Recovering Cosmological Information from Galaxy Spectra

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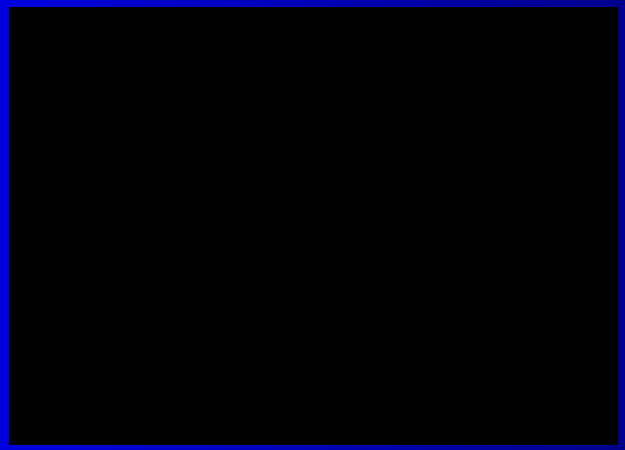
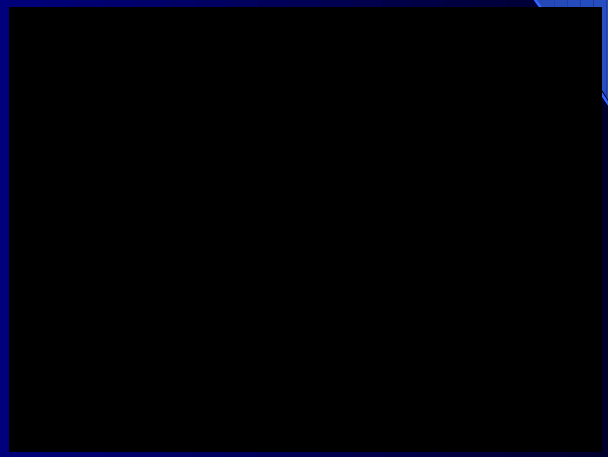
Why do we want to know?

- Galaxy formation and evolution – detailed data constraints on models
- Cosmology – galaxy age < Universe age; independent check on cosmological parameters

Star formation rate

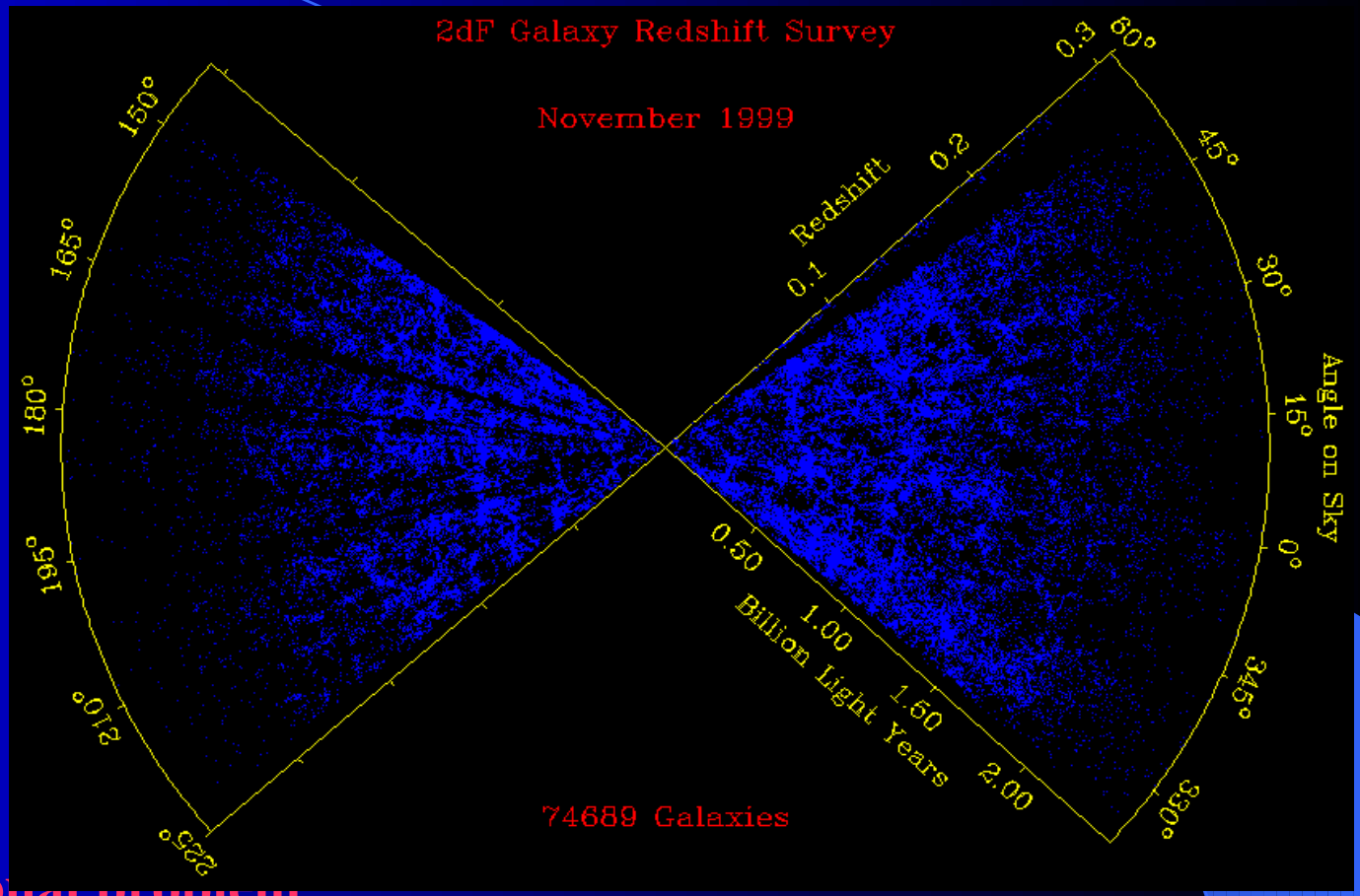
- Measure at varying z





- Deduce past SFR by analysing spectrum

Galaxy spectra → Physical parameters (age, brightness, star formation history...)



BUT with 2dF (or SDSS) Galaxy Redshift Surveys

↓

Large computational problem:

~ 10^3 pixels

~ 10^6 template spectra

up to ~10^6 galaxies

Data compression: 1000 pixels → a few numbers

Data Compression

Aim: To reduce the number of NUMBERS without losing INFORMATION

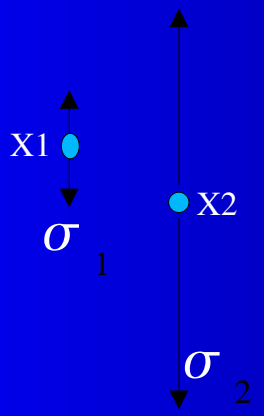
Lossless if successful, i.e. Preserving as much information as possible about the physical parameters.

Method: look for optimal linear combination of the data **x**

$$y = \mathbf{b} \cdot \mathbf{x}$$

for some vector **b**.

e.g. Estimate mean



$$y = \frac{\left(\frac{1}{\sigma_1^2}x_1 + \frac{1}{\sigma_2^2}x_2\right)}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)} = \boldsymbol{b}\boldsymbol{x}$$

Data vector: $\mathbf{x}=(x_1,x_2)$

Weights: $\boldsymbol{b} = \left(\frac{1}{\sigma_1^2}, \frac{1}{\sigma_2^2}\right)$

Optimal method

Which is best **b**? Maximise Fisher Information Matrix

$$F_{\alpha\beta} = -\langle \partial^2 \ln L / \partial\theta_\alpha \theta_\beta \rangle$$

where L=likelihood (function of parameters θ_α).

Problem is intractable: mean spectrum μ , and noise C both depend on the parameters.

