Constrained Pseudorandom Functions for Unconstrained Inputs

Apoorvaa Deshpande

Brown University

Joint work with: Venkata Koppula and Brent Waters

TEXAS
The University of Texas at Austin

Pseudorandom Functions

[GGM'84]

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F: {Keys} x {Input} ⟶ {Output} is a PRF if it is *indistinguishable* from a truly random function

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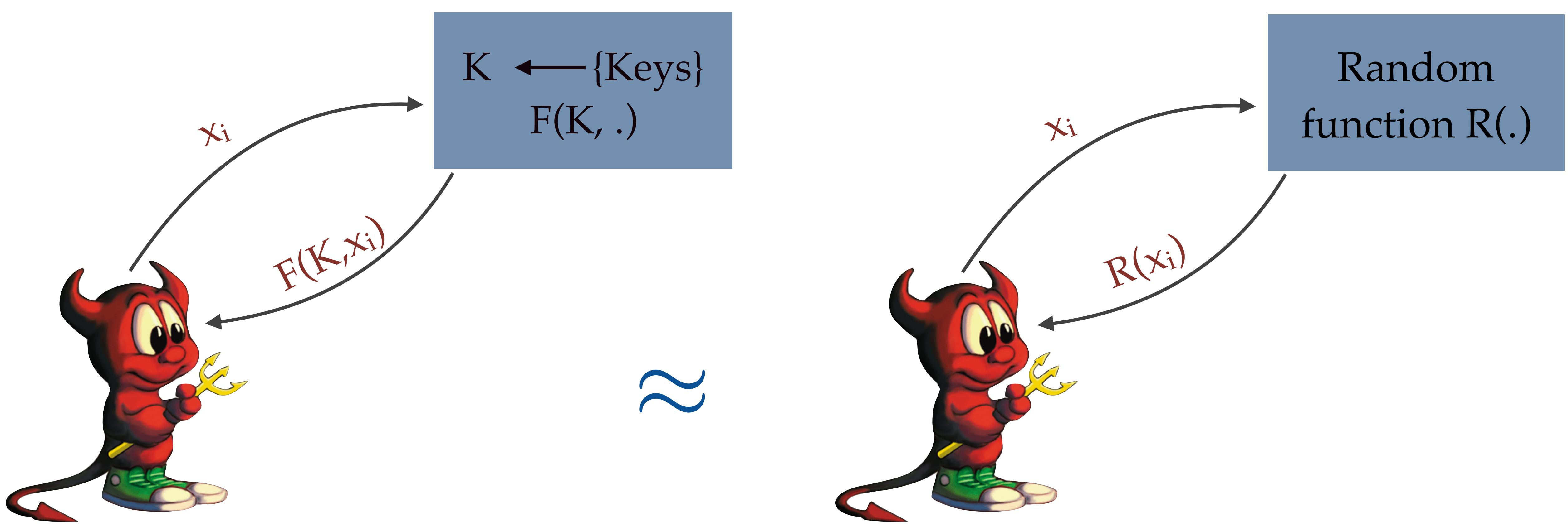

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Setup(1^k) → K

Eval(x) → F(K,x)

Pseudorandom Functions

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Setup(1^k) ⟶ K

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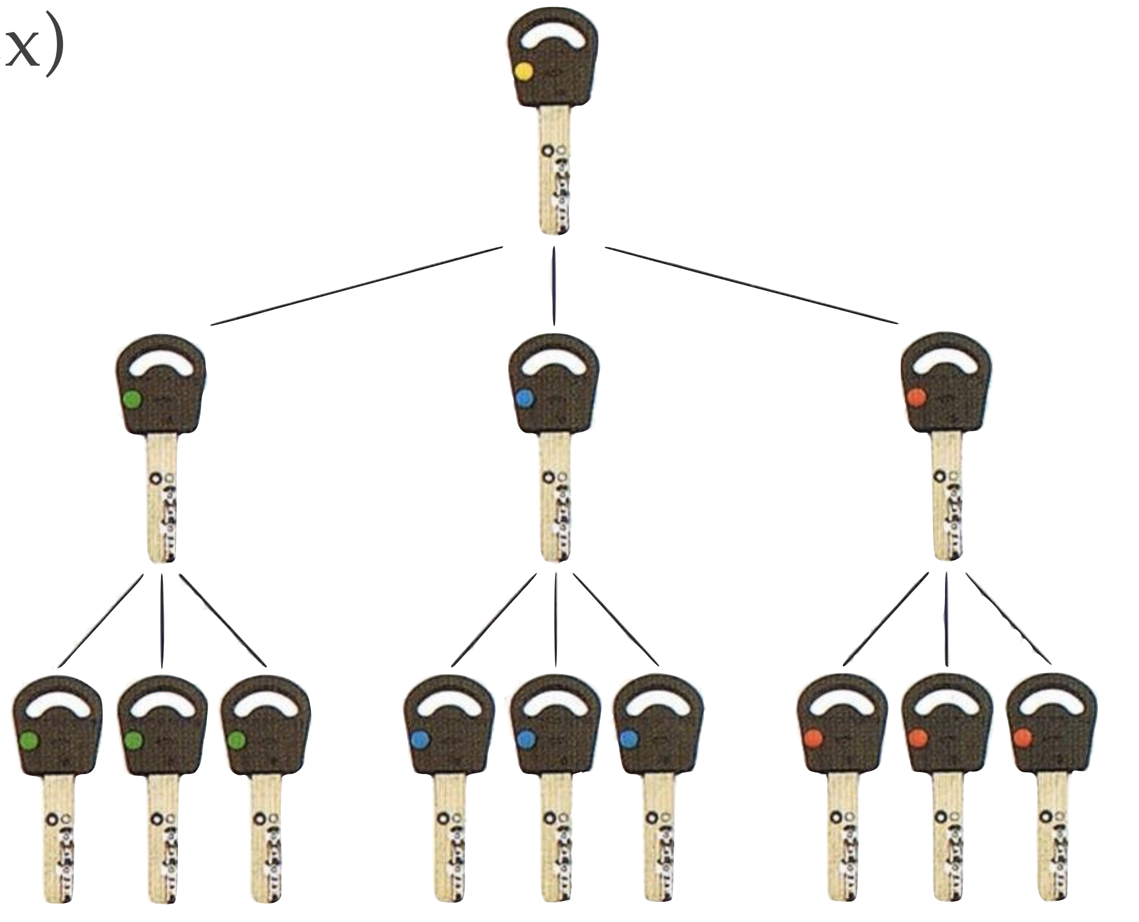
- What if we don't want to give away our PRF key completely?
- What if we want to give a key that lets us evaluate PRF only on *some* points?



Constrained PRFs

Punctured PRFs: A Type of Constrained PRFs

[SW'14]

Setup → K

Constrain(K,x^*) → K{x^*}

Eval(K{x^*},x) → F(K,x) if x not x^*, fail otherwise

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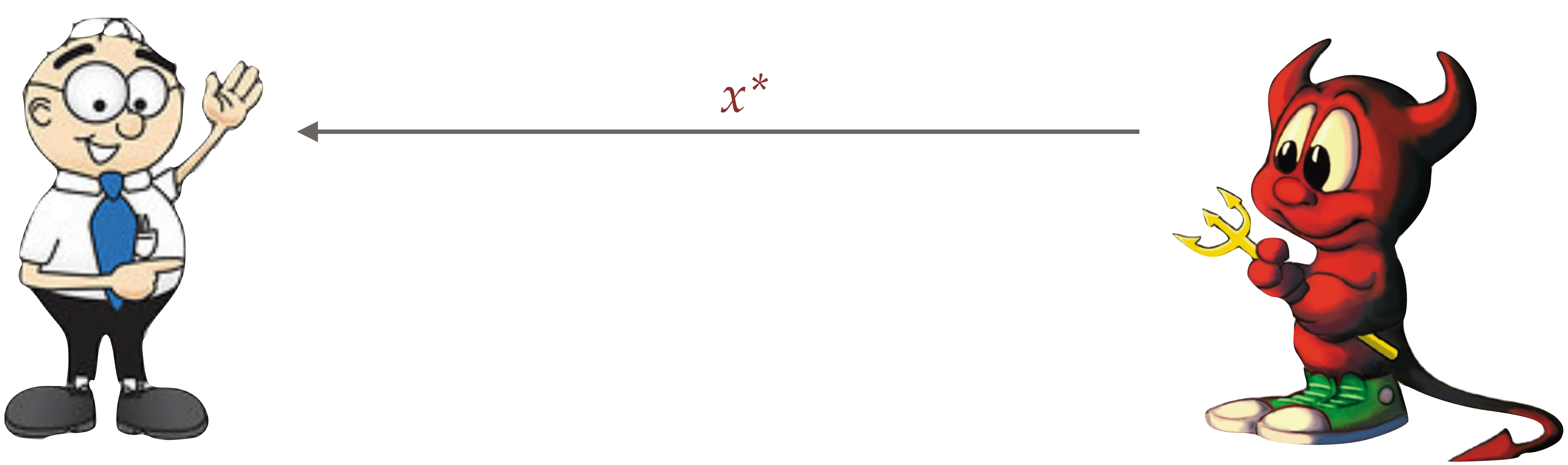

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<u>**Selective Security**</u>

← x^*

b = 0: K{x*}, F(K,x*) →

b = 1: K{x*}, Random y

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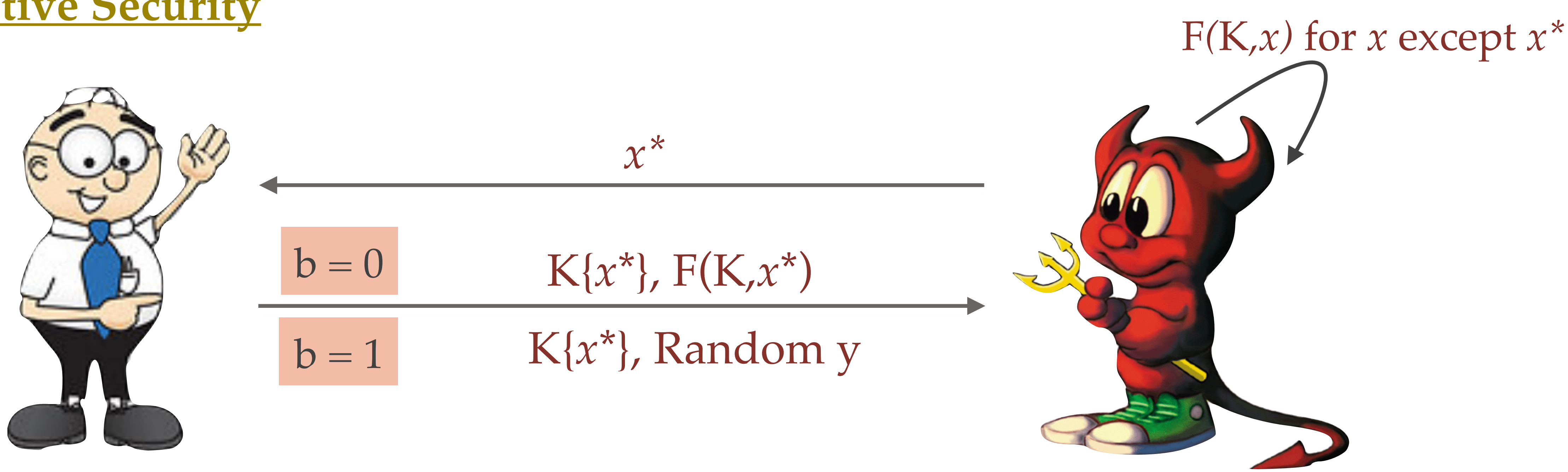

