DIMACS Series in Discrete Mathematics
and Theoretical Computer Science
Volume **00**, 19xx

Overview and Abstracts

As noted in the **Preface**, this volume makes the case that discrete mathematics should be included in K–12 classrooms and curricula, and provides practical assistance and guidance on how this can be accomplished. The organization of this volume parallels these two goals. After the **Introduction** the articles are arranged in the following eight clusters:

Section 1. The Value of Discrete Mathematics: Views from the Classroom

Section 2. The Value of Discrete Mathematics: Achieving Broader Goals

Section 3. What is Discrete Mathematics: Two Perspectives

Section 4. Integrating Discrete Mathematics into Existing Mathematics Curricula, Grades K–8

Section 5. Integrating Discrete Mathematics into Existing Mathematics Curricula, Grades 9–12

Section 6. High School Courses on Discrete Mathematics

Section 7. Discrete Mathematics and Computer Science

Section 8. Resources for Teachers

Everyone's first question is of course, "What is discrete mathematics?" Everyone's second question is, "Why should I use discrete mathematics?" Explicit discussion of the first question is delayed until Section 3, and the focus of the **Introduction** and Sections 1–2 is the second question. These sections make the case for discrete mathematics — from the perspective of teachers in the classroom, and from the perspective of researchers involved in improving mathematics education. These articles encompass a variety of agendas — implementing the four NCTM process standards (problem-solving, reasoning, communicating mathematical ideas, and making connections), improving the public's perception of mathematics, conveying the usefulness of mathematics, and providing a new start for students, teachers, and curricula.

Everyone's third question is, "How can I use discrete mathematics in my classroom?" This question is addressed in Sections 4–7. One set of responses

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involves incorporating discrete mathematics into existing curricula; these responses appear in Sections 4 and 5, arranged by grade level. Another set of responses involves introducing new courses, typically at the high school level, and these are addressed in Section 6. Section 7 addresses the role of computer science in the high school curriculum, as well as the role of discrete mathematics in the teaching of computer science.

Section 8 describes resources available to teachers who decide to enrich their classrooms with discrete mathematics.

Following are abstracts of the articles in this volume, prepared by the editors. The abstracts are arranged by section, and within each section are presented alphabetically, as are the articles in the volume.

Introduction

Joseph G. Rosenstein's article **Discrete Mathematics in the Schools: An Opportunity to Revitalize School Mathematics** serves as an introduction to this volume and describes why discrete mathematics can be a useful vehicle for improving mathematics education and revitalizing school mathematics. He provides rationales for introducing discrete mathematics in the schools, noting that discrete mathematics is applicable, accessible, attractive, and appropriate, and argues that discrete mathematics offers a "new start" in mathematics for students. This article is based on a concept document distributed to participants prior to the October 1992 conference, and on the opening presentation of the conference.

Section 1. The Value of Discrete Mathematics: Views from the Classroom

Bro. Patrick Carney's article **The Impact of Discrete Mathematics in My Classroom** describes anecdotally how the author aroused in his students an interest in mathematics, and developed in his students a more "positive attitude toward mathematics and their ability to do it".

Nancy Casey's article **Three for the Money: An Hour in the Classroom** describes the excitement generated in a class of high school students, participating in a special summer program, when they are presented with an unsolved mathematical problem, and the mathematical journeys that they take to learn what the problem is and to try to solve it. It also provides a vivid description of how the teacher's role in the classroom changes when the class embarks on an uncharted adventure of mathematical discovery.

Janice C. Kowalczyk's article **Fibonacci Reflections: It's Elementary!** is an account of her experiences giving a workshop on the Fibonacci sequence (1, 1, 2, 3, 5, 8, ...) to a fourth-grade class. She gives a detailed description of the workshop activities, including student investigations of the classical rabbit population problem that leads to the sequence, and spiral-counting in pinecones, sunflowers, shells, and other objects whose growth patterns exhibit the sequence. The article illustrates how using a topic with a strong visual appeal, along with a focus on student exploration, can bring out the strengths in many students who have had difficulties in the traditional elementary mathematics curriculum.

