NEW JERSEY MATHEMATICS COALITION

A Baseline Report on Trends in New Jersey Mathematics Education

Are we measuring Up?

Philip E. Mackey

1997

The New Jersey Mathematics Coalition

The New Jersey Mathematics Coalition, organized in 1991, draws together all sectors of the community—education, public policy, business and industry, and the public—in a sustained, multi-faceted statewide effort to improve mathematics education in the state and to increase public awareness of the importance of mathematics to the future of New Jersey's children. The mission of the Coalition is to enable all New Jersey students to move into the 21st century with the mathematical skills, understandings, and attitudes they will need to be successful in their careers and daily lives.

During the first five years of its existence, the Coalition focussed much of its energy on advocating, developing, and gaining widespread support for the mathematics standards adopted by the State Board of Education in May 1996; and developing and publishing the *New Jersey Mathematics Curriculum Framework*, a 688-page guide to implementing the standards, for teachers and administrators.

In the past three years, the Coalition has published and distributed over 40,000 copies of a parents' guide entitled *Mathematics to Prepare Our Children for the 21st Century*; a new parents' guide, entitled *Helping Your Child Reach the New Standards in Mathematics, Science, and Technology*, will be distributed during the summer of 1997. The Coalition has also coordinated activities associated with Mathematics, Science, and Technology Month (MSTM) each April since 1993; MSTM is a public outreach component of the NJ Statewide Systemic Initiative for Excellence in Mathematics, Science, and Technology Education (NJ SSI). MSTM 1996 brought together 89,000 parents and children at over 450 New Jersey sites for hands-on activities and distribution of information about efforts to improve mathematics, science, and technology education in the state.

With adoption of standards and publication of the *Framework*, the Coalition's focus has shifted to promoting implementation of the standards. Our current goals are to:

- inform parents and the general public of the meaning and significance of the standards;
- advocate adoption and implementation of public policies that enhance the goal of success for all students in mathematics;
- promote high-quality professional development activities throughout the state; and
- support district curriculum development and teacher implementation of the standards.

The Coalition recently received a $1.6 million grant from the National Science Foundation for a three-year project, titled FANS of Math, Science, and Technology (Families Achieving the New Standards), to inform parents about the new standards and to involve them in achieving the standards.

The Coalition is an autonomous organization, responsible to an independent Board of Governors (members are listed on the inside back cover). It is a partner in the NJ SSI and is based at the Rutgers University Center for Mathematics, Science, and Computer Education.

Are We Measuring Up?

A Baseline Report on Trends in New Jersey Mathematics Education

1997

by Philip E. Mackey

New Jersey Mathematics Coalition

A Project Funded by The Fund for New Jersey

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Contents

Executive Summary

The New Jersey Mathematics Coalition's *Are We Measuring Up? A Baseline Report on Trends in New Jersey Mathematics Education, 1997* is the first volume in what will be an annual series. This baseline edition provides a snapshot of where our public school children stood in the years 1991–96 relative to the state's newly adopted Core Curriculum Content Standards in Mathematics and to the kinds of mathematics instruction they will need in a technological, information-based 21st century.

Indicators of Student Achievement

New Jersey standardized test results show that only about 40 percent of the state's eighth-graders demonstrate "clear competence" in mathematics on the Early Warning Test (EWT) and that a huge gap exists between the performance levels of students in the state's richest and poorest districts.

At the eleventh-grade level, about 86 percent of the state's students are able to pass the High School Proficiency Test (HSPT), one of the requirements for a high school diploma. (The HSPT is not directly comparable to the EWT, and the 86 percent passage rate does not indicate vast improvement between the eighth and eleventh grades.)

Testing by the National Assessment of Educational Progress (NAEP) shows that New Jersey fourth- and eighth-graders outperform their peers in most other states, but are still far below desirable levels. Only about 25 percent of New Jersey fourth- and eighth-graders score at or above NAEP's "Proficient Level." Analysis of results for individual NAEP Items shows that large percentages of New Jersey students are unable to solve relatively simple and straightforward mathematics problems. For example, only 25 percent of fourth-graders and 58 percent of eighth-graders were able to use a ruler to draw a rectangle 2 inches wide and 3½ inches long.

Indicators of Quality of Content and Instruction

Data collected by NAEP and by the U.S. Department of Education's Schools and Staffing Survey (SASS) show that:

> Though all New Jersey students are required to complete three years of high school mathematics in order to graduate, many never progress beyond Algebra I, or in some cases Geometry or Algebra II. These students are apparently completing their three-year requirement by taking lower-level courses like General Mathematics or Consumer Mathematics.
>
> Too few New Jersey mathematics classrooms regularly engage in specific practices that tend to enhance student performance and prepare students for life in a technological society. In 1992, no more than 63 percent of fourth-graders and 52 percent of eighth-graders were assigned to work in groups at least weekly; only about 42 percent of fourth-graders worked with manipulatives at least weekly; only about 30 percent of fourth- and eighth-graders were required to write a few sentences about how to solve a mathematics problem at least weekly; no more than 35 percent of fourth- and eighth-graders were asked to work and discuss problems reflecting real-life situations almost every day; and only about 27 percent of fourth-graders and 51 percent of eighth-graders used calculators in class at least weekly.
>
> By the same token, relatively few New Jersey students are exposed to the kinds of student assessment practices that educators think are beneficial in developing students' abilities. In 1992, about 24 percent of fourth-graders and 15 percent of eighth-graders were assessed by means of projects, portfolios or presentations at least monthly; and about 50 percent of fourth-graders and 59 percent of eighth-graders were given questions requiring short written responses at least monthly.

Indicators of Teacher Supply and Preparation

New Jersey is one of the few states in which 100 percent of teachers assigned to teach mathematics in grades 9–12 are certified mathematics teachers. Yet too many teachers of grades 4 and 8 report that they have had little or no exposure to such critical content areas as geometry, probability/ statistics, and calculus. About five percent of fourth-graders and 35 percent of eighth-graders are taught by teachers with college majors in mathematics; about 82 percent of mathematics teachers in grades 9–12 have a major in mathematics.

Recommendations

These findings give rise to a number of recommendations for policymakers, school and college administrators, educators, parents, and community members:

1. School districts should use the *New Jersey Mathematics Curriculum Framework* to help them implement standards rapidly, efficiently, and thoughtfully.

2. State and local policymakers should increase efforts to close the performance gap between students from advantaged and disadvantaged districts.

3. Districts devoting less class time to mathematics than the current state average should increase the amount of mathematics class time.

4. Districts should develop high school courses that address the core curriculum described by the grades 9–12 cumulative progress indicators of New Jersey's Core Curriculum Content Standards and ensure that all students take such courses.

5. Districts should provide opportunities for qualified eighth-graders to take high-school-level mathematics courses that address the core curriculum for grades 9–12.

6. High schools should provide opportunities for qualified students to take advanced mathematics courses (e.g., Advanced Placement Calculus).

7. Teachers should provide frequent opportunities for students to: work in small groups; work with manipulatives; write about how to solve mathematics problems; work and discuss problems reflecting real life situations; and use calculators, computers, and other technological tools in class. These practices should be implemented thoughtfully and appropriately, as part of a coordinated approach to improving teaching and learning.

8. Teachers should make more use of such assessment techniques as questions requiring short written responses, projects, portfolios, and presentations.

9. College administrators and faculty should ensure that all mathematics teacher candidates are adequately trained in key content areas in the discipline—including geometry, probability/statistics, and calculus—and in proper use of the instructional and assessment techniques listed above.

10. School administrators should ensure that teachers with inadequate backgrounds in key content areas, or with inadequate knowledge about proper use of the instructional and assessment techniques listed above, receive appropriate professional development as soon as possible.

11. Parents and community-members should become active in encouraging improvement in mathematics education in their communities' schools.

The Coalition believes that it must start the important task of measuring progress relative to the state's new mathematics standards, even though indicators in this edition have many limitations and may give an incomplete picture of what is happening across the state. In future editions, we will provide updated and expanded data to inform policymakers, educators, parents, and community-members about the state's progress in developing a system of mathematics education capable of preparing *all* of our students for life, careers, and productive citizenship in the 21st Century.

Preface

The New Jersey Mathematics Coalition presents *Are We Measuring Up? A Baseline Report on Trends in New Jersey Mathematics Education, 1997* as an initial effort to provide important information to policymakers and the general public. The report is a snapshot of where our public school children stand relative to the state's newly adopted Core Curriculum Content Standards in mathematics (see Appendix) and to the kinds of mathematics instruction they will need for life in a technological, information-based 21st century.

This first annual edition appears soon after New Jersey's adoption of mathematics standards and distribution to teachers and administrators of the *New Jersey Mathematics Curriculum Framework*, a guide to implementing the standards. As school districts make changes required to achieve the standards—changes in curriculum, teacher training, classroom instruction, and student assessment—future editions of this report should show substantial improvement in standards-based practices, in comparison to baseline data presented here. Student performance should also improve, though planned changes to the state's Early Warning Test and High School Proficiency Test will require us to establish new baselines for relevant indicators. Finally, we hope to see progress in closing the performance gap that now exists between advantaged and disadvantaged districts in our state.

We have sought to present data that are valid, that facilitate comparison over time, and that facilitate comparison with external data. We are aware that indicators in this edition have many limitations and may give an incomplete picture of what is happening in mathematics education across the state, but we believe that we must start the important task of measuring progress.

We have omitted some valuable indicators because necessary data is unavailable; New Jersey, for example, does not collect information about enrollments in various high school mathematics courses, does not release questions from past statewide assessments, and does not release information about student performance on individual questions. Future editions of this report will add new indicators, as data become available. To this end, the Coalition will seek input from readers regarding additional indicators and sources of data, monitor relevant research and publications, urge the New Jersey Department of Education to collect and provide additional data, and consider conducting its own surveys of teachers and administrators. Of course, future editions will also add more recent data to existing indicators, permitting readers to gauge progress in relation to baseline information presented in this report.

The National Assessment of Educational Progress (NAEP) has found few gender differences in mathematics performance over the years. In 1996, the only statistically significant difference in scale scores was that males outperformed females by a small margin in grade four (average scores were 226 and 222, respectively). Therefore, we do not report differences among gender subgroups. On the other hand, NAEP has found consistent statistically significant differences at all grade-levels among racial / ethnic subgroups. Because these differences, in NAEP's words, "are almost certainly associated with a broad range of socioeconomic and educational factors," we have provided breakdowns, where available, by District Factor Group (DFG), a socioeconomic grouping of New Jersey communities (see Indicators 1 and 4).

The Coalition intends that annual editions of *Are We Measuring Up?* will encourage policymakers and the public to support implementation of the mathematics standards and development of a system of mathematics education capable of preparing all New Jersey students for productive, fulfilling lives in the 21st Century.

The Coalition acknowledges the efforts of many people who contributed to the planning and data gathering for this report. Special thanks are due to the Coalition's Assessment Committee (members listed on inside back cover). The Coalition also thanks The Fund for New Jersey for its financial support for development and publication of this report.

Joseph G. Rosenstein
Director
New Jersey Mathematics Coalition

Introduction

New Jersey policymakers and educators are engaged in a long-term reform effort to insure that all of the state's students are able to develop the mathematical skills, understandings, and attitudes they will need to be successful in their careers and daily lives. At the state, district, classroom, and individual teacher level, they are embarked on efforts to reshape the entire structure of mathematics education.

At the state level, policymakers have carried out bold steps, enacting Core Curriculum Content Standards for what all New Jersey students must know and be able to do (see Appendix) and developing statewide assessments based on these standards. Also at the state level, the New Jersey Mathematics Coalition has published the *New Jersey Mathematics Curriculum Framework*, a guide to implementing the standards. At the district, school, and classroom level, many teachers and administrators are beginning to make key changes in mathematics curriculum, teacher training, classroom practices, and means of evaluating students.

Reform in New Jersey comes at a time when international studies suggest that the United States continues to lag behind other developed nations in the quality of its mathematics education. The November 1996 report of the Third International Mathematics and Science Study (TIMSS) ranked the United States 28th out of 41 countries in mathematics performance by eighth-graders (top-scoring countries were Singapore, Korea, and Japan). U.S. students spend enough time on the subject, the TIMSS study found: they receive far more hours of mathematics instruction than their Japanese counterparts, for example, and spend as much time studying mathematics out of class as Japanese students. The real difference seems to be in how students spend their time. TIMSS researchers noted, for example, that American teachers tend to stress skill acquisition, while Japanese instructors emphasize understanding mathematics concepts.