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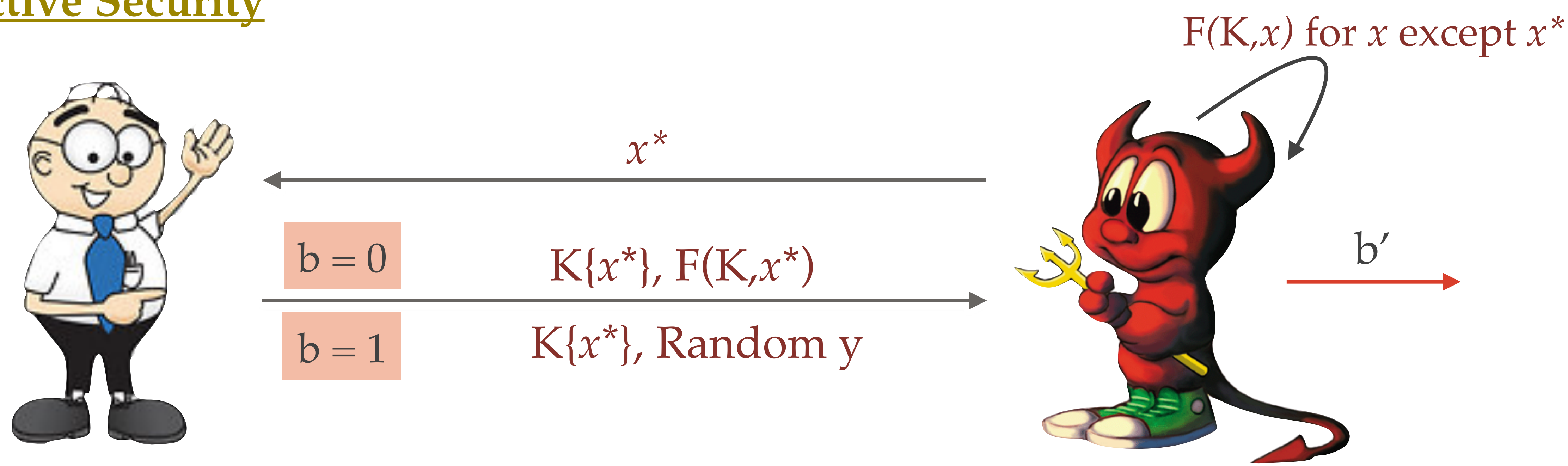

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x^* ←

b = 0: K{x^*}, F(K,x^*) →

b = 1: K{x^*}, Random y →

F(K,x) for x except x^*

b′ →

Pr[$\mathcal{A}$ wins] = Pr[b = b′] = 1/2 + negligible

Constrained PRFs

[BW'13], [BGI'14], [KPTZ'13]

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Constrain(K,*C*)	→ K{C}
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x^* ←

b = 0 y = F(K,x^*) →

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C s.t. C(x^*) = 0 ←

K{C} →

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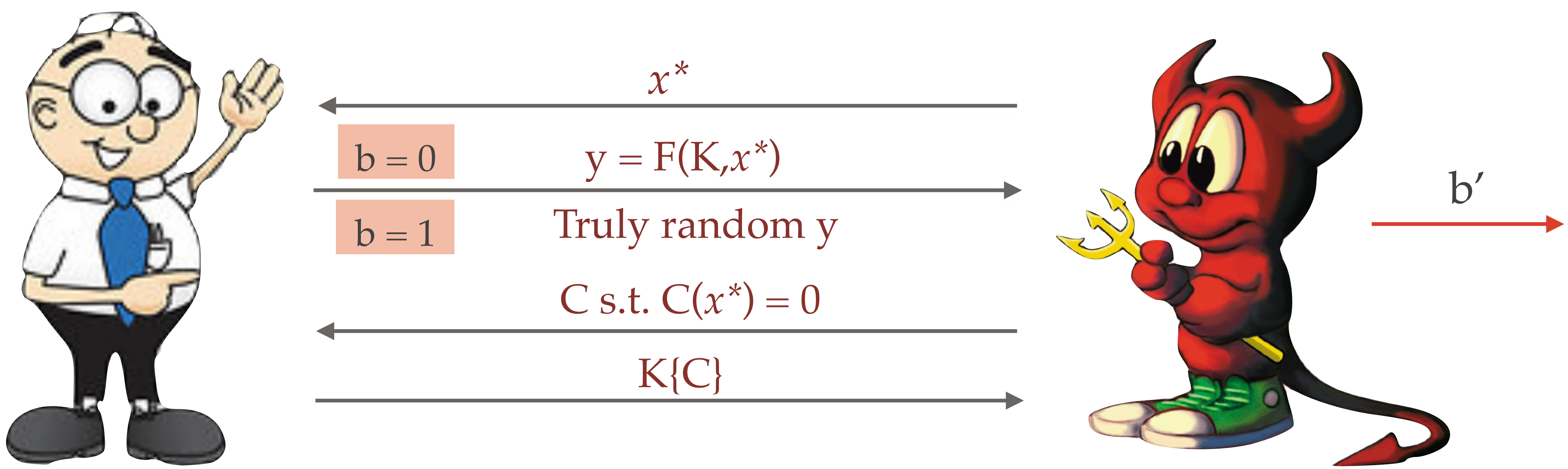

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Motivating Example

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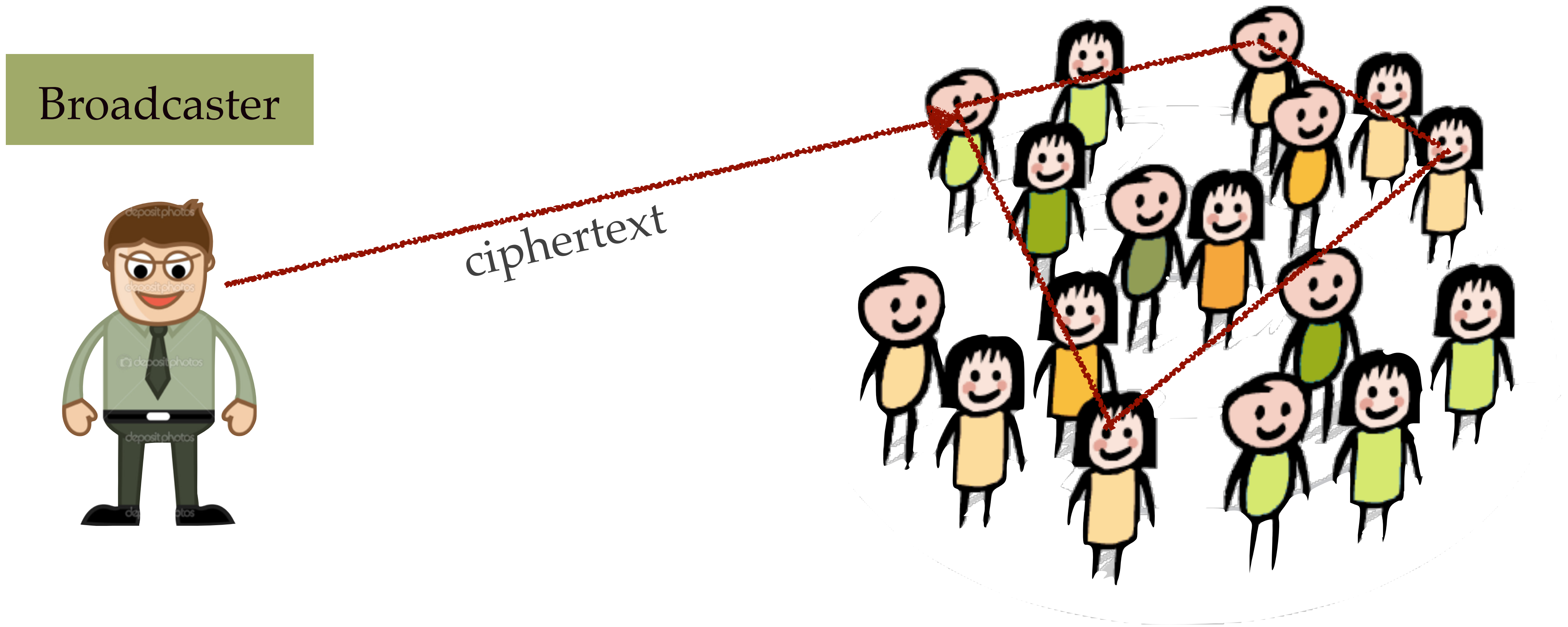
Broadcaster

