Construction of combinatorial structures, a theoretical approach

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Abstract

Group actions are reviewed as a tool for constructing combinatorial objects up to isomorphism. The isomorphism classes in these cases are group orbits. The objective is a general algebraic theory for constructing representatives of isomorphism types [?],[?],[?]. Homomorphisms of group actions allow to reduce problem sizes step by step. Construction and isomorphism testing can follow the opposite direction, enlarging objects step by step. In some cases, homorphisms also have to be used to construct smaller objects from bigger ones.

In many of the single steps, a new object is constructed by glueing smaller parts of it together. These parts had been the objects of previous steps. Such a glueing formally is an identification of isomorphic partial structures. All glueings have to be classified with respect to automorphisms of the partial structures. A group theoretic interpretation by double cosets allows to give a unified solution to such problems [?].

Using homorphisms to break up the construction process into many steps often results in situations where only the Identity group remains acting. Of course then all constructed objects are non-isomorphic. So, an explicit listing is not needed in such cases. The isomorphism types then are determined, not constructed but also not only counted. This strategy may result in larger numbers of objects determined up to isomorphisms than the computer can process bits in that time. We give an example of a fast graph generator of that kind [?].

A particularly important homomorphism of group actions is the mapping onto the stabilizer of an object. Using the general approach with this mapping leads to classifying by stabilizer type, i.e. the automorphism group of the objects [?]. This is a constructive version of Burnside's table of marks technique. In particular, for the action on sets of mappings as considered in Polya's theory of counting weighted by automorphism groups, a constructive version is presented.

keywords: Graph generation, isomorphism problem, double coset.

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