Solution: assume that noise does not depend on the parameters. Then the best weighting is

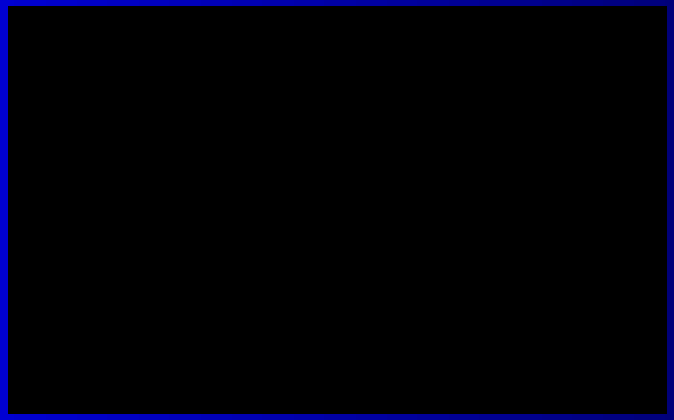
$$b \propto C^{-1} \mu_{,\alpha}$$

Derivative of spectrum w.r.t. parameter

Weight pixels which are sensitive to the parameter, and those which are least noisy.

This is not PCA! (cf Connolly & Szalay 1998/9, Folkes et al 1999)

Details: Fisher Matrix



$$F_{\alpha\beta} \equiv -\left\langle \frac{\partial^2 \ln L}{\partial\theta_\alpha \partial\theta_\beta} \right\rangle$$

$$F_{\alpha\beta} = \frac{1}{2} Trace\left[\mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial\theta_\alpha} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial\theta_\beta} + \mathbf{C}^{-1} \left(\frac{\partial\mu}{\partial\theta_\alpha} \frac{\partial\mu^T}{\partial\theta_\beta} + \frac{\partial\mu^T}{\partial\theta_\alpha} \frac{\partial\mu}{\partial\theta_\beta} \right) \right]$$

Fisher matrix gives best error you can expect:

Error on parameter q_a: $\sigma_\alpha \geq \frac{1}{\sqrt{F_{\alpha\alpha}}}$

- Compute Fisher matrix (scalar) for $y_1=b_1.x$ for general b_1
- Maximise it
- Subject to constraint $b^tCb=1$

$$F_{11} = -\left\langle \frac{\partial^2 \ln L}{\partial \theta_1^{\,2}} \right\rangle$$

Best weighting is

$$b \propto C^{-1} \mu_{,\alpha}$$

Weights pixels which are most sensitive to the parameter, and those which are least noisy. It decides.

More than one parameter

Construct $y_2=b_2.x$ such that y_2 is statistically orthogonal to y_1

- Compute Fisher matrix component for parameter 2, and maximise it

$$F_{22} = -\left\langle \frac{\partial^2 \ln L}{\partial \theta_2^2} \right\rangle$$

- etc

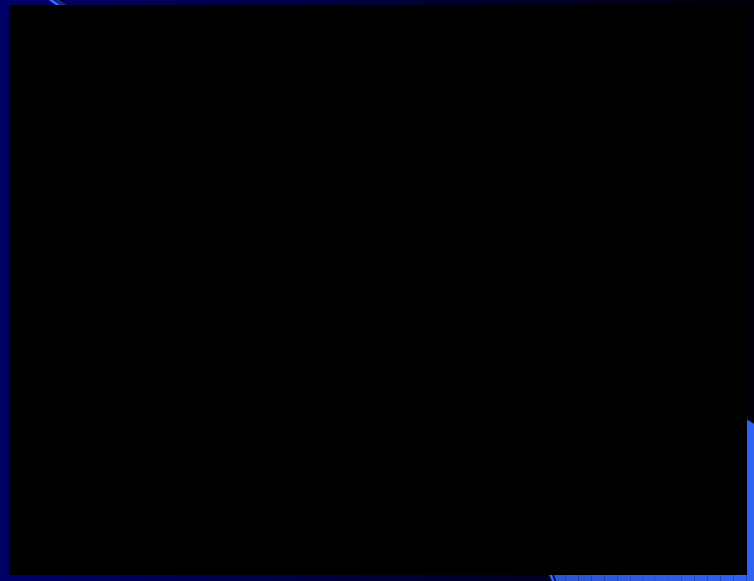
Gram-Schmidt orthogonalisation in a 1000-dimensional curved space

$y = b.x$

Many Parameters

Find $\mathbf{b_1}$ so that y_1 captures most information about age.

Find $\mathbf{b_2}$ so that y_2 captures most information about stellar mass, and which is not already contained in $\mathbf{b_1}$



Each parameter provides ***one*** number - MASSIVE DATA COMPRESSION

METHOD IS LOSSLESS*

* if errors are independent of q

Optimal Method

Find **b1** which captures most information about age

$$b \propto C^{-1}\,\mu,_{\alpha}$$

Find **b2** which is orthogonal to b1 and which captures most information about brightness

$$b_m = \frac{C^{-1}\,\mu,_{\alpha} - \sum_{q=1}^{m-1} (\mu^t,_m b_q) b_q}{\sqrt{\mu,_m C^{-1}\,\mu,_m - \sum_{q=1}^{m-1} (\mu^t,_m b_q)^2}}$$

(Schmidt orthogonalisation in #-dimensional curved spacetime, C is metric tensor)

Etc.

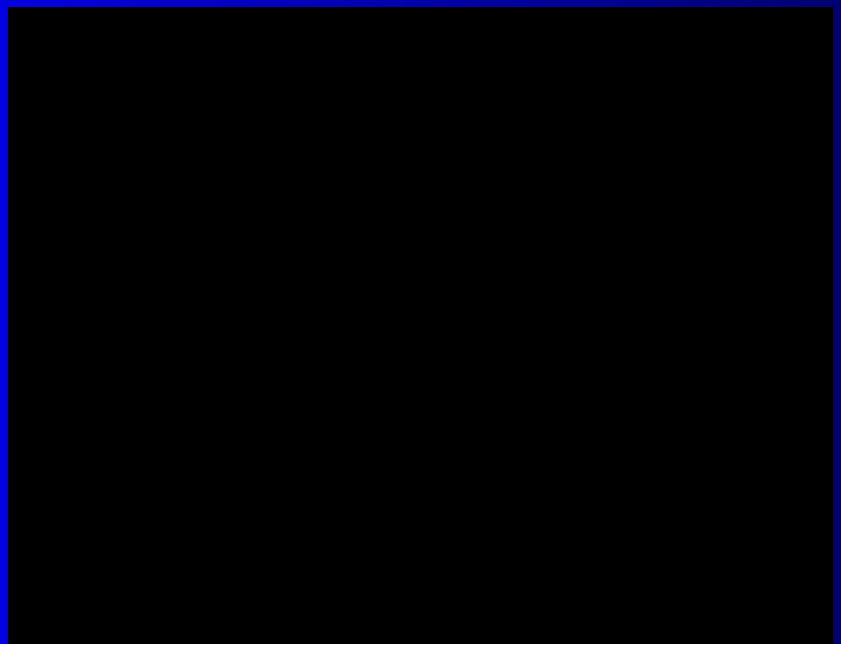
Each parameter provides one number

1000 data points → One number per parameter

Process is lossless: $F(y_1,\ldots,y_m) = F(\text{original data})$

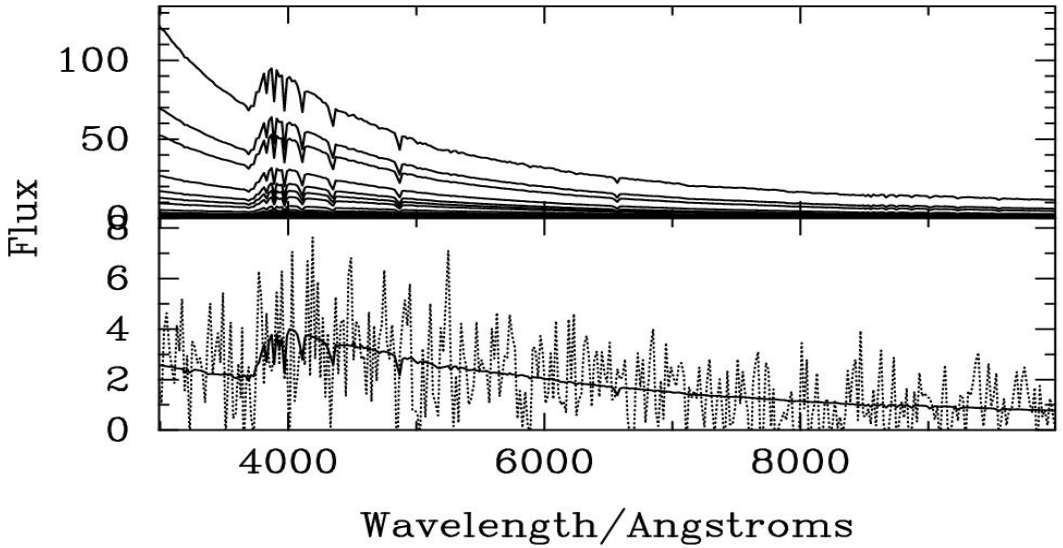
Spectrophotometric models

- Build up spectrum by combining output at different ages of stellar models
- Possible to explore simple models (exponential decaying SFR; burst models) in a reasonable time