Motivating Example



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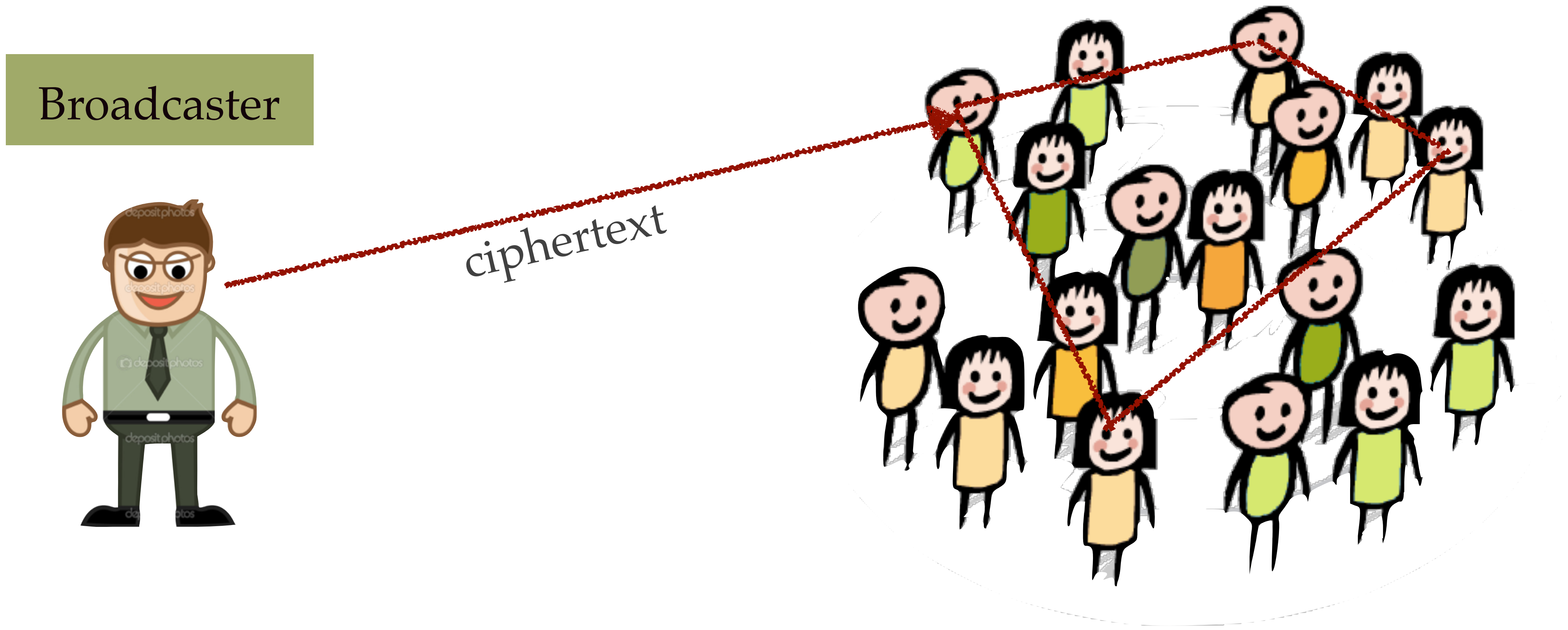
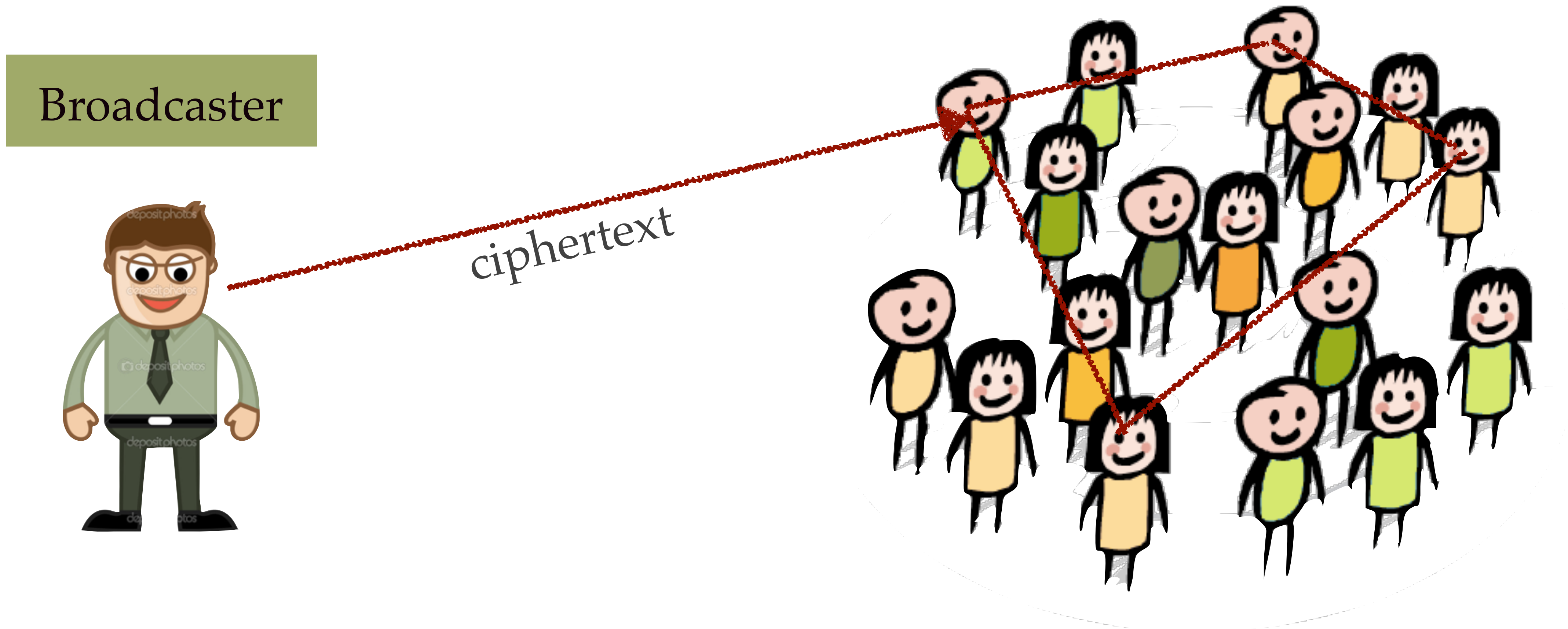


Broadcast Encryption [FN'93]

Motivating Example



Broadcast Encryption [FN'93]

- Lets you encrypt messages for a specific subset
- No one outside the subset can learn the message
- Can be constructed from constrained PRFs (bit-fixing PRFs)

[BW'13], [BWZ'14]

GGM as a Constrained PRF

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$$F(s, x = x_1 x_2 \ldots x_n) = G_{x_n}(\ldots G_{x_2}(G_{x_1}(s)) \ldots)$$

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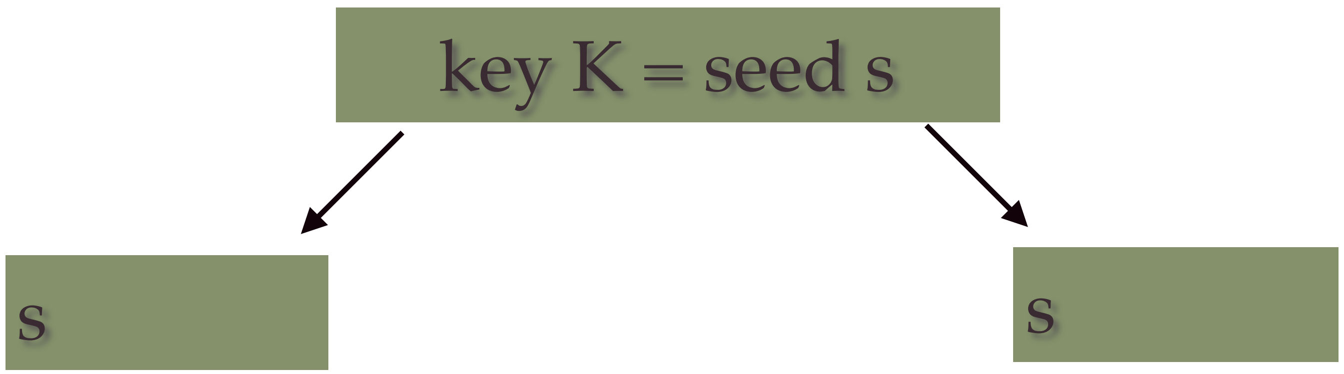

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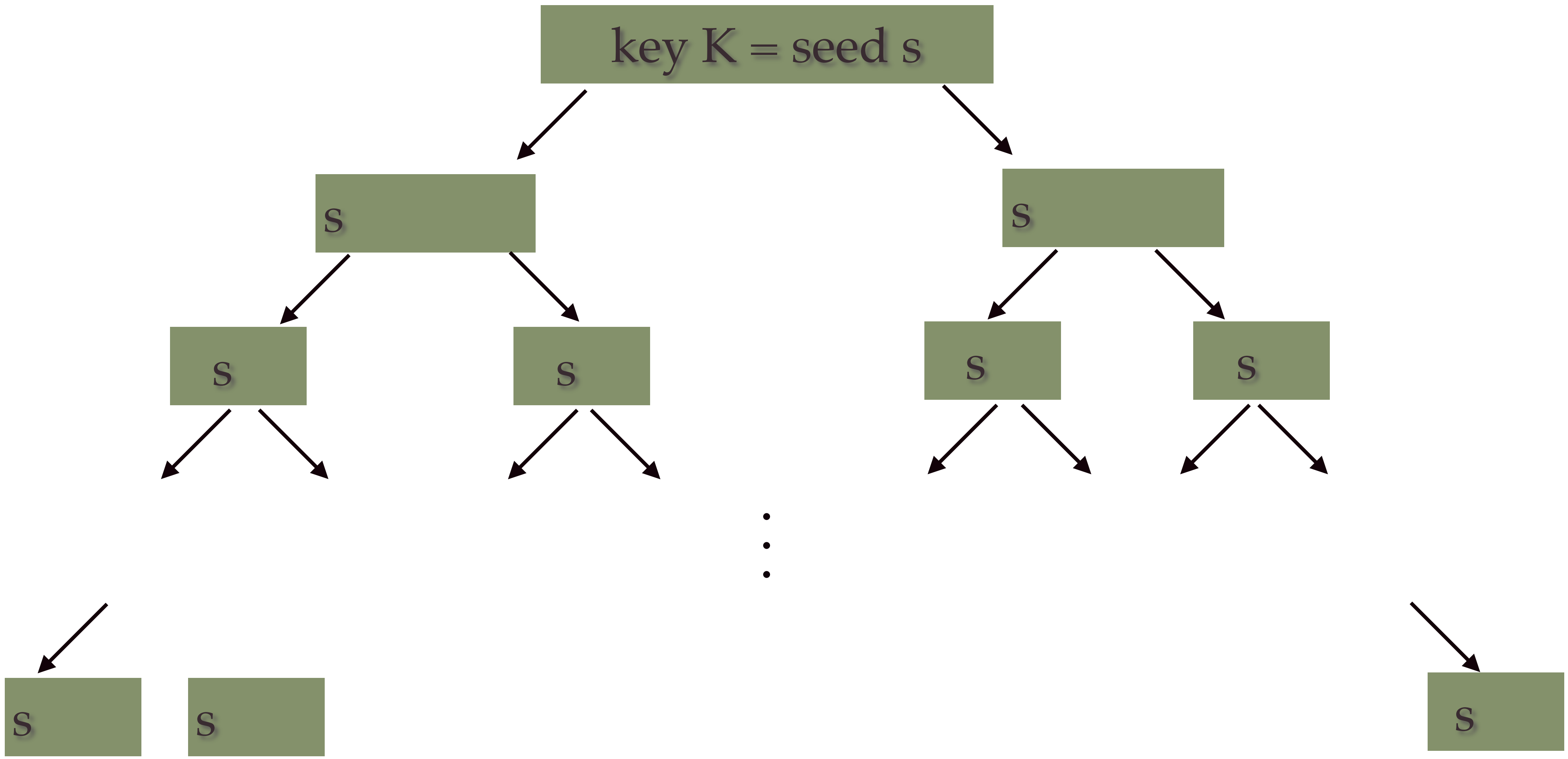

