Machine Learning & Mechanism Design:

Dynamic and Discriminatory Pricing in Auctions

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Goal: design mechanism to optimally price discriminate.

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Goal: understand how quality and incentives of learning distribution affect profit.

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1. *Unlimited supply* of stuff to sell.

2. bidders with private *valuations* for stuff.

3. make each bidder an *offer*.

4. revenue is *incentive compatible* function of offer and valuation.

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   (a) Random Sampling Solution

   (b) Retrospective bounds.

   (c) Software Versioning Example.

2. Online Auction Problem

   (a) Expert Learning based Auction.

   (b) Expert Learning with non-uniform bounds.

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Auction Problem

The *Unlimited Supply Auction Problem*:

Given:

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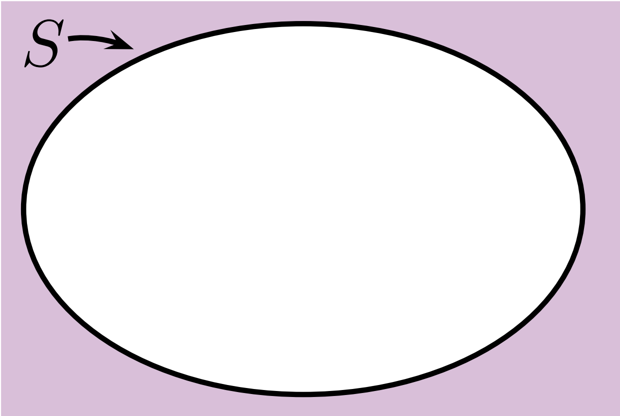
Notation:

- $g(i) =$ payoff from bidder i when offered g.
- $g(S) = \sum_{i \in S} g(i)$.
- $\mathrm{opt}_{\mathcal{G}}(S) = \mathrm{argmax}_{g \in \mathcal{G}}\, g(S)$.
- $\mathrm{OPT}_{\mathcal{G}}(S) = \max_{g \in \mathcal{G}}\, g(S)$.

Random Sampling Auction

Random Sampling Optimal Offer Auction, $\text{RSOO}_{\mathcal{G}}$

1. Randomly partition bidders into two sets: S_1 and S_2.
2. compute g_1 (resp. g_2), optimal offer for S_1 (resp. S_2)
3. Offer g_1 to S_2 and g_2 to S_1.



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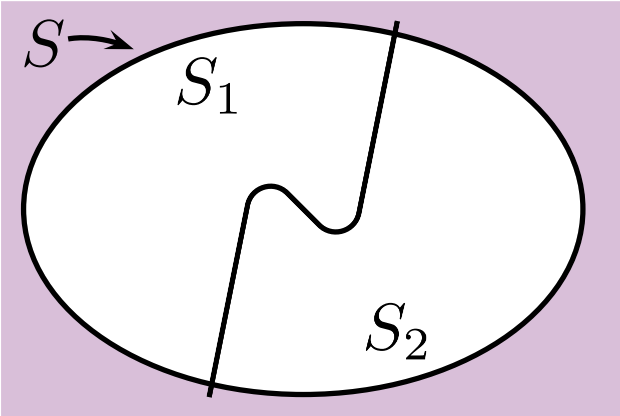

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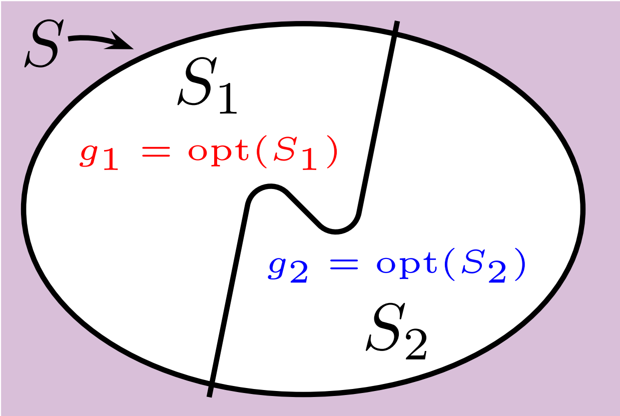

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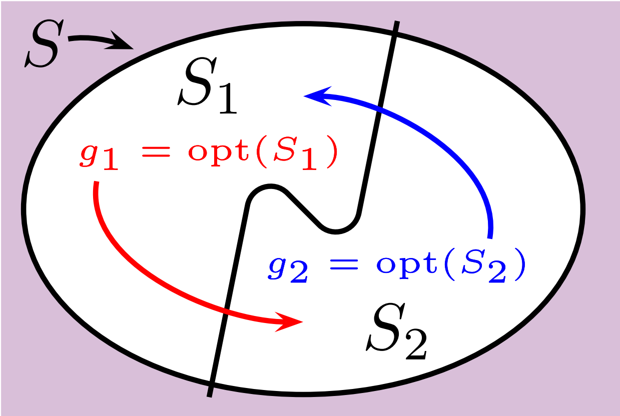