Determine best fit by maximum-likelihood:

L(age,…)



Model spectra

Observed spectrum

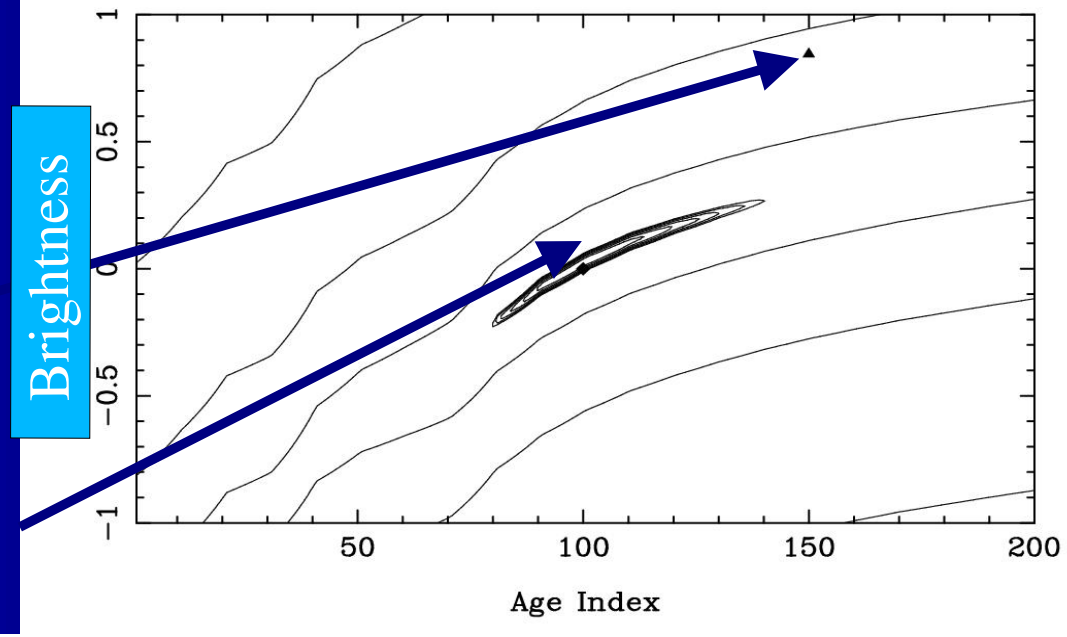
352 pixels

S/N ~ 2

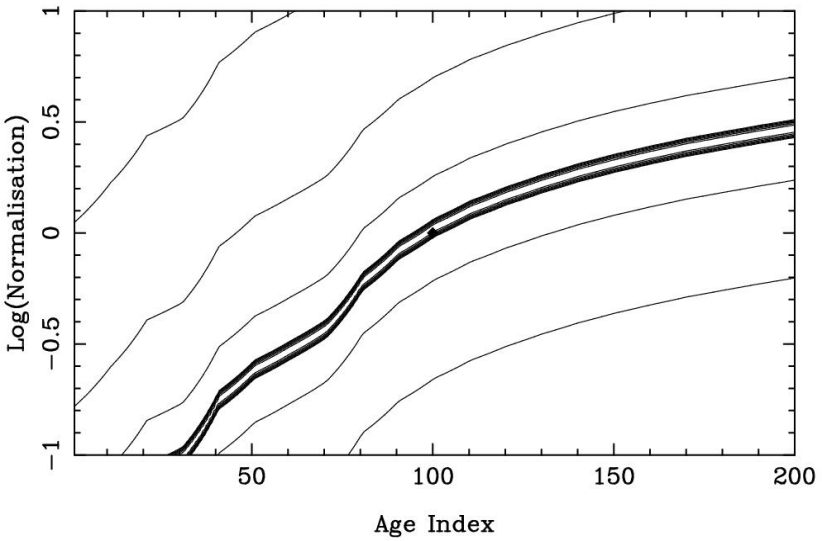
Best: use all the data:

A fiducial model sets the weights **b**

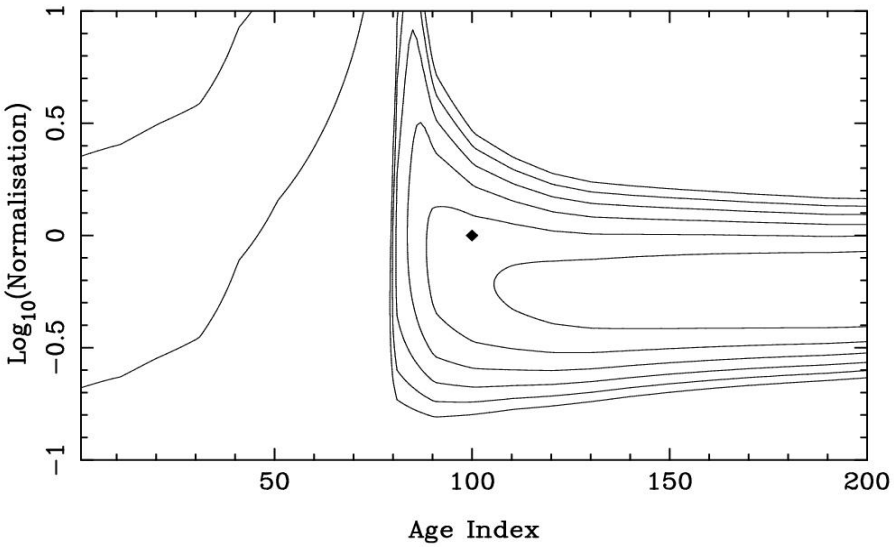
True solution



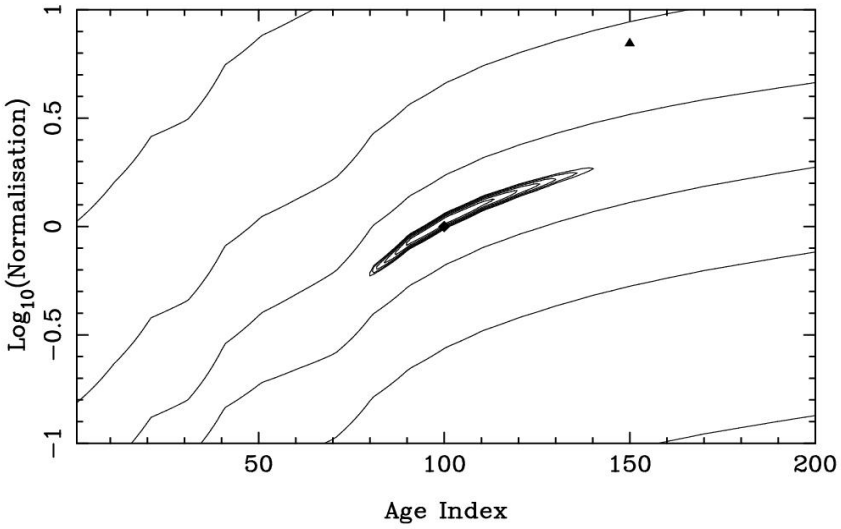
Age eigenvector:



