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Recommended Resources for Teaching Discrete Mathematics

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1. Introduction

If you are teaching or planning to teach discrete mathematics, or if you are a mathematics supervisor assisting teachers in using discrete mathematics, you may be wondering what sorts of classroom materials are available. What books should you own or ask your school to buy for the library? Where can you get ideas for interesting student activities? Are there good videos or software? What have other teachers used successfully? This article addresses these questions, and provides recommendations for a selection of the best resources known to us. This is not a comprehensive listing, but rather a description of a core library recommended for anyone teaching discrete mathematics in grades K-12. Several of the resources mentioned here (especially videos or stand-alone activities) are also useful for convincing parents, school boards, or administrators of the value of teaching discrete mathematics

We have collected these recommendations from a number of sources. The most important of these are teachers and instructors from the Leadership Program in Discrete Mathematics (LP).[1] Both of us have been involved in the LP for over two years, Janice Kowalczyk as an alumna, lead teacher, workshop leader, and program evaluator; Deborah Franzblau as instructor and summer institute organizer. Kowalczyk maintains a resource list for the program, and edits a resource column, "The Discrete Reviewer," in the LP newsletter, *In Discrete Mathematics: Using Discrete Mathematics in the*

1991 *Mathematics Subject Classification.* Primary 00A35.

[1]This NSF-funded program has been in operation since 1989, beginning with high school teachers. Middle school and K-8 teachers have joined more recently. Further details on the program can be found in [**32**]. Suggestions for and comments on resources are gathered at alumni sharing sessions, through email to a group bulletin board, and individual contacts.

Classroom (see section 3), which Franzblau has edited. Several of the resources described here have been reviewed in the newsletter. Kowalczyk has taught mathematics in middle school. Franzblau has taught mathematics at Vassar College and is currently teaching at the College of Staten Island; she also edits the "Education Forum" of the *SIAM Activity Group on Discrete Mathematics Newsletter*.

Both of us believe strongly in activity-centered teaching methods which focus on student exploration and discovery first, and abstraction and precision later. Although we agree that hard work is essential for understanding, and drill and practice are useful at the right time, we believe that student motivation and problem-solving must always come first. Thus, our focus is on materials and activities which either provide motivation for learning or lead to interesting mathematical discoveries. For this type of teaching, it is essential that the teacher have sufficient background and confidence to recognize and help students articulate their discoveries, so we have included resources providing background and breadth for teachers.

We believe that teachers are the best judges of what is appropriate for the ages or grade-level of their students. In our experience, the best activities and ideas work at many levels, and skillful teachers can adapt them to the level of their own classroom. We do however give suggested grade-level ranges for each resource, where appropriate, based on comments from teachers who have used the materials.

An important resource to teachers at all grade levels is [**31**], which presents the discrete mathematics chapter from the *New Jersey Mathematics Curriculum Framework*. This chapter is the first comprehensive attempt to describe what activities and topics from discrete mathematics are appropriate at each grade level cluster from K-2 to 9-12. The materials in the *Framework* are also based on the experiences of LP teachers.

Many of the resources listed here were developed for grades 7-12 (or the college level). Until recently, there has not been much available material that is labeled "discrete mathematics" at the elementary level. Nevertheless discrete mathematics often appears in publications such as *Wonderful Ideas* or *The Elementary Mathematician* (see Section 3). Moreover, there are many activities and topics that can be adapted from materials written for a higher level, as well as some excellent children's literature (see Section 4) that can be used to to introduce these activities.

In addition, we recommend the following catalogs for browsing; a number of the resources mentioned here can be found in them. The first three are good sources for physical models ("manipulatives") and software. Addresses of the publishers are given in the Appendix.

- Creative Publications
- Cuisenaire
- Dale Seymour
- Key Curriculum Press
- Mimosa Publications (Primarily K-8)

- COMAP (*Consortium for Mathematics and its Applications*) (Primarily 9-college)

The resource descriptions are organized by category, as follows.

Section 2: **general textbooks** and curriculum materials, suitable for high school courses or as teacher resources;
Section 3: other sources for **activities** which can be integrated into new or existing courses;
Section 4: literature and periodicals that are recommended for **student reading**;
Section 5: **supplementary reference** works primarily for teachers;
Sections 6, 7, and 8: **videos**, **software**, and **World-Wide Web** (Internet) sites for supplementary activities and materials.

Each title is followed by a suggested grade-level range and the resource topic is given (in ***bold italics***) if it is not clear from the title or the context. Within each subsection, unless stated otherwise, resources are arranged by approximate grade-level.

For each resource, we list the publisher and/or a distributor (as of 1995/96). Print resources with no distributor listed can usually be found at large bookstores. The prices given are approximate retail cost, based on 1995 or 1996 catalogs or bookstore quotes (prices may vary a few dollars between different sources).

The article is accompanied by several appendices to assist the reader in locating resources:

Appendix A: addresses and contact information for most of the publishers and distributors;
Appendix B: an index of all resource titles (except those mentioned in passing), arranged alphabetically, by type;
Appendix C: an index of titles appropriate for each of the grade-level ranges K–2, 3–5, 6–8, and 9–12;
Appendix D: a list of major topics accompanied by recommended titles.

2. Discrete Mathematics Textbooks

When teachers name the texts they like best for teaching discrete mathematics, there are four titles that are mentioned over and over.[2] In roughly descending order of popularity, the texts are as follows: *Mathematics, a Human Endeavor*; *For All Practical Purposes*; *Excursions in Modern Mathematics*; and *Discrete Mathematics through Applications.* These have been

[2]Since separate courses on discrete mathematics don't make much sense at levels K-8, recommendations in this section are primarily from high school teachers; however, middle and elementary school teachers have enjoyed using these texts as resources. At the elementary level, the *Everyday Math Program* from the Everyday Learning Corp. (Janson), *Math Their Way* from Creative Publications, or materials from Mimosa Publications can be used to create a mathematics curriculum which incorporates many discrete concepts.

used not only as texts for discrete mathematics courses, but as supplementary reading for teachers who are introducing discrete topics in traditional courses. In the remainder of this section we give further details on content, describe how teachers have used the books, and give teachers' comments.

There are also several mathematics curriculum development projects at the high school level which contain many discrete topics. One is the NSF-funded Core-Plus Mathematics Project, which is discussed in [**15**]; these materials are being published under the title *Contemporary Mathematics in Context* by Everyday Learning Corporation. Another is the NSF-funded ARISE project, being developed by the Consortium for Mathematics and its Applications (COMAP); these materials are being published under the title *Mathematics: Modeling Our World* by South-Western Educational Publishing Company.

Mathematics, a Human Endeavor *(7–12)*
Harold Jacobs; W.H. Freeman, 3rd Ed., 1994; $49.

Mathematics, A Human Endeavor was clearly a textbook ahead of its time. After going through its third revision in 1994, it is as popular today as a classroom text and a teacher resource as it was back in the late 60s. It is a recommended favorite of teachers in the Leadership Program in Discrete Mathematics. Much of the mathematics is connected to real world applications, and there are many science and mathematics connections. The book is jammed with problem-solving activities and "What if ...?" questions, and emphasizes mathematical thinking. It is also full of photos, drawings (e.g., from Escher), and mathematical cartoons (e.g., from *Peanuts*, *BC*, and *The New Yorker*). The following background is adapted from information provided to us by the publisher:

> In the late 1960s, Harold Jacobs, teaching at Grant High School in Southern California, began exploring various mathematical applications for use in the classroom. He decided to write a textbook, something original, for those students who had not done well in mathematics. Neither the author or the publisher anticipated the response of instructors to the book when it was first published. Much to his surprise, Harold Jacobs found that he had taken the mathematical community by storm. And the author is still teaching students and finding ways to introduce them to the beauty of mathematics—to motivate them to see beyond its apparent difficulty. (W. H. Freeman, private communication.)

Content. Technically, this is not a "discrete mathematics" text, but many of the topics included are in fact discrete. The book has an eclectic list of topics, including the following.

- inductive and deductive reasoning;
- patterns and sequences (e.g., Fibonacci numbers);

- symmetry and regular polygons;
- combinations, permutations, and Pascal's triangle;
- probability and statistics;
- logic and puzzles;
- network (graph) problems—including Euler paths and trees.

Classroom Use. This book is most often used as a supplementary text or resource for bright 7-10th grade students, in a variety of courses, including mainstream 7th- or 8th-grade mathematics. (One 8th-grade teacher's students requested that she use the book all the time instead of their regular text.) One teacher used it in an Applied Mathematics course, which also included computing. Another used it for students off the "calculus track"; the course proved to be popular enough that "on-track" students took it as well. Other teachers have used it as a resource for themselves.

Comments. This book invariably draws enthusiastic comments from teachers. One comment was especially strong:

> I've been using this book for over 20 years. I've used it for a senior elective for college-bound students who have had a rough time with the traditional SAT curriculum. I've used several topics in a general math class (inductive reasoning, sequences, combinatorics, elementary probability). I've used activities from it in traditional courses (functions, conics, logs). No matter what school I teach in, I make sure that I have a classroom set of this book. (Marilyn Goldfarb LP '93), private communication.)

The following are comments from other teachers who have used this text.

