Impact of Periodic and Constant Proportion Harvesting Policies On TAC-Regulated Fisheries Systems

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Emerging Ocean Diseases

Disease is increasing among most marine organisms (Ward and Lafferty, 2004).

Examples: Recent epizootics (epidemics in animals) of Atlantic Ocean bottlenose dolphins and endangered Florida manatees.

Contributing Factors include

- global warming
- habitat destruction
- human overfishing
- etc

Overfishing Implicated In Sea Urchin Epidemics

- Sea urchin epidemics have risen over the last 30 years, and diseases have decimated urchin populations in many parts of the world.
- In the early 1980s, an epidemic killed more than 95 percent of the long-spined sea urchins (*Diadema antillarum*) in the Caribbean. After the urchins died, prevalence of seaweeds increased dramatically; today, many coral reefs there are dead.
- Biologists have suggested that overfishing urchin predators such as toadfish (*Opsanus sp.*) and queen triggerfish (*Balistes vetula*) may have played a role in this epidemic.

World's Fish Supply Running Out, Researchers Warn (Journal of Science)

By Juliet Eilperin
Washington Post Writer, November 3, 2006

- Economists’ and ecologists’ warning: No more seafood as of 2048
- Based on 4-year study of
  - Catch data
  - Effects of fisheries collapses
- Causes
  - Overfishing
  - Pollution
  - Other Environmental Causes
- Loss of Species affects oceans’ ability to
  - Produce seafood
  - Filter nutrients
  - Resist the spread of disease
  - Store CO_2

Total Allowable Catch (TAC)

- Many fisheries are regulated using TAC.
- A TAC within a system of individual transferable quotas (ITQs) is currently used to manage the Alaskan halibut fishery.
- The Alaskan halibut is one of the few success stories in the book on US fisheries management. The TAC did a reasonable good job of preventing overfishing, but created another set of problems.
- Regulated open access: If TAC is imposed on a fishery where access to the resource is free or of minimal cost, fishers have an incentive to "race for the fish," trying to capture as large a share of the TAC for themselves before the cumulative harvest reaches the TAC and the season is ended.
- Regulated open access may result in a severely compressed fishing season where vast amounts of "fishing effort" are expended in a few day (halibut derby…Prior to 1995…one or two day season).

  -fishers sit idle or re-gear and cause overfishing in other fisheries.

Periodic Proportion Policy (PPP)

At start of year t,

$x(t) =$ estimated fish stock (biomass)

$\mathrm{y(t)} =$ total allowable catch (TAC)

$\mathrm{y(t)} = a(t)x(t) \qquad \text{(PPP)}$

$$\mathrm{a(t)} = \frac{F(t)(1 - me^{-F(t)})}{m + F(t)}$$

$\mathrm{F(t)} =$ fishing mortality

$\mathrm{m} \quad =$ natural mortality

Under Pulse Fishing, fishing mortality is periodic and

$$\mathrm{F(t+p) = F(t).}$$

Therefore, $\mathrm{a(t+p) = a(t).}$

Constant Proportion Policy (CPP)

$$y(t) = ax(t), \qquad \text{(CPP)}$$

$$\text{where} \qquad a = \frac{F(1 - e^{-m-F})}{m+F}.$$

CPP is transparent,
easy to implement and
acceptable to fishers.

Harvested Fish Stock Model

- Escapement

$$S(t) = x(t) - y(t) = (1 - a(t))x(t)$$

- Model

$$x(t+1) = f(S(t)) = (1-m)S(t) + S(t)g(S(t))$$

$$or$$

$$x(t+1) = (1-a(t))x(t)((1-m) + g((1-a(t))x(t)))$$

Compensatory Dynamics and CPP Without Allee Effect

$$f(x) = (1-a)x((1-m) + g((1-a)x)).$$

When the *Allee effect* is missing,

$g:[0,\infty) \to [0,\infty)$ *is* a *strictly* decreasing smooth function, and

$$a = \frac{F(1-me^{-F})}{m+F}.$$

Compensatory Dynamics and CPP (Continued)

$If\ a > \dfrac{g(0)-m}{1+g(0)-m}$, then the stock size approaches zero for any initial stock level.

If $a < \dfrac{g(0)-m}{1+g(0)-m}$ and the dynamics is compensatory, then the steady state biomass is the fixed point

$$x^{\infty} = x^{\infty}(a) = \frac{1}{(1-a)} g^{-1}\left(\frac{1}{(1-a)} - (1-m)\right).$$

Example: Beverton-Holt Model and Constant Harvesting

$$f(x) = (1-a)x\left((1-m) + \frac{\alpha}{1+\beta(1-a)x}\right),$$

where $1 - m + \alpha > 1$.

The stock is depleted when

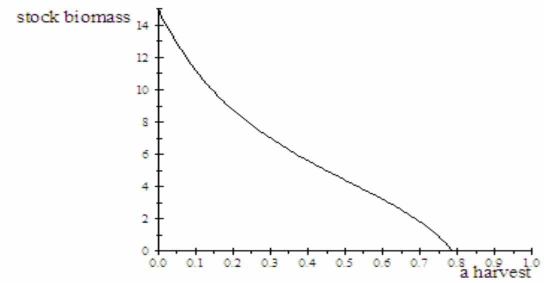
$$a > \frac{\alpha - m}{1 - m + \alpha}.$$

The stock persists on a globally attracting fixed point at

$$x^{\infty} = \frac{(1-a)(\alpha + 1 - m) - 1}{\beta(1-a)(1-(1-a)(1-m))}$$

whenever $a < \dfrac{\alpha - m}{1 - m + \alpha}$.

Alaskan Halibut m = 0.15



Allee Effect (Critical Depensation) in Real Populations

- Stoner and Ray-Culp showed evidence of the Allee effect in natural populations of the Caribbean queen conch Strombus gigas, a large motile gastropod that supports one of the most important marine fisheries in the Caribbean region.
- There is experimental evidence of the Allee effect in urchins.
- In fisheries systems, the Allee mechanism is relevant to issues of species extinction, conservation, fisheries management and stock rehabilitation.

Strong Allee Effect

The exploited stock has a strong Allee effect if there exists a critical positive stock level $\min x^{\infty}$, such that $\lim_{t \to \infty} f^{t}(x) = 0$ for all x in $[0, \min x^{\infty})$, and the stock persists uniformly on a subset of $(\min x^{\infty}, \infty)$.

Compensatory Dynamics and CPP With Strong Allee Effect (critically depensatory net growth function)

When the Allee effect is present, we assume that $g:[0,\infty)\rightarrow[0,\infty)$ is a smooth one-hump map that increases from zero to a maximum positive value that is bigger than 1, and then decreases so that

$$\lim_{x\rightarrow\infty} g(x) < 1.$$

Compensatory Dynamics and CPP With Strong Allee Effect (Continued)

Modified Beverton-Holt Model:

$$f(x) = (1-a)x\left((1-m) + \frac{\alpha(1-a)x}{1+\beta(1-a)^2x^2}\right),$$

$$where \quad \frac{\alpha(1-a)}{1-(1-a)(1-m)} > 2\sqrt{\beta}.$$

Then

$$\min x^{\infty} = \frac{\dfrac{\alpha}{1-(1-a)(1-m)} - \sqrt{\left(\dfrac{\alpha}{1-(1-a)(1-m)}\right)^2 - \dfrac{4\beta}{(1-a)^2}}}{2\beta}$$

and

$$x^{\infty} = \frac{\dfrac{\alpha}{1-(1-a)(1-m)} + \sqrt{\left(\dfrac{\alpha}{1-(1-a)(1-m)}\right)^2 - \dfrac{4\beta}{(1-a)^2}}}{2\beta}$$

Modified Beverton-Holt Model and CPP

$$Theorem:$$

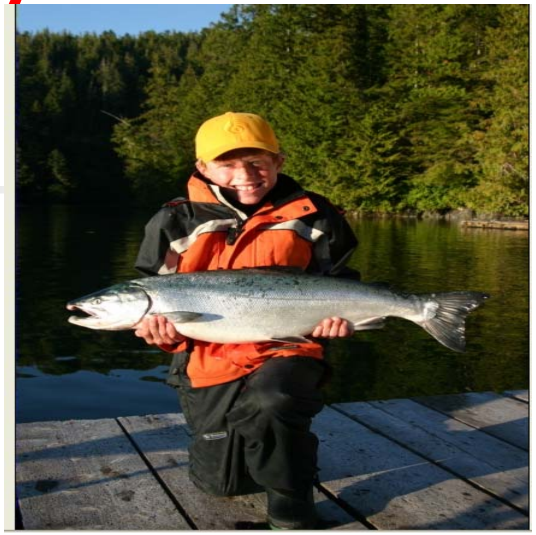
$$f(a,x) = (1-a)x\left((1-m) + \frac{\alpha(1-a)x}{1+\beta((1-a)x)^2}\right)$$

exhibits the fold bifurcation.

