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Introduction

Discrete Mathematics in the Schools: An Opportunity to Revitalize School Mathematics

Joseph G. Rosenstein

This article serves as an introduction in four different but overlapping ways:

- As an introduction to a volume advocating discrete mathematics in the schools, it outlines the case for this position.
- As an introduction to a collection of thirty-four diverse articles, it provides some context for those articles.
- As an introduction to the 1992 conference which led to this volume, it provides information about the conference and its themes.
- As an introduction to my perspective as conference organizer, author, and editor, it summarizes the main reasons for my involvement in this enterprise.

The author's perspective

Starting at the end, which is of course the beginning, there are two major reasons for my ongoing efforts to promote discrete mathematics in the schools — that in two major ways, discrete mathematics offers an opportunity to revitalize school mathematics.

- Discrete mathematics offers a new start for students. For the student who has been unsuccessful with mathematics, it offers the possibility for success. For the talented student who has lost interest in mathematics, it offers the possibility of challenge.
- Discrete mathematics provides an opportunity to focus on *how* mathematics is taught, on giving teachers new ways of looking at mathematics and new ways of making it accessible to their students. From this perspective, teaching discrete mathematics in the schools is not an end in itself, but a tool for reforming mathematics education.

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These two themes first appeared in a concept document that I developed in January 1991 and that grew out of the first two years of my experience directing the Leadership Program in Discrete Mathematics, an NSF-funded teacher enhancement program for high school teachers, at Rutgers University.[1] Participants reported changes in their classrooms, in their students, and in themselves. Their successes taught us that discrete mathematics was not just another piece of the curriculum. Many participants reported success with a variety of students at a variety of levels, demonstrated a new enthusiasm for teaching in new ways, and proselytized among their colleagues and administrators.

These two themes are discussed further in this article in sections entitled **Discrete mathematics: A new start for students** and **Discrete mathematics: A vehicle for improving mathematics education.**

The October 1992 Conference

These two views of discrete mathematics — as a new start for students and as a vehicle for improving mathematics education — seemed to me to establish an agenda for those interested in both discrete mathematics and mathematics education. If discrete mathematics could have a significant impact on mathematics education, how can that impact be actualized? This question led to a conference entitled "Discrete Mathematics in the Schools: How Do We Make an Impact?"

The Conference took place on October 2-4, 1992 at Rutgers University and was sponsored by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS), an NSF-funded Science and Technology Center. It brought together thirty-three educators who had been involved in a variety of ways in introducing discrete mathematics in the schools; see Appendix A for a list of conference participants. The concept document containing the two themes described above was distributed in advance of the conference and was reflected in the opening presentation at which I welcomed and challenged the conference participants.

The conference program was designed to inform the participants about various perspectives of discrete mathematics and its role in K–12 education, and about all of the various activities taking place that promoted discrete mathematics in the schools. An abbreviated version of the program, showing presentations and session titles, appears in Appendix B. Presentations were followed by extended discussions.

[1]The NSF-funded Leadership Program in Discrete Mathematics is co-sponsored by the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS) and the Rutgers Center for Mathematics, Science, and Computer Science Education (CMSCE). Although originally (in 1989-1991) for high school teachers, the Leadership Program subsequently (beginning in 1992) also enrolled middle school teachers, and now (since 1995) focuses on K–8 teachers. See the article by Rosenstein and DeBellis in this volume for further information about the Leadership Program.

One outcome of the discussions at the conference was the **Vision Statement** which appears at the beginning of this volume. Two major points of the Vision Statement were that "discrete mathematics is an exciting and appropriate vehicle for working toward and achieving these goals" (referring to the goals of those striving to improve mathematics education), and that "discrete mathematics needs to be introduced into the curriculum for its own sake" because of the increasing importance and prevalence of its applications.

What is discrete mathematics?