- A wonderful all-around reference. Perfect for self-teaching. Good combinatorics section.
- This is my favorite math text book.
- A "must" for every library.
- It was written with "people who don't like math" in mind. It has a nice tone, develops the topics nicely, and goes into some depth.
- The supplemental transparency package is a definite "keeper." The material in it is excellent.

For All Practical Purposes (FAPP) (*9–College*)
Solomon Garfunkel (Project Director), et al.; W.H. Freeman (for COMAP), 3rd Ed., 1994 (4th Ed., 1997); $45.

Although written for a college-level audience, the material in this book has been used successfully at many grade levels. It is a great reference that is fun to read; everyone should have this book on their shelf. It has a wealth of contemporary topics, and each edition has added new topics. Of special interest are *Spotlights*—profiles of mathematicians or computer scientists who have contributed substantially to the solutions of problems mentioned in the book. There is an accompanying series of 26 half-hour video tapes which give overviews and applications of the topics; text supplements and an instructor's guide are available.

Content. This book is packed with topics and problems; there is enough material for a year-long course on the uses of mathematics. Many different semester courses could also be designed around this book. The third edition has five parts, as follows:

1. *Management Science*: graph models and problems (Eulerian paths, Traveling Salesman Problem, spanning trees, and scheduling), linear programming;
2. *Statistics/Data*: data collection and representation, probability and statistics;
3. *Coding Information*: identification and error-correcting codes;
4. *Social Choice*: voting, fair division, apportionment, game theory;
5. *Size and Shape*: growth models, measurement, astronomy, fractals, symmetry, tilings.

The focus is on real-world problems and models. Note that a number of standard discrete mathematics topics are not covered, such as shortest path problems, graph coloring, and permutations, and some of the mathematics used is continuous rather than discrete.

Classroom Use. This book was intended for introductory college courses, but teachers have reported using it successfully with a wide range of younger students, from gifted 7th- and 8th-graders in an enrichment program, to 11th- and 12th-graders (including non-college-bound students). Most high school teachers find the reading level too high for their students; instead, they use it as a resource for themselves, adapting the problems for their classes. Specific examples of the use of the text are given in this volume [**17, 34**].

Comments. The following is an excerpt from a review of the first edition (1988), by Anthony Piccolino, on the mathematics faculty at Montclair State University, who used the book for three years in teaching an elective mathematics course to high school seniors [**28**].

> [This is] a textbook which addresses real-life situations, emphasizes mathematical modeling, encourages students to make mathematical connections, and devotes extraordinary efforts to changing students' narrow view of mathematics. ...
>
> I found this book to be most effective when used in conjunction with the 26-program video series of the same name. The animation and the delivery style are wonderfully motivating and give students an excellent sense of the "big picture" for each chapter before having students delve into various detailed aspects of the chapter. ...
>
> I recommend this book with great enthusiasm!

Joseph Malkevitch, on the mathematics faculty of York College of CUNY, and one of the authors of the section on Management Science, believes the text has played a key role in the evolution of college "mathematics-for-liberal-arts" courses [**24**]. He wrote,

> [Its] approach underlines the importance for students of analyzing and understanding real-world situations and building mathematical models rather than gaining facility with solving exercises based on the models.
>
> [Texts like this can] show how the ideas being developed in discrete mathematics are essential for emerging new technologies such as robotics, computer vision, data compression, and medical imaging.

Excursions in Modern Mathematics (*9–College*)
Peter Tannenbaum and Robert Arnold; Prentice-Hall, 2nd Ed., 1995; $57.

This book, first published in 1992, was evidently inspired by *For All Practical Purposes* (FAPP), and, like FAPP, is intended for an introductory college-level course. The title conveys its goal of providing interesting "trips" through the realm of contemporary mathematics. The style is less cluttered than FAPP, and many find that it makes a better textbook than FAPP (which is a better reference, however). Many high school teachers use it as a resource for themselves, finding the reading level too high for their students (as with FAPP). The publisher also offers a supplement of *New York Times* articles connected to the text topics. The following description and comments are based on the first (1992) edition.

Content. *Excursions* covers a subset of the topics of FAPP (there is no section on codes, and fewer topics in each section), and introduces topics in a different order. The topics covered are divided into four parts, as follows:

1. *Social Choice*: voting theory, fair division, apportionment;
2. *Management Science*: Eulerian tours, traveling salesman problem, minimum-cost spanning trees, scheduling;
3. *Growth and Symmetry*: Fibonacci numbers and sequences, population growth, transformation geometry, fractals;
4. *Statistics*: collecting and describing data, probability, and the normal distribution.

Classroom Use. Teachers have reported using this as a text for a discrete mathematics elective at the 12th-grade level, and as a resource or supplement for discrete mathematics courses for grades 9–10 and 10–11. It has also been used to introduce discrete topics in calculus at grade 12.

Comments. One teacher, who used the book as supplementary reading and as a resource for herself, felt it was the most useful of the four books in terms of the topic coverage and quantity of material. She found the material on fair division and voting very strong, but was disappointed that graph coloring and codes were not included.

The following comments are taken from a review [**2**] by high school teacher and mathematics supervisor, Ethel Breuche LP '91, who used the book as a resource.

> I feel that this text is wonderfully *rich.* It is rich with examples and simple and thorough explanations. It is rich in discussion and explorations. Most of all it is rich in exercises at the end of each chapter ..., which increase in level of difficulty and creative problem solving.
>
> Some chapters are followed by an additional appendix. ... For example, in the voting theory section, the voting scheme for the nominations for the Academy Awards is described in detail. Every chapter offers references for further research and readings. The text is written with deliberate thoughtfulness with regard to racial and gender equity. ...
>
> Whether using the book as a resource or as a classroom text, I cannot praise it enough. Although written for a college-level liberal-arts math course, [college-bound high-school] juniors and seniors whose reading ability is average or above can use this text as well.

Discrete Mathematics Through Applications (*7–12*)
Nancy Crisler, Patience Fisher, and Gary Froelich;
W. H. Freeman (for COMAP), 1994; $36.

This book was developed to fill the need for a high-school level discrete mathematics text [**8**]. It is addressed to the student, and is less sophisticated mathematically than either FAPP or *Excursions.* Teachers of students in grades 7–9 (or average students in grades 10–12) have been very pleased with it; but teachers of more advanced students have found it less useful. Overall, the comments on this book have been more mixed than those on the previous three. However, the book can provide a good introduction for a teacher who has not seen discrete mathematics previously.

Content. The book includes the following chapters:

1. Election Theory;
2. Fair Division;
3. Matrix operations and applications;
4. Graphs and applications;
5. Recursion.

Each chapter begins with a group exploration and a set of exercises to introduce the topic. This is followed by short lessons and exercises. In another article in this volume [**8**], the authors give more detail on the content and development of the text.

Classroom Use. One teacher used it as a text for a course called "Math in the Real World", for students at the 11th- and 12th-grade level with poor mathematics backgrounds. Another used it as a text for a discrete mathematics course for grades 9–10, while another used a draft version for an 11th-grade discrete mathematics course.

Comments. A teacher who used it as a text, who felt that FAPP and *Excursions* were too abstract and notation-heavy for her students, said:

> It's an excellent book. It's very concrete, interesting, and to the point. Starting with the first chapter, where there's an election activity on soda preferences, there are many good activities that involve the students. The exercises are conducive to group work, and range from very basic to challenging. (Diane DePriest LP '93, private communication.)

She supplemented the text with logic puzzles, and material on permutations and combinations which she created.

A teacher who chose *not* to use the book for a 12th-grade course felt that it "looked too easy," and feared it could have a "bad effect on the reputation of the course." A college faculty member was disappointed that the book "does not point out where the mathematical structure is," and does not do more "summing up" of the mathematics topics learned.

On the other hand, an elementary school teacher advises teachers of elementary or middle grades to use the text:

> It lends itself best of all the texts to adapting ideas to the elementary and middle grades. Also, the instructor's manual is user-friendly for teachers who are not familiar with discrete math, and would otherwise be afraid to try it. I also encourage them to order the COMAP modules on graphs.[3] (Penni Ross LP '94, private communication.)

3. Sources for Student Activities

This section contains recommended newsletters and books which provide activities on a diverse collection of topics, as well as modules or books providing resources on special topics. The resources within each subsection are listed in order of suggested grade levels.

3.1. Newsletters and General Collections.

Math Their Way (*Book, K–3*)
Mary Baretta-Lorton; Addison-Wesley, 1976; $39 (available from Creative Publications); blackline masters are included.

An old favorite. This book contains an activity-centered mathematics program which focuses on patterns. It can be used as a self-contained curriculum or for enrichment.

Wonderful Ideas (*Newsletter, K–6*)
Wonderful Ideas, 8/yr; $26/yr (individuals), $38/yr (schools).

[3]These modules are *Drawing Pictures with One Line*, *The Mathematician's Coloring Book*, and *Problem Solving with Graphs*, described in Section 3.

This newsletter provides interesting elementary-level activities, often developed around a single theme. Sample topics include the "Monty Hall" problem (probability) and making a pinwheel (geometry and origami).

The Elementary Mathematician (*Newsletter, K–6*)
COMAP, 4/yr; only available with $16/yr membership (which includes other publications and discounts).

This is another good source of classroom activities, many of which integrate mathematics with science, health, history, and other areas. The pull-out section in each issue contains a complete classroom-ready lesson. Also available is **The Elementary Mathematician Pull-out Book** (Laurie Aragon, Ed.; $9), a collection of pull-out lessons from past newsletters.