Reacting to the TIMSS report, U.S. Secretary of Education Richard W. Riley said that Americans "need to examine what's actually going on in the school and the classroom." This report seeks to establish baseline data about how New Jersey's mathematics reforms are affecting "what's actually going on" in the state's schools and classrooms.

Indicators of Student Achievement

INDICATOR 1: Mathematics Proficiency on New Jersey Standardized Tests

Each year, the New Jersey State Department of Education administers the Early Warning Test (EWT) to the state's public school eighth-graders and the High School Proficiency Test (HSPT) to eleventh-graders. The EWT is designed to identify students who may need special assistance to complete high school requirements. The HSPT is one of the requirements for a high school diploma. The Department is currently developing a fourth grade assessment, the Elementary School Proficiency Assessment (ESPA), which was piloted in May 1997.

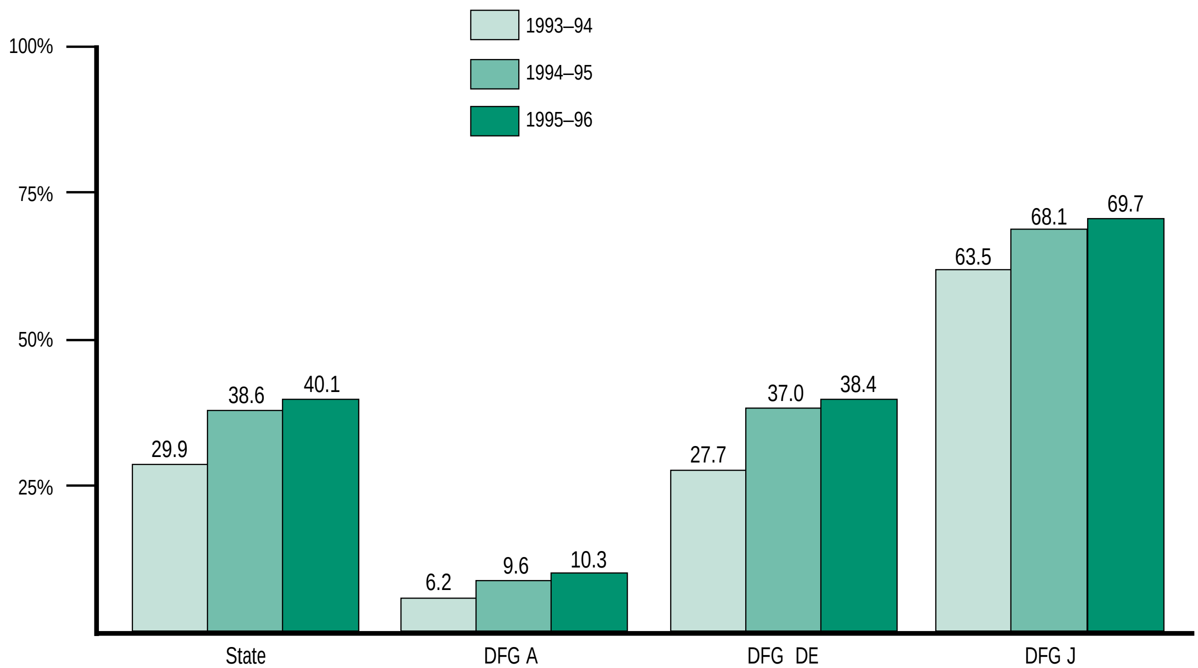
The EWT tests reading, mathematics, and writing and reports the number of students achieving three proficiency levels: Level I ("clear competence" in critical thinking skills), Level II ("at least minimal competence"), and Level III ("below the state level of proficiency"). The HSPT tests the same three subjects and reports the number of students passing. Because passing the HSPT is a requirement for graduation, students are given repeated opportunities to pass it. Results for the state, various subgroups, and every district are published in annual state summary reports. Results reported here are for mathematics only and exclude scores of special education and limited English proficient students; also excluded are scores of students retaking the HSPT.

In future years, the Department plans to align all three tests with the state's new Core Curriculum Content Standards. Because the tests will change, with inclusion of new content areas and expectation of higher-level problem-solving, future editions of this book will need to establish new baselines to permit comparing student performance from year to year.

District Factor Groups (DFGs) are indicators of socioeconomic status of residents in New Jersey school districts; they are derived from demographic data collected in the most recent United States Census—data on high school and college completion, occupational status, population density, income, unemployment, and poverty. DFGs range from A, the lowest socioeconomic group, to J, the highest socioeconomic group, and are labeled A, B, CD, DE, FG, GH, I, and J. *Figures 1.1* to *1.4* report results for students in the lowest, the highest, and one of the middle DFGs. DFG A includes such districts as Camden, New Brunswick, and Newark. DFG DE includes such districts as Ocean City, Woodbridge, and Ridgefield. DFG J includes such districts as Montgomery Township, Harding Township, and Millburn.

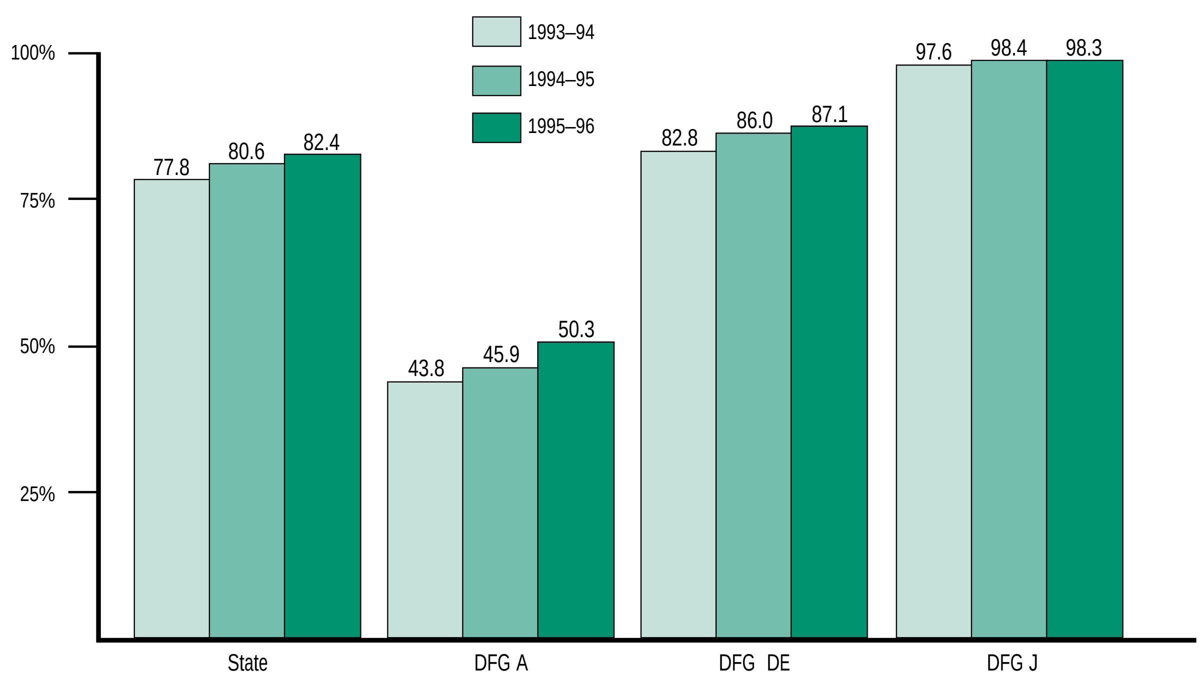
Figures 1.1–1.4 show that growing percentages of students at all socioeconomic levels are achieving competency on, or are passing, state tests. The absolute level of performance, however, remains far too low. Comparison of results by DFG illustrates New Jersey's educational equity gap. No New Jerseyan should feel satisfied with the quality of the state's mathematics education until far greater percentages of students are able to demonstrate clear competence and the equity gap begins to show steady and substantial reduction.

Figure 1.1: Percentage of Students at Level I ("Clear Competence") in Mathematics on the EWT, for the State and for DFG Groups A, DE, and J, 1993–94 to 1995–96



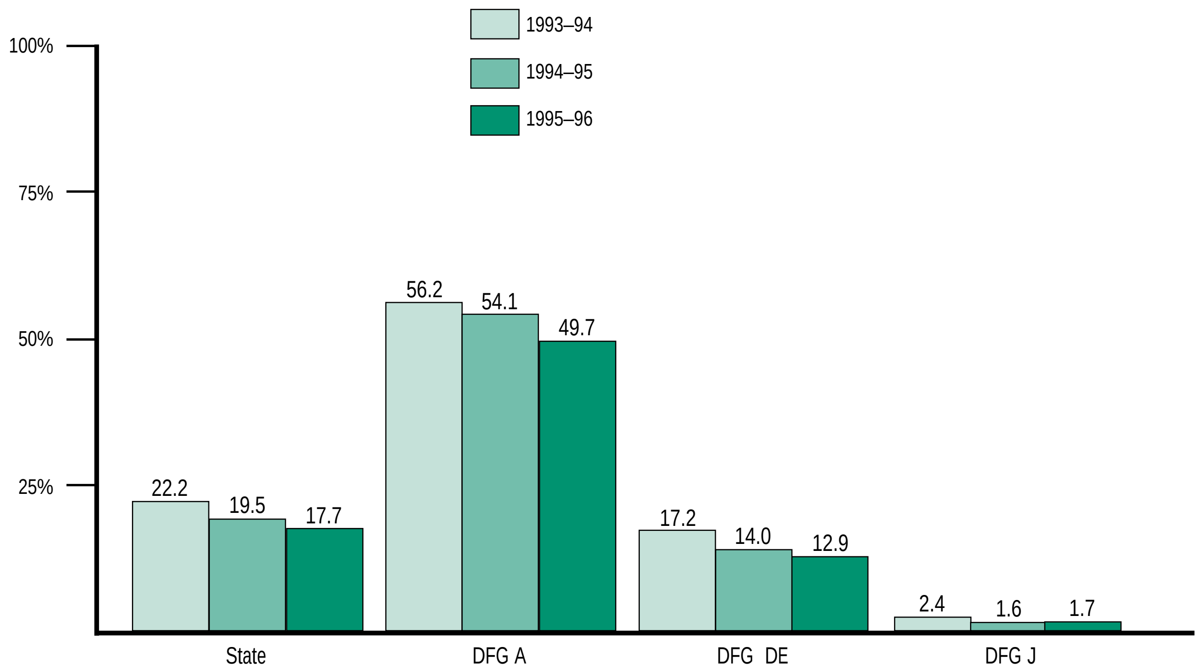
Source: NJ State Department of Education

Figure 1.2: Percentage of Students at Levels I and II ("Clear Competence" and "Minimal Competence") in Mathematics on the EWT, for the State and for DFG Groups A, DE, and J, 1993–94 to 1995–96