It is, of course, natural for K–12 teachers and administrators, as well as parents and the press, to ask this question. Unfortunately, it is not an easy question to answer. The problem is that the phrase "discrete mathematics" does not refer to a well-defined branch of mathematics — like algebra, geometry, trigonometry, or calculus — but rather encompasses a variety of loosely-connected concepts and techniques. Moreover, it is not a branch of mathematics which is generally familiar to the public. At the dedication ceremony of DIMACS as a Center in 1989, then-Governor Thomas Kean (NJ) quipped that, before participating in this ceremony, his impression was that discrete mathematics was what accountants did behind closed doors. That may be a common initial impression of discrete mathematics.

I have found that one effective way of answering the question is by giving lots of examples of the kinds of situations where the mathematics that is used is "discrete". Though not actually defining discrete mathematics, the examples give a flavor of what comprises discrete mathematics, and also helps to demystify the phrase. Here is the list that we are currently using in one of the brochures of the Leadership Program in Discrete Mathematics; this list contains examples that we anticipate will make sense to the teachers that we hope to attract to the program.

- What is the quickest way to sort a list of names alphabetically?
- Which way of connecting a number of sites into a telephone network requires the least amount of cable?
- Which version of a lottery gives the best odds?
- If each voter ranks the candidates for President in order of preference, how can a consensus ranking of the candidates be obtained?
- What is the best way for a robot to pick up items stored in an automated warehouse?
- How does a CD player interpret the codes on a CD correctly even if the CD is scratched?
- How can an estate be divided fairly?
- How can ice cream stands be placed at various street corners in a town so that at any corner there is a stand which is at most one block away?
- How can representatives be apportioned fairly among the states using current census information?

These problems — and many others from different areas within discrete mathematics — share several important characteristics. They are easily understood and discussed, readily seen as dealing with real-world situations, and can be explored without extensive background in school mathematics. This is discussed in more detail in the following section.

Although I have used this "definition-by-examples" of discrete mathematics for a number of years, in the spring of 1996, as the New Jersey Department of Education was preparing to present its recommendations for mathematics standards to the State Board of Education, I was told that I had to provide a "real definition" for the document. So here is discrete mathematics as it appears in New Jersey's *Core Curriculum Content Standards*:

> Discrete mathematics is the branch of mathematics that deals with arrangements of discrete objects. It includes a wide variety of topics and techniques that arise in everyday life, such as how to find the best route from one city to another, where the objects are cities arranged on a map. It also includes how to count the number of different combinations of toppings for pizzas, how best to schedule a list of tasks to be done, and how computers store and retrieve arrangements of information on a screen. Discrete mathematics is the mathematics used by decision-makers in our society, from workers in government to those in health care, transportation, and telecommunications. Its various applications help students see the relevance of mathematics in the real world.

In This Volume. Two articles in Section 3 of this volume address directly the question, "What is discrete mathematics?" Stephen Maurer's article explores a number of possible charactizations of discrete mathematics, none of which proves to be fully satisfactory. Joseph Rosenstein's article provides an extended elaboration of the description above, as it appears in the *New Jersey Mathematics Curriculum Framework.*

Why introduce discrete mathematics into the curriculum?

A number of different arguments have been presented for including discrete mathematics in the school curriculum; these arguments can each be viewed against the backdrop of the problems posed above. Discrete mathematics is:

Applicable: In recent years, topics in discrete mathematics have become valuable tools and provide powerful models in a number of different areas.

Accessible: In order to understand many of these applications, arithmetic is often sufficient, and many others are accessible with only elementary algebra.

Attractive: Though easily stated, many problems are challenging, can interest and attract students, and lend themselves to exploration and discovery.

Appropriate: Both for students who are accustomed to success and are already contemplating scientific careers, and for students who are accustomed to failure and perhaps need a fresh start in mathematics.

In This Volume. A number of articles in this volume illustrate and elaborate on these reasons for incorporating discrete mathematics into the curriculum. Several articles that particularly address each of the above themes are provided below.

Applicable: The articles by Henry Pollak, Fred Roberts, John Dossey, and Eric Hart address the applications of discrete mathematics and how it provides models for real-world situations.

Accessible: The articles by Janice Kowalczyk, Susan Picker, Nancy Casey and Michael Fellows, Joseph Rosenstein, Valerie DeBellis, Robert Jamison, and Evan Maletsky show, for example, how discrete mathematics can be used in elementary and middle school grades.