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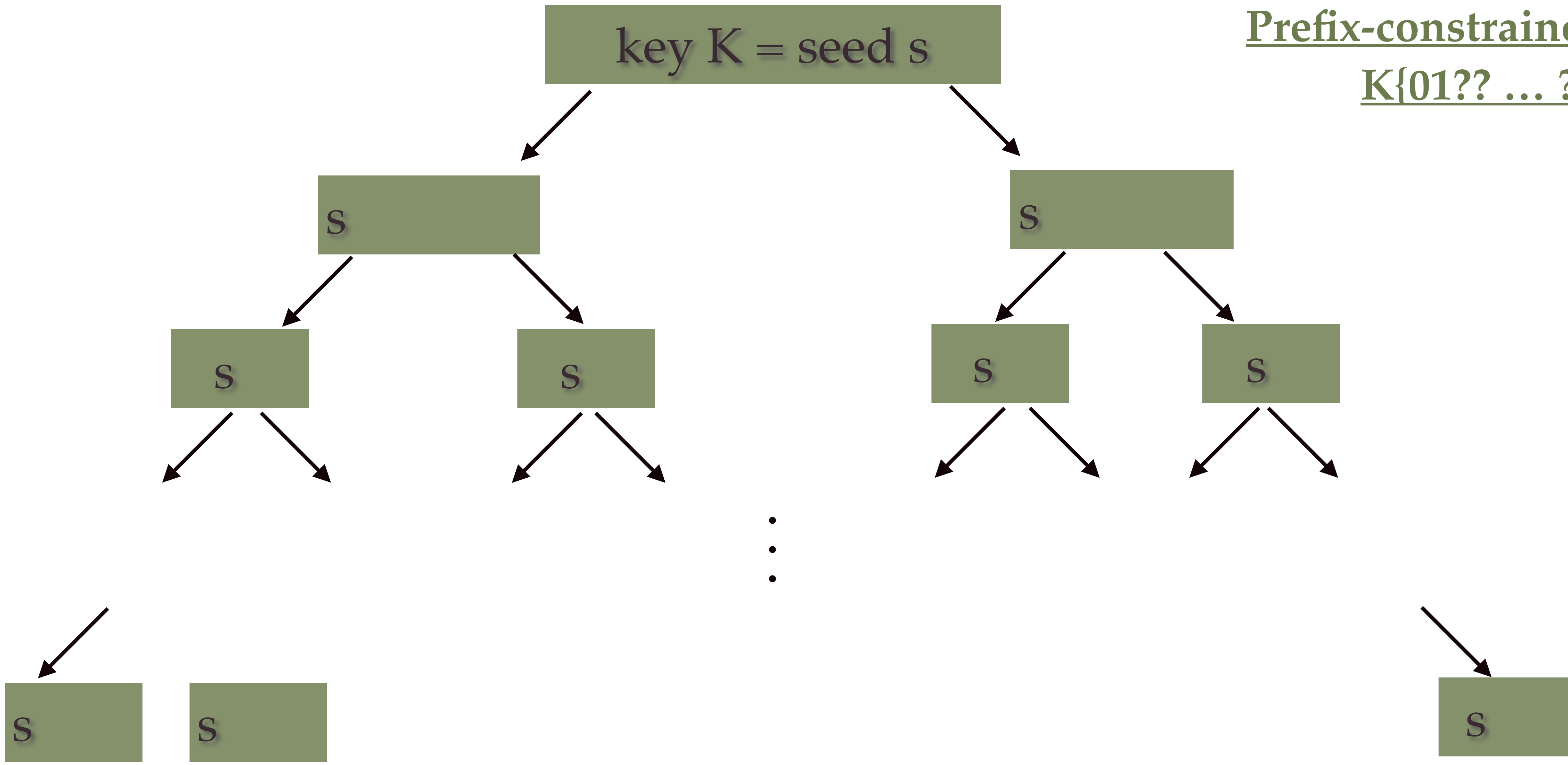

Prefix-constrained key:
K{01?? … ?}

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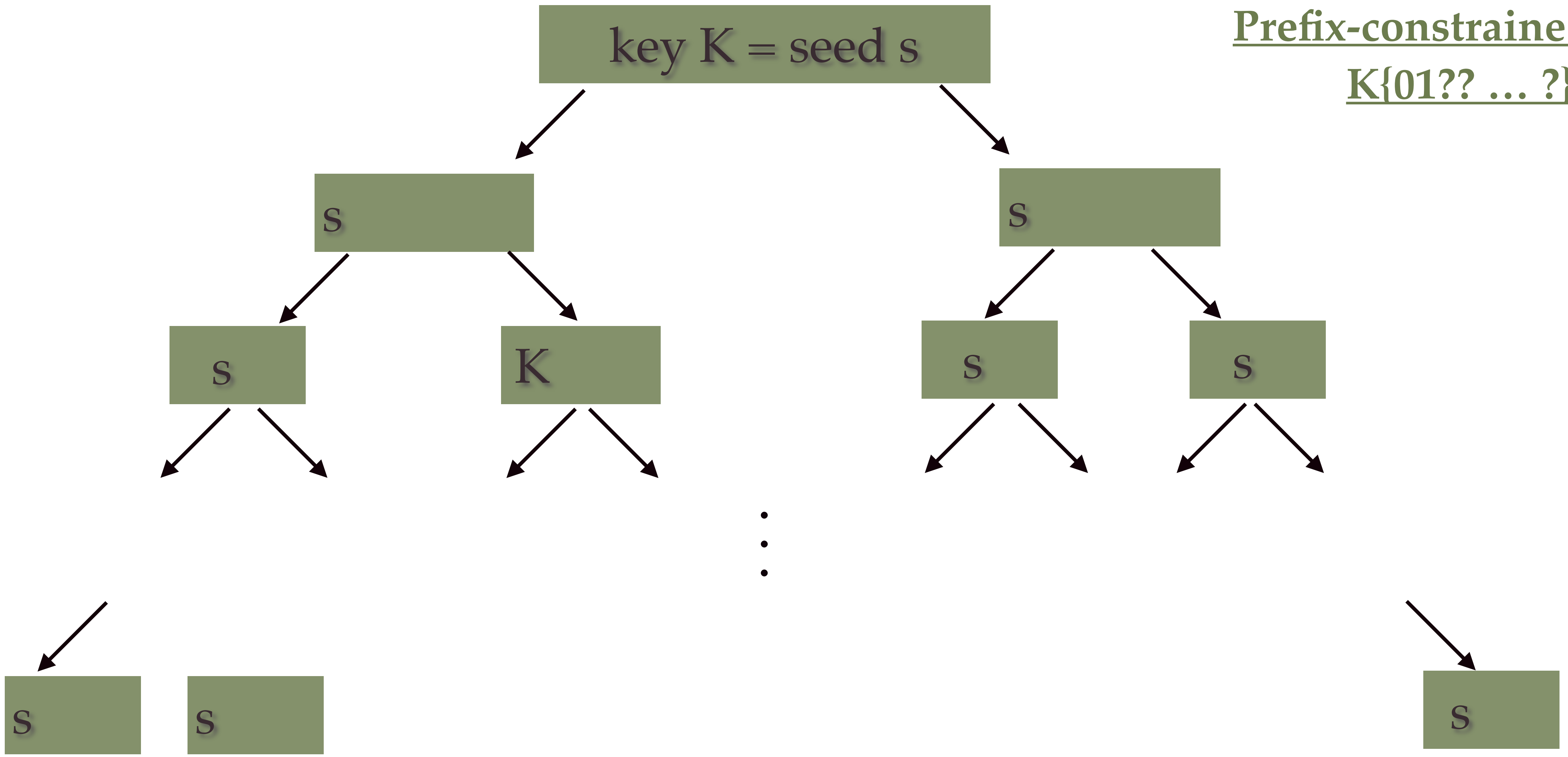