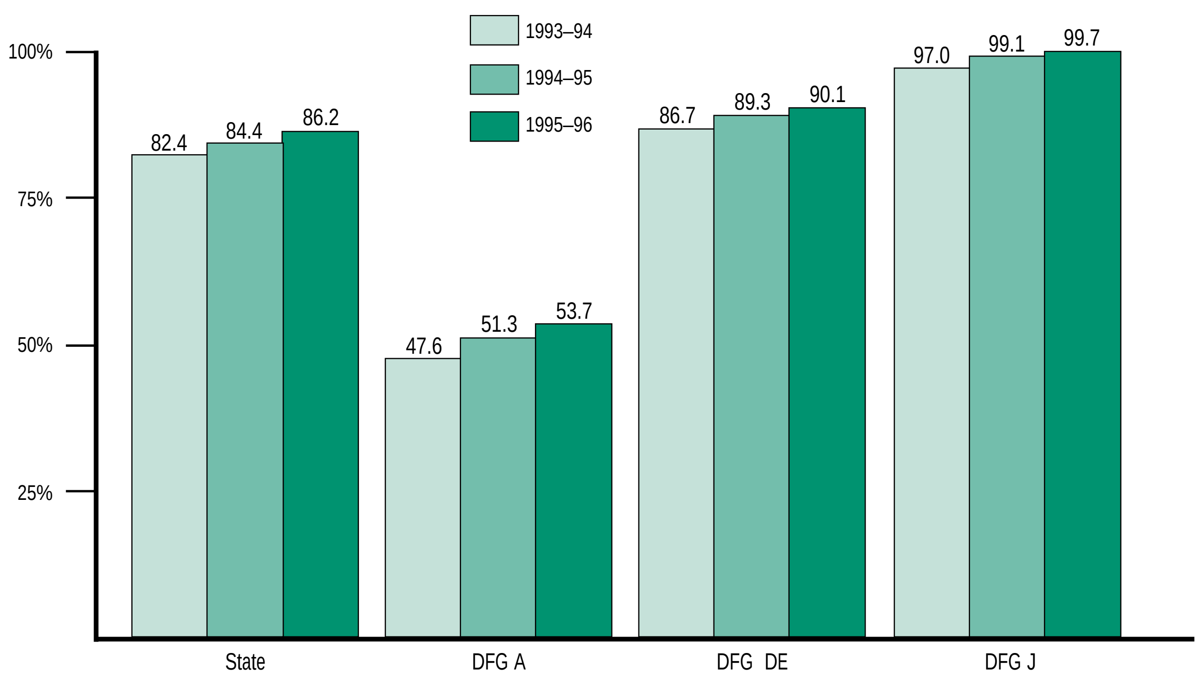
SOURCE: NJ State Department of Education

Figure 1.3: Percentage of Students At Level III ("Below the State Level of Proficiency") in Mathematics on the EWT, for the State and for DFG Groups A, DE, and J, 1993–94 to 1995–96



SOURCE: NJ State Department of Education

Figure 1.4: Percentage of Students Passing the Mathematics Portion of the HSPT at First Attempt, for the State and for DFG Groups A, DE, and J, 1993–94 to 1995–96



SOURCE: NJ State Department of Education

INDICATOR 2: Mathematics Proficiency on National Assessment of Educational Progress (NAEP) Tests

The National Assessment of Educational Progress (NAEP) is charged by Congress to conduct periodic assessments of what samples of students in grades 4, 8, and 12 know and can do in a variety of academic subjects. For national reports, NAEP assesses a sample of about 250,000 fourth-, eighth-, and twelfth-grade students attending about 10,000 schools across the country. For state reports, NAEP assesses a sample of about 2,000 students per state in grades four and eight. State assessments in mathematics were conducted in 1990, 1992, and 1996 and are scheduled for 2000 and 2004.

It is important to note that, because of the size of the samples in NAEP testing, year-to-year, state-to-state, and state-to-nation differences are sometimes not statistically significant. Where such instances occur, we have been careful to say so. (We use the 95 percent confidence level, the level employed in NAEP's own reports.) Sometimes, in fact, the number of students actually tested in a given state is not large enough to permit any conclusions about performance, in which case no data are reported. Unfortunately, this occurred in the eighth-grade sample in New Jersey in 1996, so the most recent results available for that grade are those for 1992.

The NAEP mathematics assessment tests students in five content areas: numbers and operations; measurement; geometry; data analysis, statistics, and probability; and algebra and functions. At the fourth-grade level, algebra and functions are treated in informal and exploratory ways, often through the study of patterns.

NAEP results are reported in terms of average proficiency scores and in percentages of students at three levels of proficiency—Basic, Proficient, and Advanced—and those below the Basic Level.

Very few students perform at the Advanced Level. In 1996, at the fourth-grade level, only three percent of New Jersey test-takers and two percent of national test-takers were able to demonstrate advanced proficiency. Among eighth-graders, in 1992, three percent of New Jersey and national test-takers scored at this level.

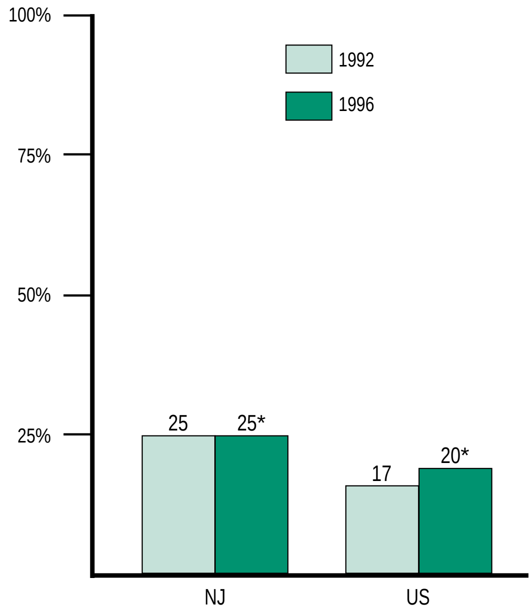
The percentage of students at or above the Proficient Level is the figure usually used to gauge progress and compare states, and the National Education Goals Panel has chosen it as a key indicator of educational progress. *Figures 2.1* and *2.2* compare the performance of New Jersey and national public school students on this measure in 1992 and 1996, though New Jersey's eighth-grade figure for 1996 is missing, as explained above. While New Jersey fourth- and eighth-graders appear to be outperforming their national counterparts, it is important to understand that some of the differences are not statistically significant (noted by asterisks) and that 75 percent of the state's students were unable to perform at the Proficient Level. It is also worth noting that New Jersey fourth-graders made no progress in this category from 1992 to 1996, while students in 26 other states did (though in only 7 states was the increase statistically significant).

A majority of students are capable of performing at the Basic Level or above. In the nation, 62 percent of public school fourth-graders and 61 percent of eighth-graders scored at this level in 1996. In New Jersey, 68 percent of public school fourth-graders achieved this level in 1996. The eighth-grade figure for New Jersey in 1992 was 63 percent.

The lowest level of performance is "Below Basic," and disturbingly large percentages of students are in this category. *Figure 2.3* shows that New Jersey fourth-grade students made no improvement in this regard from 1992 to 1996. In the absence of 1996 NAEP data for New Jersey eighth-grade students, we do not know if there has been improvement at this level, as there was for students across the nation; see *Figure 2.4*.

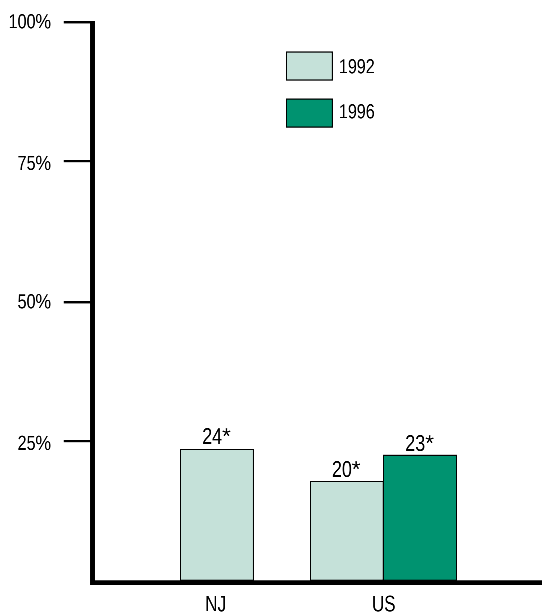
The NAEP testing program is designed to facilitate comparison of performance among states. *Figures 2.5* and *2.6* show that New Jersey students, though their performance is far from satisfactory in absolute terms, are more likely than students in most other states to perform at or above the Proficient Level.

Figure 2.1: Percentage of Grade Four Students at or Above NAEP Proficient Level, New Jersey and U.S., 1992 and 1996



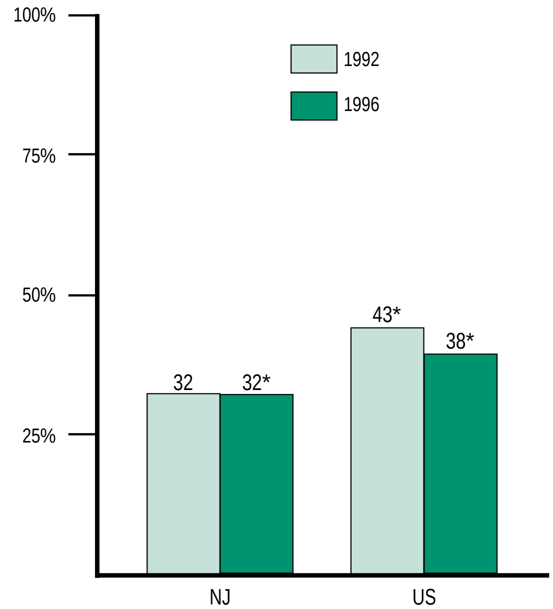
* Differences between NJ and US in 1996 and between US in 1992 and 1996 are not statistically significant and should be interpreted with caution
SOURCE: Reese, et al., *NAEP 1996 Mathematics Report Card*, p. 49

Figure 2.2: Percentage of Grade Eight Students at or Above NAEP Proficient Level, New Jersey, 1992, and U.S., 1992 and 1996



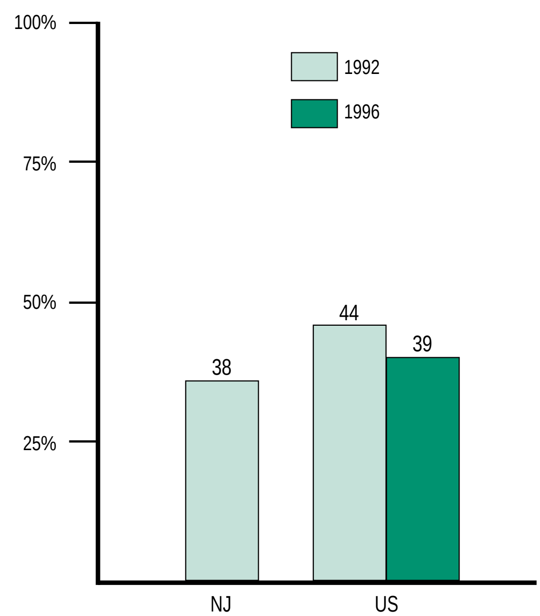
* Differences between NJ and US in 1992 and between US in 1992 and 1996 are not statistically significant and should be interpreted with caution
SOURCE: Reese, et al., *NAEP 1996 Mathematics Report Card*, p. 51; "Revised [1990 and 1992] Mathematics Assessment Data for Grade 8," NAEP

Figure 2.3: Percentage of Grade Four Students Below NAEP Basic Level, New Jersey and U.S., 1992 and 1996