Compensatory Dynamics and CPP (Continued)

Under compensatory dynamics and CPP, the stock size exhibits a discontinuity at $a=a_{cr}$ when the strong Allee effect is present. The stock size suddenly jumps to zero as a exceeds a_{cr}.

Overcompensatory Dynamics and CPP



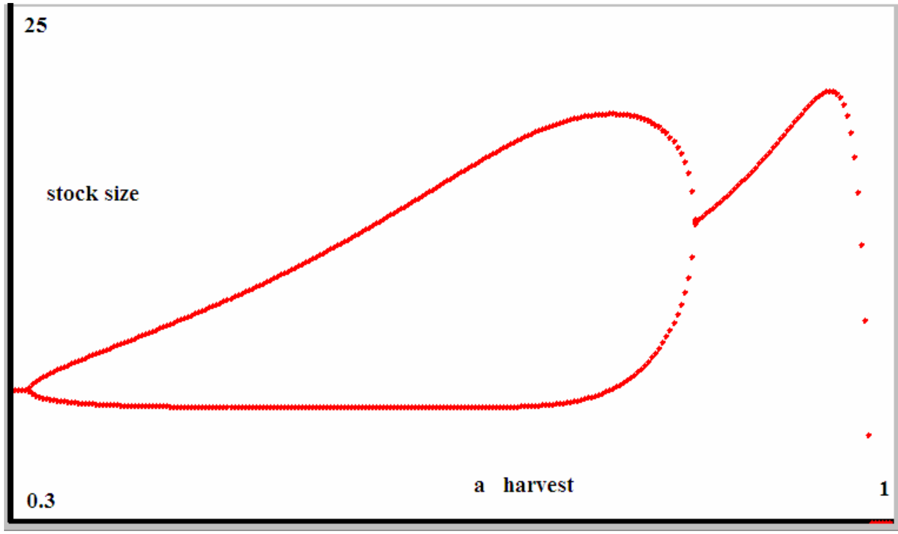
- Ricker Model:

$$x(t+1) = (1-a)x(t)\left(1 - m + e^{r - (1-a)x(t)}\right),$$

$$m = 0.2\ (\text{salmon})$$

Ricker Model and CPP Without Allee Effect

Under overcompensatory dynamics via the Ricker model (no Allee effect) and CPP, the stock size decreases smoothly to zero with increasing levels of harvesting.



- Period-doubling reversals

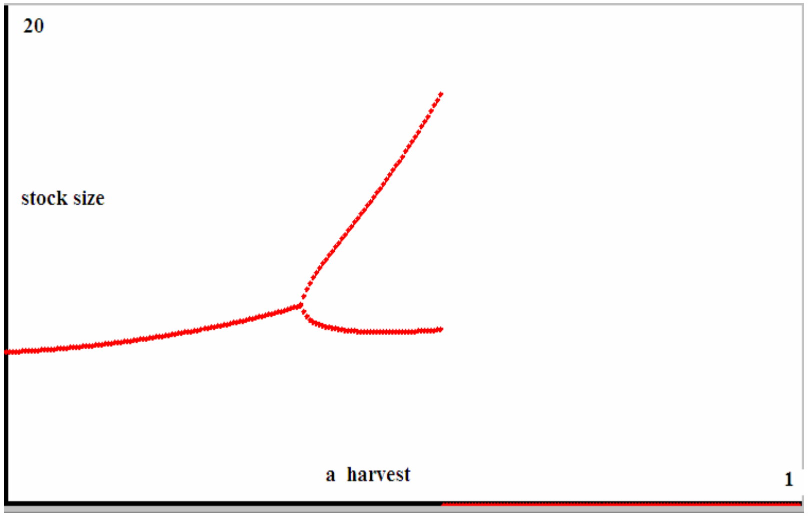
L. Stone, Nature 1993.

Modified Ricker Model With Allee Effect and CPP

Theorem:

$$f(a,x) = (1-a)x\left(1-m+(1-a)xe^{r-(1-a)x}\right)$$

exhibits the fold bifurcation.



Allee Effect and CPP

Under CPP, the Allee mechanism generates a sudden discontinuity at $a=a_{cri}$, with the stock size suddenly jumping to zero as a approaches the critical value (fold bifurcation), when the stock dynamics is either compensatory or overcompensatory.

Stock Dynamics and Periodic Proportion Policy(PPP)

$$We\ assume\ a\ k-periodic\ \ fishing\ mortality\ (F(t+k)=F(t)),$$

$$so\ that$$

$$f(t,x)=(1-a(t))x\big((1-m)+g((1-a(t)x)\big),$$

$$where$$

$$a(t+k)=a(t).$$

Compensatory Dynamics and PPP

Theorem :

For each $j \in \{0,1,2,...,k\text{-}1\}$, *let*

$$f_j(x) = (1-a(j))x\big((1-m)+g((1-a(j))x)\big)$$

be an increasing *concave down map under compensatory dynamics in* $(0,\infty)$, *where* $a(j+k)=a(j)$. *Then the stock population under period-k harvesting exhibits a globally asymptotically stable r-cycle, where r divides k.*

Proof : *Use the general result of Elaydi-Sac*ker *(JDEA*'05*)*, *a period-k extension of the result of Cushing-Henson (JDEA*'01*).*

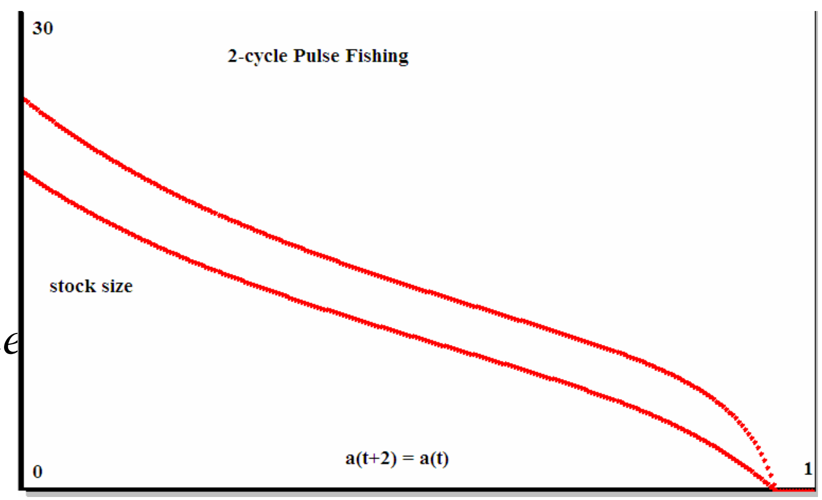
Beverton-Holt Model (Without Allee effect) and PPP

Corollary: *For each* $j \in \{0,1,2,\ldots k-1\}$, *let*

$$f_j(x) = (1-a(j))x\left((1-m) + \frac{\alpha}{1+\beta(1-a(j))x}\right),$$

where

$(1-a(j))(1-m+\alpha) > 1$, $\beta > 0$ *and* $a(j+k) = a(j)$. *Then, the stock population under period-k harvesting exhibits a globally asymptotically stable k-cycle.*



Compensatory Dynamics, Strong Allee Effect and PPP

Theorem:

For each $j \in \{1,2,\ldots,k-1\}$*, let*

$$f_j(x) = (1-a(j))x\big(1-m+g((1-a(j))x)\big)$$

exhibit the Allee effect, where f_j *is an* increasing *concave down map under compensatory dynamics in* $[\min x^{\infty}, \infty)$*, and* $a(j+k) = a(j)$*. Then, the stock under period* $-k$ *harvesting exhibits two* coexisting *attractors; zero and a globally asymptotically stable positive r-cycle in* $(\min x^{\infty}, \infty)$*, where r divides k.*