Attractive: The articles by Patrick Carney, Nancy Casey, Reuben Settergren, and Margaret Cozzens discuss how discrete mathematics excites student interest.

Appropriate: The articles by Nancy Casey, Susan Picker, Bret Hoyer, and L. Charles Biehl discuss how discrete mathematics is appropriate for students who need a fresh start in mathematics. Other articles in this volume discuss how discrete mathematics can be combined with and enhance existing topics like algebra (Bret Hoyer, Philip Lewis), precalculus (John Dossey, Joan Reinthaler, James Sandefur), calculus (Robert Devaney), and computer science (Peter Henderson, Vera Proulx).

Discrete mathematics: A new start for students

The traditional topics of school mathematics — arithmetic, algebra, geometry, etc. — are of course important; without a good grounding in these topics, students will be seriously disadvantaged in career options. And the nation will continue to have a serious shortfall in technically skilled personnel.

However, many students find school mathematics to be a serious stumbling block, and ultimately give up. The most frequently prescribed remedy for students who have failed in school mathematics appears, unfortunately, to be more of the same. And "more of the same" usually means not only repetition of content, but also repetition of method. Thus, many students come to see school mathematics only as a set of unintelligible procedures, which is not surprising since they were never given an opportunity to explore concepts meaningfully and apply them in new situations.

At the other end of the spectrum, many talented students also find school mathematics to be uninteresting and irrelevant, and thus opt for other careers. For these students, who are looking for a spark of life and challenge in mathematics, a frequent response is "wait until you get to calculus"; but many have lost interest by the time they get to calculus.

Discrete mathematics offers a new start. For the student who has been unsuccessful in mathematics, discrete mathematics offers the possibility of success. Students who have encountered mathematics which they can do successfully are encouraged to take another look at the mathematics at which they have failed. Students who have found that they can solve meaningful problems gain a sense of empowerment. Teachers in the Leadership Program have reported that, for students who have a history of failure in mathematics, being able to use terminology and solve problems in areas with which other school personnel — teachers and guidance counselors, as well as students — are unfamiliar is a very heady experience.

The ranks of students who have been unsuccessful in mathematics contain a disproportionate number of minorities and women. Such students, who have given up hope of ever learning school mathematics, can become interested in and can learn discrete mathematics since they do not associate it at the outset with routine school mathematics. Teachers in the Leadership Program in Discrete Mathematics have used discrete mathematics successfully with these students in all types of schools, including those in urban areas.

For the talented student who has lost interest in mathematics, discrete mathematics offers the possibility of challenge. Discrete mathematics serves as a natural context for many of the puzzle-like questions that intrigue the talented student, offers open-ended problems which quickly lead to the frontiers of knowledge, and provides easy access to applications which mathematicians are now making in a variety of real-life situations. One can imagine students engaged in discrete mathematics saying "This is how I would like to spend my professional life", as well as "This is fun".

In This Volume. See the articles cited under "accessible", "attractive", and "appropriate" in the previous section.

Discrete mathematics: A vehicle for improving mathematics education.

The introduction of new material into the curriculum affords a particular opportunity to infuse new instructional techniques at the same time. When there is no specific body of material that districts and teachers feel obligated to "cover", there is clearly "time" for experimentation — with computers, with group learning, with problem solving. When the problems are new to the teachers, and close to the cutting edge of knowledge, there is greater acceptance of a classroom open to discussion, to reasoning together, and to the excitement of discovering new solutions which are not "in the book".

Moreover, as teachers become familiar with these techniques and see that they work with their students in their own classrooms, they will adapt them for use in their other classes. Those teachers who have taken the time from traditional teacher-oriented instruction to try these learner-oriented techniques know that the time is well spent. The difficulty is in getting them to try.

Discrete mathematics offers a wealth of new material and, more important in this context, consists of many topics which lend themselves readily to approaches to learning that are recommended in the national reports: discovery learning, experimentation, problem solving, cooperative learning, use of technology. With discrete mathematics, students can easily become involved in the doing of mathematics, can see themselves as "mathematicians" rather than as followers of routine instructions.