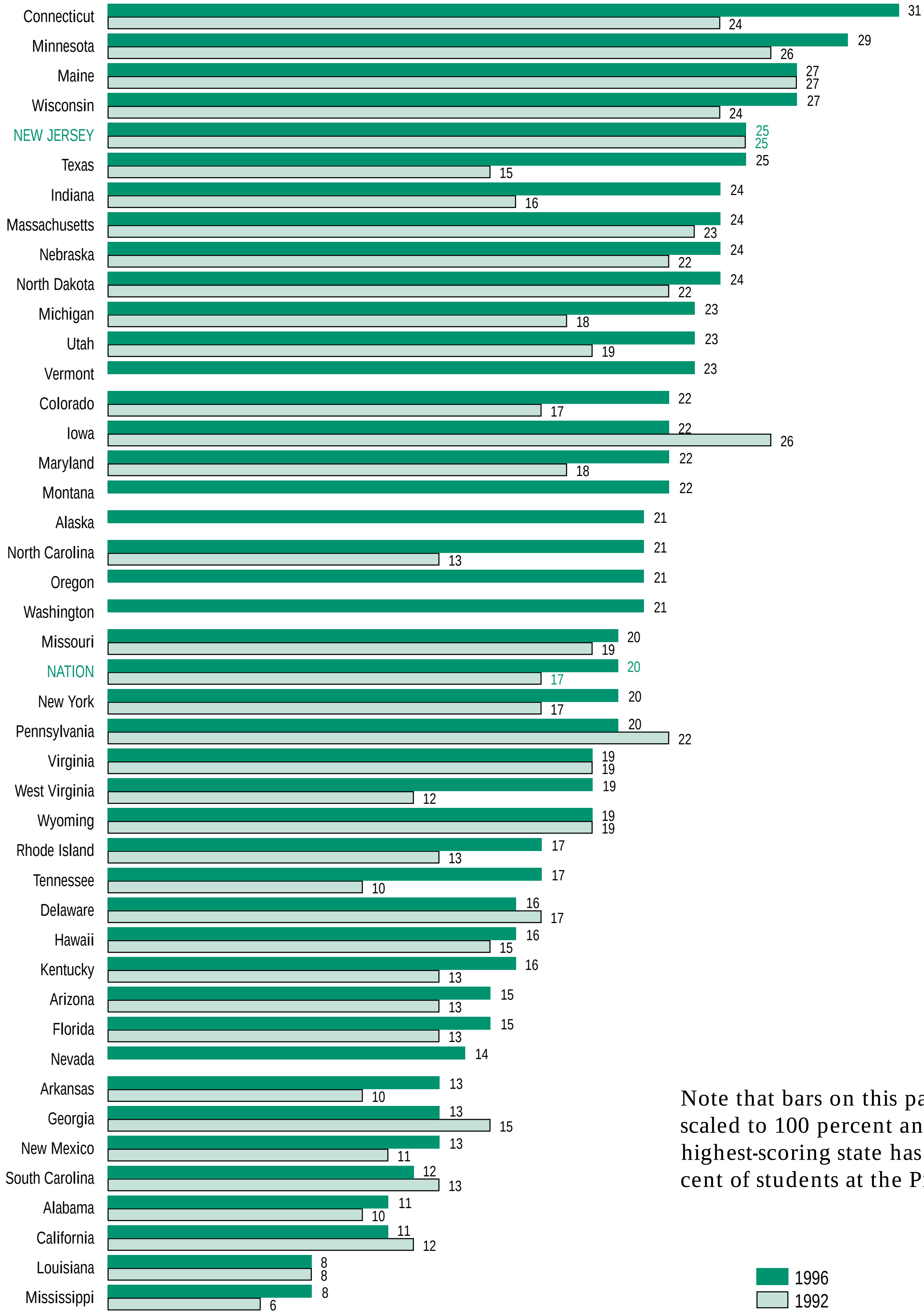
* Differences between NJ and US in 1996 and between US in 1992 and 1996 are not statistically significant and should be interpreted with caution
SOURCE: Reese, et al., *NAEP 1996 Mathematics Report Card*, p. 49

Figure 2.4: Percentage of Grade Eight Students Below NAEP Basic Level, New Jersey, 1992, and U.S., 1992 and 1996



SOURCE: Reese, et al., *NAEP 1996 Mathematics Report Card*; p. 51; "Revised [1990 and 1992] Mathematics Assessment Data for Grade 8," NAEP

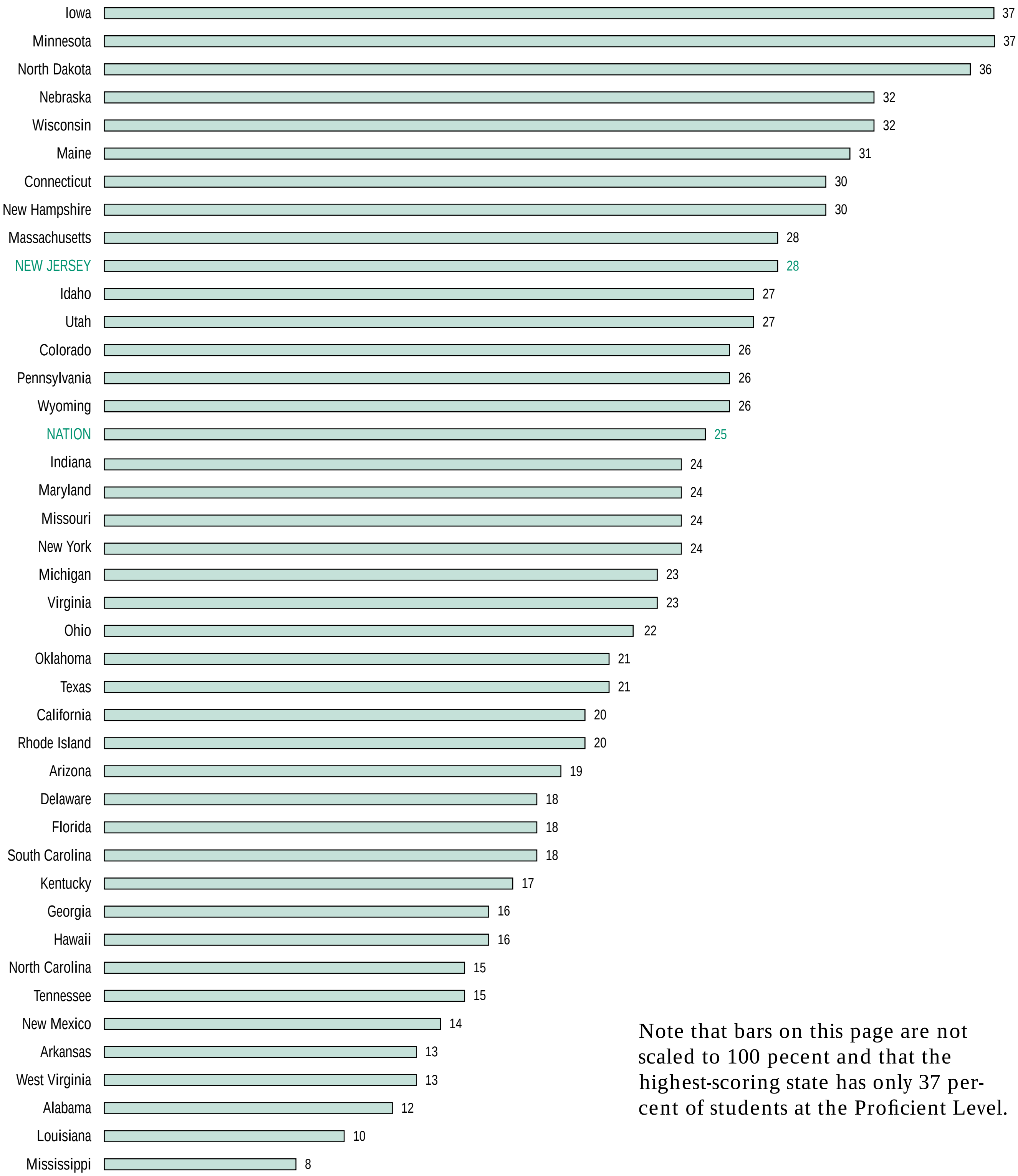
Figure 2.5: Percentage of Grade Four Students At or Above NAEP Proficient Level, 1992 and 1996



Note that bars on this page are not scaled to 100 percent and that the highest-scoring state has only 31 percent of students at the Proficient Level.

SOURCE: Reese, et al., *NAEP 1996 Mathematics Report Card*, p. 49

Figure 2.6 Percentage of Grade Eight Students At or Above NAEP Proficient Level, 1992



Note that bars on this page are not scaled to 100 pecent and that the highest-scoring state has only 37 percent of students at the Proficient Level.

SOURCE: NAEP *1996 Mathematics Report Card*; p. 51; "Revised [1990 and 1992] Mathematics Assessment Data for Grade 8," NAEP

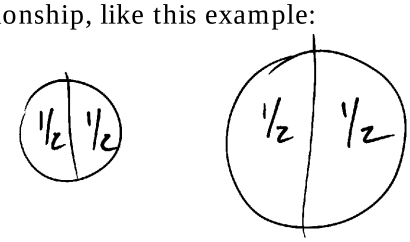
INDICATOR 3: Mathematics Proficiency on Individual NAEP Items

NAEP results take on a new meaning if we consider the performance of New Jersey students on individual test questions. These results help us understand the nature and difficulty of the test and provide insights into strengths and weaknesses in mathematics curricula.

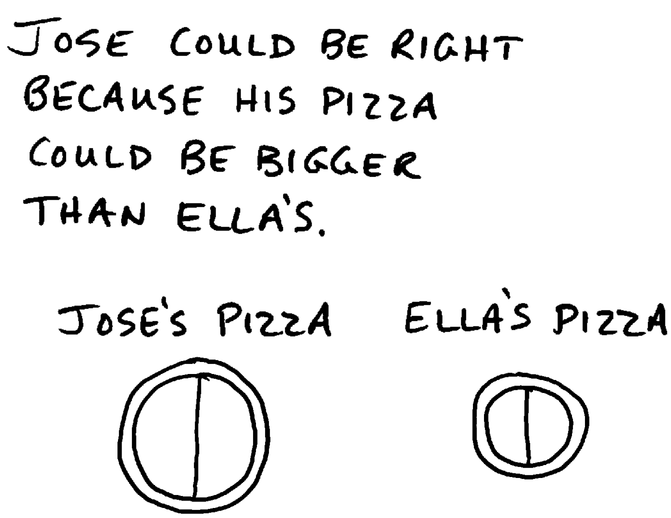
Results of individual questions on the 1996 test are not yet available, but here are five questions from the 1992 test and reports of how New Jersey students responded to them:

Problem A: Jose ate ½ of a pizza. Ella ate ½ of another pizza. Jose said that he ate more pizza than Ella, but Ella said that they both ate the same amount. Use words and pictures to show that Jose could be right.

Answer: Jose would be right if the size of his pizza was larger than the size of Ella's pizza. Satisfactory answers were those that used diagrams to show pizzas of different sizes, even though they did not clearly explain the relationship, like this example:



and those that used diagrams and wrote a clear and accurate description of the situation, like this example:



New Jersey students, grade four, whose answers were judged "satisfactory" or better, 1992—22 percent

SOURCE: *NCES, Data Compendium*, 1992, p. 274

Problem B: In the space below, use your ruler to draw a rectangle 2 inches wide and 3½ inches long.

New Jersey students, grade four, answering correctly, 1992—25 percent
New Jersey students, grade eight, answering correctly, 1992—58 percent

SOURCE: NCES, *Data Compendium*, 1992, pp. 235–36

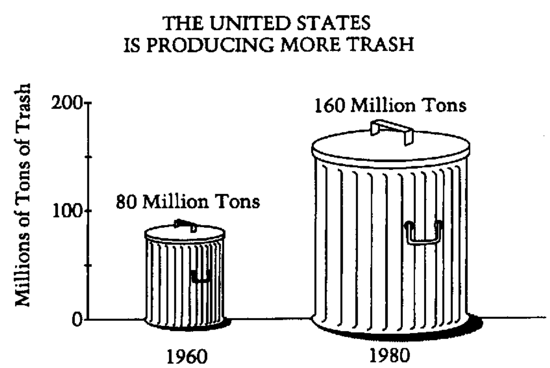
Problem C: Raymond must buy enough paper to print 28 copies of a report that contains 64 sheets of paper. Paper is only available in packages of 500 sheets. How many packages of paper will he need to buy to do the printing?

Answer: 4

New Jersey students, grade eight, answering correctly, 1992—59 percent

SOURCE: NCES, *Data Compendium*, 1992, p. 252

Problem D: The pictograph shown below is misleading. Explain why.



One possible correct answer: The size of the second trash can is much more than twice the size of the first one. Both its width and height have been doubled. Only the height should have been doubled.

New Jersey students, grade eight, answering correctly, 1992—11 percent

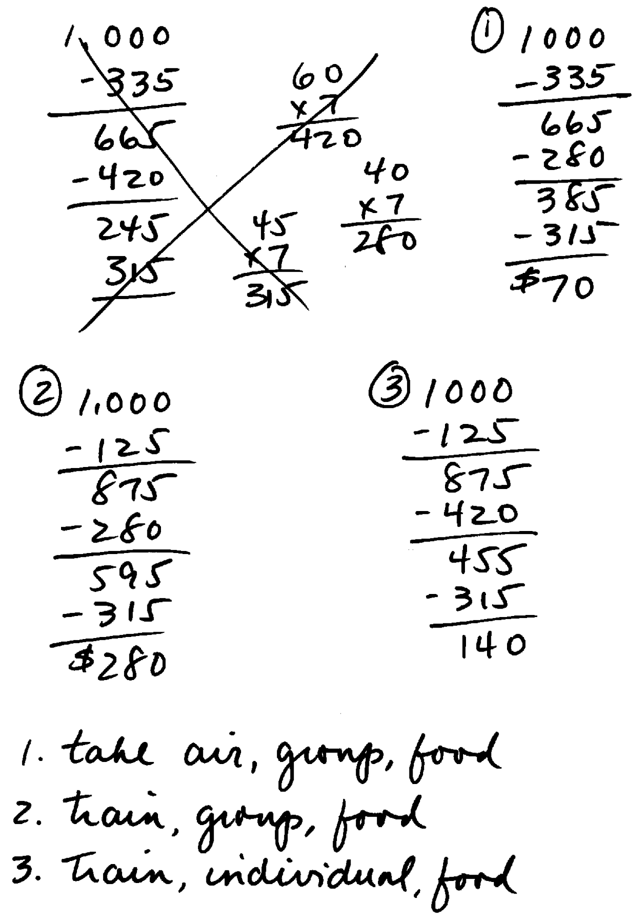
SOURCE: NCES, *Data Compendium*, 1992, p. 222

Problem E: This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show *all* your work.

