APPLICATION KIT

The RUTGERS YOUNG SCHOLARS PROGRAM in DISCRETE MATHEMATICS

July 5 - July 30, 2010

"Our Twentieth Summer"

"We're looking for students who are interested in math. Is that you?"

The Rutgers Young Scholars Program in Discrete Mathematics is sponsored by the Rutgers University Department of Mathematics and the Center for Discrete Mathematics and Theoretical Computer Science [DIMACS] -- a consortium of Rutgers and Princeton Universities, AT&T Laboratories, Bell Labs, Bellcore, and NEC Research Institute.

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2010 READ FIRST

Do you ...

... *like to work on challenging and puzzling problems*
... *like to look at a problem in many different ways*
... *get involved, make conjectures, and take risks*
... *enjoy the critical exchange of ideas with other students*
... *look forward to living in the math world for a month*
... *enjoy and feel compelled to do mathematics*
... *thrive on complexity and problems which have more than one answer*
... *like to reason out problems and play with concepts*
... *learn new material quickly and apply this learning to new situations*
... *concentrate on something you like for a long period of time*

Then this program is for you. Here is your opportunity to ...

... *experiment with some interesting mathematical problems*
... *spend four weeks on a college campus doing mathematics with people like yourself*
... *meet a variety of professional mathematicians and learn about what they do*

Come to the Rutgers Young Scholars Program ...

... *come join 30 high school students who share your fascination with mathematics.*

... *attend a variety of mathematical programs and activities ... including sessions on discrete mathematics, discussions, hands-on computer activities, and field trips.*

... *learn about careers in the mathematical sciences; these programs will involve industrial as well as academic mathematicians.*

The instructional staff includes well-known mathematicians. Teaching assistants who are graduate and undergraduate Rutgers students majoring in mathematics or computer science provide additional academic support. You will spend a lot of time with this staff; we hope that as a result you will understand what mathematicians do and why.

The focus of the program will be discrete mathematics, although the program will **not** be conducted like a course. Success will be measured in terms of your enthusiasm for the subject, not by the amount of material covered.

Your daily schedule will be full and will include various recreational and social activities.

Remember, this will be an intensive program. We will expect you to participate actively in all of the activities. It will be fun and exciting, but it will require your concentrated effort. Be sure that you want to make this commitment!

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2010 INSTRUCTIONS

COMPLETING APPLICATION MATERIALS

A completed application includes all of the following:

1. TYPED OR PRINTED (in black ink) responses to the questions on the Student Application Form;

2. a completed parent signature form;

3. a completed financial assistance form if applying for financial assistance;

4. a letter of recommendation from your teacher, who should mail the recommendation directly to our office, as well as a copy of your transcript of your high school record;

5. you will also be expected to submit your solutions (partial or complete) to the 2010 Problems to Explore (at the end of the application materials). Please make sure to **show all work** when working on your solutions and send that in with your solutions. We recommend that you make a photocopy of your solutions prior to sending them in for safe-keeping and also in case we call to discuss your responses.

MAILING INSTRUCTIONS

Please send the completed forms via First Class Mail. Because of the various weights of the enclosures, it would be wise to verify the correct postage to assure prompt delivery.

*Applications will be reviewed on a first-come first-serve basis until the program is filled. All materials should be returned to the following address **as soon as possible**.*

RETURN MATERIALS TO: ***Rutgers University***
Young Scholars Program
DIMACS – CORE Building
96 Frelinghuysen Road
Piscataway, New Jersey 08854-8018

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2010 STUDENT APPLICATION FORM

Student Name.....

Student Address .....Phone ( ).....

City ..... State.....Zip.....

Social Security Number ..... Date of Birth ..... Class of ☐ 2011 ☐ 2012 ☐ 2013
*** - ** -

Female ..... Male ..... Email Address.....

High School ..... School phone ( ).....

School Address.....

City.....State..... Zip .....

Name of Teacher who will provide the Recommendation.....

List the math courses you will have completed by July:.....

.....

List the math courses you plan to take next year:.....

.....

If you have previously attended a summer academic program, please name the program[s] and date[s]:

.....

Optional (*used for statistical purposes*)
How would you best describe yourself (please check one):

☐Native American
☐Asian or Pacific Islander (including Indian subcontinent)
☐African American
☐Hispanic(including Puerto Rican)
☐White, Anglo, Caucasian
☐Other (Specify)

List any friends or relatives who have attended the Young Scholars Program:

Name:..... Year.....