Susan H. Picker's article **Using Discrete Mathematics to Give Remedial Students a Second Chance** is an account of her experiences introducing discrete mathematics to a class of remedial tenth-grade students in Manhattan, and their success in solving complex graph-coloring problems. More than that, it is an account of the impact that this course had on the students' perceptions of mathematics and their own abilities, as well as on their subsequent school careers. The author learned from this experience the extent to which students' dislike of arithmetic serves as an obstacle to their progress and success in mathematics.

Reuben J. Settergren's article **"What We've Got Here is a Failure to Cooperate"** describes a cooperative game, based on the classical Prisoner's Dilemma, that the author played with twelve-year-old students in a summer program. The game gave students insight into why individuals are sometimes motivated to behave in a way that harms the larger community, providing an opportunity to discuss moral and social issues in a mathematics class.

Section 2. The Value of Discrete Mathematics: Achieving Broader Goals

Nancy Casey and Michael R. Fellows' article **Implementing the Standards: Let's Focus on the First Four** argues that in order to properly address the NCTM process standards — reasoning, problem-solving, communications, and connections — in the elementary school classroom, new content must be introduced into the K–4 mathematics curriculum. The authors show by example how elementary versions of problem situations that arise in computer science and discrete mathematics make it possible to realize the goals of the process standards. They describe their approach to teaching mathematics as parallel to the "whole language" approach to teaching reading.

Margaret B. Cozzens' article **Discrete Mathematics: A Vehicle for Problem Solving and Excitement** provides examples of discrete mathematics activities from several curriculum development projects funded by the NSF division that the author heads. The author argues that discrete mathematics can motivate students to think mathematically, to become better problem solvers, and to increase their interest in mathematics.

Susanna S. Epp's article **Logic and Discrete Mathematics in the Schools** argues that logical reasoning should be a component of the discrete mathematics that is discussed at all grade levels. Students should not have to wait until they are college students to explore the reasoning involved in "and", "or", and "if-then" statements, or to understand how quantifiers are used. This need not be done formally (e.g., through truth tables) but through concrete activities which ultimately will support the students' transition to abstract mathematical thinking. The author illustrates the value of explicit discussion of logic with experiences from a discrete mathematics course she has taught at DePaul University.

Rochelle Leibowitz' article **Writing Discrete(ly)** argues that discrete mathematics serves as an excellent vehicle for teaching students to communicate mathematically. Through describing carefully simple proofs and algorithms (e.g., instructions for building a Lego model), students acquire technical writing skills that will be useful in a variety of career and life situations.

Joseph Malkevitch's article **Discrete Mathematics and Public Perceptions of Mathematics** contrasts the kinds of problems typically discussed in high school mathematics classes, usually involving extensive manipulation of symbols, with the kinds of problems that manifest the ways in which mathematics influences daily life. Malkevitch argues that the negative perceptions that the general public has about mathematics arise in part from an unbalanced mathematical diet — too much of the former, too little of the latter — and notes that problems from discrete mathematics can play an important role in changing these perceptions.

Henry O. Pollak's article **Mathematical Modeling and Discrete Mathematics** discusses mathematical modeling in general, noting that "applied mathematics", "problem solving", and "word problems" all start with an idealized version of a real world problem, and so normally omit the initial and final parts of the modeling process. The author notes that in discrete mathematics situations, however, it is often possible to introduce the entire mathematical modeling process into the classroom; he provides five examples of modeling situations which lead to discrete mathematics and which can be made accessible to high school students.

Fred S. Roberts' article **The Role of Applications in Teaching Discrete Mathematics** notes that "one of the major reasons for the great increase in interest in discrete mathematics is its importance in solving practical problems." The author introduces several "rules of thumb" about the role of applications in teaching discrete mathematics, and illustrates those by providing many applications of the Traveling Salesman Problem, graph coloring, and Euler paths.

Section 3. What is Discrete Mathematics: Two Perspectives

Stephen B. Maurer's article **"What is Discrete Mathematics?" The Many Answers** provides and discusses a variety of proposed definitions and descriptions of discrete mathematics, along with several proposed goals and benefits for including discrete mathematics in the schools. The article concludes with a set of goals and topics for discrete mathematics in the schools on which the author thinks there might be general agreement.