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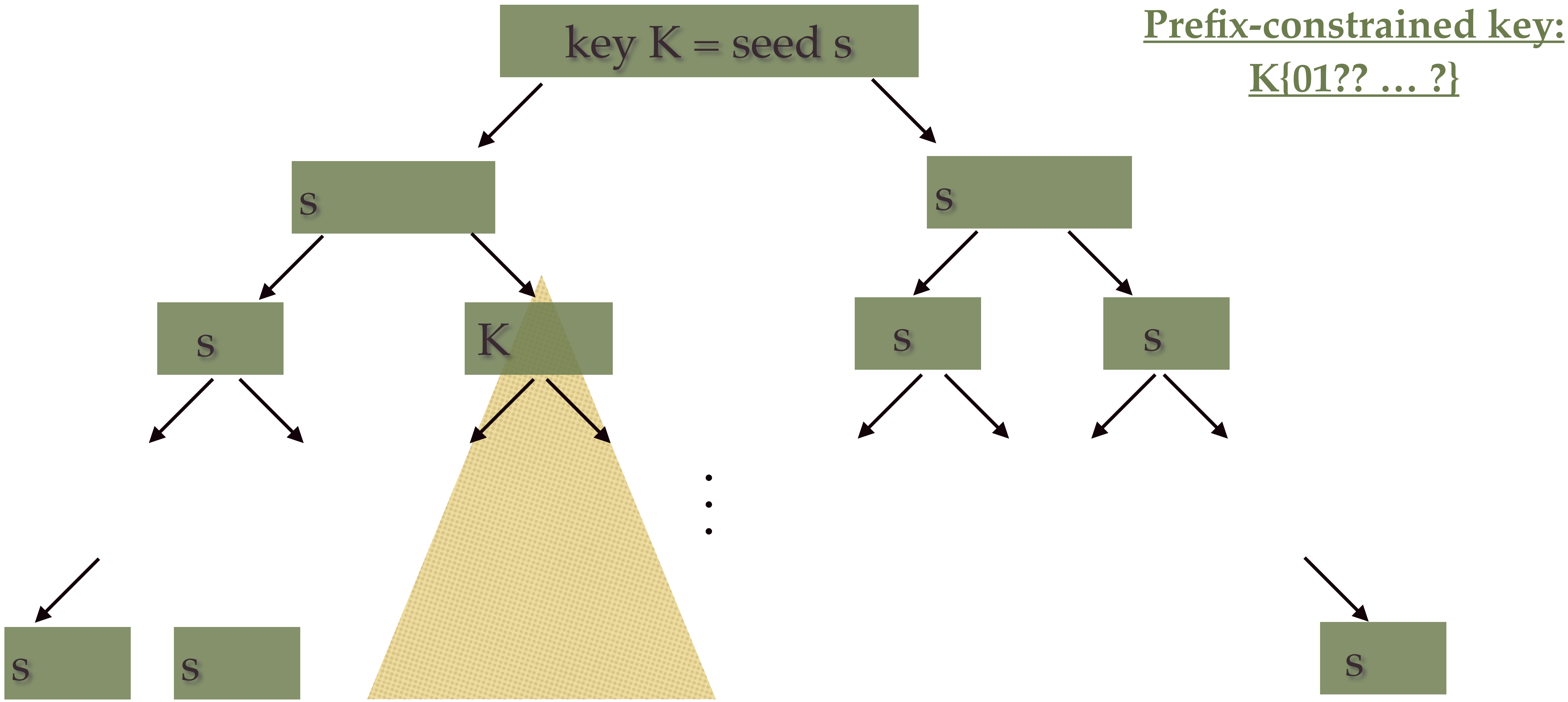

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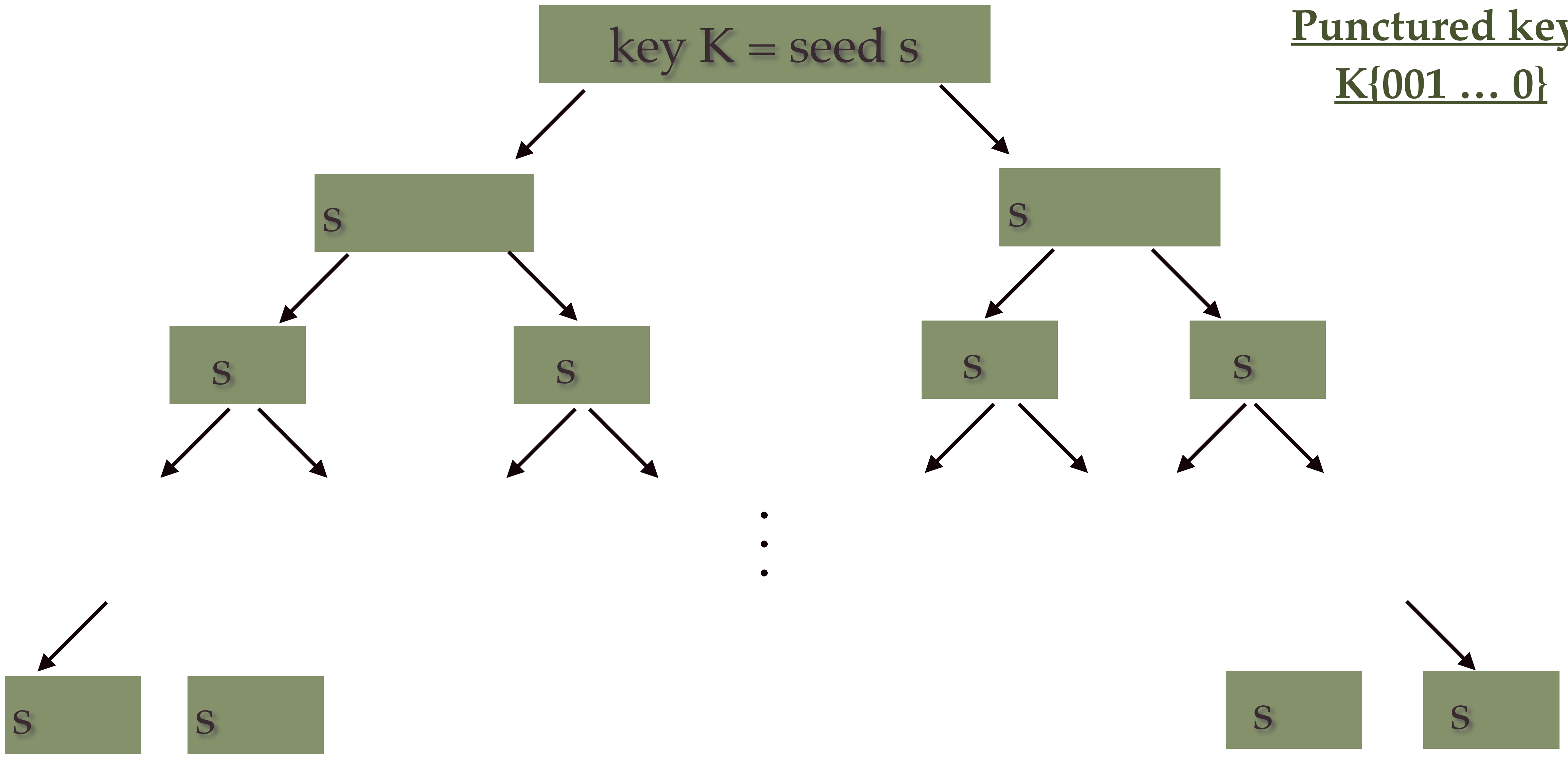

Punctured key:
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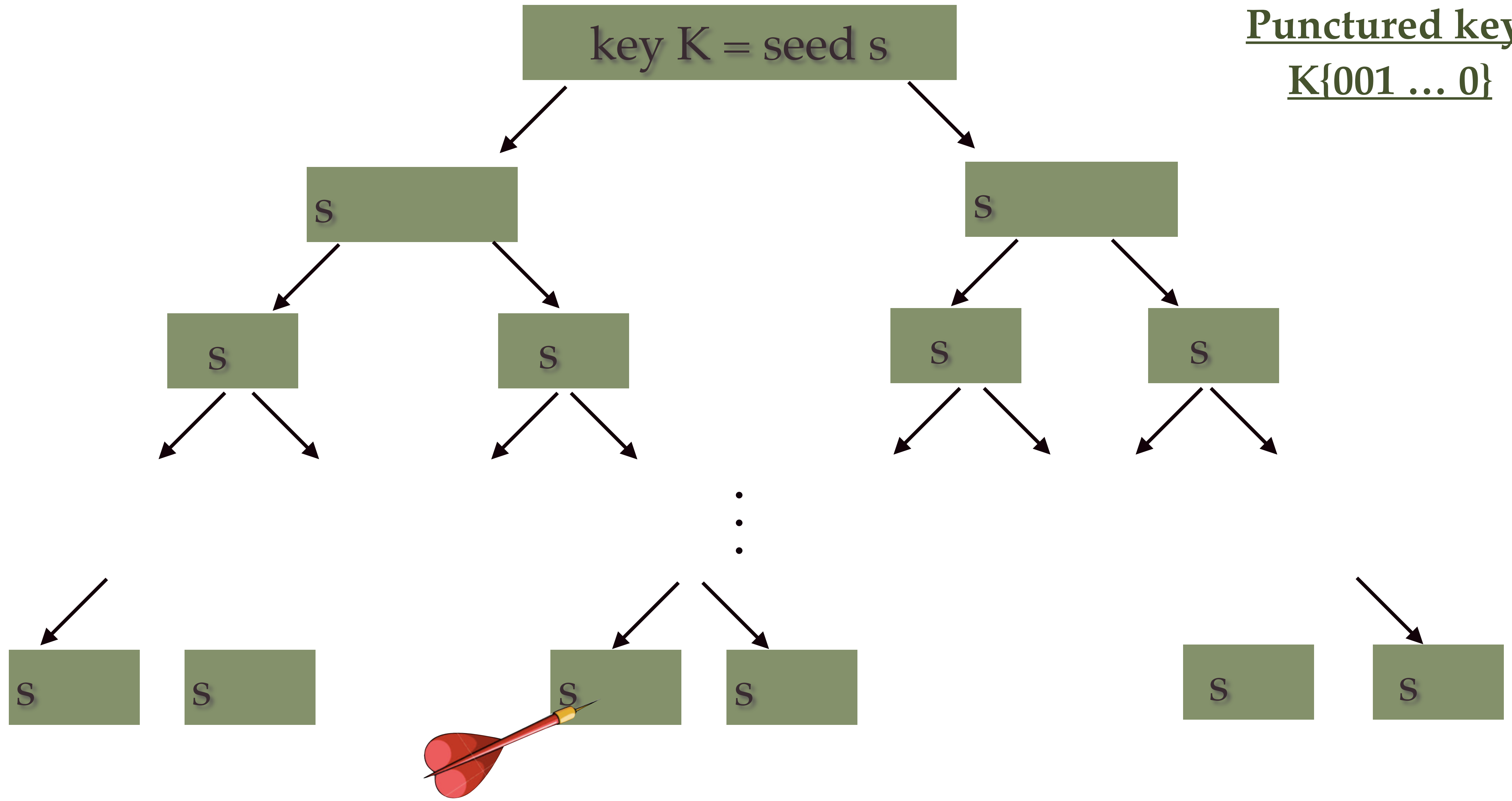