How did you learn about the program? (You may check more than one)

☐YSP Website
☐Teacher Recommended
☐Guidance Counselor
☐Parents/Relatives
☐Former Program Student/Alumni
☐Brochure
☐Newspaper or Publication (name and section).....
☐Summer Program Guide (name and section).....
☐Other (please explain).....

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2010 PARENT SIGNATURE FORM

To The Candidate

Please print your name below and give this form to a parent or guardian to complete. Both parents may complete the form together, if they wish.

_____

CANDIDATE'S NAME

To The Parent or Guardian

Your daughter/son is applying for admission to the Rutgers Young Scholars Program in Discrete Mathematics, a four-week, summer, residential program for mathematically talented high school students. The program will take place on the Rutgers University Busch Campus in Piscataway between the dates of July 5 - July 30, 2010.

The cost of the program will be $3,500; this will cover tuition, materials, meals, and lodging. A few partial scholarships are available. **No student should be discouraged from applying because of financial considerations**. If applying for financial assistance please complete the enclosed application form for financial assistance.

Please indicate here any special considerations (medical, physical, emotional, psychological, etc.) we should be aware of in terms of our responsibility to your child's education and general well-being for a month. This information will be kept strictly confidential. (You may use the back side of this page if you need additional space)

Special Considerations:_____

_____

Permission

"My daughter/son has permission to attend the Rutgers Young Scholars Program for the entire four week program from July 5 - July 30, 2010"

_____

Your Name (please print) Relationship to Candidate

_____

Home Address

_____

City State Zip

_____

Daytime telephone Evening telephone

_____

Email

Signature of parent or legal guardian.....

RUTGERS YOUNG SCHOLARS PROGRAM :: SUMMER 2010 RECOMMENDATION FORM

INSTRUCTIONS:

On this form, please tell us about the student you are recommending for the Rutgers Young Scholars Program in Discrete Mathematics.

Include information about the student's performance (including cumulative average and PSAT/SAT scores if available) and attach a transcript of the student's high school record.

Tell us about your student's abilities and interests, about the personal characteristics that make her or him most likely to benefit from and enjoy a four-week intensive exposure to a program in the mathematical sciences. Include specific examples drawn from your own experiences with the student.

Please write your remarks on the other side of this page and continue on a separate sheet of paper. Please be sure to write your student's full name in your recommendation.

STUDENT NAME:_____

In view of this student's interest and ability in mathematics, I recommend that s/he be accepted to the Rutgers Young Scholars Program in Discrete Mathematics.

Signature of recommending teacher _____

Name (please print)_____

City_____State_____ Zip_____

School_____School Phone Number ( )_____

School Address_____

City_____State_____Zip_____Email:_____

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Problems to Explore

Rutgers Young Scholars Program

Summer 2010

PROBLEM 1: THE AGES OF WOMEN?

Fifteen years ago a woman's age was half her mother's age. Fifteen years from now the woman's age will be twice her daughter's age. Right now the sum of the ages of the woman, her mother and her daughter is 100 years. How old are the woman, her mother and her daughter?

PROBLEM 2: A TRIP IN THE SLOW LANE

A man made a trip of 432 miles by automobile. If his average speed had been 6 miles per hour faster, the trip would have taken one hour less. How long did the trip take and what was his average speed?

PROBLEM 3: COOPERATIVE BRICKLAYING

A building contractor employs a master mason, a journeyman mason and an apprentice mason. The master mason and the journeyman working together can finish a brick wall in 10 hours. It takes the master and apprentice 12 hours to do the same job, while the journeyman and apprentice require 15 hours to complete the task.

If all three work together, how long does it take to build the wall?

PROBLEM 4: EVENS AND ODDS

In the multiplication shown at the right, the O's represent odd digits and the E's represent even digits. The O's do not all represent the same digit, nor do the E's. Also, as usual, the left-most digit in any number is not zero. Find all possibilities for the digits that give a multiplication with this pattern of even and odd numbers.

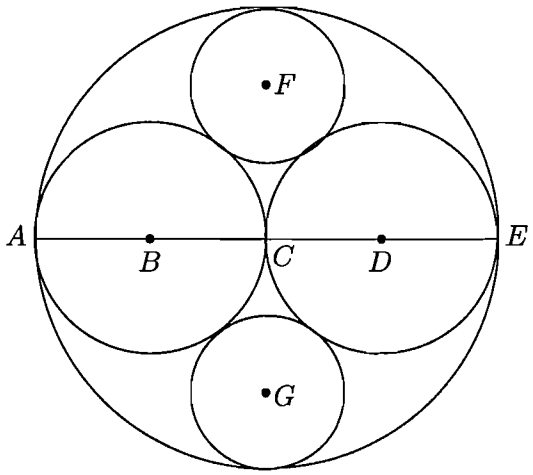
```
  O O E
    O E
  -----
  O O E
O E E
-------
O E E O E
```

PROBLEM 5: ATTACKING QUEENS

Recall that in chess a queen attacks any square that is on a straight line — horizontally, vertically, or diagonally — from the square on which the queen stands. Regard the queen as also attacking the square which she occupies. What is the smallest number of queens required to attack all of the squares of a 6×6 chessboard?