In This Volume. Nancy Casey and Michael Fellows argue in their article that only if they use discrete mathematics will K–4 teachers have sufficiently rich mathematical content to properly address the process standards of "reasoning, problem-solving, communications, and connections" stressed in the NCTM Standards.[2] Other articles focus on how discrete mathematics can help teachers achieve educational objectives such as teaching students mathematical communication (Rochelle Leibowitz), reasoning (Susanna Epp), and problem-solving (Margaret Cozzens, Peter Henderson), and change public perceptions of mathematics (Joseph Malkevitch). The article by Joseph Rosenstein and Valerie DeBellis discusses the impact of the Leadership Program in Discrete Mathematics on the activities of its participants.

Resources for introducing discrete mathematics in the schools

At the time of the conference, there were relatively few resources available to teachers interested in including discrete mathematics in their classrooms and curricula. Increasingly in recent years, in part because discrete mathematics is addressed in the NCTM Standards, more effort has been placed both on developing materials related to discrete mathematics and to incorporating discrete mathematics activities in textbooks. As a result of the efforts of the Leadership Program in Discrete Mathematics and the "Implementation of the NCTM Standard in Discrete Mathematics Project" program directed by Margaret Kenney at Boston College and other sites across the country, there are now nearly 2000 teachers who have had extensive exposure to discrete mathematics; many of them have been taking leadership roles, developing curriculum materials and making presentations at conferences.

In This Volume. The article by Deborah Franzblau and Janice Kowalczyk, based on recommendations of teachers in the Leadership Program in Discrete Mathematics, provides an extensive review of available print and

[2] *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, 1989, Reston, VA.

video resources. Two articles, one by Eric Hart and the other by Nancy Crisler, Patience Fisher, and Gary Froelich, discuss texts for high school students which include discrete mathematics. Two articles, one by Nate Dean and Yanxi Liu, and the other by Mario Vassallo and Anthony Ralston, discuss discrete mathematics software. Two articles, by Harold Bailey and L. Charles Biehl, discuss high school courses in discrete mathematics. And the article by Joseph Rosenstein and Valerie DeBellis discusses the Leadership Program in Discrete Mathematics.

Conclusion

Speaking for the editors, the conference participants, and the authors, we hope that this volume will be a major contribution both to facilitating the use of discrete mathematics in K–12 schools and to demonstrating the potential of discrete mathematics as a vehicle to improve mathematics education and revitalize school mathematics.

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Appendix A.

Discrete Mathematics in the Schools: How Do We Make an Impact? October 2–4, 1992

Conference Participants

NAME	STATE	AFFILIATION (at time of conference)
Bailey, Harold F.	NY	College of Mount Saint Vincent
Biehl, L. Charles	DE	McKean HS, Wilmington
Carrs, Marjorie		University of Queensland, Brisbane, Australia
Crisler, Nancy	MO	Pattonville School Dist., St. Louis County
Dance, Rosalie	MD	Ballou Science/Math HS, Takoma Park
DeBellis, Valerie	NJ	Rutgers University
Dean, Nathaniel	NJ	Bellcore
Epp, Susanna	IL	DePaul University
Fellows, Michael		University of Victoria, British Columbia, Canada
Froelich, Gary	ND	Bismarck HS
Hart, Eric	IA	Maharishi International University
Henderson, Peter	NY	SUNY Stony Brook
Hoover, Mark	NJ	Educational Testing Service
Hoyer, Bret	IA	John F. Kennedy HS, Cedar Rapids
Kenney, Margaret	MA	Boston College
Kowalczyk, Janice	RI	Teacher Education and Computer Center
Lacampagne, Carol B.	DC	U.S. Department of Education
Leibowitz, Rochelle	MA	Wheaton College
Lewis, Philip G.	MA	Lincoln Sudbury Regional HS
Malkevitch, Joseph	NY	York College (CUNY)
Maltas, James	IA	Malcolm Price Laboratory School, University of Northern Iowa
Maurer, Stephen	PA	Swarthmore College
McGraw, Sue Ann	OR	Lake Oswego HS
Piccolino, Anthony	NJ	Montclair State College
Picker, Susan	NY	Office of the Superintendent, Manhattan Public Schools
Pollak, Henry	NJ	Columbia University
Proulx, Viera	MA	Northeastern University
Reinthaler, Joan	DC	The Sidwell Friends School
Roberts, Fred	NJ	Rutgers University
Rosenstein, Joseph G.	NJ	Rutgers University
Saks, Michael	NJ	Rutgers University
Vassallo, Mario	NY	SUNY Fredonia
Yunker, Lee	IL	Community HS Dist. 94, West Chicago

Appendix B.