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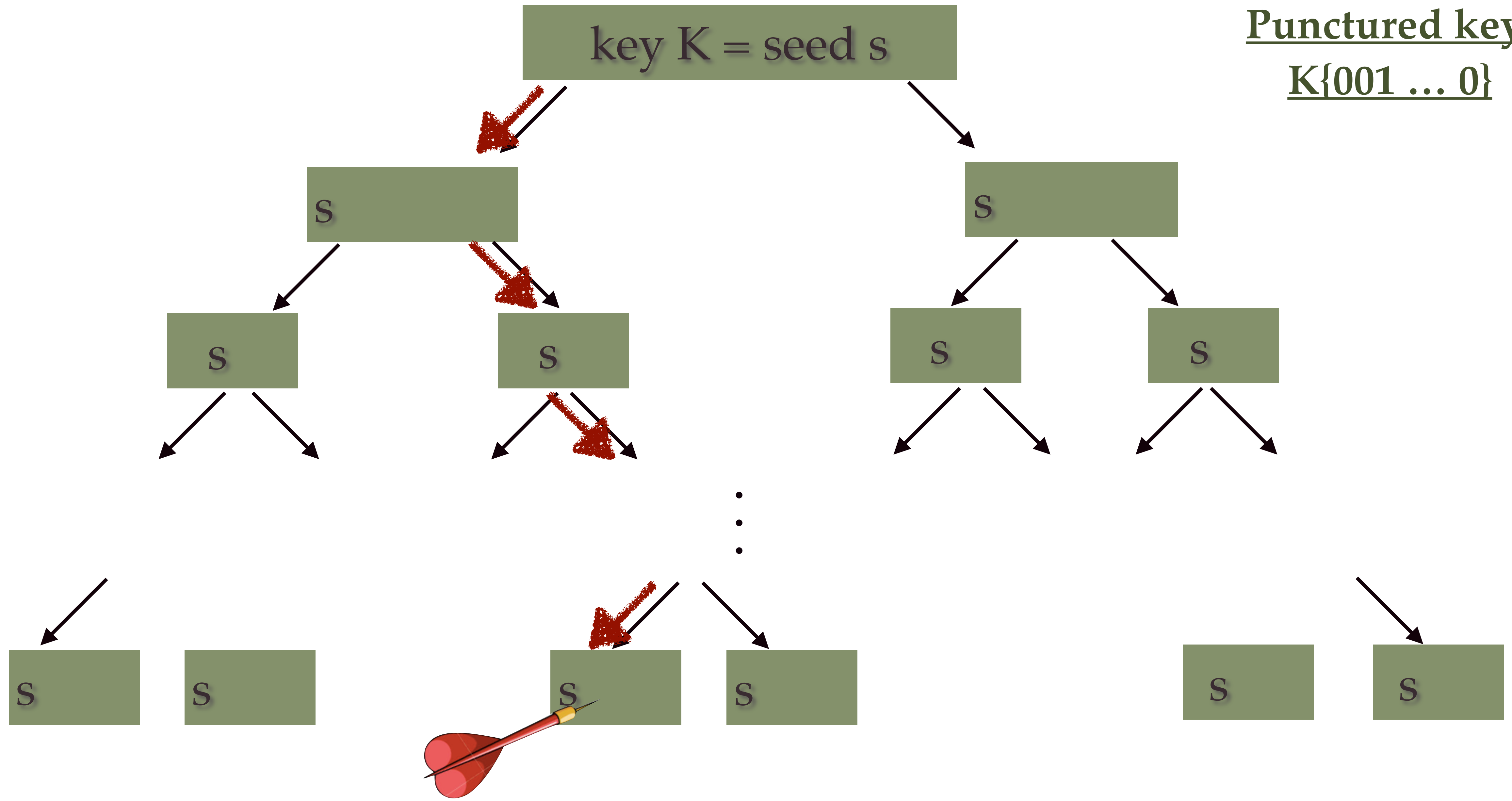

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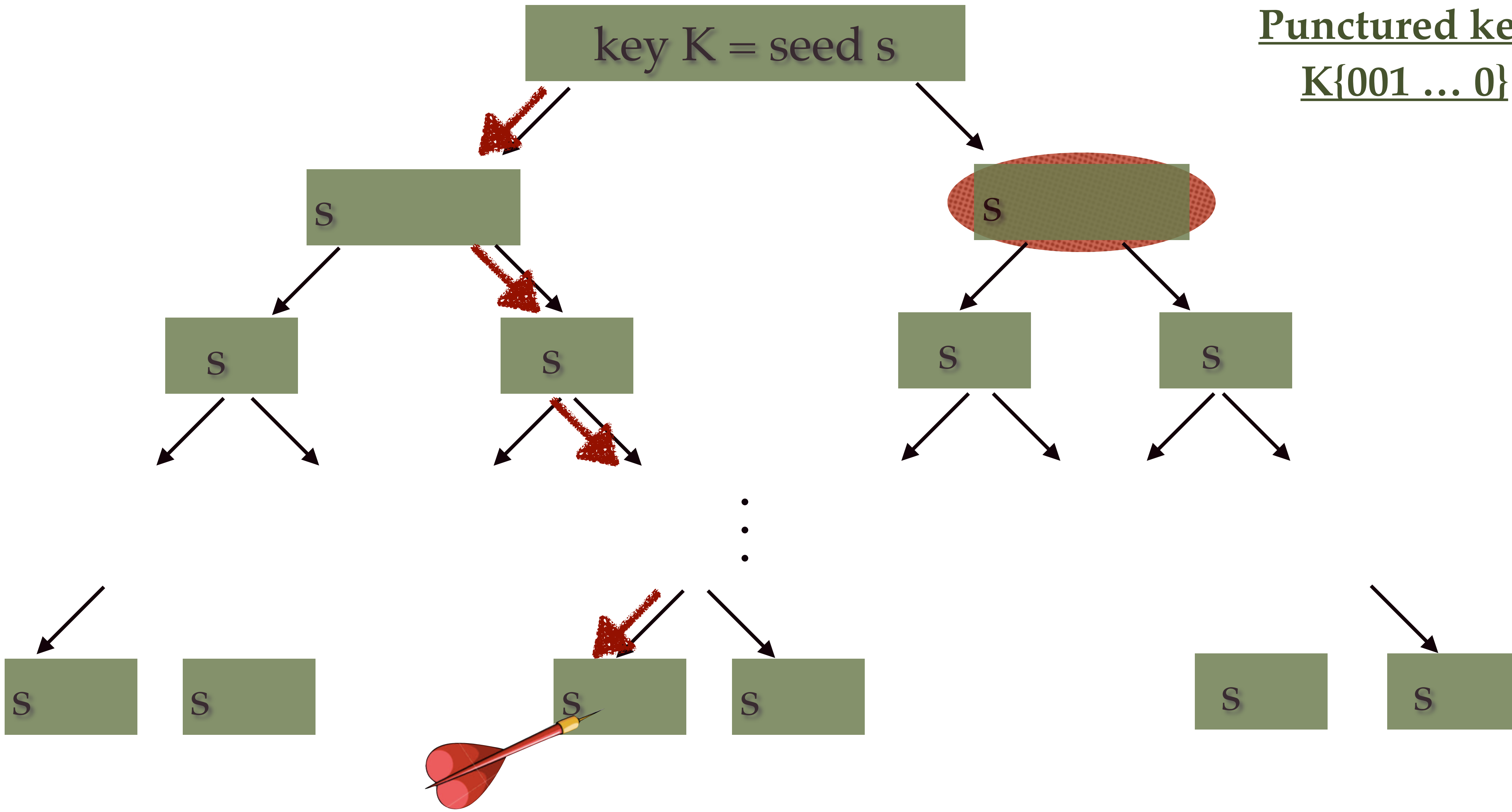

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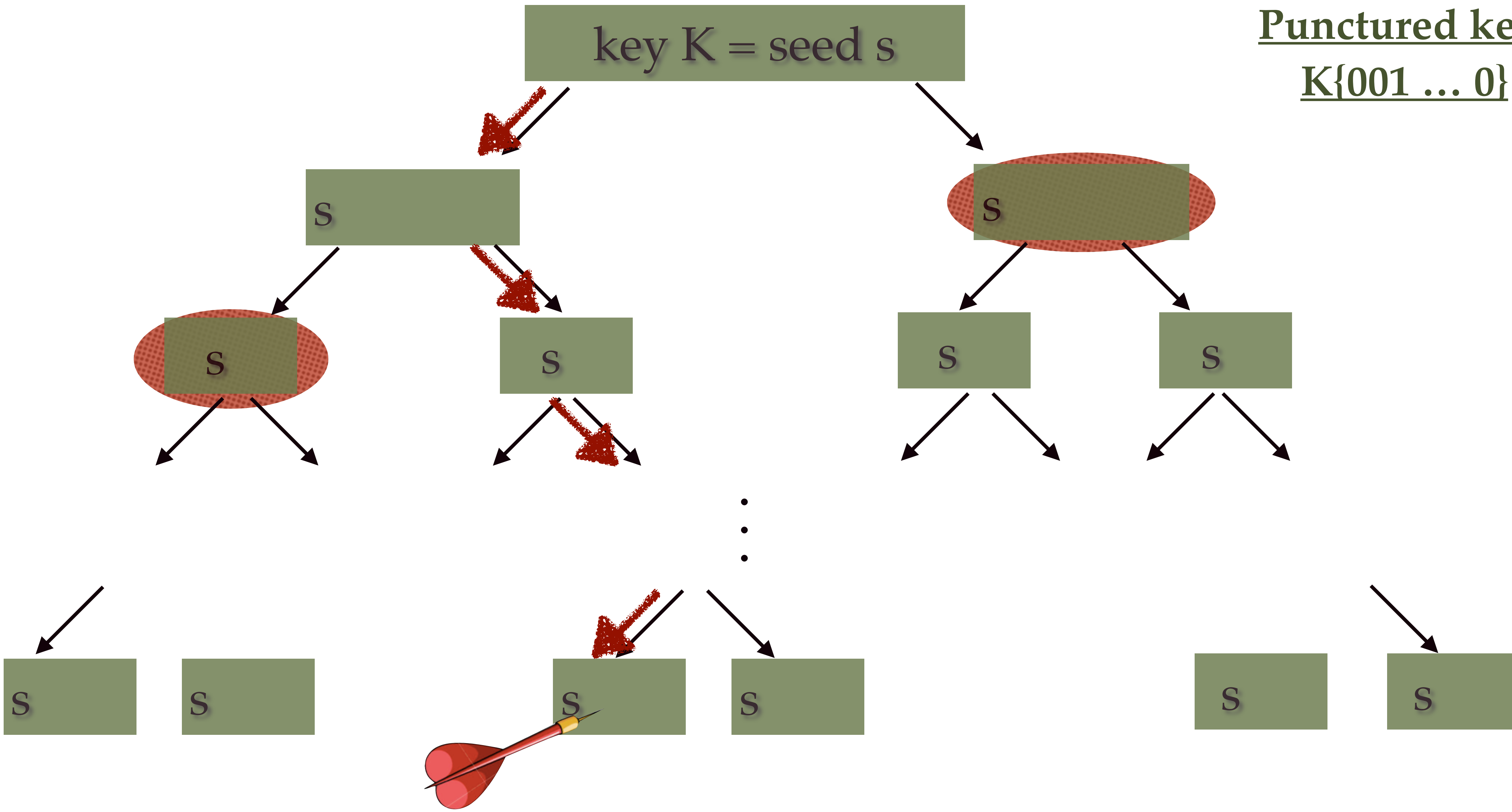