INsides, OUTsides, LOOPS and LINES (*Book, K–8*)
Herbert Kohl, W.H. Freeman, 1995; $13.

One of us (Kowalczyk) recently discovered this excellent book, which introduces concepts from discrete mathematics and topology.

> If I had to recommend one resource book to elementary and middle grade teachers in discrete mathematics this would be it. ... [T]his book provides playful introductory activities [based on concepts in graph theory and topology]. The five chapter titles give you the best flavor for its contents: *Lost in the Garden*—simple closed curves; *Map Coloring*—figuring out the rules; *Tracings*—simple beginnings lead to complicated patterns; *Stretching, Bending and Twisting*—a new way to look at shapes; and *Mobius Strips*—some thoughts on doing mathematics with a twist. [**21**]

Math on the Wall (*Looseleaf collection, 1–6*)
Linda Holden; Creative Publications, 1987; three sets are available, for grades 1–2, 3–4, and 5–6, $8.50/set.

This is a collection of problems and puzzles designed to decorate a classroom bulletin board. Blackline masters for cut-out designs are included. It has lots of combinatorial puzzles, posing such problems as "How many different bouquets can you make with these flowers?" (This includes pictures of flowers and vases to cut out and color).

Discrete Mathematics Across the Curriculum, K–12
(*Collected Articles*) NCTM, 1991; see [**19**].

This edited collection is one of the NCTM *Yearbook* series, written to supplement the Discrete Mathematics Standard in the 1989 NCTM *Standards* [**27**]. Most of the articles focus on a single topic, such as graphs or recursion, illustrated with activities suggested for specific grade levels.

Teaching Children Mathematics (*Journal, K-4*)
Mathematics Teaching in the Middle School (*Journal, 5-8*)
Mathematics Teacher (*Journal, 9-12*)
National Council of Teachers of Mathematics, available with membership (which includes other publications and discounts).

These journals often publish articles or include pullout sections with discrete mathematics activities. The newsletters of local mathematics teachers associations may also publish useful activities.

In Discrete Mathematics: Using Discrete Mathematics in the Classroom (*Newsletter, K–12*)
DIMACS DM-Newsletter, 2/yr; for a subscription (no cost — supported by DIMACS), write to DIMACS-DM Newsletter (address in Appendix A).

This newsletter features articles by teachers (primarily participants in the Leadership Program in Discrete Mathematics) describing their experiences in creating and/or using discrete mathematics topics in the classroom. There are articles on special topics, such as elections or game theory, along with bibliographies for background reading, as well as resource recommendations. Puzzles and drawings by readers (or their students) also appear. The newsletter was previously edited by Joseph Rosenstein and Franzblau (and currently by Robert Hochberg); Kowalczyk edits a resource review column. Judy Brown (see Section 8) has introduced a new Internet resource column.

Discrete Mathematics in the Schools *(Book, K-12)*; see [**33**].

This edited collection of articles (which includes this article on "Recommended Resources") is based on a conference which took place in October 1992 at Rutgers University, under the sponsorship of DIMACS, the NSF-funded Center for Discrete Mathematics and Theoretical Computer Science. The articles present different perspectives of discrete mathematics and how it can be reflected in the school curricula. Many activities and topics are presented as illustrations of the authors' perspectives.

A Comprehensive View of Discrete Mathematics: Chapter 14 of the New Jersey Mathematics Curriculum Framework *(Article, K-12)*; in [**33**]

This chapter from the *New Jersey Mathematics Curriculum Framework* presents a comprehensive discussion of discrete mathematics topics and the grade levels at which they can be appropriately presented. The grade-level overviews are illustrated by several hundred activities appropriate for various grade levels.

HiMap and HistoMap Series (*Topic modules, 4–12*)
Various authors; COMAP; \$9 - \$12 (catalog available).

These are self-contained supplements on specific topics that can be used in a number of courses. They contain background reading, as well as blackline masters for student activity sheets and transparencies. A few of the modules are accompanied by special-purpose software, such as an implementation of the *Moving Knife* strategy to accompany a module on fair division. Suzanne Foley LP '95 (private communication.) mentioned that, "One useful aspect of the HiMap books that I enjoy is the history behind the mathematics," and said that she appreciates the practical activities. There are many discrete topics in the large collection, including voting, fair division, graph problems, recurrence relations, and codes. (See also the specific modules recommended in Section 3.2.)

Consortium (*Newsletter, 9–College*)
COMAP, 4/yr; only available with \$32/yr membership (which includes other publications and discounts).

This newsletter is intended primarily for teachers of undergraduates, but continues to add more items of interest to high-school teachers. It focuses on applications of mathematics, and features pull-out modeling activities for the classroom. Lists of current and future HiMap module titles are given in each issue.

Problem Solving Strategies: Crossing the River with Dogs (*Book, 9–College*)
Ted Herr and Ken Johnson; Key Curriculum Press, 1993; \$25 (teacher's resource book, \$20).

This book, written to be used as a course text, uses problem-solving to encourage reasoning skills across the curriculum. Used mainly as a teacher resource, the problems have been used with students ranging from gifted fifth-graders to undergraduates.

3.2. Activities on Special Topics: This section lists books and modules which each give activities related to one special topic. The activities can usually be integrated into a wide variety of courses, on many levels.

The Mathematician's Coloring Book (*Module, 4–10*)
Richard L. Francis; COMAP, HiMAP Module **13**, 1989; \$12.

This entertaining unit, developed around the Four-color Theorem, is especially appropriate for general math or geometry students. It includes maps, pictures, and worksheets to explore map coloring (which have been used in the LP K–8 programs). Many of the coloring activities are appropriate for young students. At the end there are challenging problems on coloring maps on doughnut-shaped and other surfaces, suitable for advanced students.

Drawing Pictures with One Line: Exploring Graph Theory (*Module, 4-10*)
Darrah Chavey; COMAP, HistoMAP Module **21**, 1992; $12.

This popular module introduces Eulerian tours and other topics. Here is an excerpt from a teacher's review [**29**]:

> [The book takes the reader] through the historical beginnings of graph theory as recreational puzzles, to the array of applications for which graph theory is used today. Included are multicultural aspects[4] of graph theory as it exists in cultures in Africa and Oceania as part of a heritage of sophisticated story-telling.

Problem Solving Using Graphs (*Module, 6–10*)
Margaret B. Cozzens and Richard Porter; COMAP,
HiMAP Module **6**, 1987; $10.

This is an excellent source of problems on a variety of graph topics, including Eulerian tours, shortest paths, minimum spanning trees, and the Traveling Salesperson Problem. The problems can be used for teaching modeling, problem-solving, or algorithm design. Many of these problems have been used successfully in the Leadership Program, and subsequently by the participating teachers when they return to their classrooms.

Fractals for the Classroom: Strategic Activities (*Workbook, 7–12*)
Heinz-Otto Peitgen, Hartmut Jürgens, Dietmar Saupe, Evan Maletsky, Terry Perciante, and Lee Yunker;
Springer-Verlag, 1991; two volumes, $20/vol. (available from NCTM).
Vol. 1 includes slides of 2- and 3-D fractal images.

This is a set of ready-to-use classroom activities, designed to introduce the concepts of *self-similarity*, fractal generation via the *Chaos Game*, and fractal *complexity* (or *dimension*). The book is the result of a collaboration between researchers on the mathematics of fractals (the first three authors) and specialists in mathematics education.

Authors Maletsky (see [**23**]) and Perciante have each used some of these activities successfully with teachers at all levels in the Leadership Program. William Bowdish LP '92 reported using the material in a classroom activity in which his 9th-grade Honors Algebra students estimated the fractal dimension of the coastline of Martha's Vineyard [**1**].

Codes Galore (*Module, 9–12*)
Joseph Malkevitch, Gary Froelich, and Daniel Froelich;
COMAP, HistoMAP Module **18**, 1991; $12.

This module begins with a historical account of codebreaking and its impact on WWII. Its main focus is on common error-detecting and error-correcting codes (such as the zipcode and ISBN numbers), and their many

[4] See also **Ethnomathematics** in Section 5.3.

applications (such as compact discs, or transmitting images from spacecraft to Earth). These codes use the idea of *check digits*, and are a good introduction to modular arithmetic. There are many activities, many of which require a calculator. A number of the activities lend themselves to cooperative group work—like decoding a simulated picture from space. A good complement to this module is the segment on codes in the COMAP *Geometry* video, described in Section 6.

4. The Students' Bookshelf

4.1. Literature. A number of teachers have found that one way to interest and engage students in mathematical thinking is through literature. At the primary level, weaving together literature and mathematics may be a necessity, since so much classroom time at this level is needed to develop literacy. This section contains a short list of books that teachers have found connect well to topics in discrete mathematics. There are a number of publications that provide more complete lists of literature/mathematics connections, such as [**4**]; however we are not aware of any that highlight discrete math topics.

Dr. Seuss books (*pre-K–2*) ***Iteration and Recursion***
"Dr. Seuss"; Random House; \$8–\$15.

A number of Dr. Seuss books contain the seeds for thinking about iteration and recursion. **The Cat in the Hat** (1957) and **Green Eggs and Ham** (1960) are just two that come to mind. In these stories, events or activities are repeated over and over but in each repetition a new event is added. Another Dr. Seuss book, **The Lorax** (1971), is a variation on this pattern, with an environmental theme. As events occur in *The Lorax* they trigger other events, eventually throwing the environment out of balance.