PROBLEM 6: TANGENT CIRCLES

In the diagram to the right, the outer circle, centered at C, has diameter AE. The middle-sized circles, centered at B and D, have respective diameters AC and CE. The smaller circles, centered at F and G, are just tangent to the outer circle and the circles centered at B and D. If the outer circle has radius 1, what is the radius of the circle with center F?



PROBLEM 7: EASY MULTIPLICATIONS

The number 128205 ends in 5 and has the property that multiplying the number by 4 can be accomplished by moving the ending 5 to the front of the number, or $4 \times 128205 = 512820$. Find a number that ends in 6 and has the property that multiplying the number by 4 can be accomplished by moving the ending 6 to the front of the number. Can you find a second such number?

PROBLEM 8: A FIBONACCI SUBSEQUENCE

The Fibonacci sequence $f_1, f_2, f_3, f_4, \ldots, f_n, \ldots$ is defined by $f_1 = 1, f_2 = 1$ and if $n \geq 1$, by the recursion relation $f_{n+2} = f_{n+1} + f_n$. In other words, from the third term onward, each term is the sum of the previous two terms.

The resulting sequence is: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, \ldots$.

A new sequence $g_1, g_2, g_3, g_4, \ldots, g_m, \ldots$ is formed by removing the 1^{st}, 3^{rd}, 5^{th}, 7^{th}, etc. terms from the Fibonacci sequence. The resulting sequence is: $1, 3, 8, 21, 55, 144, 377, \ldots$. Since the terms are $f_2, f_4, f_6, f_8, \ldots, f_{2m}, \ldots$ it follows that $g_m = f_{2m}$. Find, if possible, constant numbers A and B so that if $m \geq 1$, the terms g_m of the sequence can be calculated by the recursion relation: $g_{m+2} = Ag_{m+1} + Bg_m$.

PROBLEM 9: TURNING PENNIES

Fourteen pennies are placed in a line with all heads up. Let k be a positive integer. The objective is to turn over the pennies k at a time, until all of the pennies have tails up. (Any individual penny may be turned over a number of times, but at the end it must be tails up.) In addition, this is to be accomplished using as few moves as possible. In your answer explain how you performed the task and why the number of moves you used was as small as possible.

(a) If $k = 3$, or in other words, the pennies are turned over three at a time.

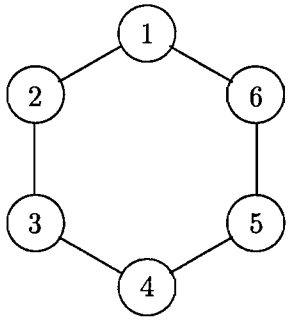
(b) If $k = 4$.

(c) If $k = 5$.

PROBLEM 10: COLORING A BRACELET

A six-bead bracelet is made out of red, yellow, and blue beads arranged so that neighboring beads have different colors. In how many different ways is this possible?

Note that for this problem two ways to color the bracelet are considered the same only when bead 1 is colored the same in both, bead 2 is colored the same in both, etc.



PROBLEM 11: SUMS OF CONSECUTIVE ODD INTEGERS

There are two different ways of expressing the number 100 as a sum of consecutive positive odd integers:

$$100 = 49 + 51$$

$$100 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

.

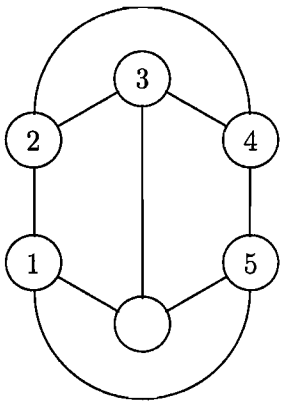
What are the ways of expressing 1000 as a sum of consecutive positive odd integers?

PROBLEM 12: CARPET CUTTING

A 7×7 rug has a 1×1 hole cut out of the center. Show how to cut it into two pieces that can be reassembled to form a 6×8 rug.