Joseph G. Rosenstein's article **A Comprehensive View of Discrete Mathematics: Chapter 14 of the New Jersey Mathematics Curriculum Framework** contains a comprehensive discussion of topics of discrete mathematics appropriate for each of the K–2, 3–4, 5–6, 7–8, and 9–12 grade levels. The author spearheaded the development of the Framework in his role as Director of the New Jersey Mathematics Coalition. Grade-level overviews are accompanied by several hundred activities appropriate for the various grade levels. The material reflects the experiences of teachers in the Leadership Program in Discrete Mathematics, discussed in a separate article in Section 8.

Section 4. Integrating Discrete Mathematics into Existing Mathematics Curricula, Grades K–8

Valerie A. DeBellis' article **Discrete Mathematics in K–2 Classrooms** describes the author's visits to several classrooms and what she learned about the reasoning and problem-solving skills exhibited by young children who are introduced to situations involving discrete mathematics. It also describes how topics in discrete mathematics can be reformulated for children at early elementary levels.

Robert E. Jamison's article **Rhythm and Pattern: Discrete Mathematics with an Artistic Connection for Elementary School Teachers** describes the material that the author has used in programs for both inservice and preservice elementary school teachers. It focuses on how elementary school teachers can use geometric activities involving drawing polygons and planar representations of polyhedra, moving in geometric patterns, and using modular arithmetic in movement and music — to provide their students with foundational experiences for future study of mathematics.

Evan Maletsky's article **Discrete Mathematics Activities in Middle School** provides a wealth of activities that are appropriate at the middle school level; these involve counting (e.g., finding the triangular numbers when you count rectangles on a folded piece of paper), graphs, and iteration (e.g., generating Sierpinski triangles). The author discusses how these can be incorporated into the activities that are already taking place in the classroom.

Section 5. Integrating Discrete Mathematics into Existing Mathematics Curricula, Grades 9–12

Robert L. Devaney's article **Putting Chaos into Calculus Courses** describes how fundamental ideas of dynamical systems, including iteration, attracting and repelling points, and chaos, can be introduced in a beginning calculus class, through an in-depth investigation of the behavior of Newton's Method, using a computer or graphing calculator. The author's approach integrates discrete with continuous mathematics and provides a connection from calculus to the fascinating world of fractals and chaos.

John A. Dossey's article **Making a Difference with Difference Equations** shows how difference equations can be used to model change in a number of real-world settings. The author recommends the use of difference equations to provide a unified development of standard sequences studied in mathematics, such as arithmetic, geometric, and Fibonacci sequences.

Eric W. Hart's article **Discrete Mathematical Modeling in the Secondary Curriculum: Rationale and Examples from the Core-Plus Mathematics Project (CPMP)** discusses the questions of what discrete mathematics belongs in the secondary curriculum, and how it should be incorporated, from the perspective of the curriculum developer. The article presents examples adapted from CPMP materials which illustrate the CPMP approach — that discrete mathematics should be woven into an overall integrated mathematics curriculum, and that the emphasis should be on discrete mathematical modeling.

Bret Hoyer's article **A Discrete Mathematics Experience with General Mathematics Students** describes how the author introduced topics in discrete mathematics first into intermediate algebra and geometry classes, and then, as a result of the students' positive experiences, into other classes as well — including general mathematics and consumer mathematics courses. The article focuses on the "Street Networks" unit on Euler paths and circuits that was woven into these courses.

Philip G. Lewis' article **Algorithms, Algebra, and the Computer Lab** describes how the author's high school students used the LOGO computer environment to explore and develop concepts in linear algebra. These explorations, which took place in a computer lab, enabled students to view linear algebra algorithmically and to learn how to construct and analyze algorithms.

Joan Reinthaler's article **Discrete Mathematics is Already in the Classroom — But It's Hiding** argues that many problems in high school courses are discussed as problems with continuous domains when a discrete perspective would be more realistic, and would lead to different investigations and solutions. Several examples are given involving standard textbook problems in algebra.