One Hundred Hungry Ants (*K–3*) ***Listing and Counting***
Elinor Pinczes, Houghton-Mifflin, 1993; \$15 (avail. from Creative Pub.).

This is a simple story told in verse about one hundred hungry ants heading towards a picnic. Different formations of 100 ants are tried in order to speed their way to the food; illustrations provide visual patterns for counting by twos, fives, and tens. In the process the author introduces the factors of 100 and the problem of counting factors.

Grandfather Tang's Story (*K–3*) ***Visual Problem Solving***
Ann Tompert; Crown, 1990; \$15 (avail. from Cuisenaire or Creative Pub.).

This is a Chinese folktale told with tangrams, a set of seven simple shapes cut from a square. Using tangrams, Grandfather Tang tells a story to his granddaughter: two shape-changing fox fairies try to best each other until a hunter brings danger to both of them. With tangrams students can be encouraged to investigate geometrical concepts and to use their visual imaginations to retell or invent their own stories.

The Tangram Magician (*K–3*) ***Visual Problem Solving***
Lisa Campbell Ernst and Lee Ernst, Harry N. Abrams, NY, 1990; $20.

This is another good book for developing visual imagination for geometric problem solving. The story involves a magician who can change shape, illustrated with tangrams. The reader is asked to supply the end of the story by creating the next shape that the magician will take on. High school teacher Eric Simonian LP '94 reports on an interesting classroom experience using this book involving students in grades 2, 4, and 10, in [**35**].

A Three Hat Day (*K–3*) ***Listing and Counting***
Laura Geringer; Harper-Collins, 1987; $5 (paper).

This amusing tale about a hat collector and his search for a perfect wife provides an opportunity for teachers to ask "How many different ways are there to ...?" The story can be used to introduce the concept of combinatorics, i.e., systematic counting.

Sam Johnson and The Blue Ribbon Quilt (*K–4*)
Tessellations and Geometric Transformations
Lisa Campbell Ernst, Wm. Morrow and Co., 1983 (paper, 1992), $5 (paper).

Sam Johnson and the Blue Ribbon Quilt raises both mathematical and social questions. While mending the awning over the pig pen, Sam discovers that he enjoys sewing patches of cloth together. However, when he asks his wife if he can join her quilting club, he meets at first with scorn and ridicule. The borders of the pages feature traditional American quilt patterns which can be used to introduce mathematical concepts such as symmetry, tessellations, and transformational geometry.

A Cloak for the Dreamer (*K–5*) ***Tessellations***
A. Friedman, Scholastic, 1995; $15 (available from Creative Pub.).

A tailor asks each of his three sons to sew a colorful cloak that will keep out wind and rain. The first two sons sew watertight cloaks made of rectangles and triangles, but the third son, the dreamer, makes a cloak out of circles, which is full of holes. This book gives a good introduction to tessellations, and contains (for parents and teachers) a section on the underlying mathematical concepts.

Two of Everything (*1-4*) ***Iteration; Exponential Growth***
Lilly Toy Hong, Albert Whitman & Co., 1993; $15 (available from Creative Pub.).

A Chinese folktale about a couple who find a magic pot that doubles everything that is put into it. The story is retold and illustrated by the author. This is a good starting point for thinking about iteration and exponential growth.

Anno's Mysterious Multiplying Jar (*1-6*) ***Listing and Counting***
Masaichiro and Mistumasa Anno; Putnam Publishing, 1983;
$17 (available from Creative Pub.).

Simple text and beautiful illustrations tell a tale about a porcelain jar with a sea inside. In the sea is one island, and on the island are two countries, and in each country are three mountains .... This book provides a rich introduction to the concept of counting by multiplication. One of our favorites!

Math Curse (*4-8*) ***General Problem Solving***
J. Scieszká and L. Smith; Viking/Penguin Group, 1995; $11.

A popular new book. A young girl wakes up one day afflicted with the "math curse"—she sees math in everything she does. The book has a wonderful crazy-quilt graphical style, and is full of engaging problems (mainly on algebra, geometry, and numbers—but with a few discrete math problems).

Jurassic Park (*6–up*) ***Fractals and Iteration***
Michael Crichton; Random House, 1990; $6 (paper).

Jurassic Park, which connects mathematics, biotechnology, and prehistoric legend, has proved to be a "student magnet" in a number of mathematics classrooms and can serve as a stepping stone to the introduction of fractals. In the beginning of each chapter, the "dragon-curve" fractal is constructed to one more level, as a mathematical foreshadowing of the events to come. An 8th-grade teacher's students asked her to teach them more about fractals after reading the book, leading to some of the best learning she'd seen in her career—sparked by a real "need to know."

4.2. Other Good Reading. You may want to keep the following periodicals and books in the classroom for students to browse through.

Aha! Insight (*Book, 6–10*)
Martin Gardner; W. H. Freeman (for *Scientific American*), 1978; $15 (paper).

This book contains short, fun problems that Gardner has collected, that can all be solved easily—if one chooses a good approach. Many of the problems involve combinatorics (counting), algorithms, or logic. Illustrated with cartoons.

Math EQUALS (*Book, 6–10*)
Teri Perl, Addison-Wesley, 1978; $20.

Each chapter contains a biography of a woman mathematician, followed by mathematics activities related to her work. There are a few discrete mathematics activities, including those in a chapter on Lady Ada Byron Lovelace (1815-1852), who wrote about Charles Babbage's design for a machine that anticipated the modern computer.

QUANTUM: The student magazine of math and science
(*Magazine, 10–College*) Springer-Verlag, 6/yr.; $20/yr.

This is a new magazine on mathematics and physics for sophisticated high school students. It has well-written, challenging articles about current math and science topics and also has a problem section. It is a collaborative effort of the NCTM and the American Association of Physics Teachers, along with their counterparts in Russia. (*Quantum* was inspired by the Russian journal *Kvant*, which in turn may have been inspired by an older Hungarian mathematics newsletter for students.)

Math Horizons (*Magazine, 11–College*)
MAA, 4/yr; $35/yr ($20/yr, MAA members).

Written for undergraduates, this also makes interesting reading for high-school students, and teachers at all levels. Articles include profiles of famous contemporary mathematicians, including John Conway (tiling, knots, and game theory), Fan Chung (graph theory), Persi Diaconis (probability and magic), and Jean Taylor (soap-film surfaces). There are articles giving information on careers in mathematics, as well as a problem section.

5. The Teachers's Bookshelf: Reference and Reading

This section focuses on print resources for teachers that are appropriate for reference, background reading, or as sources for ideas. Many of these resources are written for undergraduates and/or high-school and college faculty, although mature students may also be able to use them.

5.1. College Textbooks on Discrete Mathematics. Here are several sources for getting more background or detail on discrete math topics, as well as many applications and examples. Many of the problems, although intended for the college level, have been used successfully at lower levels.

Discrete Mathematics and its Applications (*Book, College*)
Kenneth Rosen, McGraw Hill, 2nd Ed., 1991; $70.

Moshe Vardi, a computer science professor at Rice University, asked college/university faculty which book they used to teach their introductory course on discrete mathematics. Over a dozen faculty responded: Rosen's book was the most popular text in the survey.[5] The book has a companion volume, with further applications, written by various authors: **Applications of Discrete Mathematics**, J. Michaels and K. Rosen, Editors (McGraw Hill).

[5]The survey was conducted informally using the electronic mailing list **TheoryNet**, in May 1993. Rosen's book was mentioned by faculty from U. Mass (Amherst), Columbia, U.C. Santa Cruz, U.C. Berkeley, RPI, SUNY Brockport, and U. Kansas. On the other hand, when I (Franzblau) did a similar informal survey of colleagues (mainly in mathematics departments), I found no clear favorite text, though Rosen's book was mentioned favorably [**13**].

Other texts mentioned several times in Vardi's survey were **Discrete Mathematics**, by Ken Ross and Charles Wright, Prentice-Hall, 3rd Ed., 1992; and **Discrete Mathematics with Applications**, by Susanna Epp, Wadsworth, 1990. (In [**12**], the author discusses her experience teaching basic logic in the course at DePaul University that led to this text.)

Applied Combinatorics (*Book, College–Graduate*)
Fred S. Roberts, Prentice-Hall, 1984; $70.

This is a good source of applications of discrete mathematics.[6] The book also describes many classic algorithms, such as those for finding shortest paths, minimum spanning trees, Eulerian tours, and maximum flow. I (Franzblau) have used the text several times for an intermediate-level undergraduate course; students find it somewhat difficult, so I prefer using it as a resource for myself.

Graph Theory Applications (*Book, College*)
L.R. Foulds, Springer-Verlag, 1992; paper, $49.

This text was recommended by Susan Picker LP '90 (private communication):

> The distinctive thing about this text is the variety of applications, including social sciences, economics, physics, biology, chemistry, civil engineering, operations research, circuit design, matrices, algorthms, architecture, and industrial engineering.

Graphs: An Introductory Approach (*Book, College*)
Robin Wilson and John Watkins, Wiley, NY, 1990.

One of the referees of this article strongly recommended this text, as a "great introduction to graphs."

Graphs, Models, and Finite Mathematics (*Book, College*)
Joseph Malkevitch and Walter Meyer, Prentice-Hall, 1974.

