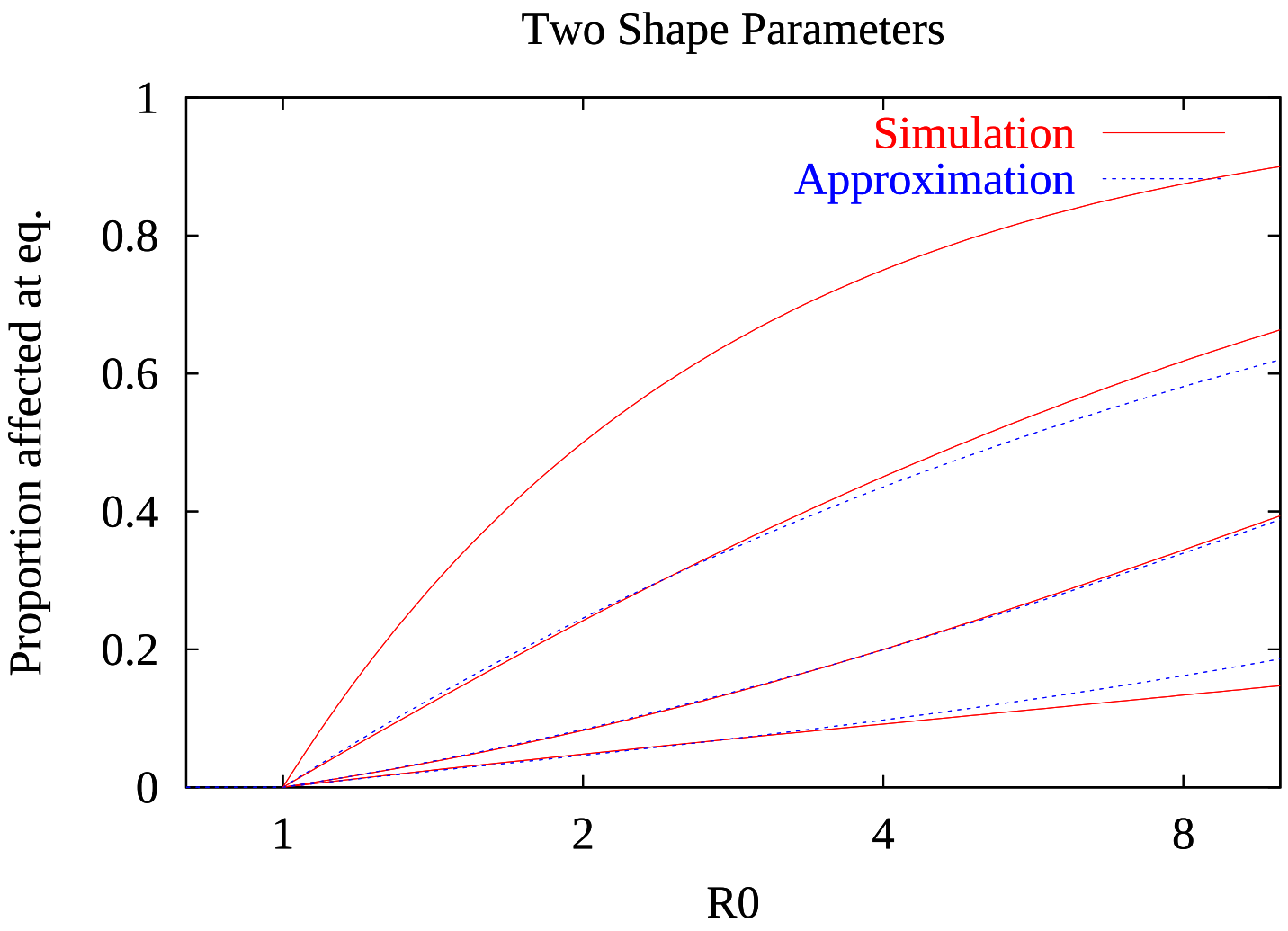


Approximations based on moments



Host heterogeneity summary

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- Disease-control strategies should take heterogeneity on a variety of scales into account.
- Simple moment-based approximations can help extend the usefulness of the endemic curve – relationships between risk factors and disease level – to heterogeneous populations.