James T. Sandefur's article **Integrating Discrete Mathematics into the Curriculum: An Example** describes how he uses the handshake problem to review with his precalculus class the notions of function, domain and range, and graphing quadratic functions. The author argues that "this approach integrates discrete mathematics into the existing curriculum, results in deeper student understanding, and can be accomplished in about the same amount of time as is presently devoted to these topics."

Section 6. High School Courses on Discrete Mathematics

Harold F. Bailey's article **The Status of Discrete Mathematics in the High Schools** reports on a survey that the author did to ascertain how many high schools offer courses in discrete mathematics, what those courses contain, and the goals of the schools in offering such courses.

L. Charles Biehl's article **Discrete Mathematics: A Fresh Start for Secondary Students** describes a project-based discrete mathematics course developed by the author for juniors and seniors of average ability. The students explored a variety of mathematical topics in real-world settings; moreover, since many topics in discrete mathematics have few prerequisites, these students were able to become successful problem solvers and to develop more positive attitudes to mathematics. The article includes an outline of the course.

Nancy Crisler, Patience Fisher, and Gary Froelich's article **A Discrete Mathematics Textbook for High Schools** describes the textbook they have co-authored, providing a discussion of its origins and development. The organization and content of the book is based on the NCTM report, *Discrete Mathematics and the Secondary Mathematics Curriculum*; it addresses five broad areas (social decision making, graph theory, counting techniques, matrix models, and the mathematics of iteration) and interweaves six unifying themes (modeling, use of technology, algorithmic thinking, recursive thinking, decision making, and mathematical induction). The article includes summaries of and examples drawn from each chapter of the book.

Section 7: Discrete Mathematics and Computer Science

Peter B. Henderson's article **Computer Science, Problem Solving, and Discrete Mathematics** addresses the role of discrete mathematics in a first course in computer science, based on the author's experience in developing a "Fundamentals of Computer Science" course at SUNY Stony Brook. Although the course described was developed originally for students planning a career in computer science, it has drawn students with a wide variety of goals. The author notes that "With its emphasis on logical reasoning and problem analysis and solution, discrete mathematics provides a catalyst for general thinking and problem-solving skills ... ," making such a course valuable for teaching computer science to high school students as well.

Viera K. Proulx' article **The Role of Computer Science and Discrete Mathematics in the High School Curriculum** identifies six key themes in computer science that the author argues should be taught to all high school students, and sketches activities for students to explore these themes. The ideas in the article grew out of the author's participation in the Association for Computing Machinery (ACM) Task Force on the High School Curriculum, which produced a "Model High School Computer Science Curriculum" in 1993.

Section 8. Resources for Teachers

Nathaniel Dean and Yanxi Liu's article **Discrete Mathematics Software for K–12 Education** describes two workshops involving teachers and software developers in which teachers solved problems using software developed for research, and shared their reflections on the features that would make such software useful in their classrooms. In the first workshop, teachers used NETPAD, written by Dean when he was at Bellcore; in the second workshop, teachers used Combinatorica, written by Steven Skiena of SUNY Stony Brook. The article also provides an annotated list of other software packages that are potentially useful to teachers.

Deborah S. Franzblau and Janice C. Kowalczyk's article **Recommended Resources for Teaching Discrete Mathematics** identifies outstanding resources, including books, modules, periodicals, literature, Internet sites, software, and videos for the K–12 mathematics teacher or supervisor building a core resource library for teaching topics in discrete mathematics. There are extensive reviews of four popular textbooks; other resources are accompanied by briefer descriptions. The list of resources, which is indexed by topic and grade level, and which includes publisher information, was developed from recommendations by participants and instructors in the DIMACS Leadership Program in Discrete Mathematics.

Joseph G. Rosenstein and Valerie A. DeBellis' article **The Leadership Program in Discrete Mathematics** describes the DIMACS-sponsored programs for K–12 teachers that have taken place for the past nine years at Rutgers University, the development and implementation of the program's goals, and how the program is serving as a continuous resource for the dissemination of discrete mathematics to K–12 schools.

Mario Vassallo and Anthony Ralston's article **Computer Software for the Teaching of Discrete Mathematics in the Schools** provides a number of criteria for judging the suitability of computer software for educational use, and then describes and evaluates three software systems (Mathematica/Combinatorica, GraphPack, and SetPlayer) against these criteria.