

**Thomas J. Peters**  
**joint work with**

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**also with**

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**Ambient Isotopy for Topological Equivalence  
in Surface Reconstruction**

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## **Seminal Surface Reconstruction Paper**

”The output of our reconstruction method produced the correct topology in all the examples. We are trying to develop formal guarantees on the correctness of the reconstruction, given constraints on the sample and the original surface.”

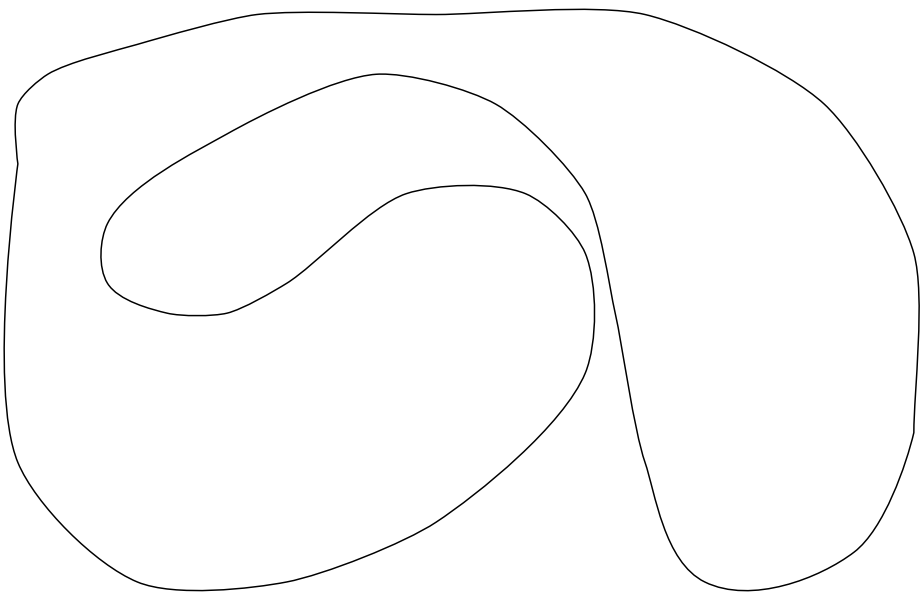
Surface reconstruction from unorganized points,  
H. Hoppe, T. DeRose, T. Duchamp,  
J. McDonald, W. Stuetzle  
Vol. 26, No. 2, July 1992, Siggraph Proceedings, ACM, p. 75

## Amenta and Bern: Homeomorphism

- Delauney Triangulation
- Use of Medial Axis to control sampling

Sampling Criterion: for every point  $x$  on  $F$  the distance from  $x$  to the nearest sampling point is at most 0.08 times the distance from  $x$  to  $MA(F)$ .

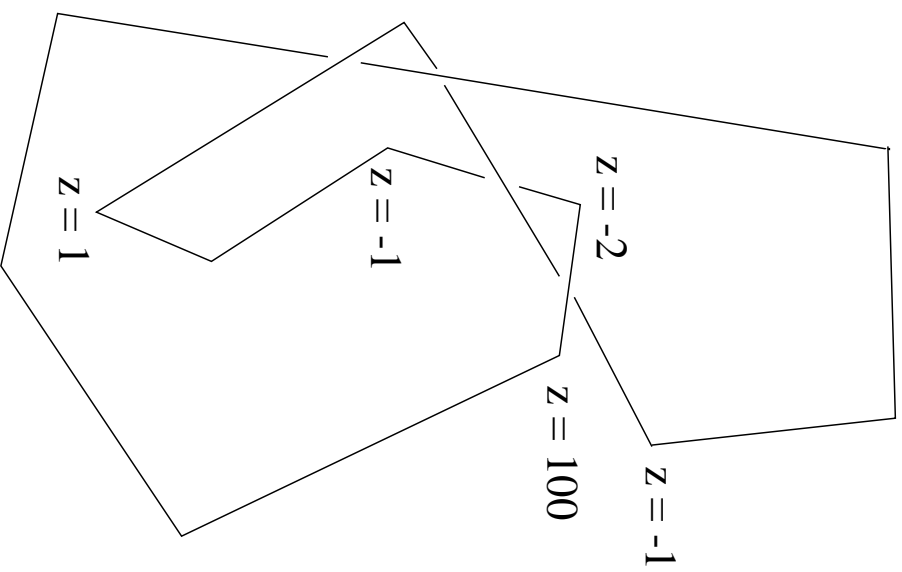
**BUT, homeomorphic may not be sufficient!**