Modified Beverton-Holt Model, Strong Allee Effect and PPP

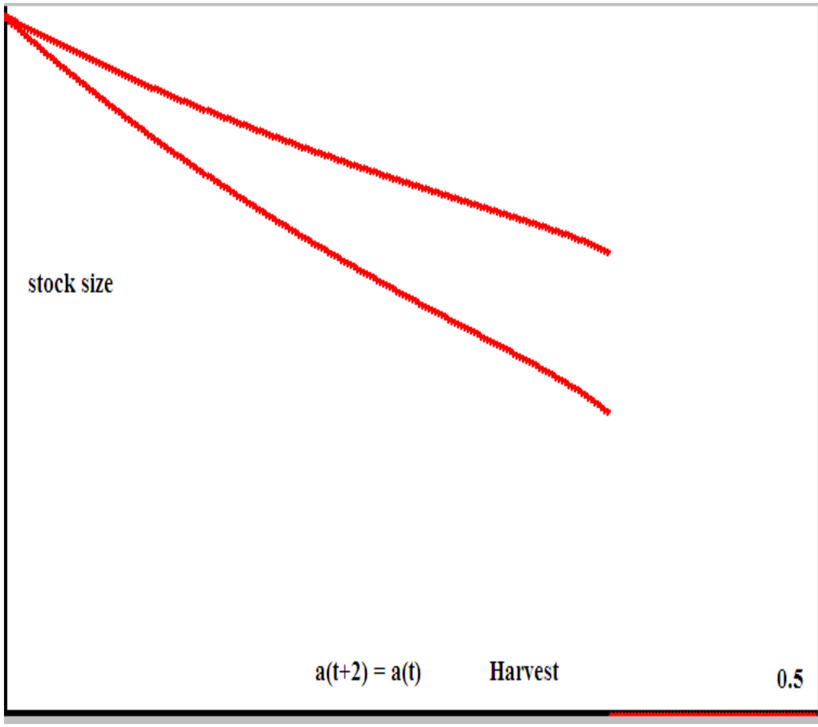
Corollary:

For each $j \in \{0,1,2,\ldots,k-1\}$, *let*

$$f_j(x) = (1-a(j))x\left(1-m+\frac{\alpha(1-a(j))x}{1+\beta((1-a(j))x)^2}\right),$$

where $\dfrac{\alpha(1\text{-}a(j))}{1\text{-}(1\text{-}a(j))(1\text{-}m)} > 2\sqrt{\beta}$ *and* $a(j+k) = a(j)$.

Then, the stock population under period-k harvesting has two coexisting attractors; zero and an asymptotically stable positive k-cycle (Allee effect).

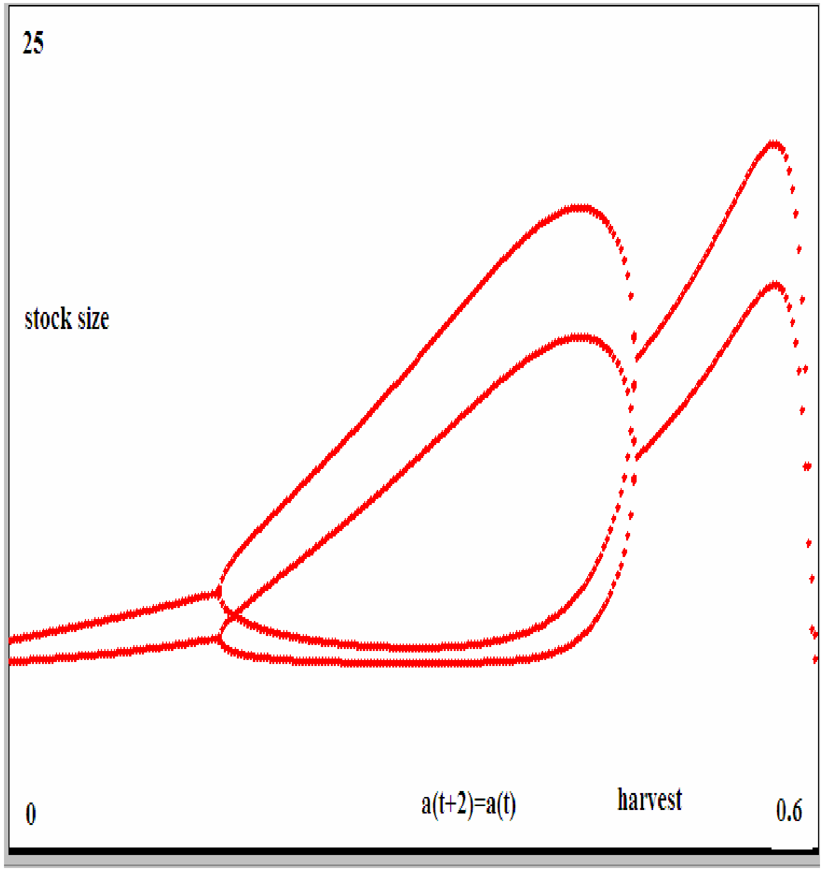


Overcompensatory Dynamics and PPP

Ricker Model and PPP

As in the case of CPP, under PPP and no Allee effects the stock size exhibits the "bubble" bifurcation as it decreases smoothly to zero.

*Attractors in periodic environments (S. M. Henson, J. M. Cushing *et al.*, Bull. Math. Biol. 1999, and J. Franke and J. Selgrade, JMAA 2003).



Overcompensatory Dynamics, Allee Effect and PPP

Under PPP and overcompensatory dynamics, low population sizes lead to the extinction of the stock, whenever the strong Allee effect occurs during each pulse fishing season.

Modified Ricker Model With Allee Effect and PPP

Halibut Data

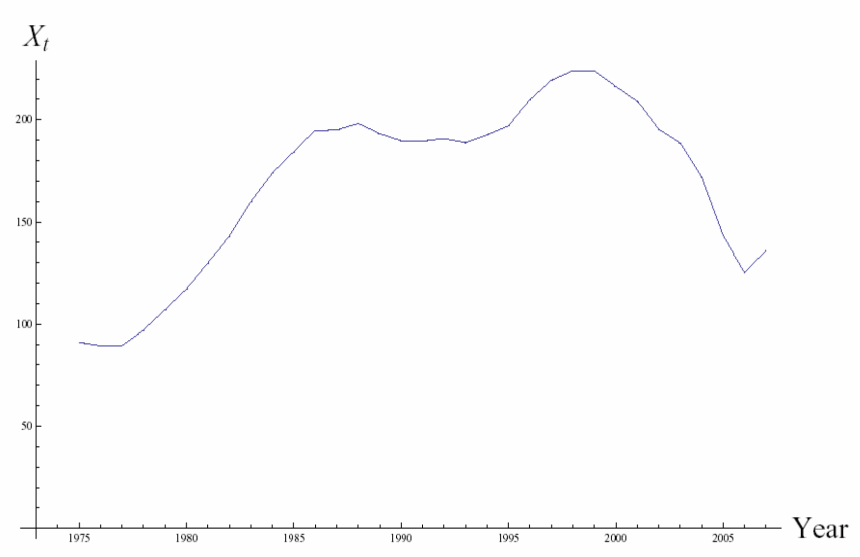
Pacific Halibut Data

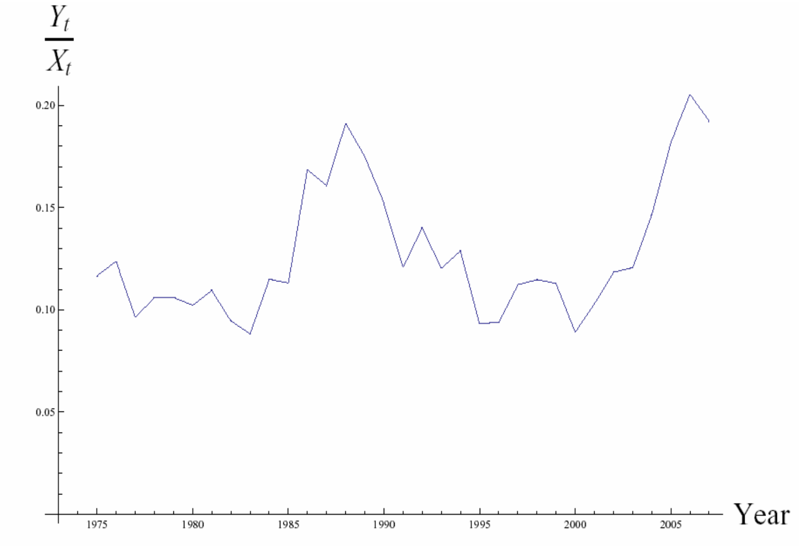
Let X_t denote the estimated halibut stock in the sea area at year t, Y_t the landed fish at year t and E_t the effort people made to fish at year t. Then the following table displays the fishing data of pacific halibut from 1976 to 2007.