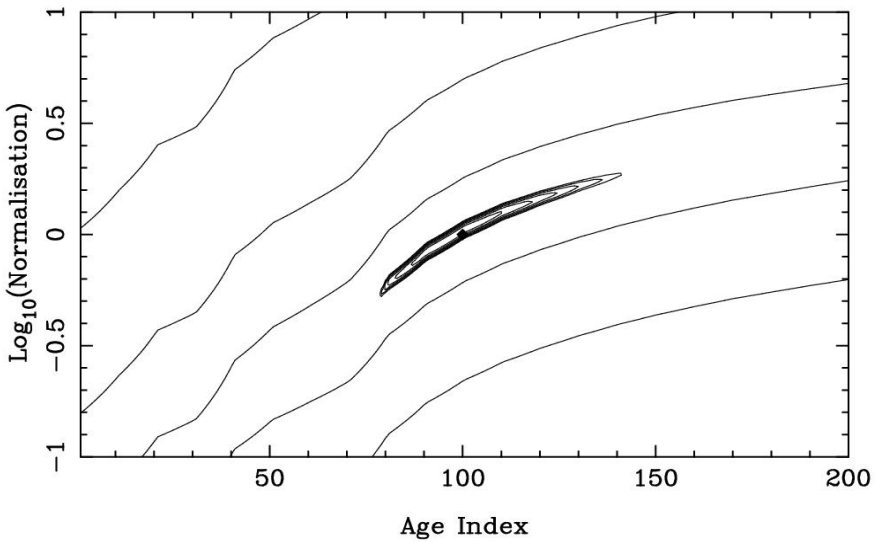
Brightness eigenvector



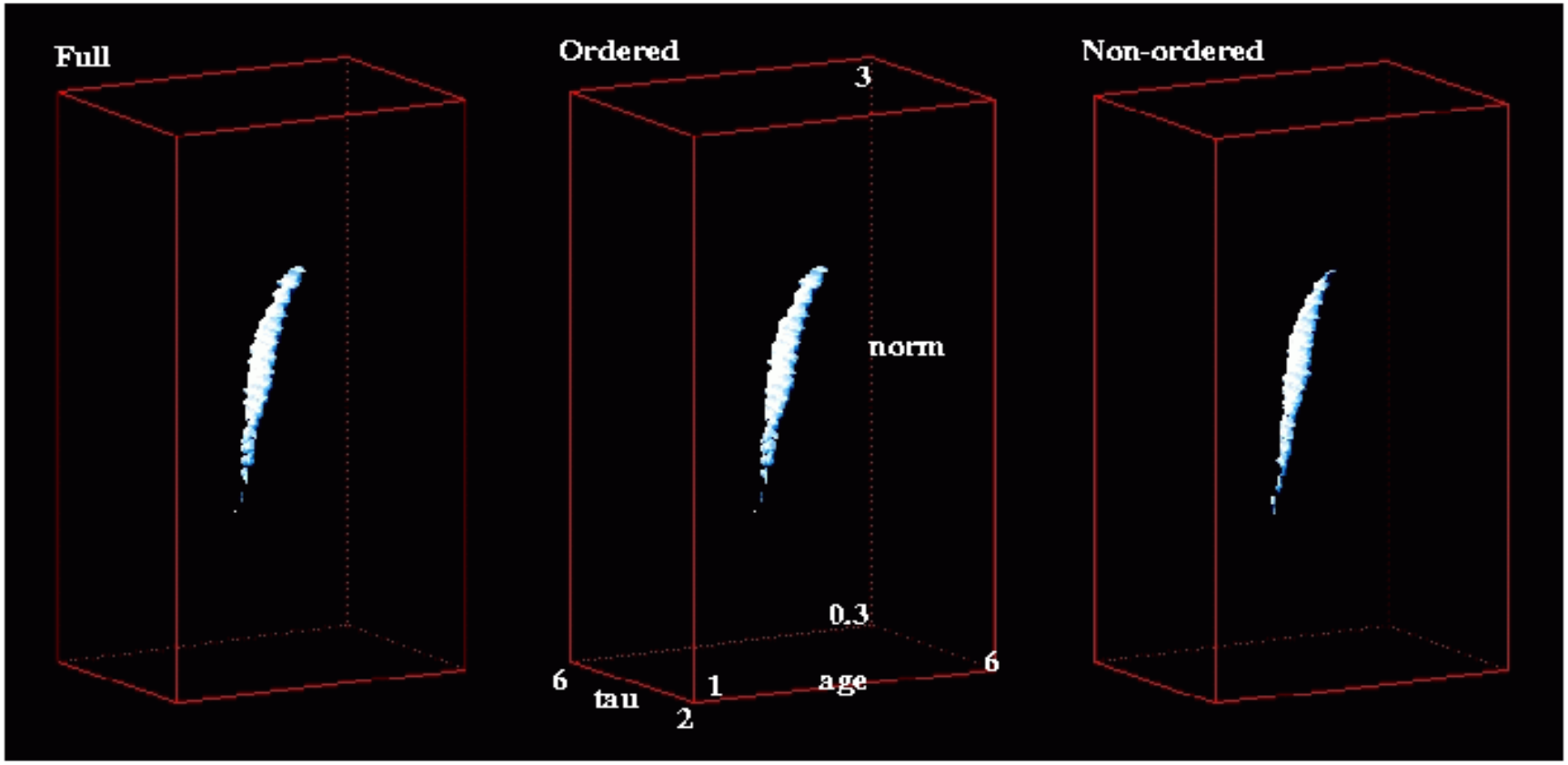
All data points:



Both:

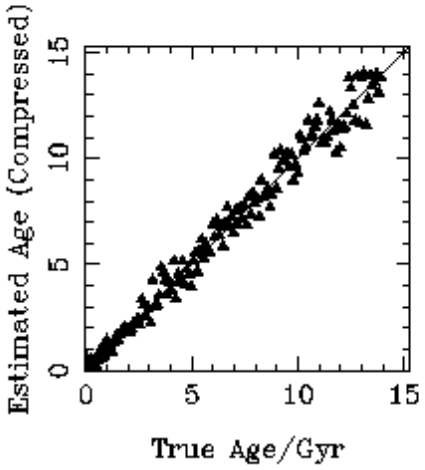


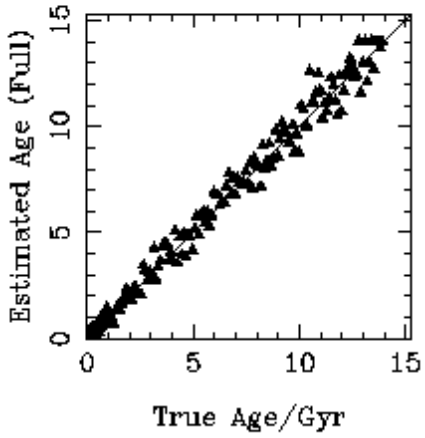
3 parameters:
SFR=Aexp(-t/τ)

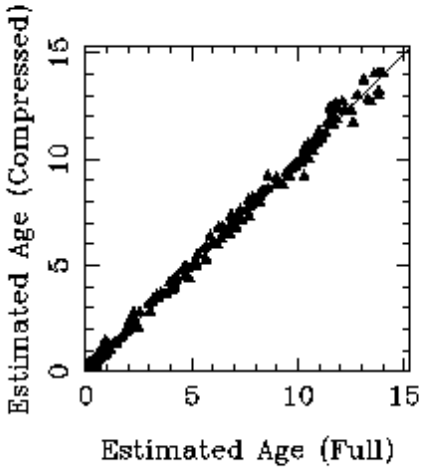


Full data
(1000 numbers)

Compressed data
(3 numbers)



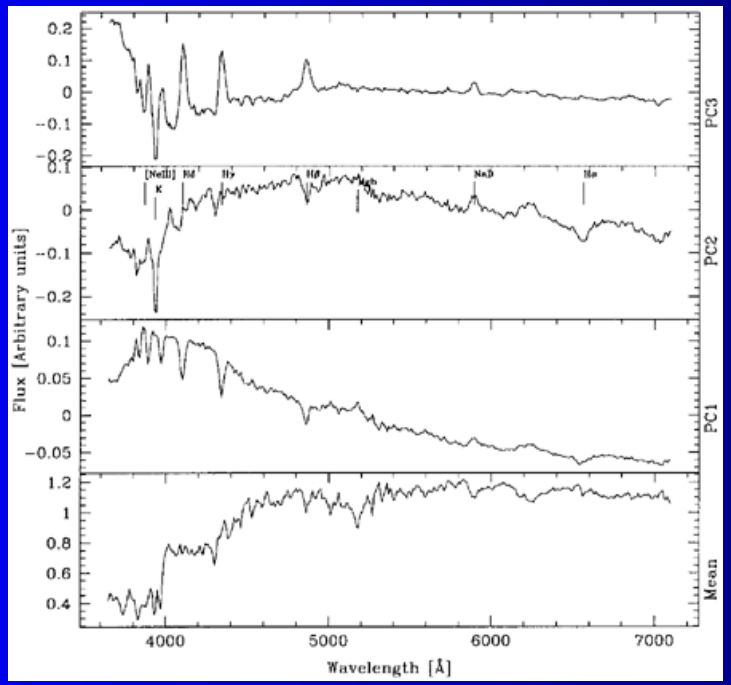




Choice of fiducial model isn't crucial

Aside…history

cf Principal Component Analysis (PCA) e.g. Folkes et al 1999, Ronen Aragón-Salamanca & Lahav 1999.