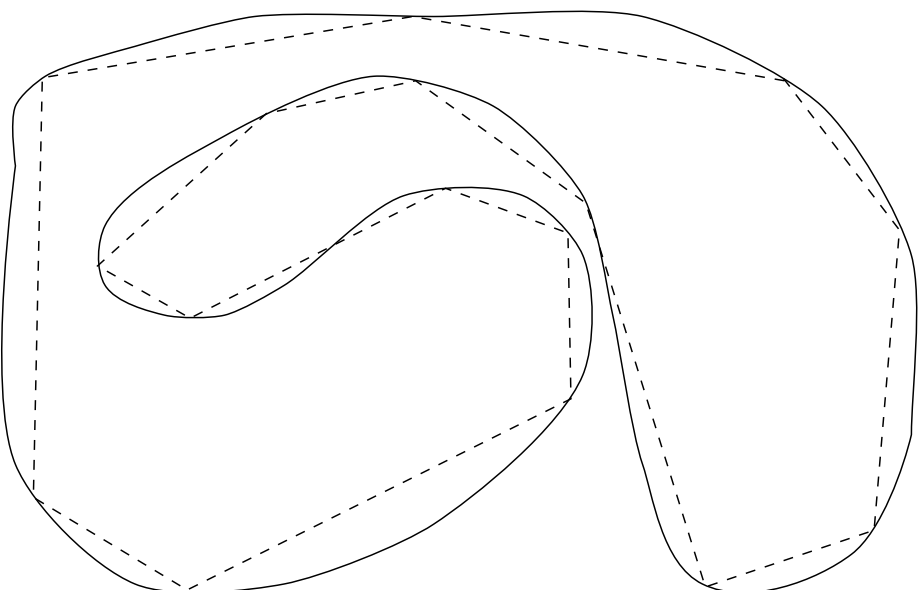
Unknot

# Approximating Knot $4_1$

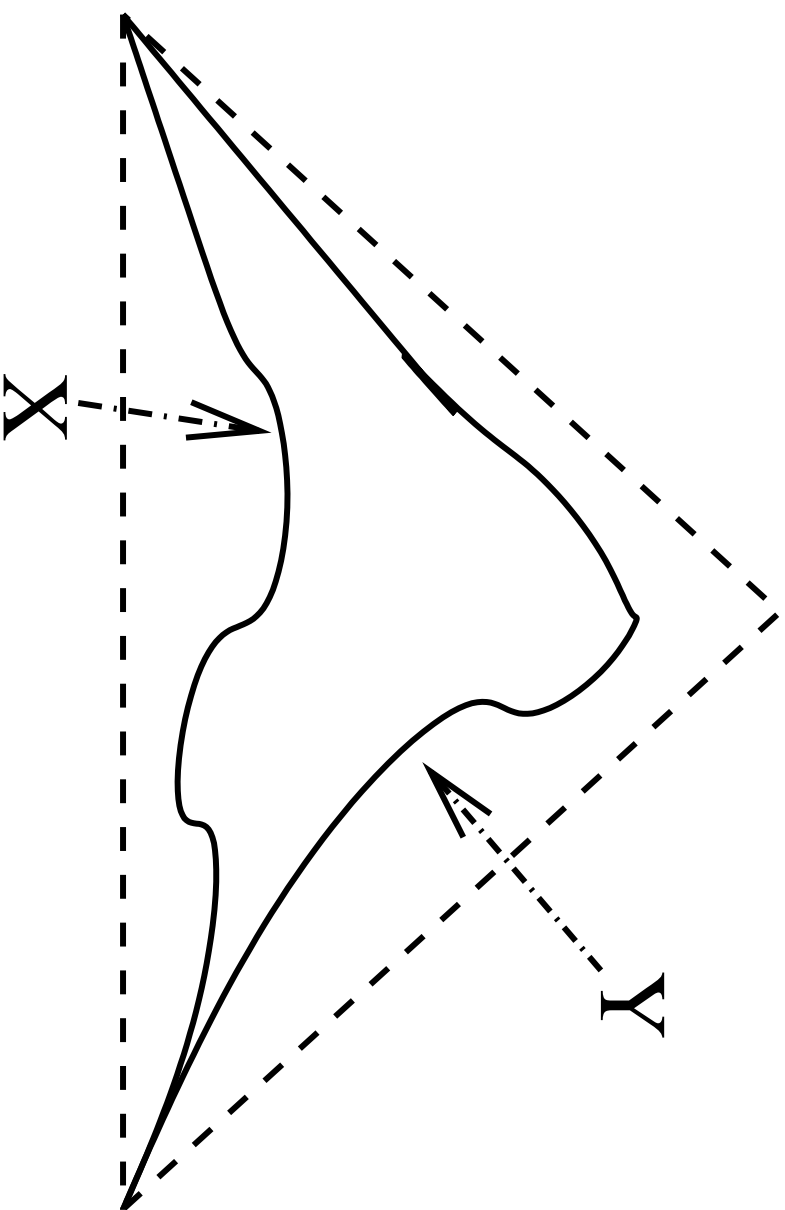
$4_1^m$



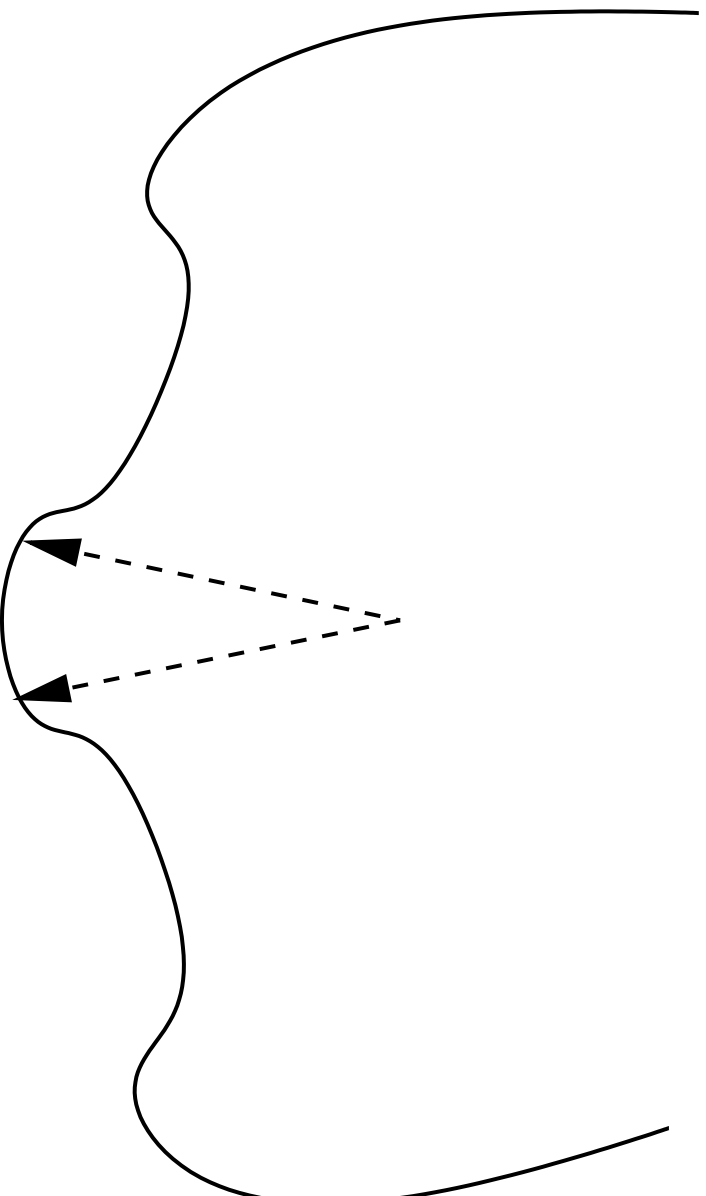
# Approximating Unknot



# Ambient Isotopy (with Compact Support)



**Main Differences Vs. Medial Axis Proof (I)**  
**Focal Point**  
**Intersection of Nearby Normals**

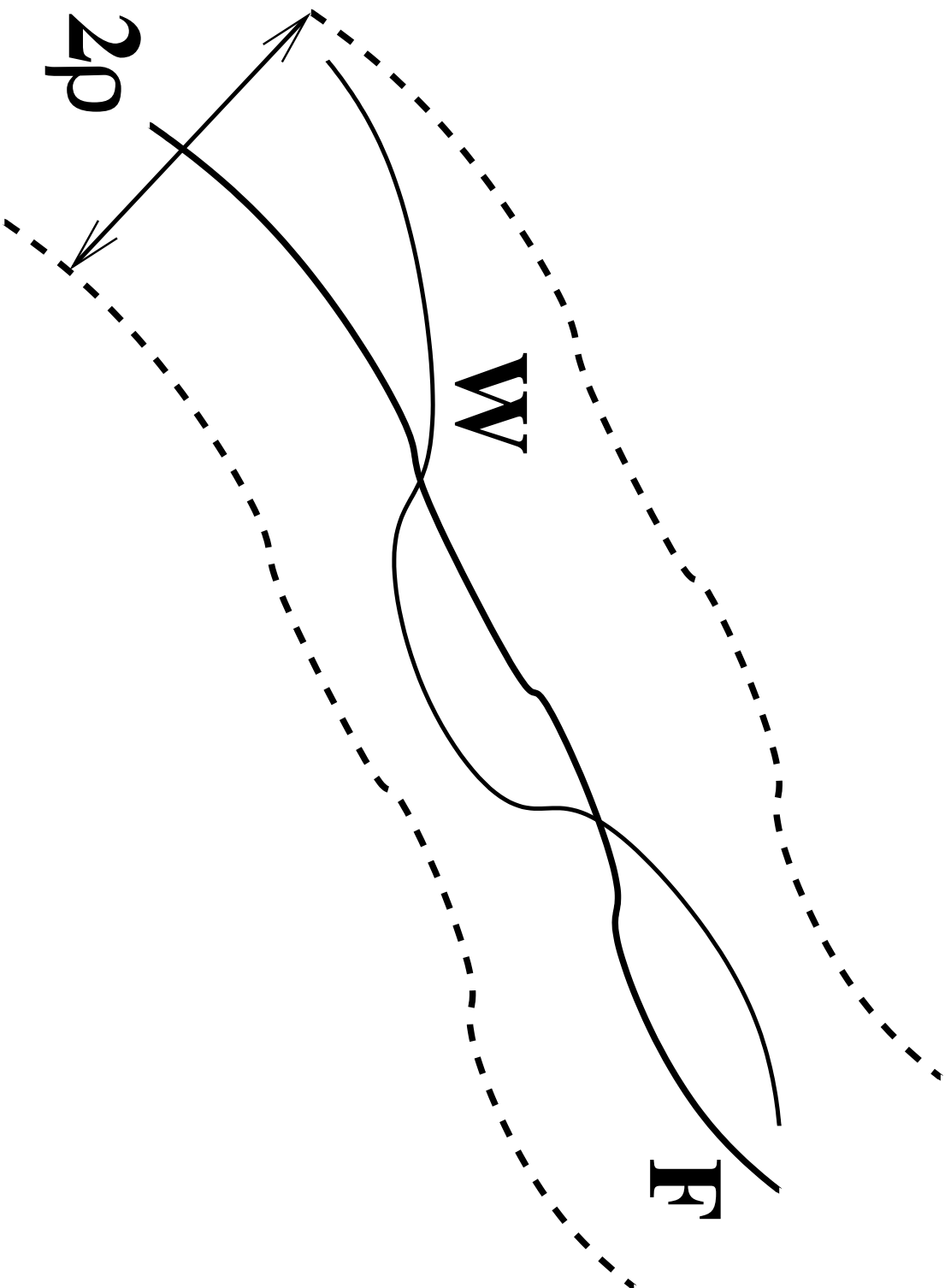




## **Role of Focal Points**

- **Each homeomorphism is well-defined.**
- **Allows injectivity.**

Not Only PL



## **Main Differences Vs. Medial Axis Proof (II)**

- **Global bound, not adaptive**
- **Implicit criterion, only.**

## Applications

- **Isotropy aspect of reconstruction, but**
- **still need explicit sampling density.**
- **However, applied directly to interval solids, and**
- **Graphics.**

## **Extends to Surfaces with Boundary**

- **Careful definition of charts at boundary.**
- **Classical use of collars.**
- **Computational upper bound.**
- **Needs bound on curvature.**

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