Table 1. Data of Pacific Halibut in Area A3 from 1976-2007.[1]

Year	X_t	Y_t	E_t
1975	90.989	10.6	192
1976	89.339	11.044	154.848
1977	89.484	8.641	141.639
1978	96.987	10.295	132.051
1979	106.831	11.335	131.86
1980	116.954	11.966	101.441
1981	129.693	14.225	100.211
1982	142.881	13.53	79.529
1983	159.637	14.112	58.629
1984	173.717	19.971	37.729
1985	184.207	20.852	40.54
1986	194.695	32.79	66.398
1987	194.991	31.316	65.258
1988	198.127	37.862	78.25
1989	193.12	33.734	77.341
1990	189.684	28.848	84.873
1991	189.582	22.926	75.455
1992	190.776	26.782	69.093
1993	188.782	22.738	58.728
1994	192.548	24.844	72.776
1995	196.91	18.342	44.375
1996	209.634	19.696	42.008
1997	219.196	24.628	53.93
1998	223.962	25.703	57.317
1999	223.847	25.292	58.192
2000	216.138	19.288	43.634
2001	208.928	21.541	46.055
2002	195.243	23.131	45.897
2003	188.546	22.748	46.858
2004	171.794	25.168	52.041
2005	143.105	26.033	58.543
2006	125.32	25.714	63.921
2007	136.344	26.2	64.024

Pacific Halibut





Parameter Estimation

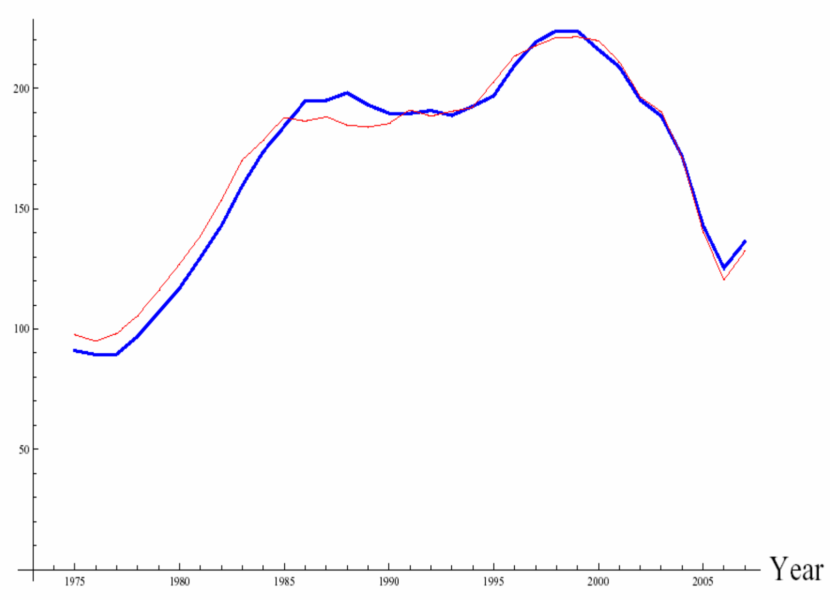
$$\textit{Minimize}\ \mathrm{MSE} = \frac{1}{32}\sum_{1975}^{2007} \{x(t+1) - s(t)(1-m+g(s(t)))\}^2$$

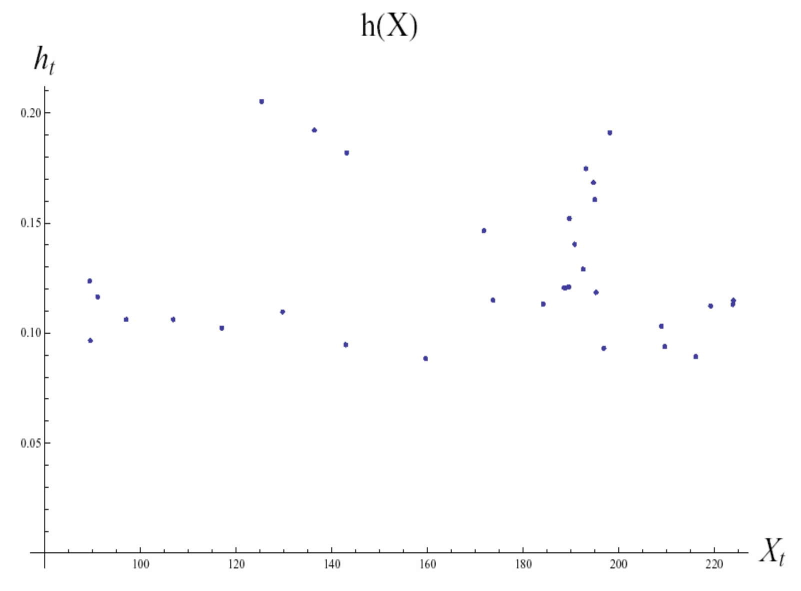
$$\textit{subject}\ \text{to s(t) = x(t) - y(t) and m = 0.15.}$$

Parameter Estimation: Continued

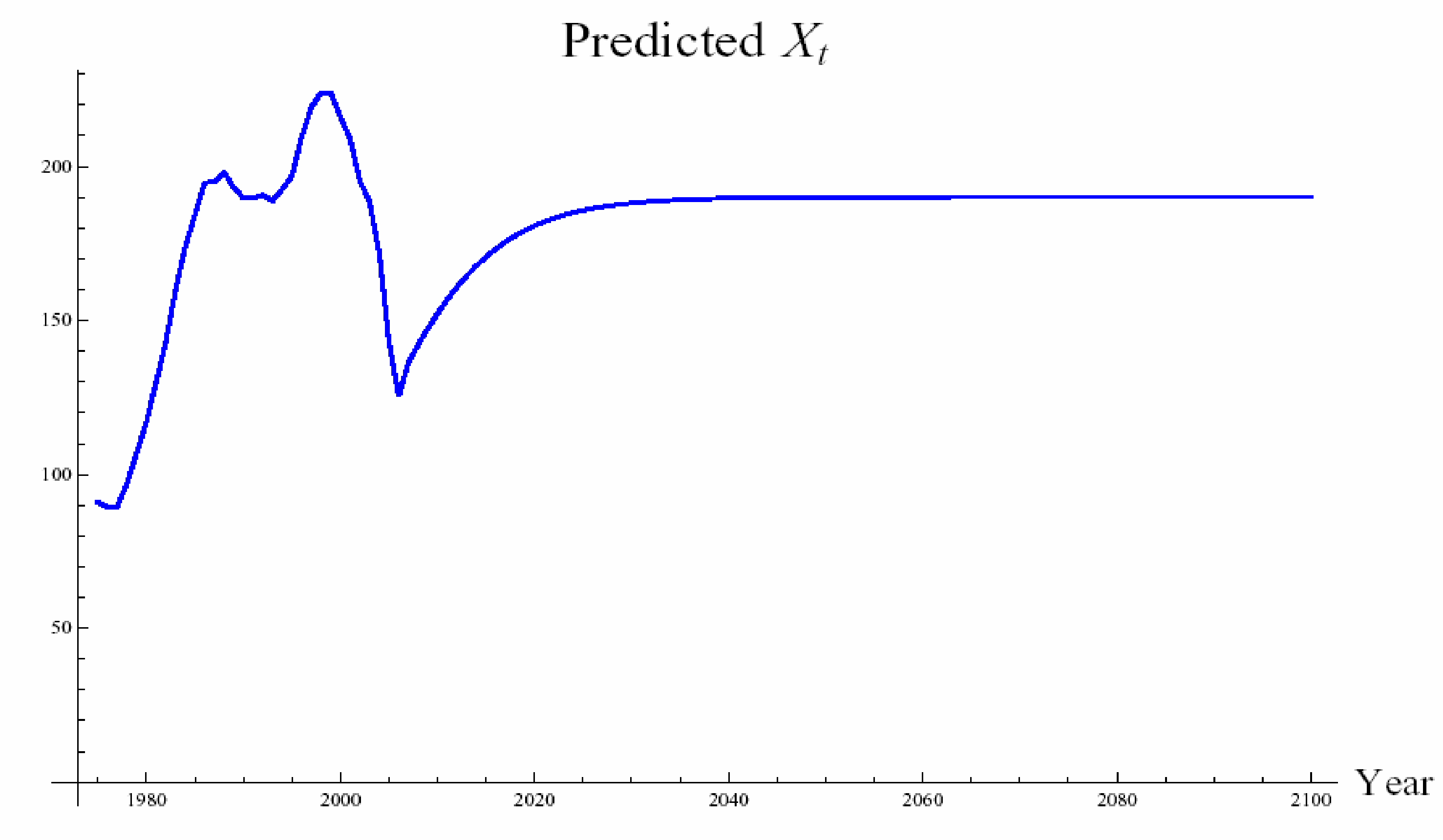
Model	**g(s)**	**Parameters**	**MSE**
1. Beverton-Holt	$\frac{\alpha}{1+\beta s}$	$\alpha = 0.5434$ and $\beta = 0.0051$	89.4449
2. Ricker	$\alpha e^{-\beta s}$	$\alpha = 0.4949$ and $\beta = 0.0031$	88.5096
3. Modified Beverton-Holt	$\frac{\alpha s}{1+\beta s^2}$	$\alpha = 0.0086$ and $\beta = 0.0001$	86.5722
4. Modified Ricker	$\alpha s e^{-\beta s}$	$\alpha = 0.0102$ and $\beta = 0.0104$	85.4096
5. Logistic	$r(1 - \frac{s}{K})$	$r = 0.4659$ and $K = 455.725$	87.6465