Treena won a 7-day scholarship worth $1,000 to the Pro Shot Basketball Camp. Round-trip travel expenses to the camp are $335 by air or $125 by train. At the camp, she must choose between a week of individual instruction at $60 per day or a week of group instruction at $40 per day. Treena's food and other expenses are fixed at $45 per day. If she does not plan to spend any money other than the scholarship, what are *all* choices of travel and instruction plans that she could afford to make? Explain your reasoning.

Answer: Treena's fixed expenses will be $45 × 7 = $315. Therefore, she has $685 to spend on travel and instruction. Travel costs are either train ($125) or plane ($335); instruction costs are either group ($40 × 7 = $280) or individual ($60 × 7 = $420). Of Treena's four options—train and group instruction; plane and group instruction; train and individual instruction; plane and individual instruction—only the choice of travel by plane and individual instruction exceeds her budget.

Satisfactory answers were those that showed either correct mathematical evidence that Treena has three options, even though they lacked complete supporting work, or correct mathematical evidence and complete supporting work for at least two options. Here is an example of what NAEP considered an outstanding answer:



New Jersey students, grade eight, whose answers were judged "satisfactory" or better, 1992—
5 percent

SOURCE: NCES, *Data Compendium*, 1992, p. 297

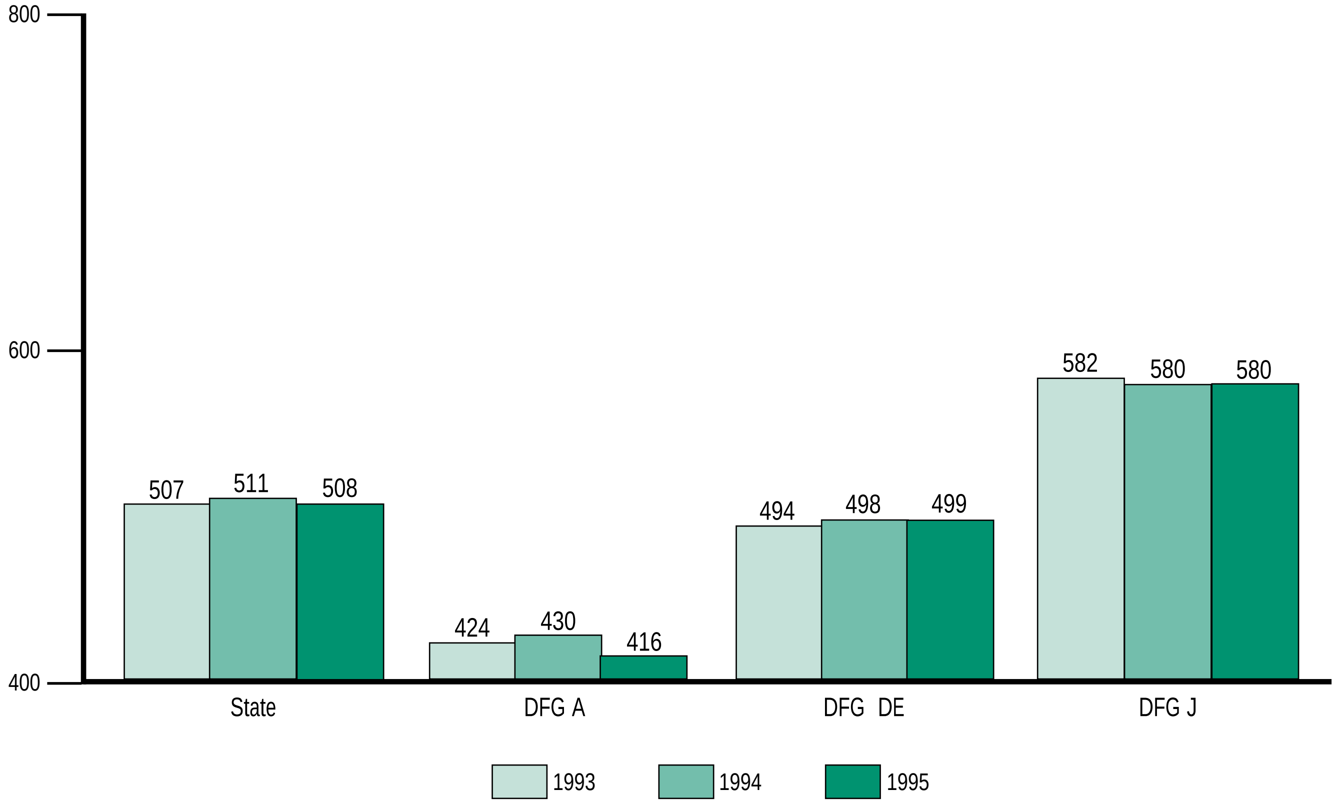
INDICATOR 4: Performance on SAT Test

The widely used Scholastic Assessment Test (SAT), formerly called the Scholastic Aptitude Test, is designed to measure students' development of verbal and mathematical abilities important for success in college, but average scores may also be used—with caution—as indicators of student progress in a school, district, or state. Caution is required because not all students take the test and the percentages and characteristics of test-takers may vary from year to year.

Figure 4.1 shows SAT mathematics scores of New Jersey high school seniors in recent years. The great disparity in performance between New Jersey's poorest and wealthiest communities is again apparent in results reported by District Factor Group (see Indicator 1); the gap actually widened from 1994 to 1995.

The Educational Testing Service revised SAT scoring in 1995, using a new "recentered" scale that results in higher scores than would have been computed under procedures used before that date. All SAT results reported here are based on the recentered scale.

Figure 4.1: Average Mathematics SAT Score for State and DFGs A, DE, and J, 1993 to 1995



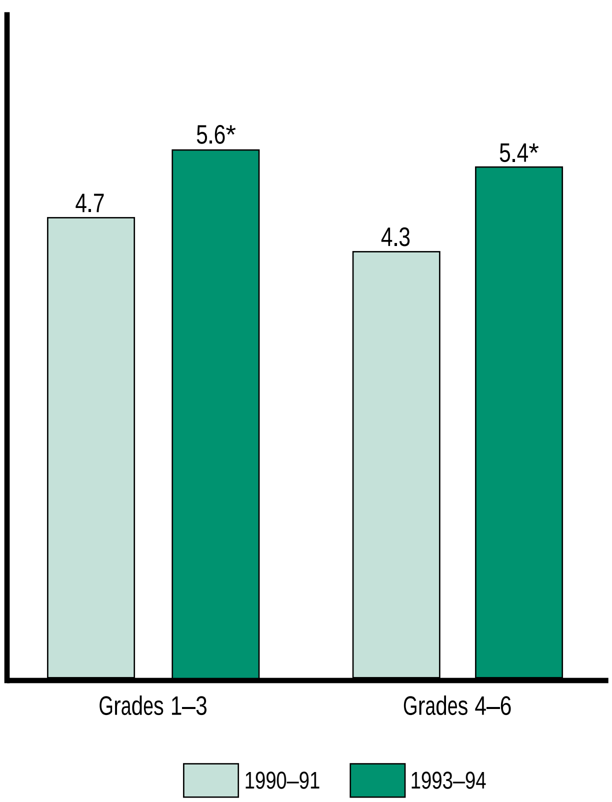
SOURCE: NJ State Department of Education Report Cards, 1995–96

Indicators of Quality of Content and Instruction

INDICATOR 5: Hours Per Week in Mathematics Instruction

In their *State Indicators of Science and Mathematics Education 1995*, Rolf Blank and Doreen Gruebel cite consistent research showing that mathematics achievement is strongly related to the amount of time devoted to instruction. The U.S. Department of Education's Schools and Staffing Survey (SASS) provides information about hours of instruction over a period of years. SASS employs questionnaires mailed to teachers, principals, and district administrators in a representative sample of public and private schools in each state.

Figure 5.1: Hours of Mathematics Instruction Per Week, New Jersey, 1990–91 and 1993–94



* Change is not statistically significant and should be interpreted with caution
Source: Blank et al., *SASS by State*, 1990–91, pp. 77–78; Bandeira and Broughman, *SASS by State*, 1993–94, pp. 141, 143

Figure 5.1 shows an apparent increase in time allotted to mathematics instruction in elementary grades in the period 1990–91 to 1993–94. School personnel and community members may wish to obtain equivalent data about their schools, compare them to statewide numbers presented here, and, where warranted, encourage school administrators to schedule more time for mathematics.

INDICATOR 6: Students Taking Specific Mathematics Courses Prior to Graduation

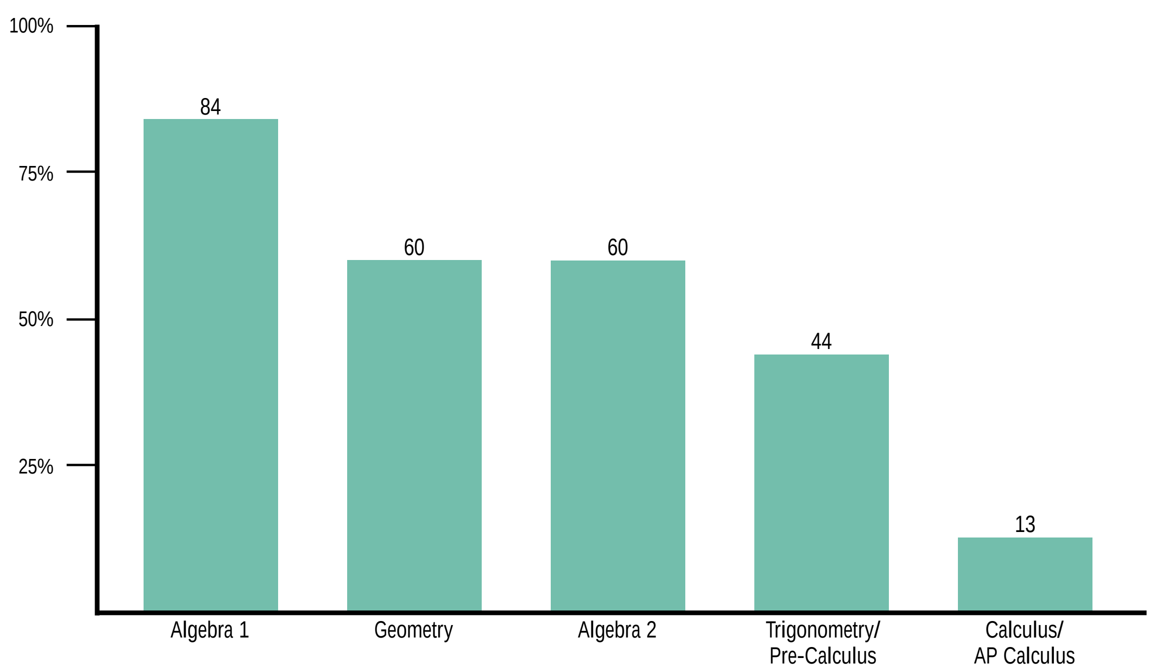
Blank and Gruebel also cite research indicating that the number of secondary school mathematics courses students take is strongly related to achievement. Yet a majority of American students never take advanced courses in mathematics. The 1992 NAEP assessment asked twelfth-graders about the highest level mathematics course they had taken in high school: 15 percent responded Algebra I, 13 percent Geometry, 44 percent Algebra II, and 16 percent Pre-Calculus or Calculus. Another 12 percent had taken no algebra or pre-algebra and, of course, some students had dropped out of school prior to the twelfth grade and were not reflected in the results at all.