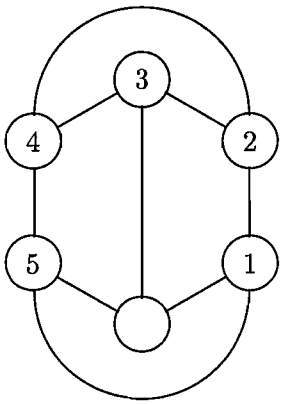
PROBLEM 13: SHUNTING LOCOMOTIVES

The diagram shows the assigned location of five locomotives in a railroad storage yard. Each number represents an engine, each circle represents a storage berth, and each line represents a track between two storage berths. The storage berth at the bottom is empty.

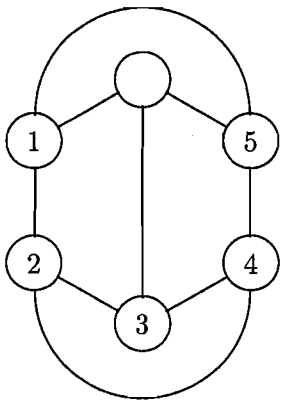


In the following diagrams, the locomotives were brought into the storage area and parked haphazardly with no regard for proper position. How can you use the empty storage berth and existing tracks to rearrange the engines into their assigned locations?

(a)



(b)

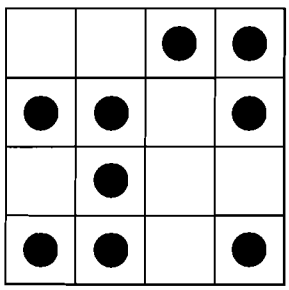


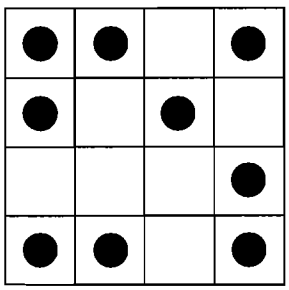
In (a), for example, your first step would be to move one of the engines 1, 3, or 5 to the empty bottom berth. If, for example, you moved 5 to this berth, you could next move 1, 4, or 5, to the berth vacated by 5. The last move, while not incorrect, is pointless, since you have simply returned to the original position and started over.

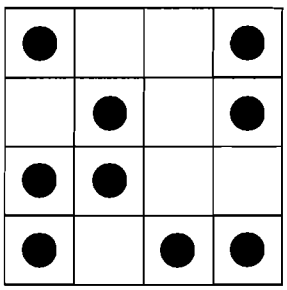
PROBLEM 14: CHECKERS

What is the largest number of checkers that can be placed on a 6×6 checkerboard so that no three consecutive checkers lie on a line – horizontal, vertical, or diagonal?

For example, pictured below are three placements of checkers on a 4×4 board. In each case there are nine checkers on the board, but the first two cases do not satisfy the condition, since there are three consecutive checkers on a vertical line in the first case and on a diagonal line in the second case. In the third case, the condition holds since no three checkers are consecutive on a line.





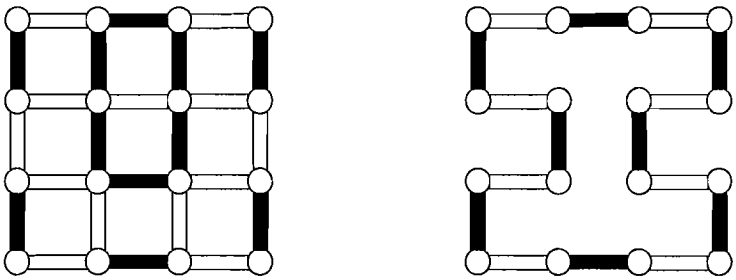


Show that nine is the smallest number of checkers that can be placed on a 4×4 board subject to the above condition. Perform a similar analysis for a 6×6 board, and try to explain why the number of checkers that you placed is as large as possible.

PROBLEM 15: VISITING ALL LOCATIONS

Below are several examples of graphs. The circles are called vertices, and the horizontal and vertical roads connecting the vertices are called edges. In the graphs below, the edges have been colored, some white and some black. In this problem, the objective is to begin at some vertex and follow a route, along the edges, that passes through every vertex exactly once and returns to the starting vertex. In addition, the edges of the route must alternate in color: black, white, black, white, etc.

For the 4×4 grid graph below at the left, such a route is shown on the right. The route has been indicated by removing all edges not in the route. Start at any vertex and follow the included edges.



How many similar routes can you find for the 6×6 grid graph below?