Pacific Halibut & Modified Ricker Model

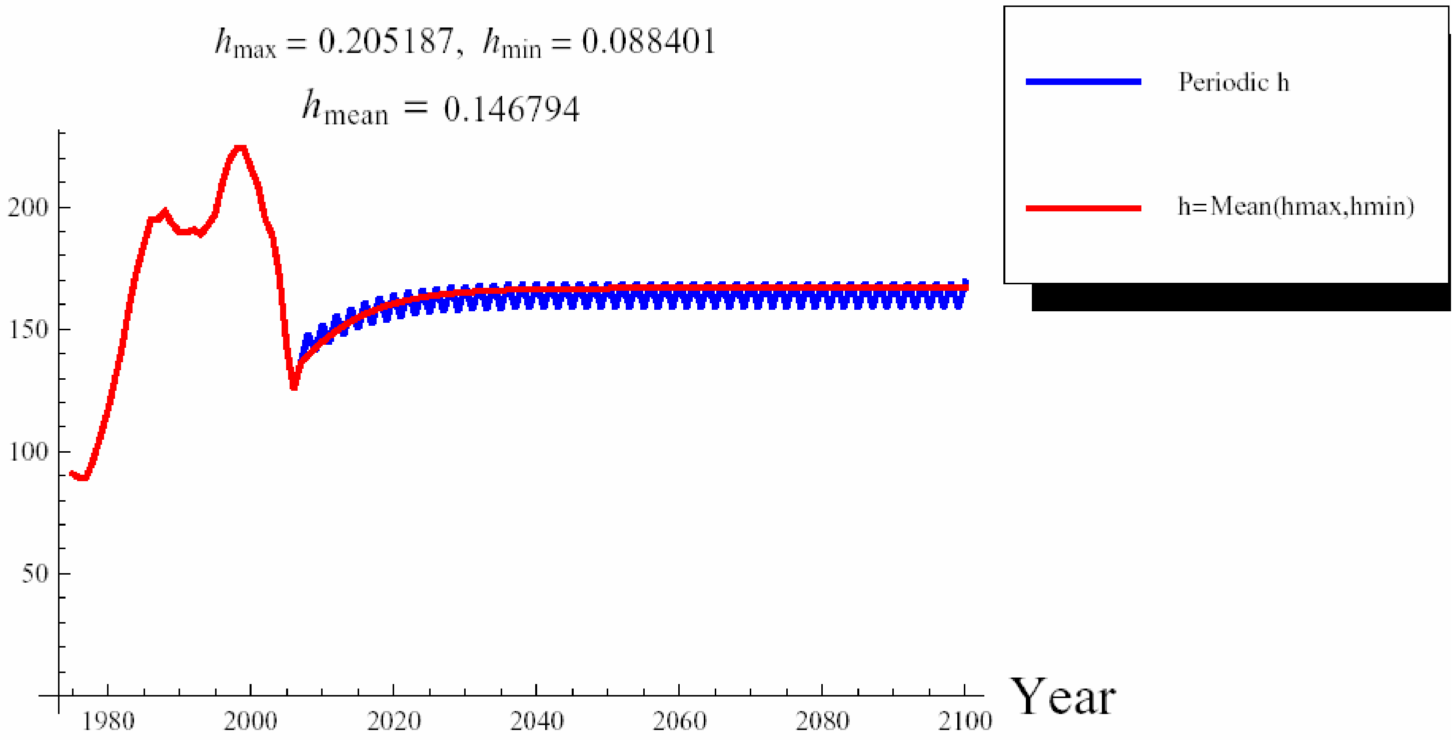




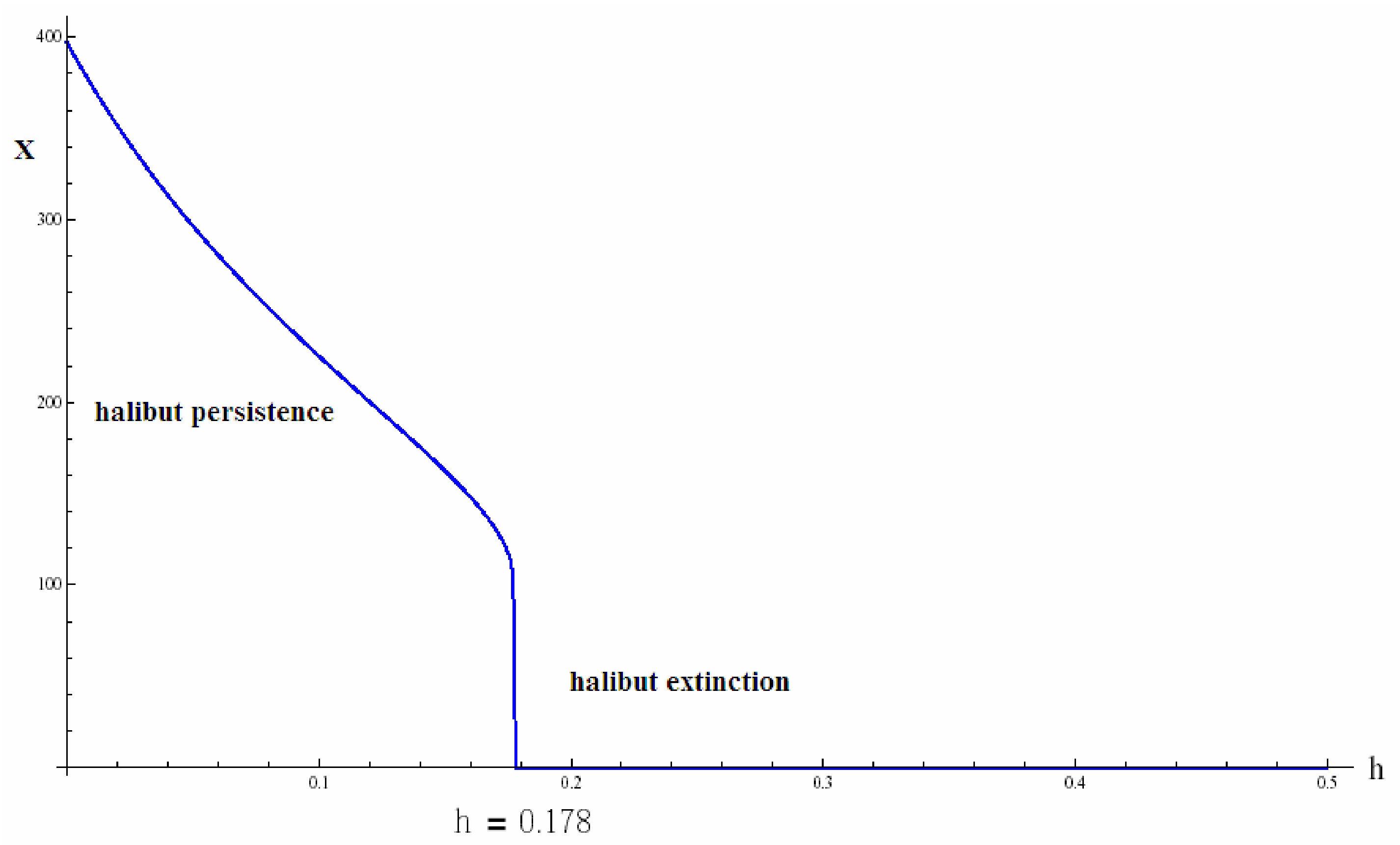
Future Of Pacific Halibut (a=0.1277)



Halibut Under Period-2 Harvest



Halibut Under High Constant Fishing Pressure



Question

What are the interactions between climate change, Allee effect and persistence of exploited species?