In New Jersey, all students must complete three years of high school mathematics in order to graduate. It would be useful to know the percentage of students taking advanced courses—and racial and gender breakdowns, as well—to determine if schools are doing enough to insure that all students are taking challenging coursework. Unfortunately, the Department of Education does not regularly collect such information. In 1994, however, the Department did gather data on the percentage of students taking the traditional mathematics courses—Algebra I, Geometry, Algebra II, Trigonometry / Pre-Calculus, and Calculus / Advanced Placement Calculus—prior to graduation; see *Figure 6.1*.

This list ignores other advanced courses, such as Probability and Statistics and Discrete Mathematics, but the results suggest that a substantial percentage of New Jersey students never progress beyond Algebra I, or in some cases Geometry or Algebra II. These students are apparently completing their three-year mathematics requirement by taking lower-level courses like General Mathematics or Consumer Mathematics. With adoption of the state's mathematics standards, it is likely that a much larger percentage of students will take higher-level mathematics courses throughout their high school years, since they will need to complete core curriculum requirements in the subject.

While the information in *Figure 6.1* is slightly dated, it can be useful as a comparison with local conditions. School personnel and community members may wish to consider what steps might increase percentages of students taking higher-level coursework in their schools; this is particularly important if their schools fall short of or barely reach these state averages.

Figure 6.1: Percentage of New Jersey Students Taking Specific Mathematics Courses Prior to Graduation, 1994



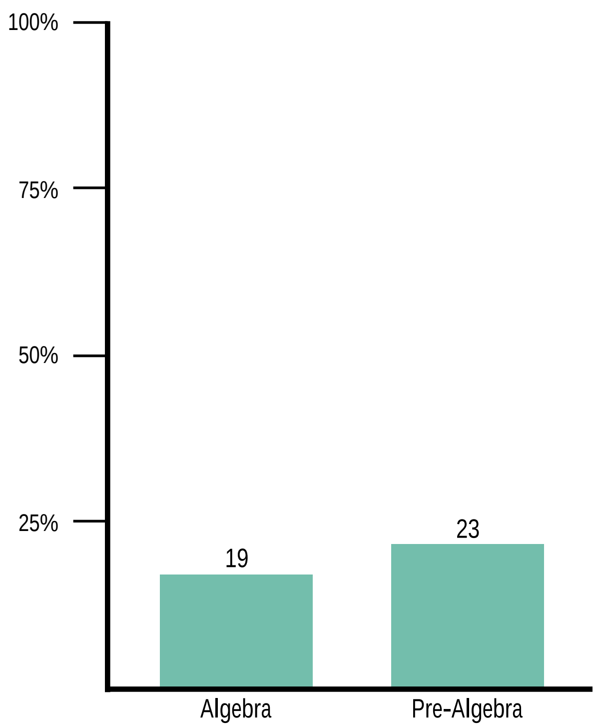
Note: Percentages reported are statistical estimates of course taking by public high school students by the time they graduate, based on total course enrollment in grades 9–12, divided by the estimated number of students in a grade cohort during four years of high school; Algebra I percentages include grade eight Algebra I.
SOURCE: Blank and Gruebel, *State Indicators*, 1995, p. 23

INDICATOR 7: Students Taking High-School-Level Mathematics in Grade Eight

Researchers Mullis, Jenkins, and Johnson (see References) have identified factors that are related to higher mathematics performance on the NAEP test. One finding (which took into account the socioeconomic and home background of students) was that schools with more students taking algebra or pre-algebra in grade eight, and planning to take geometry in grade 9, evidenced higher NAEP proficiency than schools with fewer students taking such courses. Beginning algebra at an earlier grade also increases options for the number and level of mathematics courses students can take before graduation. *Figure 7.1* shows the percentage of New Jersey eighth-graders taking algebra and pre-algebra in 1992.

Mathematics educators believe that students who are capable of taking high-school-level mathematics in eighth grade (or earlier) should be afforded opportunities to do so. However, with the adoption of Core Curriculum Content Standards in mathematics, it is likely that New Jersey districts will introduce a variety of courses that address the "core curriculum" in their high schools and offer these courses for students in earlier grades as well. Thus, in many districts, high-school-level courses other than algebra and pre-algebra will be available to students in the eighth grade or earlier. Moreover, many middle schools are introducing new curricula in grades 6–8 which incorporate high-school-level mathematics. In the future, determining the percentage of students taking high-school-level mathematics in the eighth grade will be a challenge.

Figure 7.1: Percentage of New Jersey Students Taking High-School-Level Mathematics in Grade Eight, 1992



SOURCE: NCES, *Data Compendium*, 1992, p. 407

INDICATOR 8: Incidence of Various Classroom Instructional Practices

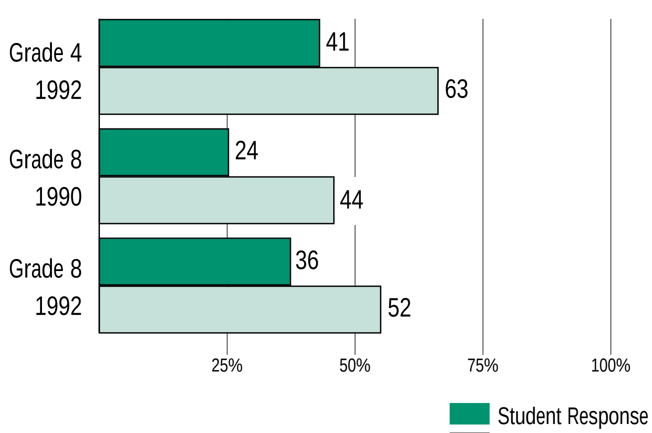
Some classroom practices—activities cited in, or consistent with state mathematics standards (see Appendix)—tend to enhance the performance of students and prepare them for life in a technological, information-based society. Educators and parents have opportunities to monitor classroom activity and encourage use of these practices.

While tracking some of these practices would require special data collections and visits to classrooms, others can be monitored through questions asked as part of the NAEP assessment program. Four of the practices that NAEP monitors—working in small groups, using "manipulatives," writing about how to solve problems, and using real-life situations—are particularly relevant to New Jersey standards.

In most cases, NAEP asked both teachers and students about these practices; where available, both sets of responses are presented here. Disparities between teacher and student responses may be due to students' inability to answer some questions accurately (younger children, for example may have difficulties estimating how often activities occur) and differences in the way teachers and students define some of the terms employed.

N. Davidson's book *Cooperative Learning in Mathematics* documents the benefits of learning that result from effective use of small group work. Moreover, the U.S. Department of Labor's *What Work Requires of Schools: A SCANS Report for America 2000* identifies the ability to work in teams as a key skill for the workplace of the future. *Figure 8.1* shows NAEP findings about the use of small groups in New Jersey Mathematics classes in 1990 and 1992 (1996 data for grade four are not yet available and problems with NAEP's sample size mean there will be no 1996 data for grade eight).

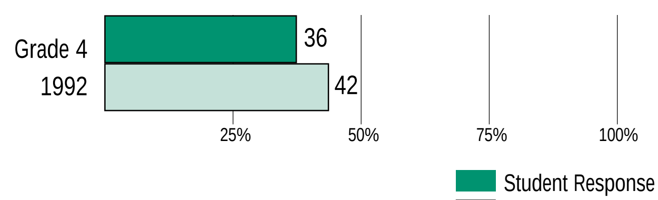
Figure 8.1: Percentage of New Jersey Students Working in Small Groups, at Least Weekly, 1990 and 1992



SOURCE: NCES, *Data Compendium*, 1992, p. 486–90

In their book *Reaching Standards: A Progress Report on Mathematics*, Mary Lindquist, John Dossey, and Ina Mullis report that research on mathematical learning supports the use of "manipulatives," i.e., concrete objects, such as colored rods, linking cubes, geometric shapes, or spinners, to illustrate concepts. Yet students seem to be given few opportunities to engage in these hands-on activities. *Figure 8.2* shows NAEP findings about the use of manipulatives in New Jersey in 1992.

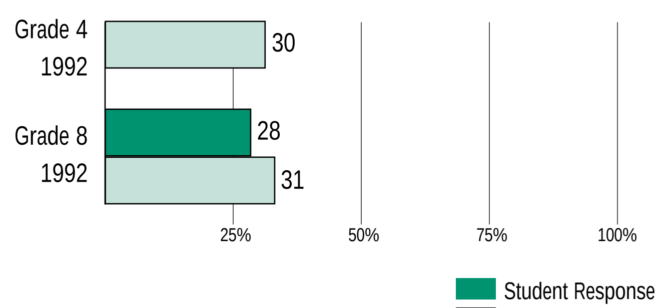
Figure 8.2: Percentage of New Jersey Students Working With Objects Like Rulers, Counting Blocks, or Geometric Shapes, at Least Weekly, 1992



SOURCE: NCES, *Data Compendium*, 1992, p. 499–500

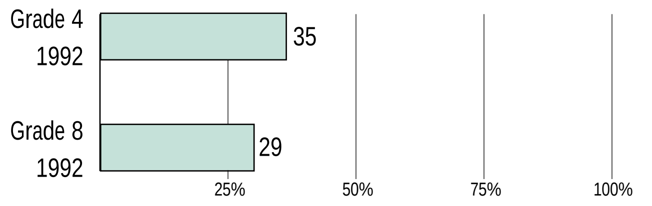
Reaching Standards also observes that American students are seldom required to write about how to solve a mathematics problem, and questions how performance can improve if students are not given opportunities to participate in activities that researchers regard as most effective. *Figure 8.3* shows NAEP findings about this practice in New Jersey in 1992.

Figure 8.3: Percentage of New Jersey Students Required to Write a Few Sentences About How to Solve a Problem, at Least Weekly, 1992



SOURCE: NCES, *Data Compendium*, 1992, p. 522–24

Figure 8.4: Percentage of New Jersey Students Asked to Work and Discuss Problems Reflecting Real-Life Situations, Almost Every Day, 1992

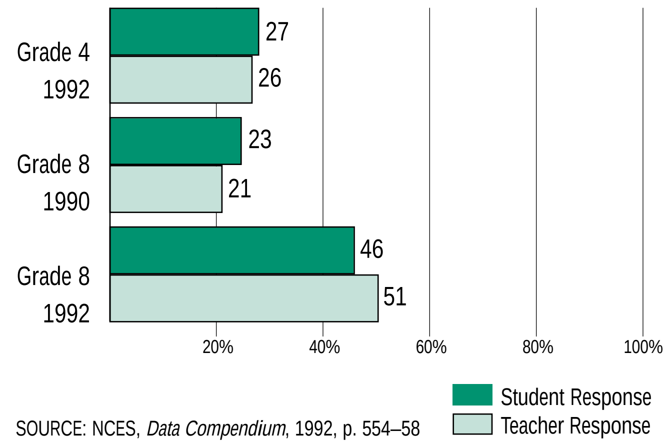


SOURCE: NCES, *Data Compendium*, 1992, p. 532–33

Similarly, students benefit from teachers who require them to work and discuss mathematics problems that reflect real-life situations that they can relate to easily. *Figure 8.4* shows NAEP findings about the incidence of such practices in New Jersey in 1992.

New Jersey mathematics standards emphasize the importance of regular use of calculators, computers, and other mathematical tools. *Figure 8.5* shows NAEP findings about use of calculators in New Jersey mathematics classrooms in 1990 and 1992.

Figure 8.5: Percentage of New Jersey Students Using Calculators in Class, at Least Weekly, 1990 and 1992



SOURCE: NCES, *Data Compendium*, 1992, p. 554–58

Of course, these five classroom practices are not ends in themselves, and a school where they are common is not necessarily implementing the mathematics standards. Nevertheless, such practices, if used appropriately and thoughtfully, can be important components of a coordinated approach to improved teaching and learning.