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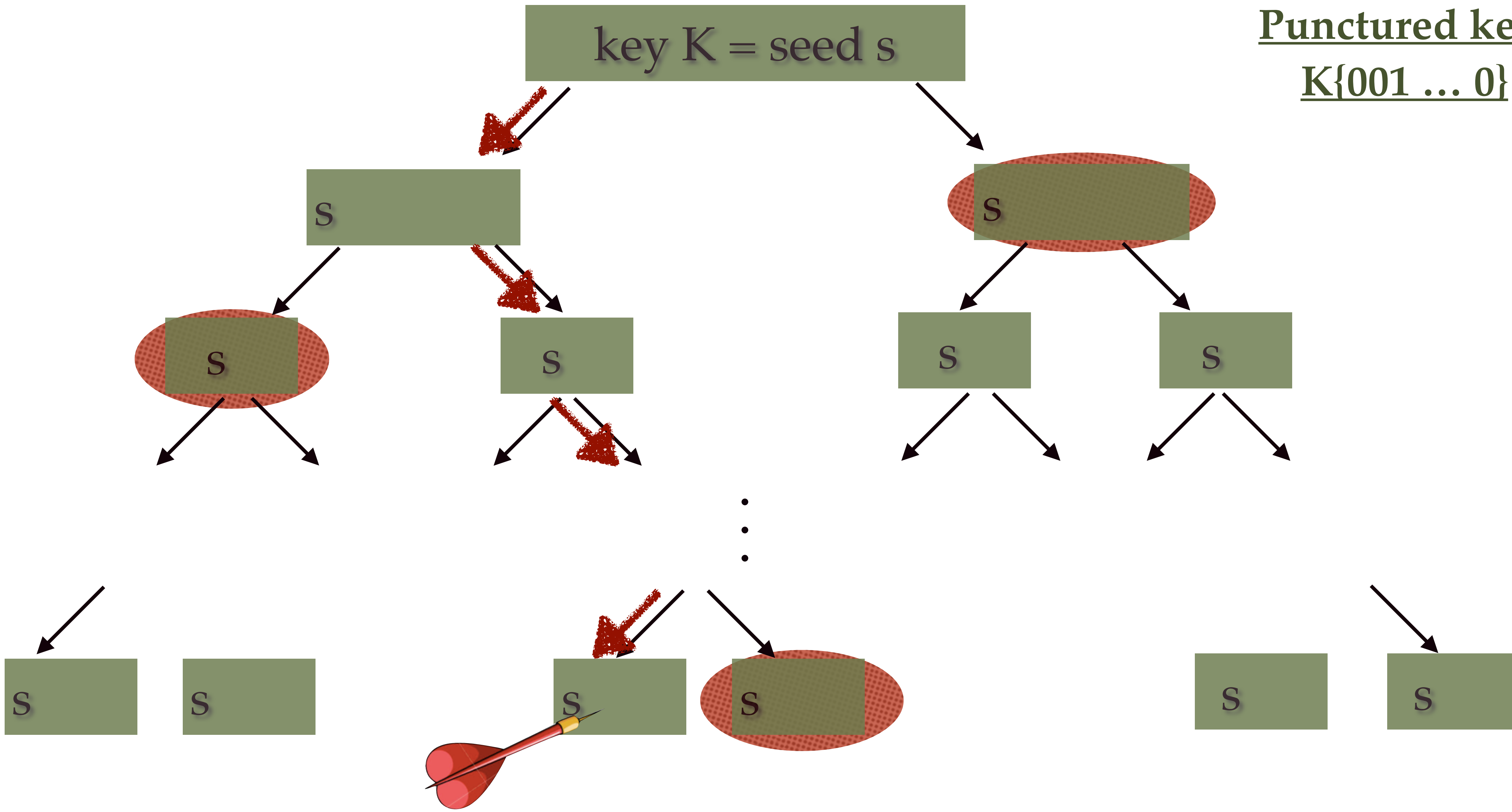

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CPRF Generically

- Generic CPRF: Constraint expressed as an arbitrary circuit
- Known only from strong cryptographic assumptions

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Possible Construction

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Possible Construction

x

↓

If C(x) = 1

output F(K,x)

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CPRF Generically

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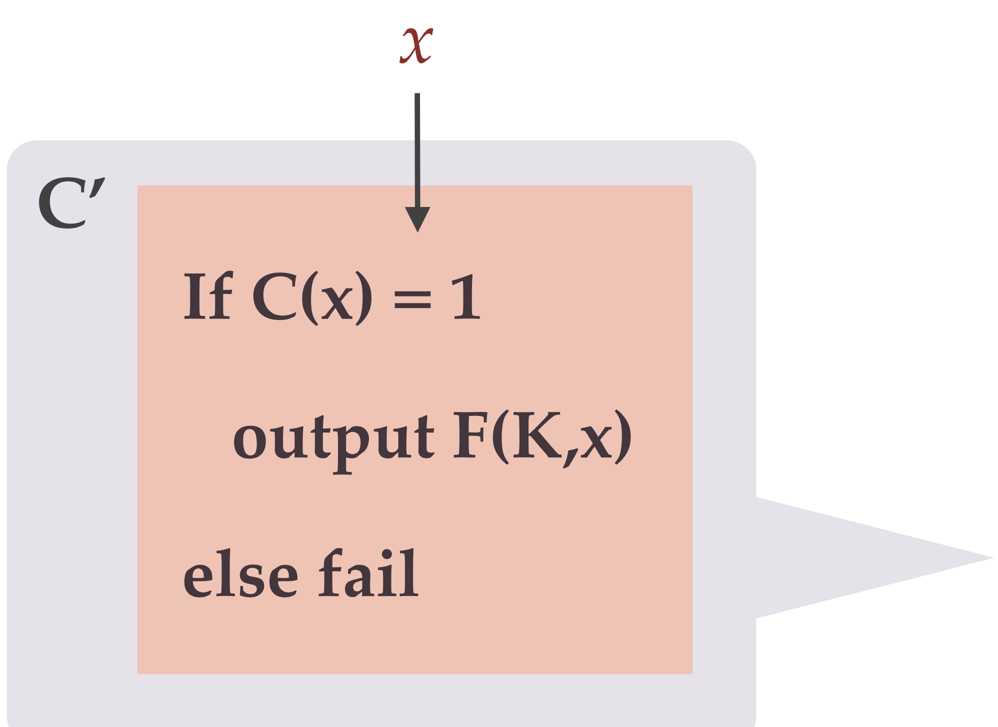

Output this circuit

CPRF Generically

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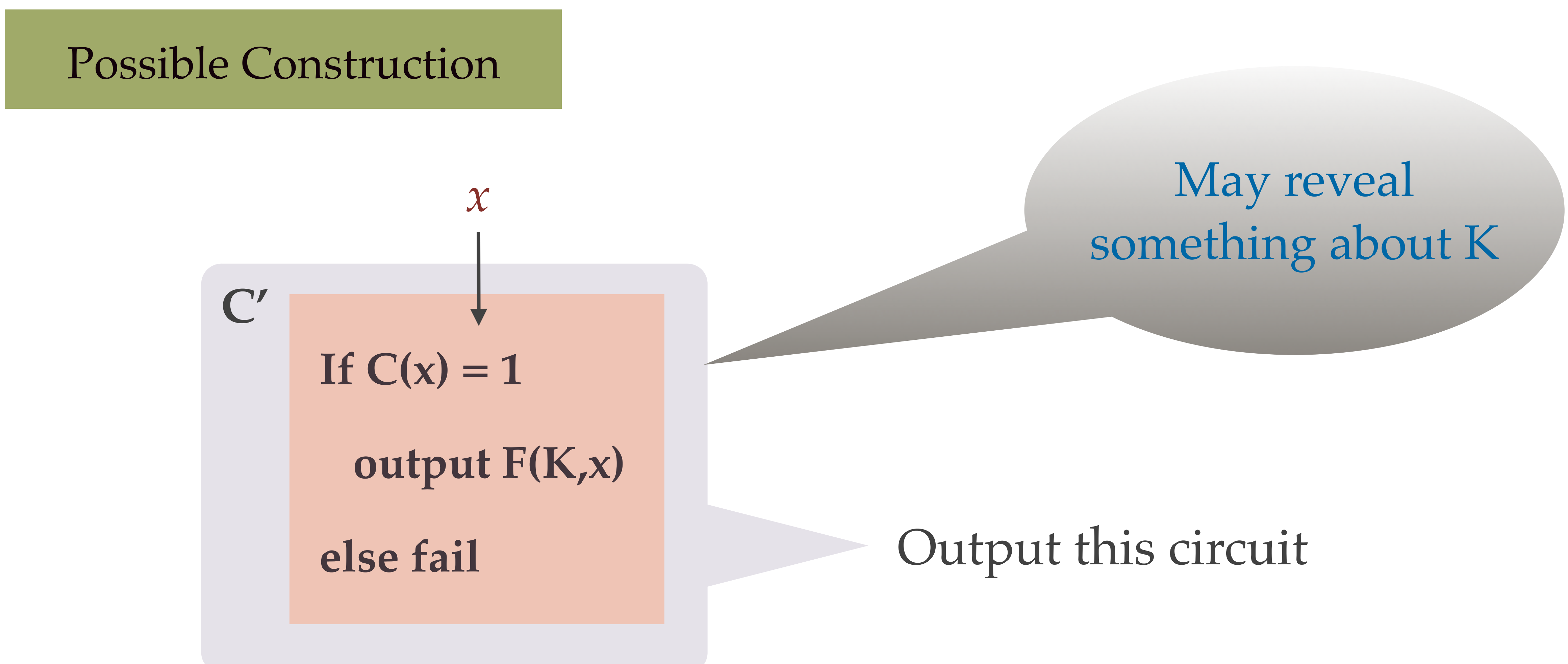