This book is the source of some of the material in the HiMAP module **Problem Solving Using Graphs** (described above in Section 3).

5.2. Periodicals. The following are high-quality newspapers and magazines that you may want to read regularly. They are good sources for current discoveries and applications of both discrete and continuous mathematics.

New York Times (*Newspaper*)
NY Times Co., daily; cost varies.

[6] In [**30**], the author provides examples of applications of discrete topics and discusses how he uses them in the classroom.

See especially the *Science Times* (Tuesdays): Gina Kolata, who recently won an award from the Joint Policy Board for Mathematics, often writes articles in this section on mathematical topics, such as recent breakthroughs on factoring large primes, the Traveling Salesperson Problem, and "DNA computing." The economics and patents columns in the business section, as well as financial and stock market data, are good sources for both applications and problems.

What's Happening in the Mathematical Sciences (*Booklet*)
Barry Cipra; AMS, annual (since 1993); $7 (ea.).

A well-written review of "hot" and accessible mathematics topics on which progress was made during the year. The author writes frequently on mathematics topics for the newsletter of the Society for Industrial and Applied Mathematics (SIAM).

Mathematical Intelligencer (*Magazine*)
Springer-Verlag, 4/yr; $33/yr.

This is an unusual journal with lively expository articles for a general mathematical audience. It has book reviews and articles about mathematicians and the history of mathematics. The text is sprinkled with intriguing and humorous quotations and pictures. A special feature is the *Mathematical Tourist* column, in which readers report on mathematically interesting sites (such as buildings or sculpture that incorporate interesting geometry or topology, or home towns of famous mathematicians), illustrated with photographs.

Scientific American (*Magazine*)
Scientific American, 12/yr; $36/yr.

This classic journal has a variety of articles on current science topics aimed at a broad, educated audience. There are often good articles involving mathematical modeling, some suitable for student reading. The column *Mathematical Recreations*, edited by Ian Stewart, which has undergone several incarnations since Martin Gardner's *Mathematical Games* column, is still a great source of fun problems. Many of Gardner's columns can be found in books, such as [**14**]. His successor, Douglas Hofstadter also published his columns [**16**], which can be used for classroom activities [**34**].

American Scientist (*Magazine*)
American Scientist, 6/yr; $28/yr.

This is the magazine of Sigma Xi, the scientific honor society. It has well-written articles on a range of scientific topics, as in *Scientific American.* It contains an extensive section of book reviews, and has good science cartoons.

5.3. Books on Special Topics . The following sources give interesting background material not usually found in textbooks. We include grade-level suggestions for use of the material (or for student reading), where appropriate.

Ethnomathematics: A Multicultural View of Mathematical Ideas ***Graph Theory and Applications***
Marcia Ascher; Chapman and Hall, 1991; $42.

This is a fascinating book, which will give you a new perspective on mathematical ideas that one often takes for granted, such as the words used for counting. Of special interest is a chapter exploring Eulerian paths, a standard topic in discrete mathematics (e.g., see [**17**]), as an artistic aid to story-telling in the South Pacific islands. This is the source of some of the material in the module *Drawing Pictures with One Line*, described on page 13. The following is an excerpt from a review by Susan Picker LP '90.

> But *Ethnomathematics* explores more than the topic of graph theory as it presents the mathematical ideas of number, kin relations, games of chance and strategy, and symmetric strip decorations. ... It provides a comprehensive look at the meaning and use of similar mathematical ideas in different cultures, illuminating both the mathematics and the culture in which it appears, and through this showing the value of the study of mathematics in a multicultural setting.

Alan Turing: The Enigma ***Codebreaking***
Andrew Hodges, Simon and Schuster, NY, 1983.

This is a popular book that tells the story of Alan Turing and his role in breaking the German ENIGMA code during WWII. It makes good background reading for teaching cryptography or computer science.

Visions of Symmetry (*7–up*) ***Escher's Tessellations***
Doris Schattschneider; W.H. Freeman, 1990; $25.

This excellent book on the work of M.C. Escher provides both historical and mathematical perspective on the evolution of his tessellation prints. Beginning with his sketches of mosaics in the Alhambra, the author guides the reader through Escher's notebooks, explaining how he developed an understanding of the mathematics of tessellation (as well as his own system of classification) in order to create his work. The book is illustrated with many color reproductions from his notebooks.

An Introduction to Tessellations (*4–12*)
Dale Seymour and Jill Britton; Dale Seymour, 1989; $21.

This is a good source of ideas and activities for students. It illustrates basic tessellation concepts and gives a variety of ways to create interesting artwork via geometric transformations of simpler tessellations.

Teaching Tessellating Art (*K–12*)
Jill Britton and Walter Britton; Dale Seymour, 1992; $21.

This book contains transparency masters and activities that are very useful for teaching Escher-style tessellation techniques, as well as transformation geometry. I (Franzblau) have used these (as well as pictures from

Visions of Symmetry) for hands-on workshops for K-8 teachers and high school students.

Orderly Tangles (*8–up*) ***Knots***
Alan Holden; Columbia U. Press, 1983; $32.

This is an excellent, accessible introduction to knots. The author begins with a discussion of highway interchanges, and goes on to discuss knots in general. It is illustrated with photographs of wonderful knot models made out of wooden dowels.

The Knot Book (*College–up*)
Colin Adams; W. H. Freeman, 1994; $33.

Adams has led many undergraduate research projects on knots; this book grew out of his notes. It develops the mathematics of knot theory, and can be read at many levels. A good source of problems about knots.

Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics
Robert Devaney, Addison-Wesley, Menlo Park, NJ, 1989.

Devaney is one of the pioneers in the applications of fractals, as well as the introduction of dynamical systems and chaos in the undergraduate and high school curriculum. See also Devaney's article in this volume [**10**], his video (section 6), and the Web site he developed (section 8).

Fractals, The Patterns of Chaos
John Briggs; Simon and Schuster, 1992; $20 (paper).

The following is a recommendation from a Leadership Program participant:

> When I try to explain what fractals are and how they relate to our world, I sometimes come up short. I was wandering through my favorite bookstore and discovered this book. It's terrific. In addition to the pretty fractal pictures, he shows fractals in the real world—decaying leaves, weather systems, lightning, cauliflower, the human body, etc. He even relates fractals to art and architecture. In the appendix, the author lists and evaluates fractal software both for the IBM and the Mac (both shareware and commercial software). He also lists several fractal publications. (Edward Polakowski LP '92, email posting.)

Game Theory and Strategy (*12–College*)
P. Straffin; MAA, 1993; $34.

This book was recommended by Joe Malkevitch [**25**]:

> This is a wonderfully rich book about the theory of games. It covers most of the major ideas in a motivated and succinct way, and has many examples.

Fair Division: From Cake Cutting to Conflict Resolution
Steven Brams and Alan Taylor, Cambridge Univ. Press, 1996. $18.

If you're interested in estate settlement, voting, auctions, and similar topics, this book comes highly recommended by high-school teacher L. Charles Biehl LP '90 (email posting). It was written by the authors of a well-known recent work on "envy-free" cake-cutting.

Basic Geometry of Voting
Donald G. Saari, Springer-Verlag, NY, 1995; $39.

This recent book provides additional mathematical background for teaching election theory and voting paradoxes, topics which are covered in the books *For All Practical Purposes*, *Excursions in Modern Mathematics*, and *Discrete Mathematics through Applications*, discussed in Section 2.

Unit Origami (*7–up*) ***Geometry of Polyhedra; Graph Problems***
Tomoko Fusé; Japan Publications, 1990; $19 (available from Cuisenaire or Dale Seymour; a classroom guide is also available).

This is an introduction to a style of origami that involves building complex shapes, including regular polyhedra, by fitting together simple (usually identical) units. It is often recommended by teachers. Many of the constructions are suitable for teaching to students, and can provide inspiration for exploring the geometry of polyhedra. Creating the shapes out of small units also leads one naturally to discrete problems: counting vertices, edges, and faces (as a prelude to Euler's formula), as well as to questions on coloring edges and faces.

Build Your Own Polyhedra (*5–up*)
Peter Hilton and Jean Pederson; Addison-Wesley, 1988; $28
(available from Dale Seymour).

This is a beautiful book with illustrated directions for constructing polyhedra. It also introduces some of the mathematics of polyhedra, such as Euler's formula. This is a good source for classroom projects at almost all grade levels. See [**18**] for a discussion of the value of introducing students to polygons and polyhedra long before beginning the formal study of geometry.

6. Video Tapes

In addition to the COMAP Videos which accompany the text FAPP (see section 2), we found only two video tapes that have been widely used and recommended, *Geometry: New Tools for Technologies* and *Powers of Ten*, which are described below. If you are not able to purchase them, you may be able to borrow them through a school or regional library.

For All Practical Purposes (*9-College*)
Twenty-six half-hour programs on 15 cassettes;
Available from Annenberg/CPB Multimedia Collection (address in Appendix A); $39.95 per cassette (or $85 per module of three cassettes)

These video tapes accompany the text FAPP discussed in Section 2; there is an introductory tape, plus five half-hour programs for each of the five sections of FAPP.

Geometry: New Tools for Technologies *(5–12)*
Graph Applications; Codes
Five units, 10-15 minutes each (1 hr. total); COMAP, 1992; $70 (user's guide, $10).