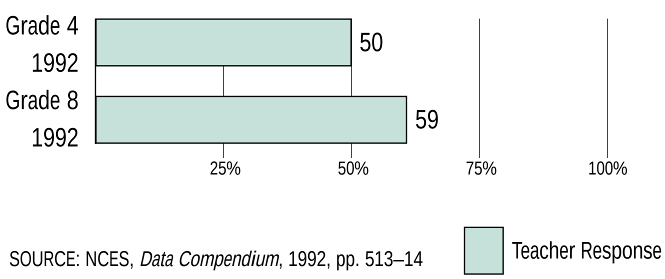
INDICATOR 9: Incidence of Various Classroom Assessment Practices

Many mathematics educators believe that certain student assessment practices are beneficial in developing students' abilities. These include use of questions that require short written responses and use of such techniques as projects, portfolios, and presentations. School personnel and community members should review current assessment techniques and, where warranted, urge adoption of these and other innovative practices.

Questions that require students to provide short written answers require them to think about their reasoning process and describe it in writing. For example, fourth-graders might be asked to estimate how many 3" × 5" index cards would be needed to cover the surface of their desks and to describe in a few sentences how they arrived at their answers. *Figure 9.1* shows NAEP findings about the incidence of this form of assessment in New Jersey in 1992 (1996 data for grade four are not yet available, and problems with NAEP's sample size mean there will be no 1996 data for grade eight).

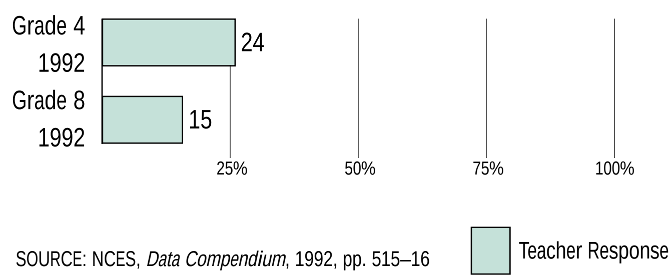
Reaching Standards cites research suggesting that such alternative forms of assessment as projects, portfolios, and presentations are most effective in reinforcing learning. As an example of the use of portfolios, students might be instructed to review all of their work during a just-completed unit and compile a portfolio exemplifying their best work and the progress they have made. The self-reflection required in preparing such portfolios can be important in helping students solidify knowledge and understandings they gained during the unit. *Figure 9.2* presents NAEP findings about the use of these practices in New Jersey in 1992.

Figure 9.1: Percentage of New Jersey Students Given Questions Requiring Short Written Responses, at Least Monthly, 1992



SOURCE: NCES, *Data Compendium*, 1992, pp. 513–14

Figure 9.2: Percentage of New Jersey Students Required to Offer Projects, Portfolios, or Presentations to Assess Student Progress, at Least Monthly, 1992



SOURCE: NCES, *Data Compendium*, 1992, pp. 515–16

Indicators of Teacher Supply and Preparation

INDICATOR 10: Supply of Certified Mathematics Teachers

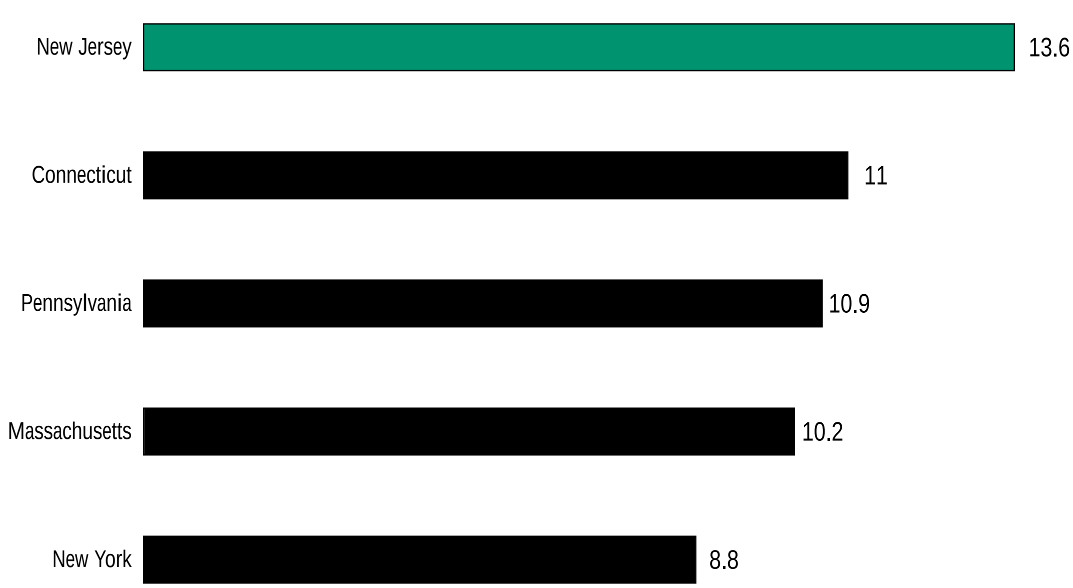
While requirements for teacher certification vary from state to state, certification implies that teachers have at least a basic level of preparation in their teaching fields. Thus, the percentage of mathematics teachers who are certified is a basic measure of teacher qualification and an indicator of teacher supply.

In New Jersey, a standard elementary certificate enables teachers to teach any subject in grades K–8. A mathematics certificate is required to teach mathematics in grades 9–12 and also licenses the holder to teach mathematics—but no other subject—in grades K–8. There are two routes to the mathematics certificate. The "traditional route" requires prospective teachers to complete a State Department of Education-approved teacher education program at a college or university, attain an acceptable score on the Praxis II examination (formerly the National Teachers Examination) in mathematics, major in mathematics, and complete one year of full-time mentored teaching. An "alternate route" grants certification to any college graduate who completes a coherent sequence of at least 30 credits in mathematics and passes the Praxis II examination in mathematics. Alternate route teachers then must complete a teacher-training program (requiring no additional study of mathematics) outside of school during their first year of teaching.

According to data collected by the Council of Chief State School Officers (Blank and Gruebel, *State Indicators*, p. 56), New Jersey, in 1993–94, was one of a very few states in the Union in which 100 percent of teachers assigned to one or more periods of teaching mathematics in grades 9–12 were certified mathematics teachers. New Jersey teachers of mathematics in grades K–6 tend to teach many other subjects as well and are virtually all certified as elementary, rather than mathematics teachers. In grades 7–8, the picture is mixed, with some teachers holding elementary certification and others holding mathematics certification and having substantial backgrounds in the subject.

The fact that 100 percent of its grade 9–12 mathematics teachers are certified implies that New Jersey has an ample supply of certified mathematics teachers at that level. In fact, data compiled by the Council of Chief State School Officers show that, in 1994, it was the best-supplied state in the Union. *Figure 10.1* compares New Jersey with other states in its region.

Figure 10.1: Certified Teachers per 1,000 Students Taking Mathematics, Grades 9–12, 1994



SOURCE: Blank and Gruebel, *State Indicators, 1995*, p. 58

INDICATOR 11: Teacher Preparation

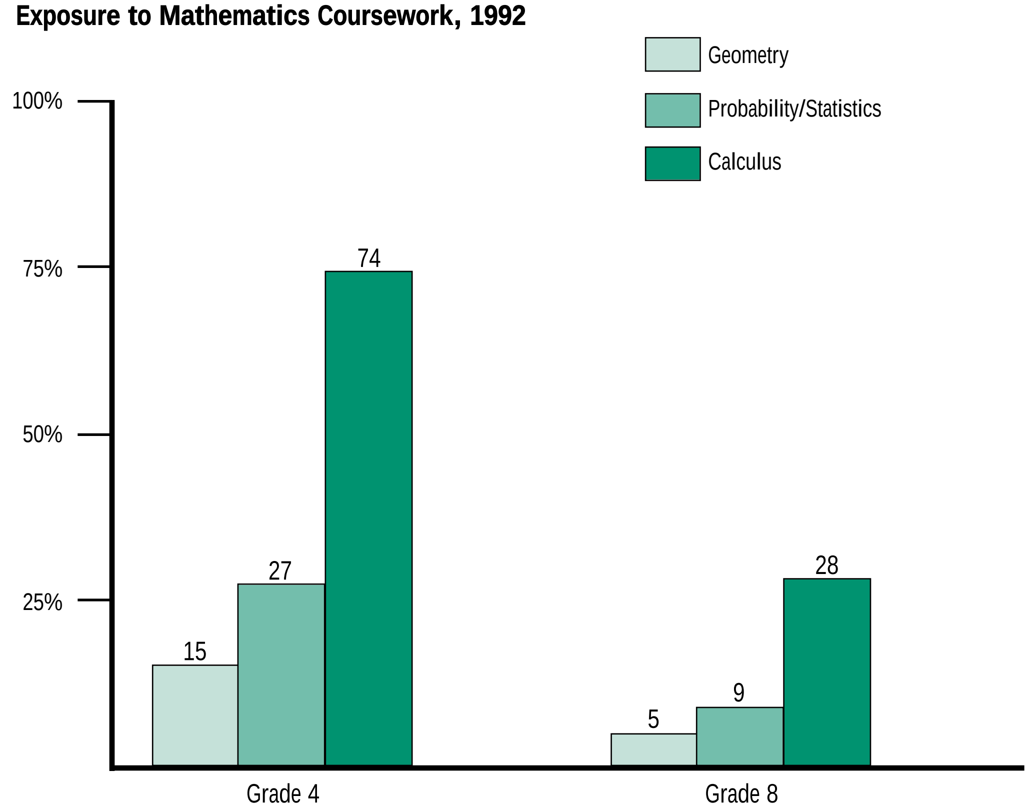
While New Jersey has an ample supply of certified mathematics teachers, the certification process does not guarantee that all teachers have been well-prepared for their task. New Jersey does not collect information about college coursework or subsequent professional development of teachers, but NAEP asks teachers about their exposure to content areas recommended by the National Council of Teachers of Mathematics.

Figure 11.1 shows percentages of New Jersey teachers who reported “little or no exposure”—no college courses, no part of a college course, and no inservice training programs—to geometry, probability / statistics, or calculus. These may be older teachers, since the State Department of Education requires new teachers to complete a college program in which they demonstrate competence in “understanding the basic concepts” of all these subjects. Since New Jersey’s mathematics standards require students at all grade levels to progress in these subject areas, colleges should ensure that all teacher candidates are trained in these subjects and schools should hasten to provide appropriate inservice training to teachers with “little or no exposure.”

Perhaps the best indicator of teacher preparation, especially at the secondary level, is the percentage of teachers with an undergraduate or graduate major in mathematics. This is because the number, variety, and quality of courses required of mathematics majors are likely to provide them with a broad and deep knowledge of the subject. Research by Shavelson, et al., and by Monk (see References) has shown a positive relationship between the amount of coursework taken by mathematics teachers and student learning in mathematics. And NAEP results for 1992 showed that the number of mathematics courses taken by teachers had a positive relationship with student proficiency (Mullis, et al., *NAEP 1992 Mathematics Report Card*).

Both the NAEP and SASS studies ask teachers whether they majored in mathematics. NAEP asks the question of fourth- and eighth-grade teachers of mathematics and reports the percentage of students taught by teachers with mathematics majors. SASS asks teachers of grades 7–12 whose main or secondary assignment is teaching mathematics and reports in terms of the teachers themselves.

Figure 11.1: Percentage of New Jersey Students Whose Teachers Report Little or No Exposure to Mathematics Coursework, 1992



SOURCE: NCES, *Data Compendium*, 1992, pp. 621–28

Most fourth-grade mathematics teachers also teach other topics and very few have majored in mathematics. The most recent NAEP results available, from 1992 (see *Figure 11.2*), indicate that only five percent of New Jersey fourth-graders—the same as the national percentage—were taught mathematics by teachers who had majored in mathematics, while 84 percent were taught by teachers who had majored in education and 10 percent by teachers with majors in other subjects (NCES, *Data Compendium*, 1992, p. 610).