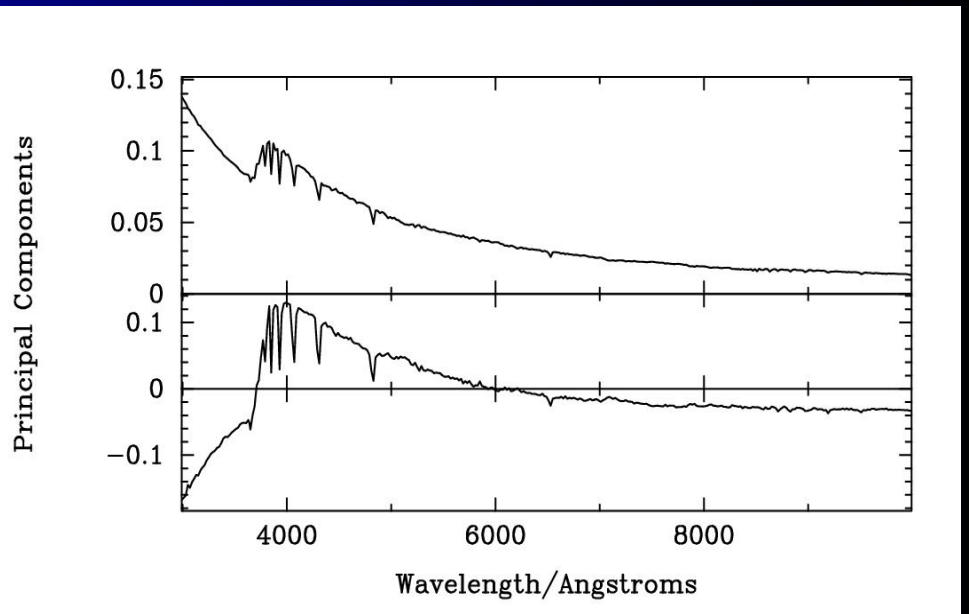


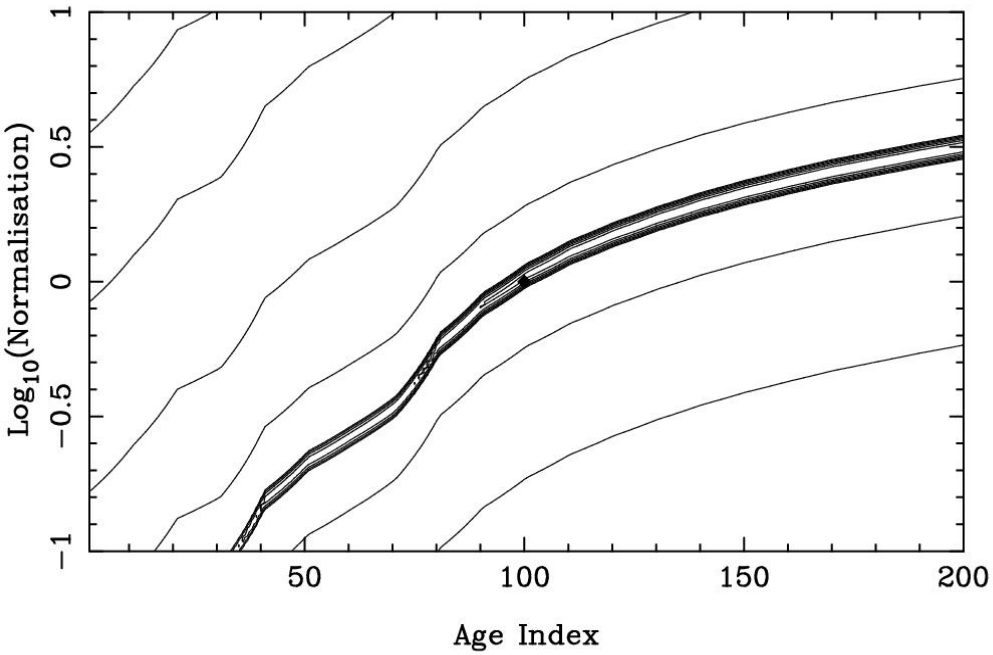
PCA very useful in absence of underlying model

PC1 = e_1.**x** **etc. (x=spectrum)**

PCs age, star formation etc. (approximately)

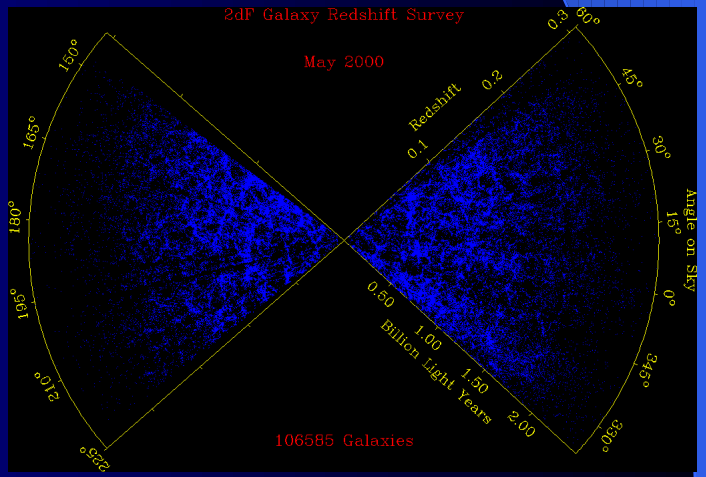
PCA does worse





More realistic modelling

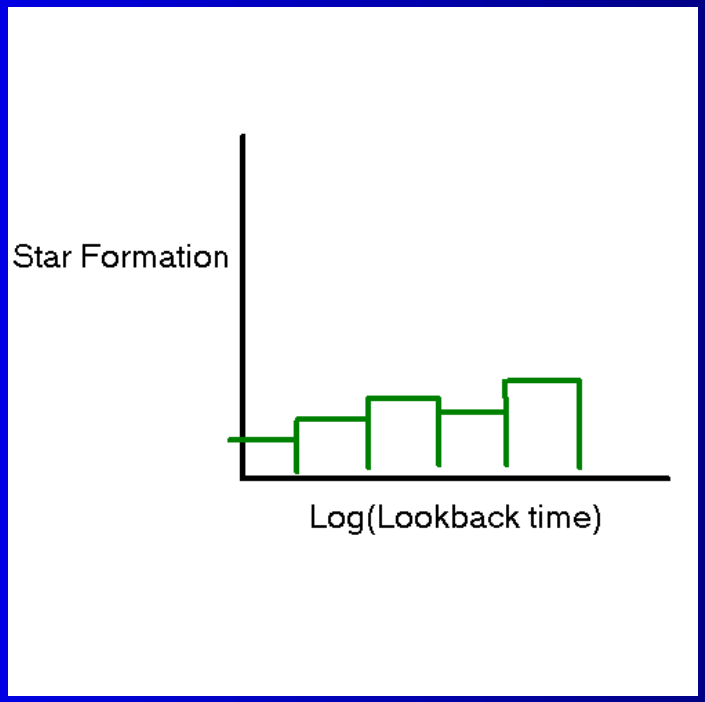
- Less parametric SF history. e.g. 8 bins of time
- Þ ~17 parameters to determine (8 SFR; 8 metallicities; 1 dust parameter)
- Computationally expensive (~5 hours per galaxy). Impractical for 2dF (250,000 galaxies), or Sloan (1,000,000 galaxies)