The following description is from a teacher's recommendation:

> This well-done ... video, complete with user's guide, illustrates the geometry of the 20th century: motion planning, error-correcting codes, Euler circuits, vertex coloring, and tomography. (Ethel Breuche LP '91 [**3**])

The five unit titles are as follows:

- *A Hero's Jouney: Motion Planning for Robots*
- *Connecting the Dots: Vertex Coloring*
- *Picture Perfect: Error-correcting Codes*
- *X-ray Vision: the CAT Scan*
- *Snowbound: Euler Circuits*

This videotape was conceived and directed by Joseph Malkevitch who says (private communication) that he "still finds parts of the tape very robust in attracting interest. For example, the piece on CAT scans shows people that math is behind tomography as well as computing and engineering and physics." (See also Malkevitch's article [**26**] on the value of addressing real-world problems in teaching mathematics.)

Several teachers have also mentioned that the tape appeals to a broad audience. For example, the segment entitled "Snowbound" begins with a charming cut of young children trying to draw a house without lifting a pencil, before getting into more practical applications. Another cut in this segment uses interesting graphics to let you "ride along" a graph. The segment "Connect the Dots", shows an application of graph coloring to creating zoo habitats. Both of these segments have been very successful as a follow-up to problems on Eulerian paths and graph coloring with all teachers in the Leadership Program in Discrete Mathematics. One of us (Franzblau) likes to use the segment on error-correcting codes as part of teaching a unit on codes, using the first half to motivate the concept of error correction.

Powers of Ten *(6–12)* ***Iteration; Exponential Growth and Decay***
10-15 minutes; W.H. Freeman; $40.

Teachers are very enthusiastic about this short video. It has been used at a number of grade levels, as well as in teacher-training programs, especially for number sense, positive and negative exponents, and scientific notation. Most find the graphics very powerful, and show it two or more times with discussion in between. It could be a good resource when discussing the exponentially growing time needed to run many exhaustive search algorithms.

Here are some comments made by teachers who have used it:

- I show the first half, go over exponents, scientific notation, etc., and then show the whole thing, introducing negative exponents.
- It's also a really great visual experience for the "trip" back down the powers of ten.
- One of the amazing things is how outer space looks so much like inner space with the vast distances between things. It's a great link to science because through the use of the negative exponents you travel to the molecular level.
- It's one of those *ahhhhhh* films!

There are other good videos available that are less well known. We list below a few that have been especially recommended.

Mathematical Eye Series (*9–12*) ***Graphs; Probability; Logic***
20 min. each (approx.); Journal Films; $270 each.

This series was recommended enthusiastically by high-school teacher Diane DePriest LP '93 (who borrowed the films from a district library). She provided the following description.

This outstanding series includes 18 titles, including the following on discrete math topics:

> *Lines and Networks* (Euler paths, subway representation, isobars);
> *Fibonacci and Prime Numbers*;
> *Logic and problem solving* (flow charts, probability, truth tables);
> *Probability*; and
> *Shapes and Angles* (includes tessellations).

Lines and Networks works well in a geometry class, and shows, for example, how the tangled mess of a real subway system can be represented much more simply with an abstract network (graph) model.

Futures Series (*9–12*)
Narrated by Jaime Escalante; PBS; $450 for the entire 12-part series.

This series was also recommended by Diane DePriest LP '93 (private communication):

> Each part deals with a different aspect of math and how it is used by real people in the real world. There are many famous guests (such as Cindy Crawford, Arnold Schwarzenegger, Jackie Joyner Kersee, and Sally Ride). The titles with some discrete math content are *Statistics and Sports performance,* and *Water Engineering and Optics.* The series is *excellent*!

Fractals: The Colors of Infinity (*8–up*)
52 min.; Films for the Humanities, 1994; $149

This film is recommended by high-school teacher Edward Polakowski LP '92 (email posting):

In addition to beautiful images of the Mandelbrot set (the best I've seen), it includes interviews with Benoit Mandelbrot, who talks about his discovery, the rise of fractal geometry as a means of looking at the world, and the "practical" uses of fractals. It is narrated by Arthur C. Clarke in an understandable fashion and even includes an interview with Stephen Hawking: "Is the world infinitely small?"

Professor Devaney Explains the Fractal Geometry of the Mandelbrot Set (*10–College*)
Key Curriculum Press, $25.

This is another good video for teaching fractals. Although recommended for higher grades, one teacher (Erica Voolich LP '94, email posting) found that her 7th-grade students responded enthusiastically. The only background concepts needed are that of a function and multiplication of complex numbers. (An article by Devaney on chaos is included in this volume [**10**] and in [**19**].)

NOVA Series (*6–up*)
NOVA/WGBH; some of the videos in the series can be purchased or rented; otherwise, check local televion listings or your library (teacher's guides and transcripts are also available).

This television series, shown regularly on PBS, has occasional programs in areas relevant to discrete mathematics. For example, "The Man Who Loved Numbers" (1988) is about the self-taught Indian mathematician Srinivasa Ramanujan, who developed many remarkable facts in number theory. More recently, a program on codes used in WWII, "The Codebreakers" (1994), was shown.

7. Software

In this section we list the programs and software that teachers have found most useful in teaching discrete mathematics. Overall, general-purpose programs, such as spreadsheets and drawing programs, or programs that allow open-ended explorations are much more useful than the many limited-use or "drill-and-kill" programs that dominate the market. As one can see in the articles [**9, 36**], there is essentially no general-purpose software designed for discrete mathematics that is appropriate in grades K-12; nevertheless software intended for other purposes has proven useful.

Teachers interested in using software may be interested in contacting **CLIME**, the *Council for Logo and Technology In Mathematics Education* (see the Appendix). The organization, which is affiliated with the NCTM, publishes a newsletter; issues have included a "top-ten mathematics-education software list" (based on a survey of teachers in a mentorship program at Stevens Institute of Technology), as well as a complementary "top-ten lessons-with-software list". In our recommendations below, we refer to the software survey as the "CLIME survey."

Spreadsheets (Excel, ClarisWorks, Lotus 123, etc.) (*4–up*)

Spreadsheets contain rows and columns of *cells* in which one can enter either numbers or algebraic formulas. Although one doesn't normally think of spreadsheets as educational software, they are in fact one of the most useful tools to have in the middle- or high-school classroom, especially for Algebra or Pre-Algebra students. Spreadsheets were rated "No. 2" in the CLIME Survey ("No. 1" was the *Geometer's Sketchpad*, described below).

Spreadsheets are ideal for explorations involving iteration and recursion, such as experimentation with population growth and other models of change, such as described in [**11, 15**]. Of course, calculators can perform many of the same functions, but spreadsheets can do many calculations in parallel, and so can be much faster and easier to use. Also, data from spreadsheets can often be exported to graphics programs to be displayed in different formats.

Classroom Use. The following are some examples of projects in which students used spreadsheets.

- (*6th grade*) Students acted as landscape consultants to students from other countries (communicating over the Internet), doing their planning, estimates, and budgeting with the spreadsheet. (Sr. Diane Mollica LP '95, private communication.)
- (*High school*) As preparation for finding probabilities mathematically, students rolled dice and entered their data on a spreadsheet; they then estimated the probabilities of various outcomes, such as rolling "7" with two dice. (Br. Patrick Carney LP '91, private communication.)
- (*High school*) In a unit on understanding the electoral college, students used a spreadsheet to compute the number of electoral votes each state would get using various methods of apportionment. Using their data, they showed how Hayes won over Tilden in 1876 in the electoral college—even though Tilden won the popular vote. (William Bowdish LP '92 and David Fogle LP '93, private communications.)

Geometer's Sketchpad (*7–up*)

Key Curriculum Press; Windows or Macintosh, \$170 (plus shipping); site licenses are available.

Sketchpad is a drawing program that allows one to do precise geometric constructions. Teachers can use it to create demonstrations and examples; students can experiment with examples to make conjectures or verify theorems. It is one of the all-time favorites of mathematics teachers everywhere. It was rated "No. 1" in the CLIME survey. In addition to being a great experimental tool for traditional geometry, it is also excellent for demonstrating tessellation concepts, or creating tessellations. One can create animations easily by saving "scripts". Sketchpad can also be used to generate fractal patterns, such as the Sierpinski Triangle or Koch Snowflake (see *Fractals for the Classroom*, discussed in section 3.2). It is also possible to create activities that illustrate graph (network) concepts, such as those described for *Netpad* in [**9**].

A similar program, which uses slightly different conventions, but is not as well known, is **Cabri** (Texas Instuments), a limited version of which is installed on the new TI-92 graphing calculator. Another recommended though less-popular program is the **Geometric Supposer** (*8–up*) (Sunburst Communications; Mac, $129; Apple/Windows, $99), developed by Dr. Judah Schwartz of the Educational Development Corporation. For lower grades (5–up), there is the **Geometric PreSupposer** (Sunburst; Apple/Windows, $99). Standard **drawing programs** such as in ClarisWorks may also be used effectively (e.g., for tessellations and fractals).

Logo (*3–up*)
Logo Computer Systems Inc. (LCSI); $70-$200 (Windows, Mac, Apple IIe); (see also the references in [**22**] for other versions).

The best description of Logo is perhaps the one given by Hal Abelson of MIT: "Logo is the name of a philosophy of education and a family of computer languages that aid in its realization." The idea behind Logo is that we learn by constucting our own knowledge. Logo, as conceived by Seymour Papert, was intended to be the educational clay that would facilitate the building of mathematical knowledge.