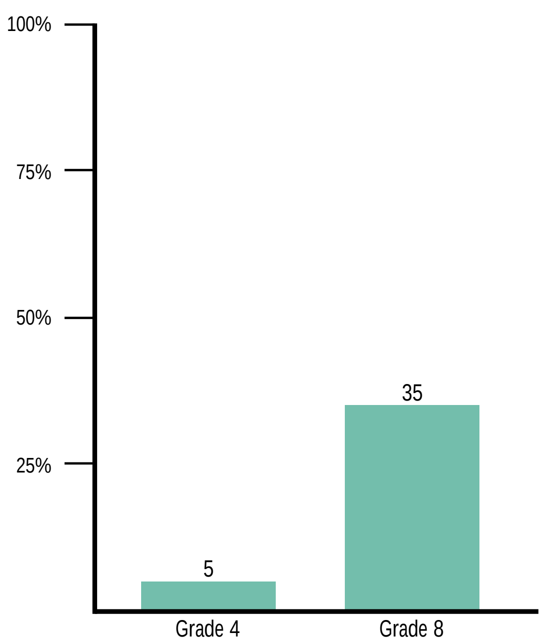
At the eighth-grade level in 1992 (see *Figure 11.2*), NAEP reports that 35 percent of New Jersey students were taught mathematics by teachers who had majored in mathematics, 13 percent by those who had majored in mathematics education, 41 percent by elementary education majors, and 10 percent by teachers with majors in other subjects. In this regard, New Jersey suffers by comparison with most other states in its region: 43 percent of eighth-graders in Connecticut were taught mathematics by teachers who had majored in mathematics, 46 percent of those in Maryland, 54 percent of those in Massachusetts, 55 percent of those in New York, and 57 percent of those in Pennsylvania (differences between New Jersey and the last three states are statistically significant; NCES, *Data Compendium*, 1992, p. 611).

Interestingly, New Jersey eighth-graders taught by teachers who had majored in mathematics or mathematics education had average NAEP proficiency scores of 279 and 281, respectively (the difference is not statistically significant), while those taught by elementary education majors had an average proficiency of 263 (this difference *is* statistically significant).

At any rate, the percentage of New Jersey fourth- and eighth-grade teachers with majors in elementary education is bound to decrease in years to come because the state now requires prospective teachers to major in academic subjects.

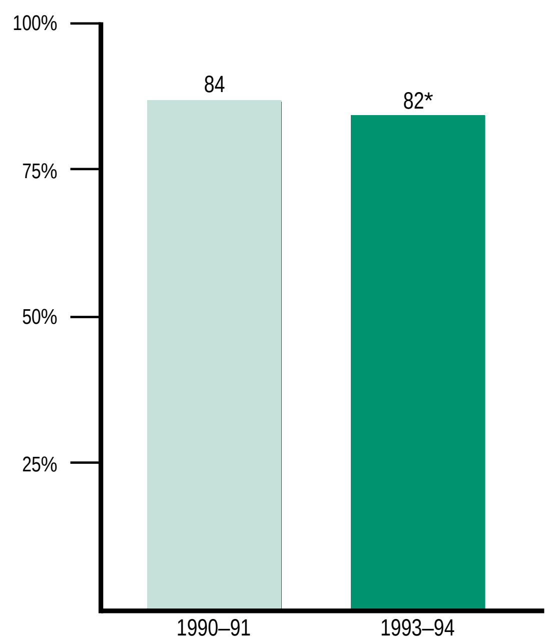
Data on the percentage of high school teachers with mathematics majors was last collected by SASS in 1993–94. In that year, according to National Education Data Resource Center calculations, 82 percent of New Jersey public school teachers in grades 9–12, with mathematics as their main assignment, reported that they had majored in mathematics or mathematics education, an apparent decrease from 1990–91; see *Figure 11.3*. Corresponding figures in other states were 83 percent in Connecticut, 83 percent in Maryland, 78 percent in Massachusetts, 77 percent in New York, and 94 percent in Pennsylvania (differences between New Jersey and other states are not statistically significant).

Figure 11.2: Percentage of New Jersey Students, Grades Four and Eight, Taught by Teacher With Major in Mathematics, 1992



SOURCE: NCES, *Data Compendium*, 1992, pp. 610–11

Figure 11.3: Percentage of New Jersey Grade 9-12 Mathematics Teachers with Major in Mathematics, 1990–91 and 1993–94



* Change is not statistically significant and should be interpreted with caution
SOURCE: NCES, National Education Data Resource Center

Conclusion

In this snapshot of mathematics instruction in New Jersey, it is clear that our state often outperforms others, but still falls far short of providing what our students will need to live fulfilling, productive lives in a highly competitive international environment. The findings reported here suggest a number of recommendations for policymakers, school and college administrators, educators, parents, and community members:

1. School districts should use the *New Jersey Mathematics Curriculum Framework* to help them implement standards rapidly, efficiently, and thoughtfully.

2. State and local policymakers should increase efforts to close the performance gap between students from advantaged and disadvantaged districts.

3. Districts devoting less class time to mathematics than the current state average should increase the amount of mathematics class time.

4. Districts should develop high school courses that address the core curriculum described by the grades 9-12 cumulative progress indicators of New Jersey's Core Curriculum Content Standards and ensure that all students take such courses.

5. Districts should provide opportunities for qualified eighth-graders to take high-school-level mathematics courses that address the core curriculum for grades 9-12 .

6. High schools should provide opportunities for qualified students to take advanced mathematics courses (e.g., Advanced Placement Calculus).

7. Teachers should provide frequent opportunities for students to: work in small groups; work with manipulatives; write about how to solve mathematics problems; work and discuss problems reflecting real life situations; and use calculators, computers, and other technological tools in class. These practices should be implemented thoughtfully and appropriately, as part of a coordinated approach to improving teaching and learning.

8. Teachers should make more use of such assessment techniques as questions requiring short written responses, projects, portfolios, and presentations.

9. College administrators and faculty should ensure that all mathematics teacher candidates are adequately trained in key content areas in the discipline—including geometry, probability/statistics, and calculus—and in proper use of the instructional and assessment techniques listed above.

10. School administrators should ensure that teachers with inadequate backgrounds in key content areas, or with inadequate knowledge about proper use of the instructional and assessment techniques listed above, receive appropriate professional development as soon as possible.

11. Parents and community-members should become active in encouraging improvement in mathematics education in their communities' schools.

Such changes—and general implementation of the state's mathematics standards—can give New Jersey a system of mathematics education capable of preparing *all* of the state's students for life, careers, and productive citizenship in the 21st Century.

Appendix: New Jersey Core Curriculum Content Standards in Mathematics

Adopted by the New Jersey
State Board of Education, May 1, 1996

Standard 1. All students will develop the ability to pose and solve mathematical problems in mathematics, other disciplines, and everyday experiences.

Problem-posing and problem-solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. By developing their problem-solving skills, students will come to realize the potential usefulness of mathematics in their lives.

Standard 2. All students will communicate mathematically through written, oral, symbolic, and visual forms of expression.

Communication of mathematical ideas will help students clarify and solidify their understanding of mathematics. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematics learners and enable teachers to better monitor their progress.

Standard 3. All students will connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the roles that mathematics and mathematical modeling play in other disciplines and in life.

Making connections enables students to see relationships between different topics and to draw on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to everyday life.

Standard 4. All students will develop reasoning ability and will become self-reliant, independent mathematical thinkers.

Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied. Mathematical reasoners are able to reflect on solutions to problems and determine whether or not they make sense. They appreciate the pervasive use and power of reasoning as a part of mathematics.

Standard 5. All students will regularly and routinely use calculators, computers, manipulatives, and other mathematical tools to enhance mathematical thinking, understanding, and power.

Calculators, computers, manipulatives, and other mathematical tools need to be used by students in both instructional and assessment activities. These tools should be used, not to replace mental mathematics and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Historically, people have developed and used manipulatives (such as fingers, base ten blocks, geoboards, and algebra tiles) and mathematical devices (such as protractors, coordinate systems, and calculators) to help them understand and develop mathematics. Students should explore both new and familiar concepts with calculators and computers, but should also become proficient in using technology as it is used by adults, that is, for assistance in solving real-world problems.

Standard 6. All students will develop number sense and an ability to represent numbers in a variety of forms and use numbers in diverse situations.

Number sense is defined as an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent, coming from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how they can best be used to describe a particular situation. Number sense is an attribute of all successful users of mathematics.

Standard 7. All students will develop spatial sense and an ability to use geometric properties and relationships to solve problems in mathematics and in everyday life.

Spatial sense is an intuitive feel for shape and space. It involves the concepts of traditional geometry, including an ability to recognize, visualize, represent, and transform geometric shapes. It also involves other, less formal ways of looking at two- and three-dimensional space, such as paper-folding, transformations, tessellations, and projections. Geometry is all around us in art, nature, and the things we make. Students of geometry can apply their spatial sense and knowledge of the properties of shapes and space to the real world.

Standard 8. All students will understand, select, and apply various methods of performing numerical operations.

Numerical operations are an essential part of the mathematics curriculum. Students must be able to select and apply various computational methods, including mental math, estimation, paper-and-pencil techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, and other kinds of numbers. With calculators that perform these operations quickly and accurately, however, the instructional emphasis now should be on understanding the meanings and uses of the operations, and on estimation and mental skills, rather than solely on developing pencil-and-paper skills.

Standard 9. All students will develop an understanding of systems of measurement and will use measurement to describe and analyze phenomena.

Measurement helps describe our world using numbers. We use numbers to describe simple things like length, weight, and temperature, but also complex things such as pressure, speed, and brightness. An understanding of how we attach numbers to those phenomena, familiarity with common measurement units like inches, liters, and miles per hour, and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.

Standard 10. All students will use a variety of estimation strategies and recognize situations in which estimation is appropriate.

Estimation is a process that is used constantly by mathematically capable adults, and that can be mastered easily by students. It involves an educated guess about a quantity or a measure, or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that decision is through estimation. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact answer.

Standard 11. All students will develop an understanding of patterns, relationships, and functions and will use them to represent and explain real-world phenomena.

Patterns, relationships, and functions constitute a unifying theme of mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena. These explorations present unlimited opportunities for problem-solving, making and verifying generalizations, and building mathematical understanding and confidence.

Standard 12. All students will develop an understanding of statistics and probability and will use them to describe sets of data, model situations, and support appropriate inferences and arguments.

Probability and statistics are the mathematics used to understand chance and to collect, organize, describe, and analyze numerical data. From weather reports to sophisticated studies of genetics, from election results to product preference surveys, probability and statistical language and concepts are increasingly present in the media and in everyday conversations. Students need this mathematics to help them judge the correctness of an argument supported by seemingly persuasive data.

Standard 13. All students will develop an understanding of algebraic concepts and processes and will use them to represent and analyze relationships among variable quantities and to solve problems.

Algebra is a language used to express mathematical relationships. Students need to understand how quantities are related to one another, and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic techniques, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation and the graph, and on what the graph represents in a real-life situation.

Standard 14. All students will apply the concepts and methods of discrete mathematics to model and explore a variety of practical situations.

Discrete mathematics is the branch of mathematics that deals with arrangements of distinct objects. It includes a wide variety of topics and techniques that arise in everyday life, such as how to find the best route from one city to another, where the objects are cities arranged on a map. It also includes how to count the number of different combinations of toppings for pizzas, how best to schedule a list of tasks to be done, and how computers store and retrieve arrangements of information on a screen. Discrete mathematics is the mathematics used by decision-makers in our society, from workers in government to those in health care, transportation, and telecommunications. Its various applications help students see the relevance of mathematics in the real world.

Standard 15. All students will develop an understanding of the conceptual building blocks of calculus and will use them to model and analyze natural phenomena.

The conceptual building blocks of calculus are important for everyone to understand. How quantities such as world population change, how fast they change, and what will happen if they keep changing at the same rate are questions that can be discussed by elementary school students. Another important topic for all mathematics students is the concept of infinity—what happens as numbers get larger and larger and what happens as patterns are continued indefinitely. Early explorations in these areas can broaden students' interest in and understanding of an important area of applied mathematics.

Standard 16. All students will demonstrate high levels of mathematical thought through experiences which extend beyond traditional computation, algebra, and geometry.

High expectations for all students form a critical part of the learning environment. The belief of teachers, administrators, and parents that a student can and will succeed in mathematics often makes it possible for that student to succeed. Beyond that, this standard calls for a commitment that all students will be continuously challenged and enabled to go as far mathematically as they can.

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Abbreviations Used in This Report:

NAEP—National Assessment of Educational Progress
NCES—National Center for Education Statistics
SASS—Schools and Staffing Survey

Technical Note

In this report, the term “statistically significant” means that the difference is significant at the 95 percent confidence level.

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