Galaxy Parameters

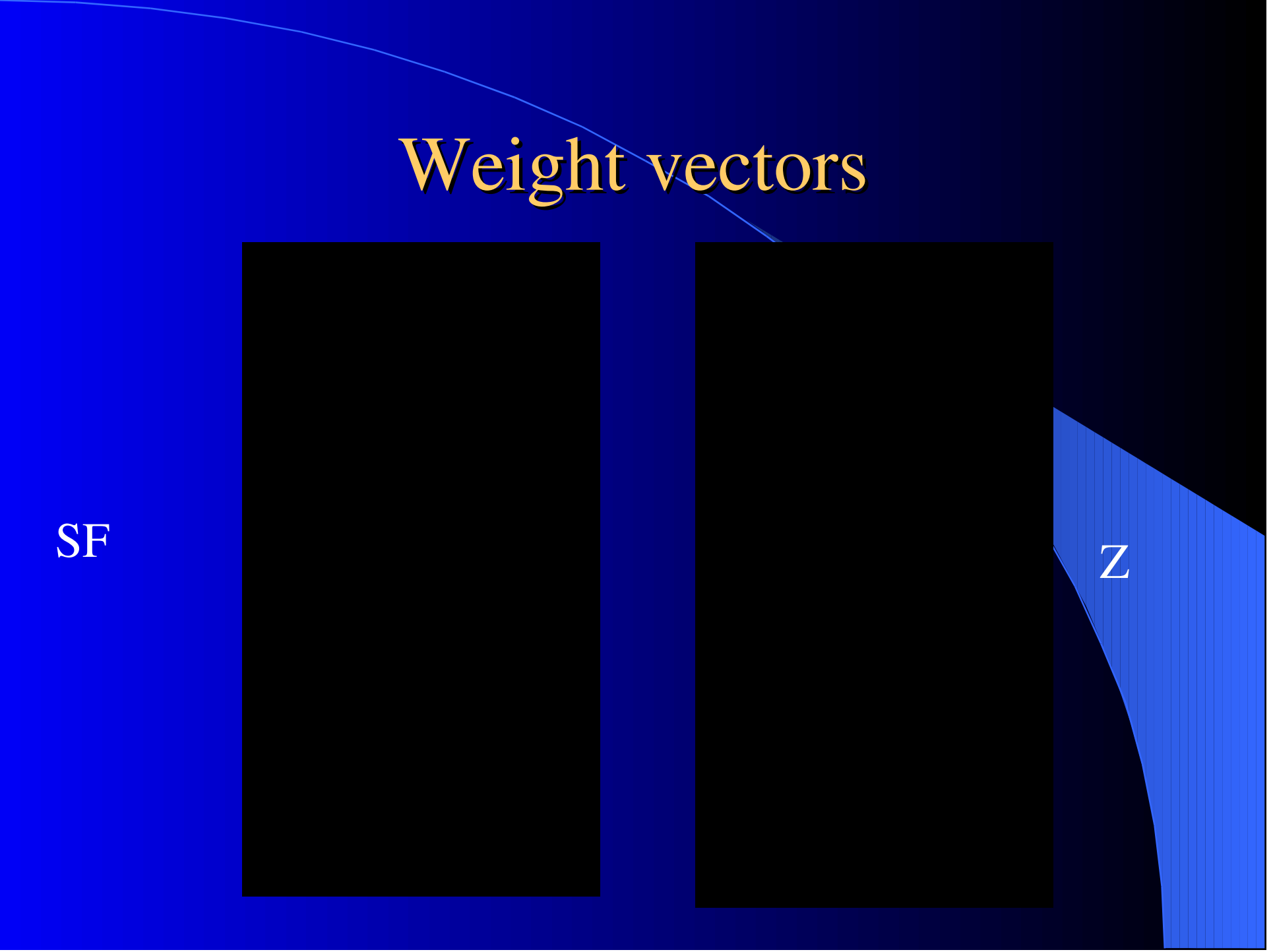
Reichardt, Jimenez & Heavens (astro-ph/0101074)

Star formation and metallicity in 8 time bins; dust



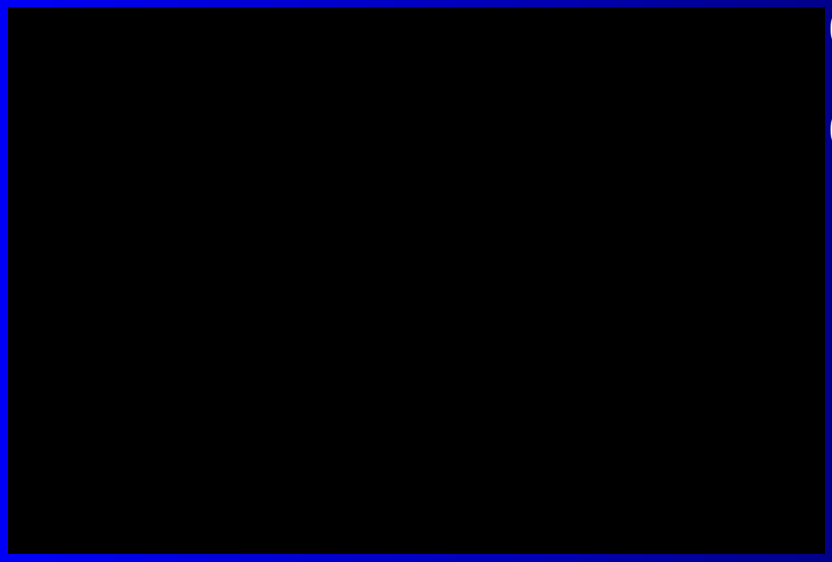
Optimal bins are almost equally spaced in log(lookback time). Choose equally-spaced bins

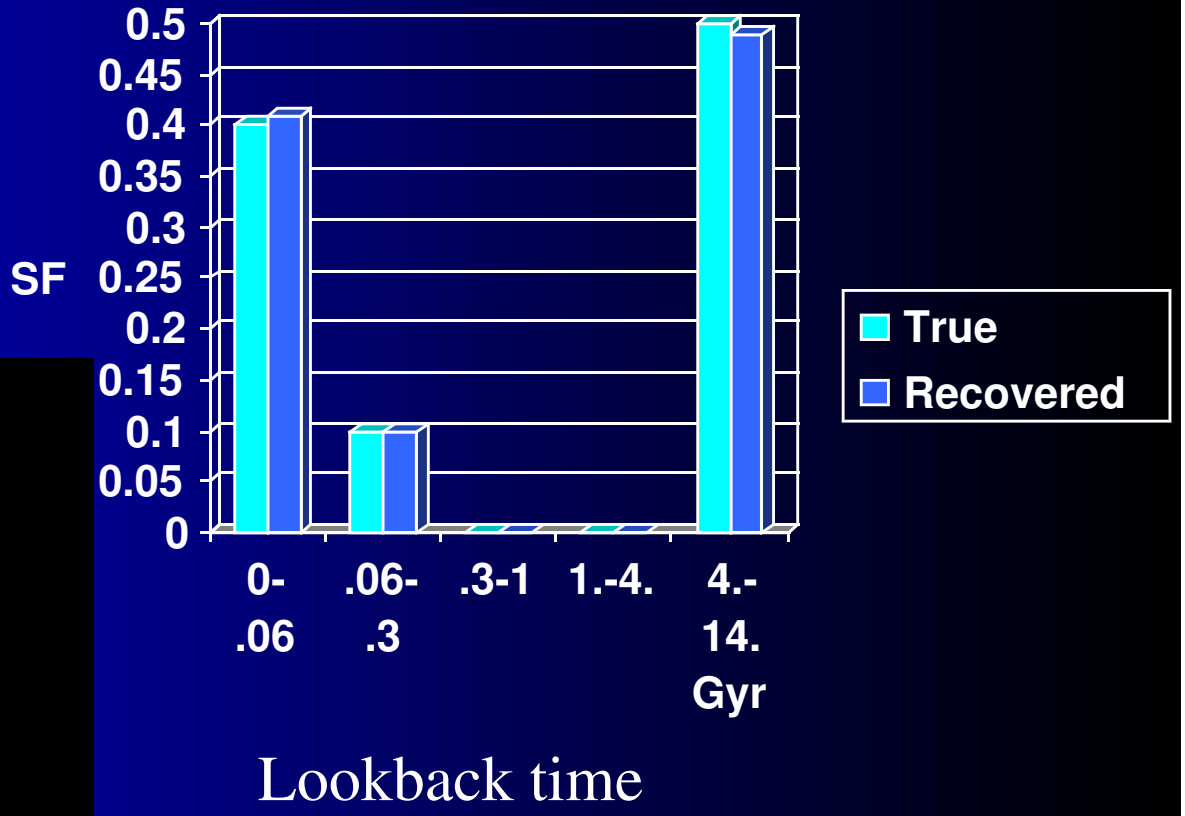
Weight vectors



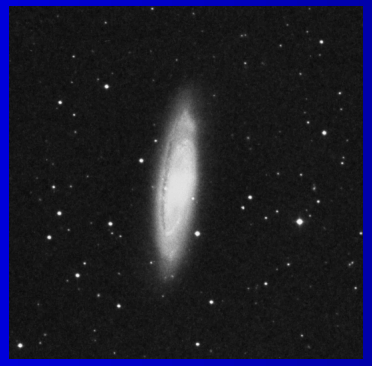
Test Results

Test data: Synthetic spectrum (Jimenez models):
5 parameters (Z fixed; no dust)