Stochastic Model
(Random Environment and Fisheries)

$Let\ \zeta(t)\sim U(1-\sigma,\ 1+\sigma)\ be\ a\ "mean-preserving\ spread"$

$uniformly\ distributed\ random\ \text{variable}.$

$Stochastic\ Model:$

$$x(t+1)=(1-a(t))x(t)\left((1-m)+\zeta(t)g((1-a(t))x(t))\right)$$

Unstructured Populations In Random Environments

$$x(t+1) = x(t)G(\zeta(t), x(t))$$

where

$$G(\zeta(t), x(t)) = (1 - a(t))(1 - m + \zeta(t))g((1 - a(t))x(t)))$$

* Lewinton and Cohen (1969)

* Birkhoff Ergodic Theorem

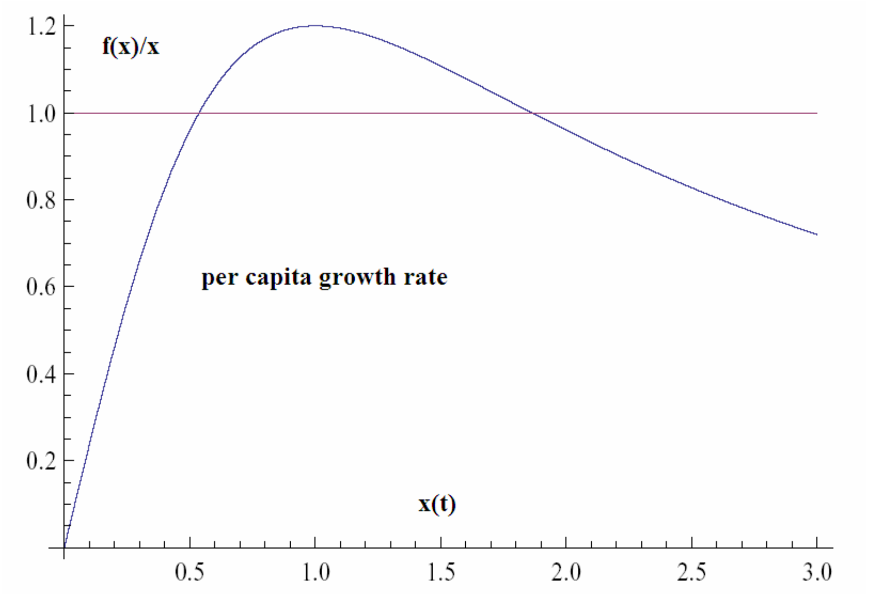
* Chesson (1982), Ellner (1984), Hardin *et al* (1988) etc

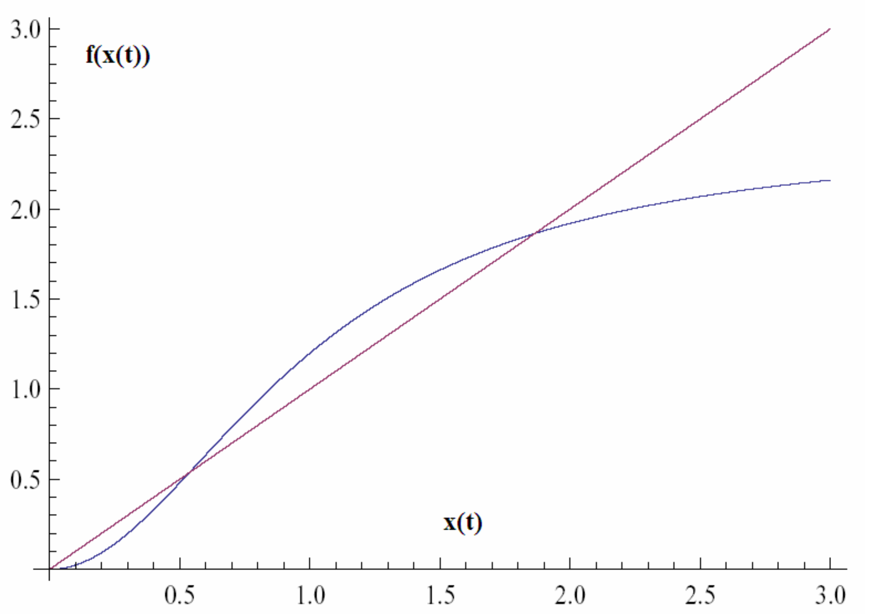
Let $\gamma = \text{Expected}\left\{\ln \text{G}(\zeta(1), 0)\right\}$.

If $\gamma < 0$ the population goes extinct with probability 1.

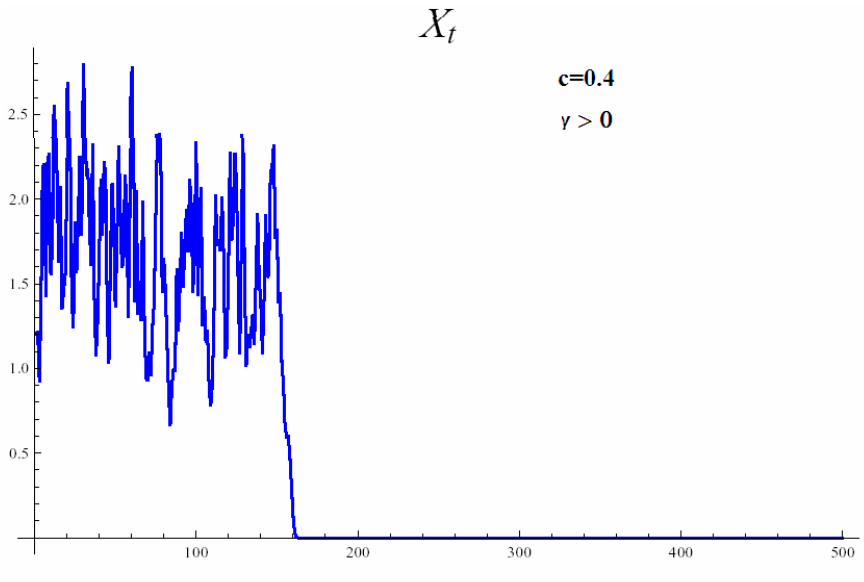
If $\gamma > 0$ the population has a low probability of reaching low abundances in the long - term.

Uncertainty and Allee Effect





Uncertainty and Allee Effect



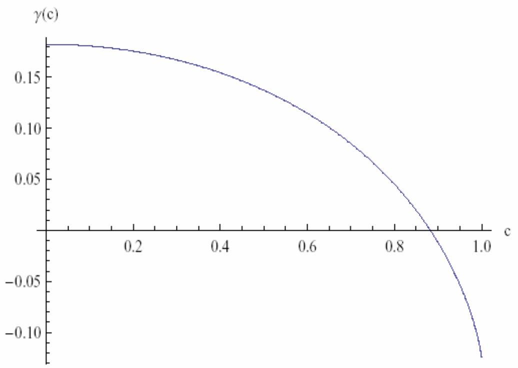
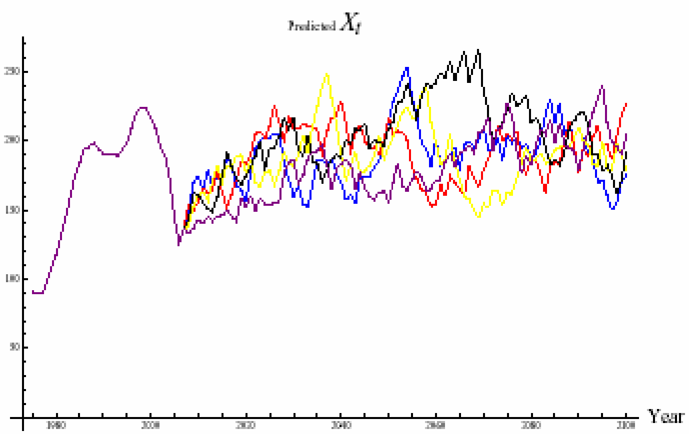


Figure 3: γ as a function of c.