Discrete Mathematics in the Schools: How Do We Make an Impact? October 2–4, 1992

Conference Program (Abbreviated)

Friday October 2
- Presentation: Joseph G. Rosenstein
  - **"Discrete mathematics as a new start for students and teachers"**
- Classroom Perspectives, Experiences, and Models — Session 1
  - L. Charles Biehl — **"Discrete mathematics for students of average ability"**
  - Susan Picker — **"Discrete mathematics: Giving remedial students a second chance"**
- Presentation: Stephen Maurer
  - **"What is discrete mathematics: The many answers"**
- Classroom Perspectives, Experiences, and Models — Session 2
  - Gary Froelich — **"A semester discrete mathematics course at the high school level"**
  - James Maltas — **"Implementing a discrete mathematics course for non-math students"**
  - Nancy Crisler — **"My experiences as a teacher and math coordinator"**
  - Philip Lewis — **"Using a computer lab: Algorithms, algebra, and axioms"**
- Presentation: Joseph Malkevitch
  - **"Discrete mathematics and the public's perception of mathematics"**
- Classroom Perspectives, Experiences, and Models — Session 3
  - Rosalie Dance - **"Integrating discrete and continuous approaches in secondary math"**
  - Lee Yunker - **"Current and future trends on discrete mathematics in the curriculum"**
- Presentation: Eric Hart
  - **"Curriculum materials for discrete mathematics in the schools"**
- An overview and a taste of ...
  - For All Practical Purposes — Joe Malkevitch and Tony Piccolino
  - COMAP Project — Nancy Crisler and Gary Froelich
  - UCSMP materials — Susanna Epp
  - CORE-PLUS — Eric Hart
  - Several textbooks — Lee Yunker

Saturday October 3
- Programs for teachers
  - Georgetown project — Rosalie Dance and Joan Reinthaler
  - NCTM project — Peg Kenney and others
  - Iowa Project — Eric Hart and others
  - Rutgers Project — Joe Rosenstein and others

Classroom Perspectives, Experiences, and Models — Session 4
- Joan Reinthaler — "**Teaching modeling to weak math students**"
- Sue Ann McGraw — "**Integrating discrete mathematics into traditional math courses**"
- Bret Hoyer — "**A discrete mathematics course using For All Practical Purposes**"

Perspectives
- Susanna Epp — "**Strengthening thinking skills using discrete mathematics**"
- Rochelle Leibowitz — "**Strengthening writing skills using discrete mathematics**"
- Anthony Piccolino — "**Discrete mathematics: Making math accessible to all**"

Presentation: Henry Pollak
- "**The role of modeling in teaching discrete mathematics**"

Presentation: Fred Roberts
- "**The role of applications in teaching discrete mathematics**"

Presentation: Mario Vassallo
- "**Computer software for teaching discrete mathematics in the schools**"

Presentation: Nate Dean
- "**What computer software is currently being developed?**"

Presentation: Michael Fellows
- "**Discrete mathematics and computer science in the elementary schools**"

"**How Do We Make an Impact?**"
- Organizing our suggestions
- Structuring Sunday's discussions

Sunday October 4

Perspectives
- Viera Proulx — "**Computer science in high school**"
- Peter Henderson — "**Computer science, discrete mathematics, and problem solving**"
- Mark Hoover — "**Assessment and discrete mathematics**"
- Harold Bailey — "**Assessing current practice in discrete mathematics**"

"**How Do We Make an Impact?**"
- Work sessions in smaller groups
- Reports from groups
- The next steps