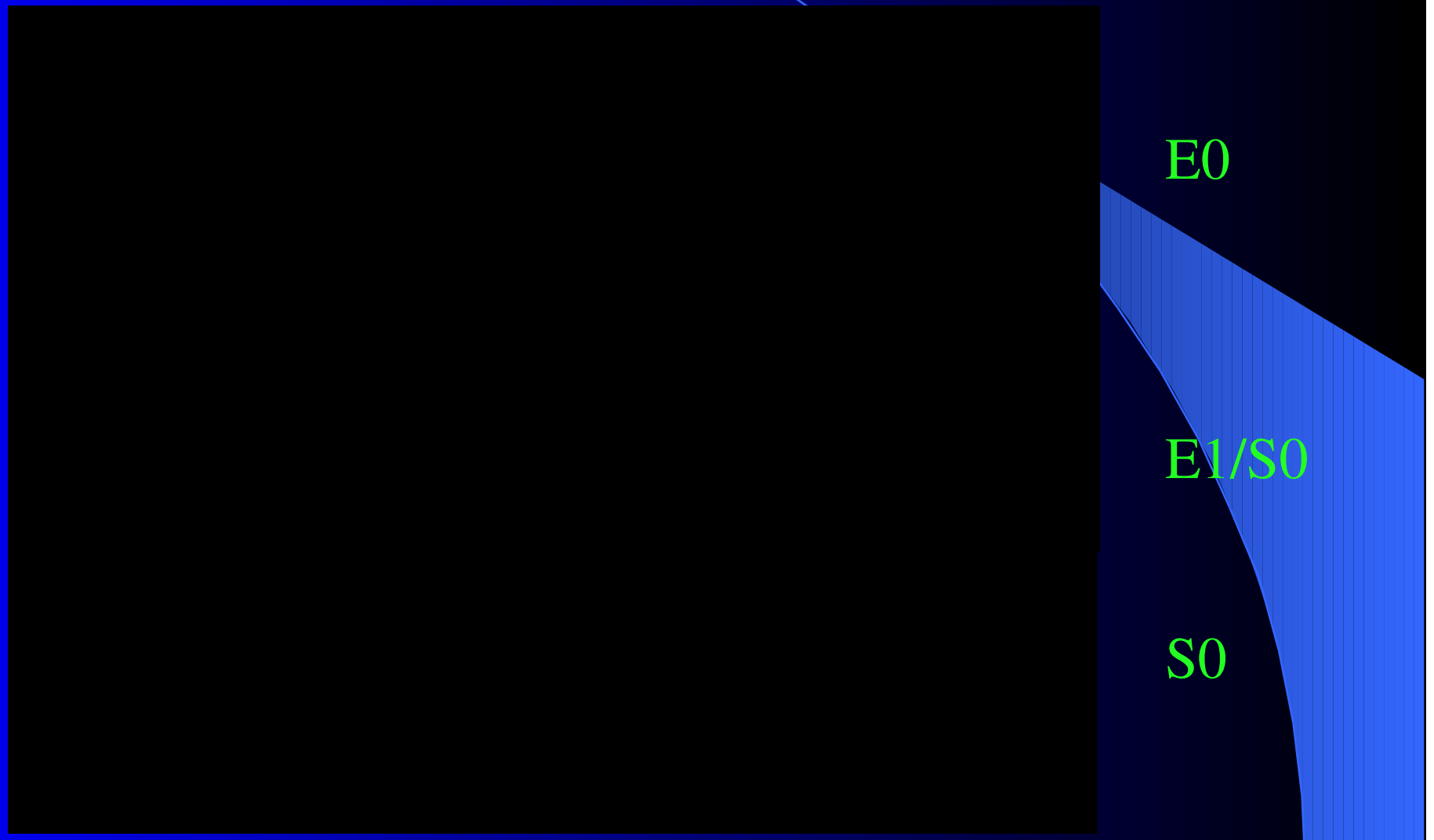
Kennicutt Atlas



NGC 3623
Sa

NGC 3310
Sbc pec

E/S0



Results

E/S0 galaxies all show signs of some relatively recent star formation (~1-5 Gyr ago)

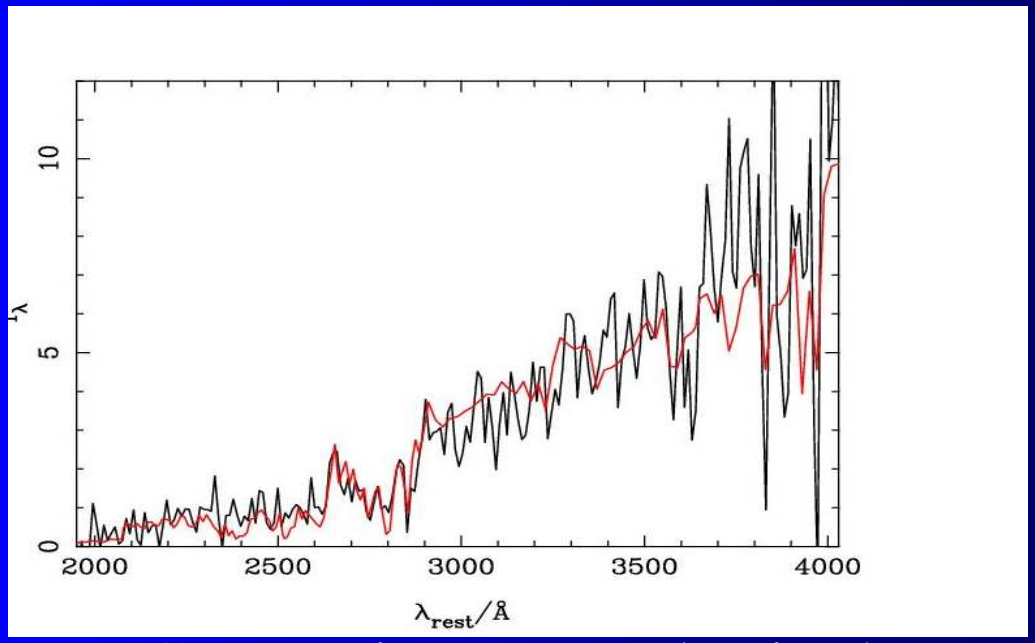
Closed-Box model doesn't work, infall model favoured

Timings: 15 seconds per spectrum (cf 2 hours)

2dF in 6 weeks; Sloan in 6 months

High-redshift galaxies

53W069: z=1.43 (Dunlop 1998) and 53W091 (Spinrad et al 1997)