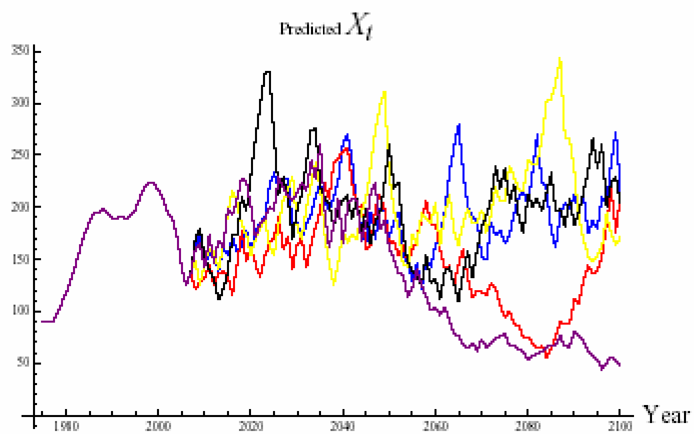
From Figure 3 we see that

$$\begin{cases} \gamma > 0, & \text{if } 0 < c < 0.87; \\ \gamma < 0, & \text{if } 0.87 < c < 1. \end{cases}$$

Stochastic Model Predictions

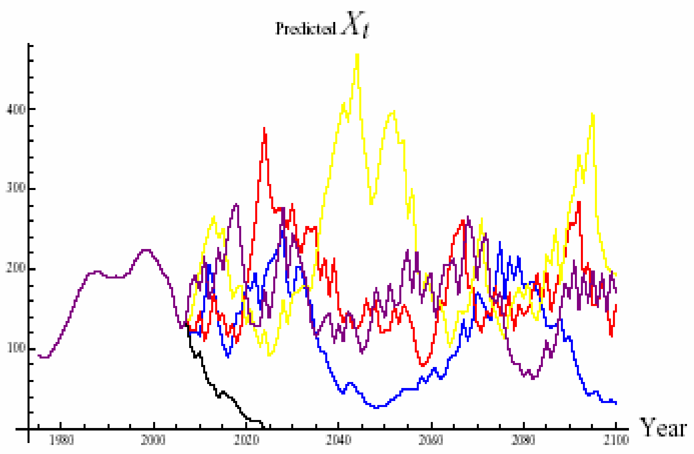


Simulations of stochastic model when a=0.1

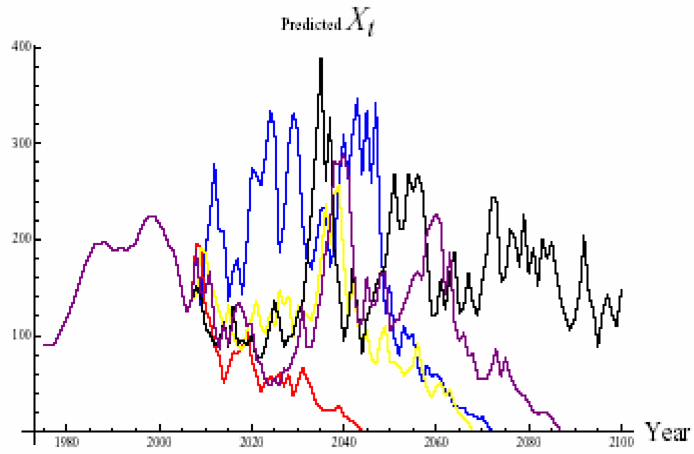


Simulations of stochastic model when a=0.2

Stochastic Model Extinctions



Simulations of stochastic model when a=0.3

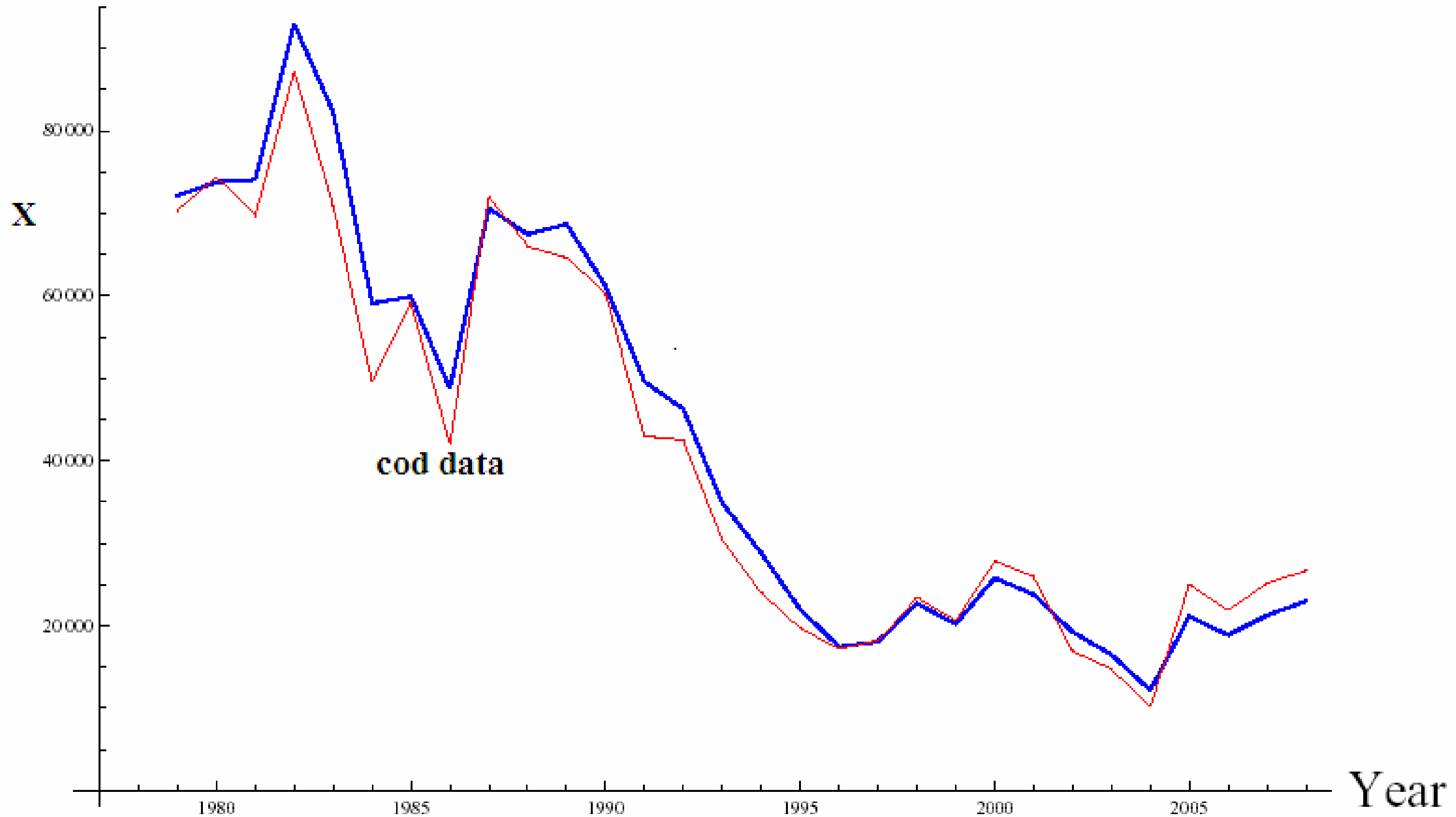


Simulations of stochastic model when a=0.4

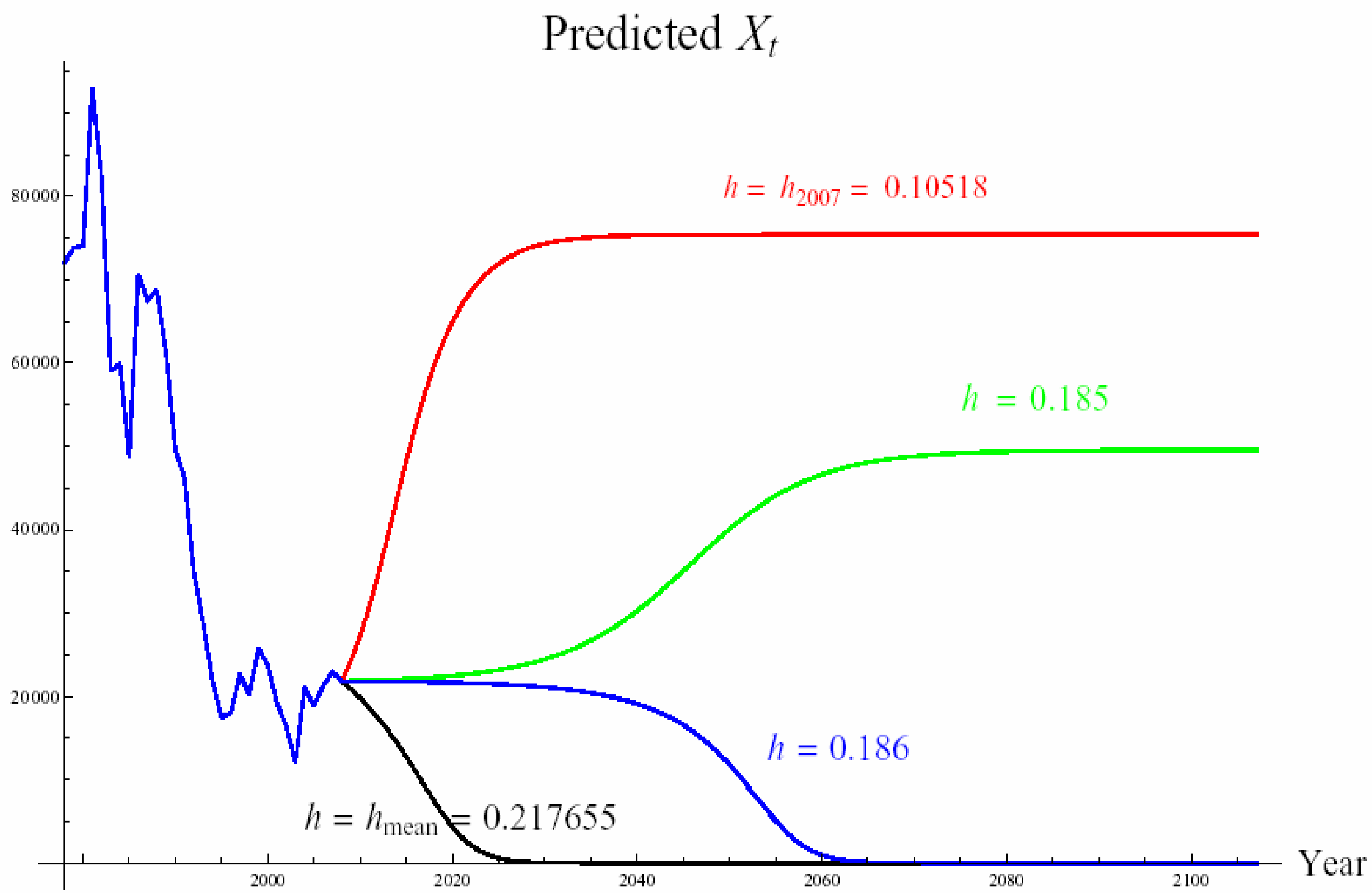
Cod Data From Georges Bank

Year	X_t	h_t
1978	72148	0.18847
1979	73793	0.149741
1980	74082	0.219209
1981	92912	0.176781
1982	82323	0.282033
1983	59073	0.34528
1984	59920	0.206545
1985	48789	0.338185
1986	70638	0.147236
1987	67462	0.19757
1988	68702	0.231541
1989	61191	0.208597
1990	49599	0.335648
1991	46266	0.295344
1992	34877	0.331848
1993	28827	0.350394
1994	21980	0.282701
1995	17463	0.199275
1996	18057	0.18781
1997	22681	0.193574
1998	20196	0.189526
1999	25776	0.170108
2000	23796	0.156601
2001	19240	0.281787
2002	16495	0.252869
2003	12167	0.255417
2004	21104	0.081034
2005	18871	0.0873972
2006	21241	0.0819517
2007	22962	0.105181
2008	21848	unknow

Modified Ricker Cod Model



Cod’s Future & Fishing Pressure



Optimal CPP and PPP

At time *t*, industry revenue is

$$R(t) = py(t) = pa(t)x(t)$$

$$E(t) = amount \;\; of \;\; fishing \;\; effort$$

$$By \text{ Cobb - Douglas production function,}$$

$$\text{cost equation is}$$

$$\mathrm{C(t)} = \mathrm{cE(t)} = c\left(\frac{a(t)}{q(x(t))^{b-1}}\right)^{\frac{1}{d}}$$

where p>0 is the dockside (or ex-vessel) price per unit for y(t),
c>0 is the unit cost for fishing effort,
q= "catchability coefficient", and
b, d >0 are the elasticities of harvest.

Optimal PPP and CPP

Net revenue function is

$$\Pi(x(t)) = pa(t)x(t) - c\left(\frac{a(t)}{q(x(t))^{b-1}}\right)^{\frac{1}{d}}$$

The optimal PPP and CPP will

$$\underset{\{a(t)\}}{Maximize}\ \Pi = \sum_{t=0}^{\infty} \rho^t \left(pa(t)x(t) - c\left(\frac{a(t)}{q(x(t))^{b-1}}\right)^{\frac{1}{d}} \right)$$