A Logo program is a set of commands to a "turtle" that moves and draws on the computer screen in response to those commands. Students can construct new commands, and in turn use them as building blocks for even more elaborate creations. For example, students can write Logo programs to generate fractal curves, such as the Koch Snowflake. Logo's structure makes it especially easy to implement iteration and recursion.

Classroom Use. I (Kowalczyk) have extensive experience using and teaching with Logo. For example, I used it to simulate "gnomon growth", which leads to Fibonacci spirals, as described in [**20**]. (The text *Excursions in Modern Mathematics* described in Section 2 has good material on this topic.) Charlie Hennessy LP '95, who teaches 6-8th grades, reports that it is a good tool for getting students comfortable with the concept of a variable, since students can change parameters and see the effects immediately. Philip Lewis, a high school teacher, describes success using Logo to teach vector algebra and elementary algebra with an algorithmic approach [**22**].

Further Information. One of the most popular versions of Logo over the last 10 years has been **Logowriter** (LCSI; Mac/Apple IIe, $199). This version is slowly being replaced by its more sophisticated relative, **Microworlds** (LCSI; Mac, $99), which features an unlimited number of turtles, parallel processing, a melody editor, drawing tools, and the ability to create hypermedia links. **MathLinks** (LCSI; $79) (rated "No. 4" in the CLIME "top-ten lessons with software" list) is an interactive set of activities designed to help students develop mathematical thinking and become mathematical problem solvers. Activities are organized around the topics of polygons, repeating patterns, permutations and combinations and transformations. **Turtle Math** (LCSI; Mac, $69),[7] another version of Logo, was developed to address the needs of the elementary classroom. It is accompanied by 36 classroom-tested activities and materials geared to grades 3–6, which involve students in such discrete math topics as geometry, similarity, patterns, and probability.

Tesselmania! (*3–12*)
Dale Seymour; $48 (Mac), $69 (Windows), $79 (CD-ROM).

This program is designed for demonstrating and creating tessellations. *Tesselmania* is easy to use and good for sparking interest. Judy Nesbit LP '94 reports using the program for a student tessellation contest: "Students have been very enthusiastic about this project. They learn a lot and many of them are quite creative!" The program includes animation that is good for illustrating the different types of polygons that tile the plane and the transformations (reflections, translations, and rotations) involved in creating the tiling. The program comes with a resource guide, which has a series of lesson plans for teaching tessellations and using the program. Judy Brown LP '92 comments: "*Tesselmania* is the perfect tool for teaching teachers about tessellations. It gives teachers the ability to experiment with a number of different types of transformations and tessellations in a quick and accurate manner."

Many, including myself (Franzblau), have found that in teaching tessellations it's much better to start by having students create tessellations by hand; but once they understand the concepts, the program may be useful as a demonstration, or to inspire student creativity. Introducing any software too early may give students the misconception that a computer is necessary to create the tessellations.

I find that paper and pencil, drawing programs, or *Geometer's Sketchpad* are better tools for creating interesting tessellation designs. Although *Tesselmania* allows one to add decorations easily, and does the transformations automatically, its drawing capabilities are limited.

8. World Wide Web Sites

Teachers with access to the Internet can now find a variety of resources through the World Wide Web. There is no way to keep up with the constant growth and change on the Web, but we have identified several well-established Web sites that are useful for teachers or students of discrete mathematics, which we expect to continue and grow. The Web addresses (URLs) that we give are current as of July, 1996, but are subject to change.

Special thanks are due to Judy Ann Brown LP '92, a mathematics teacher at Pleasant Valley Middle school in Pennsylvania, who has used the

[7] *Turtle Math* was created by Doug Clements and Julie Sarama [**7**], and is available through both Dale Seymour and LCSI.

Web extensively in her teaching over the last few years, and who provided us with much of the information in this section. She has been a lead teacher in the Leadership Program in Discrete Mathematics, as well as an adjunct professor at Allentown College, teaching a discrete mathematics course for educators. Judy's home page, which has links to many of the resources mentioned here, is at `http://dimacs.rutgers.edu/~judyann/`; it also has links to other resources that she uses.

General Resources. We first mention several addresses with general information for mathematics and/or science teachers, which also have links to further resources.

Eisenhower National Clearinghouse for Math and Science Teachers (Ohio State University) `http://www.enc.org/`

This umbrella site is a gold mine for mathematics and math education. You can read newsletters, specialized catalogs, and topical publications, or connect to other education sites. An *Online Documents* service allows you to find articles on curriculum issues in mathematics that are available in an electronic format.

A *Lessons and Activities* page includes classroom activities with projects and supporting materials. When we last checked, the page contained sixteen different activities; the following three titles give a flavor for the kind of things that can be found here.

- The CHANCE Database contains materials to help teach a CHANCE probability course or a standard introductory probability or statistics course.
- The *Good News Bears Stock Market Project* is an interdisciplinary project designed for middle school students. It is an interactive stock market competition between classmates using the New York Stock Exchange and NASDAQ.
- The *Spanky Fractal Database* is a collection of fractal images, documents, and software available for free distribution. It links to several other fractal sites on the Internet.

Mathematics Forum (Swarthmore College)
`http://forum.swarthmore.edu/`

This is a new site, extending the well-known **Geometry Forum**, which was developed several years ago at Swarthmore. The site is intended to create an on-line mathematics community; it hosts discussion groups on teaching mathematics, provides useful information and interesting math problems, and has a search engine for mathematics resources.

One of the useful features at this site is **Ask Dr. Math**. K-12 students can get answers to their mathematical questions by linking to the *Dr. Math Archive*, which is a database of previously and frequently asked questions in mathematics. Students can also send a message directly to *Dr. Math* (actually a team of mathematics students at Swarthmore) using the email address `dr.math@forum.swarthmore.edu`. Replies are usually sent within 24 hours.

NCTM Standards (National Council of Teachers of Mathematics)
`http://www.enc.org/online/NCTM/280dtoc1.html`

A complete copy of the NCTM *Curriculum and Evaluation Standards for School Mathematics* can be found here, which includes a standard for discrete mathematics in grades 9-12. This excellent resource includes a discussion of each standard at its designated grade level groups: K-4, 5-8, or 9-12. These discussions contain a wealth of ideas and materials that could be adapted for use in the classroom. This is a great starting place for anyone who is interested in mathematics education.

Special Topics. Our next set of recommendations is for Web sites which have proven valuable sources for enrichment material for students or teachers on specific topics, or for classroom activities.

MegaMath (Los Alamos National Laboratory)
`http://www.c3.lanl.gov/mega-math/`

The MegaMath site brings important mathematical ideas to elementary school classrooms with unique activities. The site was developed by Nancy Casey and Michael Fellows [**5, 6**]. It includes many discrete math activities including the following:
The Most Colorful Math of All (map and graph coloring);
Games on Graphs;
Untangling the Mathematics of Knots;
Algorithms and Ice Cream for All (an interesting problem on graphs); and
A Usual Day at Unusual School (logic and paradoxes).
This site offers teachers the opportunity to bring discrete mathematics into their classroom with engaging stories.

MacTutor for Math History Information
`http://www-groups.dcs.st-and.ac.uk:80/~history/Mathematical_MacTutor.html`

MacTutor offers the history of mathematics on the Web. The Welcome Page for the site includes a Famous Curves Index, a Biographical Index, Chronologies, a History Topics Index, a Birthplace Map, the Mathematicians of the Day, Anniversaries for the Year, a Search Form, and Search Suggestions. I (Kowalczyk) used MacTutor to find a wealth of information about Fibonacci and the Fibonacci numbers (see also the article [**20**], which I wrote before I found this site).

Dynamical Systems (Boston University)
`http://math.bu.edu/DYSYS/dysys.html`

This site is designed to help teachers bring contemporary mathematics topics—chaos, fractals, and dynamics—into the classroom and to illustrate how to use technology effectively in the process. The interactive activities at this site can also help teachers understand the mathematics behind these topics. This site is well worth exploring—especially for high school teachers. This site was developed by Robert Devaney, who has used these activities in teaching calculus [**10**] and differential equations.

After exploring the site, Judy Brown LP '92 sent a note which is excerpted below.

> I spent a lot of time with the "chaos for the classroom" section. This is in such an easy-to-read digestible format that I really can't wait to go back and investigate the Mandelbrot and Julia set information. I don't know exactly how to express my feelings, except that some of the "neat stuff" that I've done before now has taken on a more mathematical tinge. There are also a few fractal "movies" that you have got to see [under the heading *Rotations and Animation*]. My favorite is the dancing Sierpinski triangle. I'll never be able to look at a Sierpinski triangle again without imagining it dancing.

Fractal Frequently Asked Questions and Answers (Ohio State)
`http://www.cis.ohio-state.edu/hypertext/faq/usenet/fractal-faq/faq.html`.

This is another good source for those interested in fractals, with hyperlinks to many other interesting and useful fractal sites on the Web. A sample of questions addressed are as follows. "I want to learn about fractals. What should I read first?" "What is a fractal?" "What are some examples of fractals?" "What is chaos?" This site is recommended for both teachers and students who want to begin learning about fractals.