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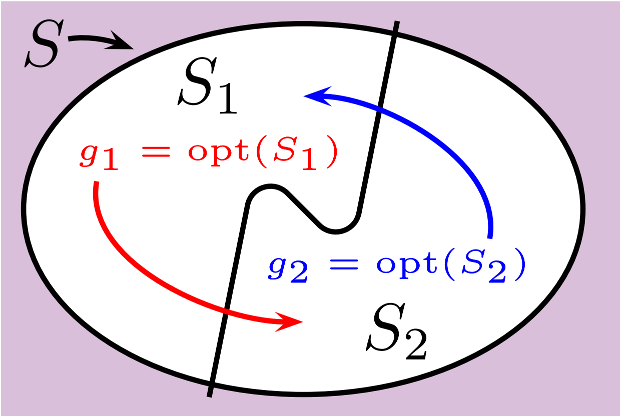

Question: when is RSOO$_{\mathcal{G}}$ good?

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Performance Analysis

Lemma: For g and random partitions S_1 and S_2:

$$\textbf{Pr}[|g(S_1) - g(S_2)| > \epsilon \max(p, g(S))] \leq 2e^{-\epsilon^2 p/2h}.$$

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- Use $p = \mathrm{OPT}_{\mathcal{G}}$.
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Interpretation: $O(h \log |\mathcal{G}|)$ is *convergence time*.

Example

Example: Selling tee shirts. (discretized prices)

- Bidders with valuations in $[1, h]$ for a tee shirt.
- Reasonable offers: $\mathcal{G} = \{$price 2^i for $i \in \{1, \ldots, \log h\}\}$.
- Convergence Time: $O(h \log |\mathcal{G}|) = O(h \log \log h)$

Overview

1. Auction Problem
   (a) Random Sampling Solution
   ⟹ (b) Retrospective bounds.
   (c) Software Versioning Example.
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   (a) Expert Learning based Auction.
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- Suppose RSOO$_{\mathcal{G}}$ on S only offers $g \in \mathcal{G}_S \subset \mathcal{G}$.
- Then RSOO$_{\mathcal{G}_S}(S)$ is same as RSOO$_{\mathcal{G}}(S)$.
- Retrospectively perform analysis on $\mathcal{G}_S$ instead of $\mathcal{G}$.

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7. Sum over M^m possible m-item sets.

Other Results

See paper for details on:

- Bounds for $\text{RSOO}_{\mathcal{G}}$ for item-pricing in combinatorial auctions.
- Bounds for $\text{RSOO}_{\mathcal{G}}$ on bidders with observable features.
- Better bounds with *ϵ-covers* of $\mathcal{G}$.
- Better random sampling auction with *structural risk minimization*.
- Using approximation algorithms in $\text{RSOO}_{\mathcal{G}}$.

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Online Auction Problem:

- unlimited supply of stuff.
- class $\mathcal{G}$ of reasonable offers.
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Conclusion: offer for bidder i based only on prior bids: $b_1, \ldots, b_{i-1}$.

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In round i:

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Weighted Majority Algorithm: (for round i)

Let h be maximum payoff. For expert j, let s_j be total payoff thus far.

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Note: Same convergence time as for $\mathrm{RSOO}_{\mathcal{G}}$.

Example

Example: Selling tee shirts. (discretized prices)

- Bidders with valuations in $[1, h]$ for a tee shirt.
- Reasonable offers: $\mathcal{G} = \{\text{price } 2^i \text{ for } i \in \{1, \ldots, \log h\}\}$.
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Better Bounds?

Can we get better bounds?

Retrospective technique like using $\mathcal{G}_S$ does not work.

Overview

1. Auction Problem
   (a) Random Sampling Solution
   (b) Retrospective bounds.
   (c) Software Versioning Example.
2. Online Auction Problem
   (a) Expert Learning based Auction.
   $\Longrightarrow$ (b) Expert Learning with non-uniform bounds.
3. Conclusions

Non-uniform Bounds on Payoff

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1. (initialization) For each expert, j, add initial score, s_j, as:

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Note: this is optimal up to constant factors.

Conclusions

1. Used machine learning techniques for auction design/analysis.
2. Prior-free discriminatory optimal mechanism design.
   (a) distinguishing between products (and selecting products to sell).
   (b) price discriminate based on observable customer features.
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5. **Open:** ϵ-cover arguments for online auctions?
6. **Open:** limited supply?
7. **Open:** general cost function on outcomes?