subject to

$$x(t+1) = (1-a(t))x(t)\big((1-m) + g((1-a(t))x(t))\big)$$

where

$0 \le a(t) < 1$ *and* $x(0) > 0$ *is given.*

$\rho = \dfrac{1}{1+\delta}$ *is a discount factor and*

$\delta > 0$ *is a discount term.*

Data

Table 1. Important species landed or raised in the Northeast, their landings, L (thousand mt), ex-vessel revenue, R ($, millions), and prices,P ($ per lb), 1995-1999

Year	L	R	P	L	R	P	L	R	P	L	R	P	L	R	P
	American lobster			Sea scallops			Blue crab			Atlantic salmon2			Goosefish		
1995	31.8	215	3.06	8	91.1	5.16	56.7	101	0.81	10	56.7	2.56	25.1	36.1	0.65
1996	32.5	243	3.39	7.9	98.2	5.64	37.7	64.3	0.77	10	46.2	2.1	25.3	32.3	0.58
1997	37.5	272	3.29	6.3	90.5	6.56	45.3	82.7	0.83	12.2	49.5	1.84	28.3	35.2	0.56
1998	36.3	255	3.19	5.6	76	6.19	39.1	90.1	1.05	13.1	60.4	2.09	26.7	33.9	0.58
1999	39.7	323	3.69	10.1	123	5.5	39	80.6	0.94	12.2	58.2	2.16	25.2	47	0.85
	Hard Clam			Surf clam			Menhadin			Squid Loligo			Cod		
1995	4.2	42.1	4.5	30.1	47.1	0.71	345	45.7	0.06	18.5	23.8	0.58	13.7	28.6	0.95
1996	3.2	35.1	4.94	28.8	42.6	0.67	283	37.9	0.06	12.5	18.6	0.68	14.3	26.7	0.85
1997	4.4	44.5	4.62	26.3	38.9	0.67	247	33.8	0.06	16.2	26.5	0.74	13	24.6	0.86
1998	3.6	41.2	5.2	24.5	33	0.61	249	44.4	0.08	19.2	32.7	0.77	11.1	25.5	1.04
1999	3.5	40.7	5.25	26.7	34.1	0.58	189	33.2	0.08	18.8	32.2	0.78	9.7	23.9	1.11

Conclusion

- Constant exploitations diminish stocks while preserving compensatory dynamics.
- Periodic and constant exploitations simplify complex overcompensatory stock dynamics with or without the Allee effect.
- In the absence of the Allee effect, stock size decreases smoothly to zero with increasing levels of constant or periodic fishing pressure.
- Constant and periodic exploitations force sudden decline in fisheries systems that show evidence of the Allee mechanism.
- The probability of extinction increases with large enough variance in the climate variable.
- Optimal control techniques for discrete-time models with exploitation.

Thank You.