AIMS Puzzle Page
`http://204.161.33.100/puzzle/puzzlelist.html`

Here is a delightful site with excellent classroom-ready activities, which are based primarily on discrete mathematics. Each month a new and challenging puzzle is posted, complete with student worksheets, which can be downloaded. As I (Kowalczyk) viewed the puzzles for the first four months of 1996, I could hardly wait to print them so I could get started.

The World of MC Escher
`http://www.texas.net/users/escher`

At this site you will come to know this fascinating artist (and mathematician) through stories, his tessellations and other art works, as well as his insights into these works. The site also offers high-quality (commercial) products featuring Escher's designs. If you are already familiar with Escher you'll have a great time just looking around, otherwise, it's time to explore and be captivated by his work. (See also the books *Teaching Tessellating Art* and *Visions of Symmetry*, described in Section 5.)

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References

[1] William L. Bowdish, "Finding the Fractal Complexity of a Coastline", *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **5** (Nov 1994), p. 3.
[2] Ethel Breuche, Book Review, *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **2** (October 1992), p. 10.
[3] ———, in "The Discrete Reviewer", *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **6** (Spring/Summer 1995), p. 7.
[4] Marilyn Burns and Stephanie Sheffield, *Math and Literature, K–3*, Math Solutions, Sausalito, CA, 1992. (Avail. from Cuisinaire.)
[5] Nancy Casey and Michael Fellows, "This is MEGA-Mathematics!", Los Alamos, 1993.
[6] ———, "Implementing the Standards: Let's Focus on the First Four", this volume.
[7] Doug Clements and Julie Sarama, "Turtle Math: Redesigning Logo for the Elementary Classroom", *Learning and Leading with Technology* (April, 1996), p. 10.
[8] Nancy Crisler, Patience Fisher, and Gary Froelich, "A Discrete Mathematics Textbook for High Schools", this volume.
[9] Nathaniel Dean and Yanxi Liu, "Discrete Mathematics Software for K-12 Education", this volume.
[10] Robert L. Devaney, "Putting Chaos into Calculus Courses", this volume.
[11] John A. Dossey, "Making a Difference with Difference Equations", this volume.
[12] Susanna S. Epp, "Logic and Discrete Mathematics in the Schools", this volume.
[13] Deborah Franzblau, "New Models for Courses in Discrete Mathematics", *Newsletter of the SIAM Activity group in Discrete Mathematics* **4** (Winter, 1993-94), p. 1-3.
[14] Martin Gardner, *The Scientific American book of Mathematical Puzzles and Diversions*, Simon and Schuster, NY, 1959. See also other books by author.

[15] Eric W. Hart, "Discrete Mathematical Modeling in the Secondary Curriculum: Rationale and Examples from the Core-Plus Mathematics Project", this volume.
[16] Douglas R. Hofstadter , *Metamagical Themas*, Basic Books, New York, 1985.
[17] Bret Hoyer, "A Discrete Mathematics Experience with General Mathematics Students", this volume.
[18] Robert E. Jamison, "Rhythm and Pattern: Discrete Mathematics with an Artistic Connection for Elementary School Teachers", this volume.
[19] Margaret J. Kenney and Christian R. Hirsch, Eds., *Discrete Mathematics Across the Curriculum*, 1991 Yearbook, NCTM, Reston, VA, 1991.
[20] Janice C. Kowalczyk, "Fibonacci Reflections: It's Elementary!", this volume.
[21] ———, resource review in "The Discrete Reviewer", *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **7** (Fall/Winter 1995), p. 7.
[22] Philip G. Lewis, "Algorithms, Algebra, and the Computer Lab", this volume.
[23] Evan Maletsky, "Discrete Mathematics Activities for Middle School", this volume.
[24] Joseph Malkevitch, "Applied Discrete Mathematics for Liberal Arts Students", *Newsletter of the SIAM Activity group in Discrete Mathematics* **4** (Summer, 1994), p. 1-2.
[25] ———, "Game Theory Bibliography", *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **6** (Spring/Summer 1995), p. 9.
[26] ———, "Discrete Mathematics and Public Perceptions of Mathematics", this volume.
[27] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, NCTM, Reston, VA, 1989.
[28] Anthony Piccolino, Book Review, *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **2** (October 1992), p. 10.
[29] Susan Picker, Resource Review, *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **3** (August 1993), p. 4.
[30] Fred S. Roberts, "The Role of Applications in Teaching Discrete Mathematics", this volume.
[31] Joseph G. Rosenstein, "A Comprehensive View of Discrete Mathematics: Chapter 14 of the New Jersey Mathematics Curriculum Framework", this volume.
[32] Joseph G. Rosenstein and Valerie A. DeBellis, "The Leadership Program in Discrete Mathematics", this volume.
[33] Joseph G. Rosenstein, Deborah S. Franzblau, and Fred S. Roberts, Eds., *Discrete Mathematics in the Schools*, (this volume), American Mathematical Society, AMS/DIMACS series, 1997.
[34] Reuben J. Settergren,"What we've got here is a failure to cooperate", this volume.
[35] Eric Simonian, "The Tangram Magicians", *In Discrete Mathematics: Using Discrete Mathematics in the Classroom* **7** (Fall/Winter 1995), p. 3.
[36] Mario Vassallo and Anthony Ralston, "Computer Software for the Teaching of Discrete Mathematics in the Schools", this volume.

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Rhode Island School of the Future, P.O. Box 4692 Middletown, RI 02842
E-mail address: `kowalcjn@ride.ri.net`

Appendix A. Publishers, producers, and distributors.

AMS (American Math. Society)
P.O. Box 6248
Providence, RI 02940-6248
800-321-4267, 401-455-4000

American Scientist
P.O. Box 18975
Research Triangle Park, NC 27709-9890
800-282-0444, 919-549-0097

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WWW: http://www.maa.org

Mimosa Publications
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800-MIMOSA-1, Fax: 415-995-7155

NCTM (National Council of Teachers of Mathematics)
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703-620-9840 (info),
Fax: 703-476-2970
Email: nctm@nctm.org

New York Times Co.
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New York, NY 10036-3959
800-698-4637,
WWW: http://www.nytimes.com

NOVA/WGBH
125 Western Ave
Boston, MA 02134
800-255-9424, 617-492-2777
Note: Distributors of NOVA video tapes change often; contact WGBH for information on distributors of particular programs.

PBS/Futures Series
1320 Braddock Place
Alexandria, VA 22314-1698
800-344-3337

Prentice Hall
Englewood Cliffs, NJ 07632
800-848-9500, 201-592-2000

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800-SPRINGER, 201-348-4033

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800-92-IDEAS

Appendix B. Index by title.

The following is a list of titles of all resources described in the text, each followed by its corresponding page number. Titles are arranged alphabetically within the main resource categories: Texts, Literature, Periodicals, Videos, Software, and Web Sites.

Texts.

Literature.

Periodicals.

Videos.

Software.

Web Sites.

Appendix C. Index by grade level.

In this section, for each of the grade-level ranges K–2, 3–5, 6–8, and 9–12, we list the titles of resources described in the text whose content seems especially appropriate. Within each subsection, titles are ordered (approximately) by reading level. Each title is followed by the number of the page that contains its description. See Appendix B for unabbreviated titles.

Grades K–2

Grades 3–5

Grades 6–8

Grades 9–12

- **Texts:**
  Math Coloring Book, 12
  Drawing Pictures/One Line, 13
  Intro to Tessellations, 20
  Build Your Own Polyhedra, 22
  Unit Origami, 22
  Fractals for the Classroom, 13
  Math/Human Endeavor, 4
  DM through Applications, 8
  Prob Solving Using Graphs, 13
  HiMap/HistoMap Series, 12
  Visions of Symmetry, 20
  Teaching Tessellating Art, 20
  Codes Galore, 13
  Orderly Tangles, 21
  Prob Solving/Crossing River with Dogs, 12
  Excursions in Modern Math, 7
  For All Practical Purposes, 5
  Ethnomathematics, 20
  Game Theory and Strategy, 21
  Fair Division, 22
- **Literature:**
  Jurassic Park, 16
  Alan Turing: The Enigma, 20
- **Periodicals:**
  Mathematics Teacher, 11
  In Discrete Mathematics, 11
  QUANTUM, 17
  Math Horizons, 17
  Consortium, 12
  New York Times, 18
  Mathematical Intelligencer, 19
  Scientific American, 19
  American Scientist, 19
- **Videos:**
  For All Practical Purposes, 22
  Geometry: New Tools ... , 23
  Powers of Ten, 23
  NOVA Series, 25
  Fractals/Colors of Infinity, 24
  Mathematical Eye Series, 24
  Futures Series, 24
  Prof. Devaney Explains Fractal Geometry, 25
- **Software:**
  Logo, 27
  Tesselmania, 28
  Spreadsheets, 26
  Geometer's Sketchpad, 26
- **Web Sites:**
  MacTutor/ Math History, 30
  AIMS Puzzle Page, 31
  World of MC Escher, 31
  Mathematics Forum, 29
  Fractal Q & A, 31
  Dynamical Systems, 30

Appendix D. Index by topic.

In this section, for each major topic, we list the titles of resources described in the text which are recommended either for background or as a source of activities. Resources are listed in order by increasing grade level within each subsection. The title of each resource is followed by the number of the page that contains its description. See Appendix B for unabbreviated titles.

Graph Problems and Applications

Iteration, Recursion, and Fractals

Geometric Patterns and Transformations