53W069

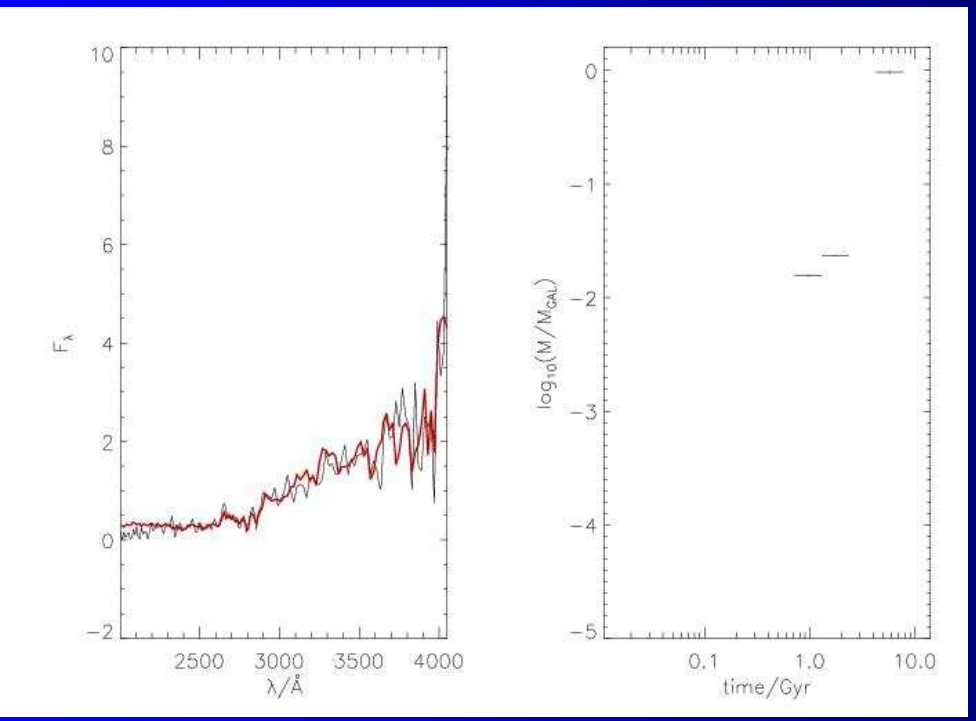
Previous work – burst models:

4.0 Gyr old

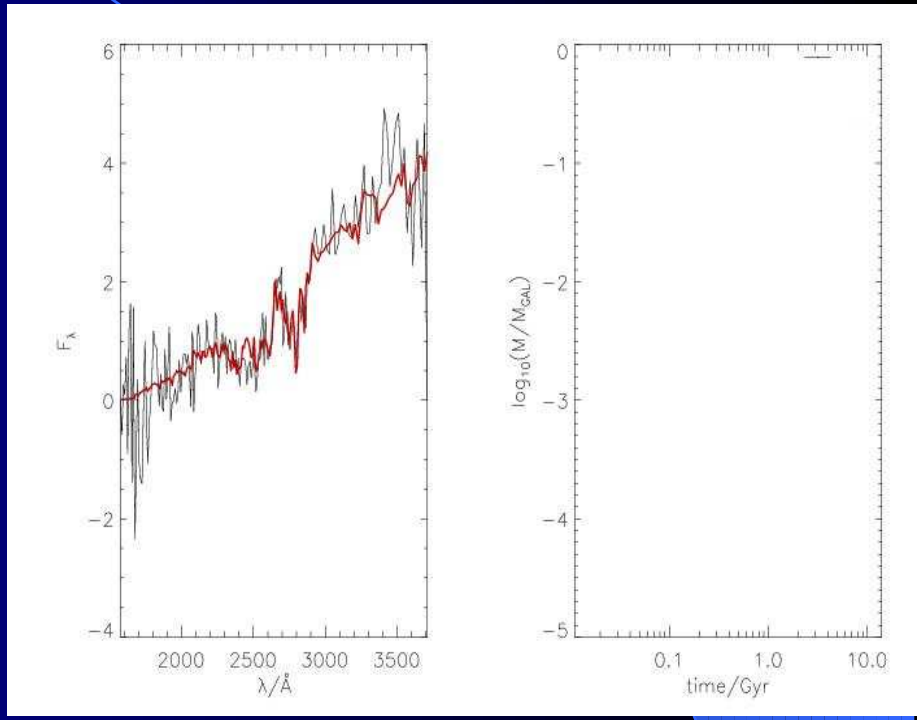
Constrains cosmological parameters:

H_0<45 if Ω_0=1 Ω_Λ>0.5 if Universe is flat and H_0=70

Results



53W069

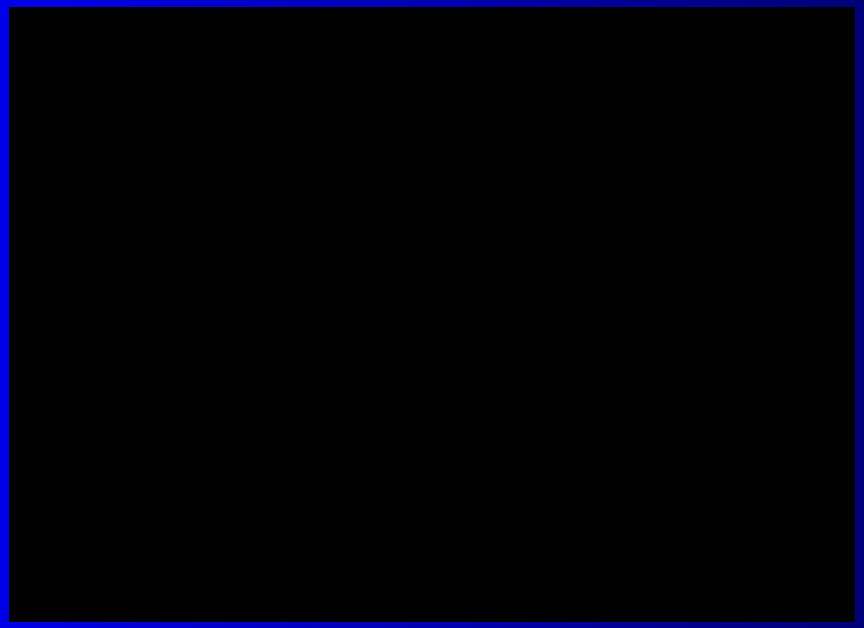


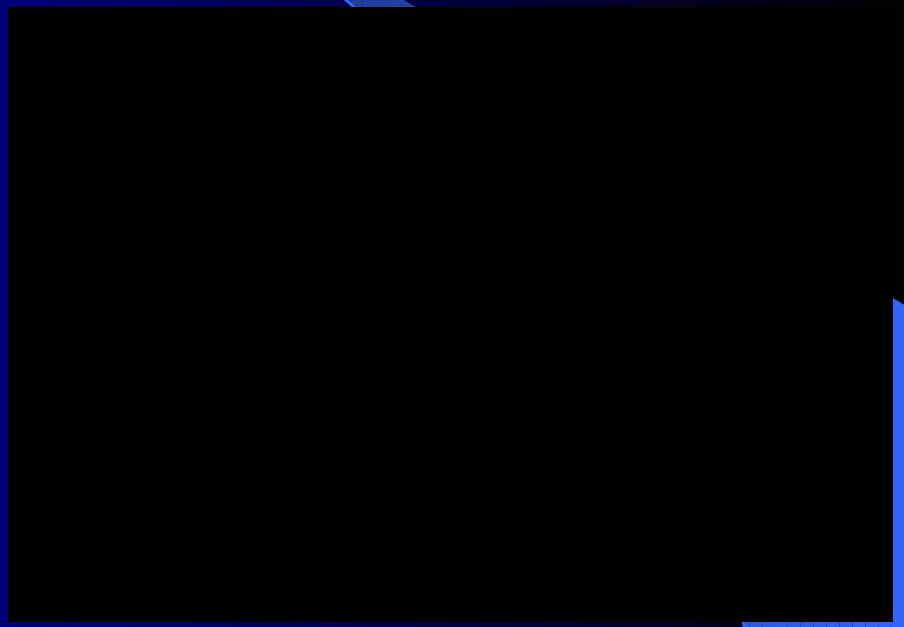
53W091

Age at least 4 Gyr (53W069) and 3 Gyr (53W091) – confirms old results

CMB Performance

Faster by factor of a billion...





Full dataset - 299 data

Compressed dataset- 3 data

Conclusions

- New data compression algorithm ~2000 fluxes
- ~17 numbers; almost no loss of information about physical parameters – as good as using all the data
- Lightning fast ~200 times faster than brute force. Allows more realistic modelling (CMB – 10^9 faster)
- Recent star formation E/S0 show some relatively recent activity
- High-z radio galaxies are old confirmed with ages ~3-4 Gyr

Heavens, Jimenez & Lahav, MNRAS, 317, 965 (2000)
Reichardt, Jimenez & Heavens, astro-ph/0101074