Program Obfuscation

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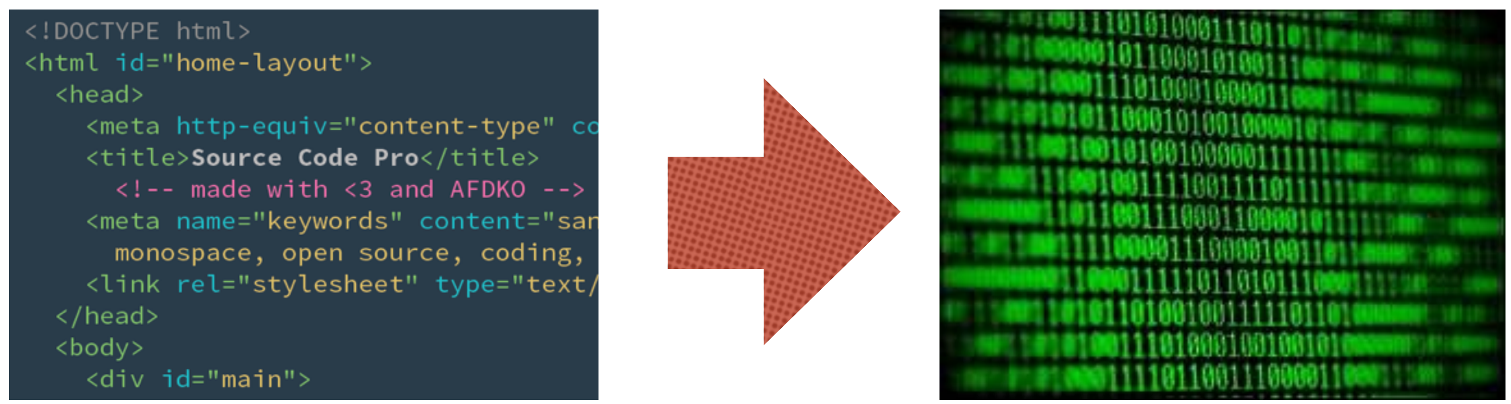
Program Obfuscation

Obfuscated Program

Program Obfuscation

Program Obfuscation

[BGI+’01]



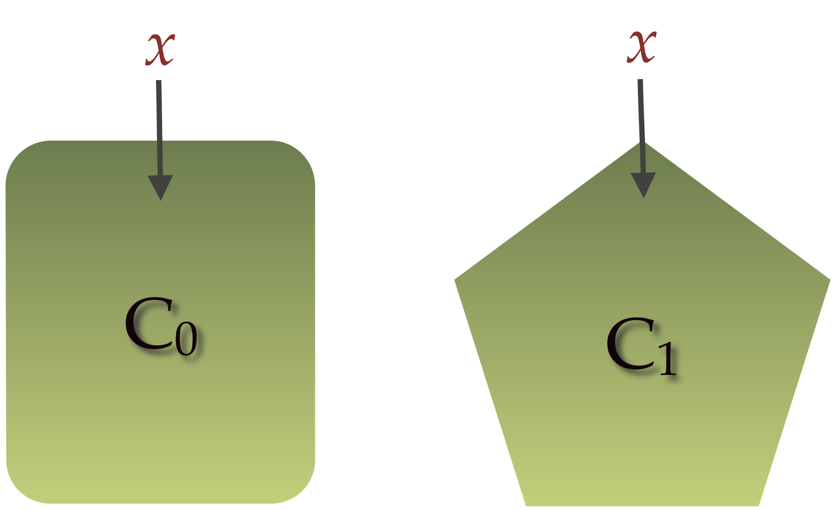
Obfuscated Program

Program Obfuscation

Program Obfuscation [BGI+'01]

Obfuscated Program

Indistinguishability Obfuscation



$C_0(x) = C_1(x)$ for all x

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[BGI+'01]

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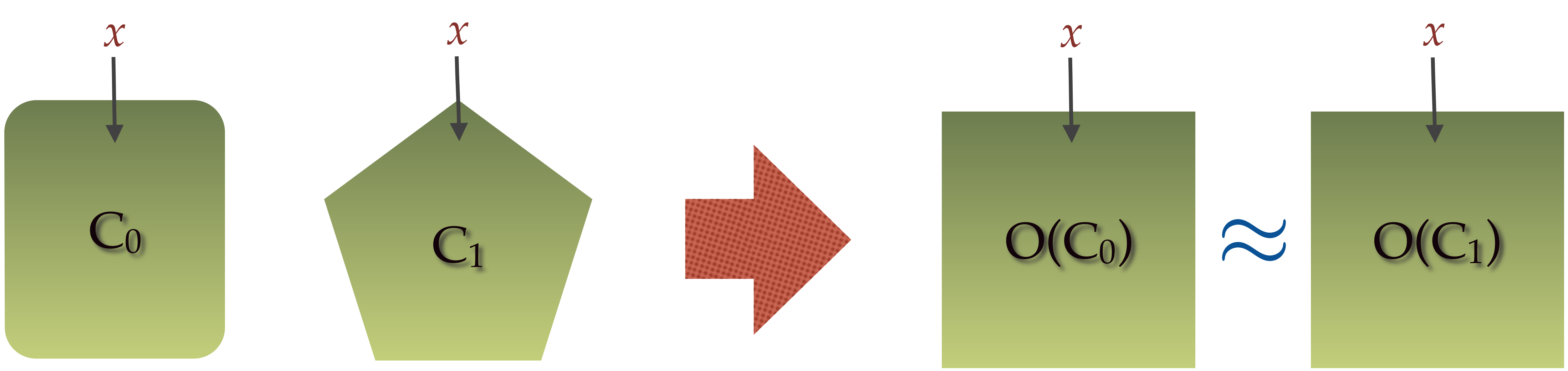

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[BGI+'01]

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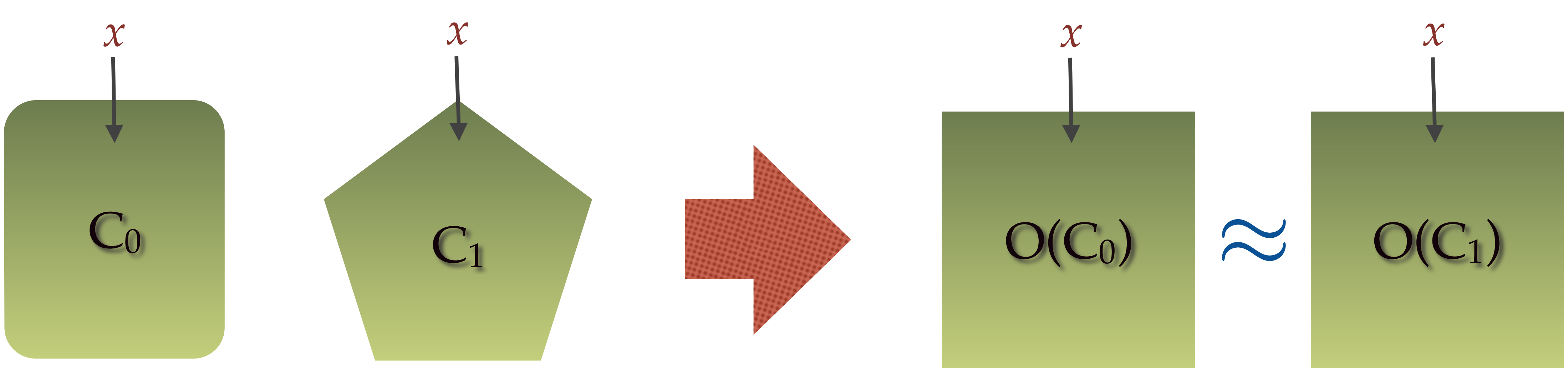
Indistinguishability Obfuscation

[GGH+'13] [Zimmerman'14] [BGKPS'14]



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CPRF Construction from iO

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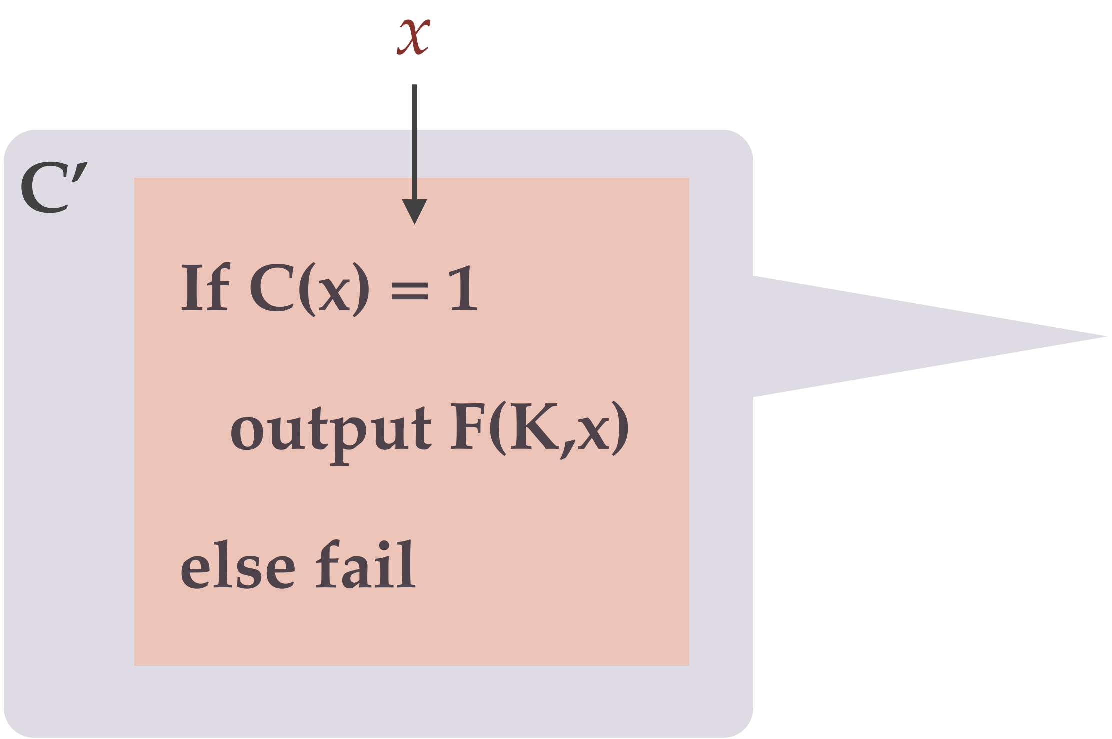

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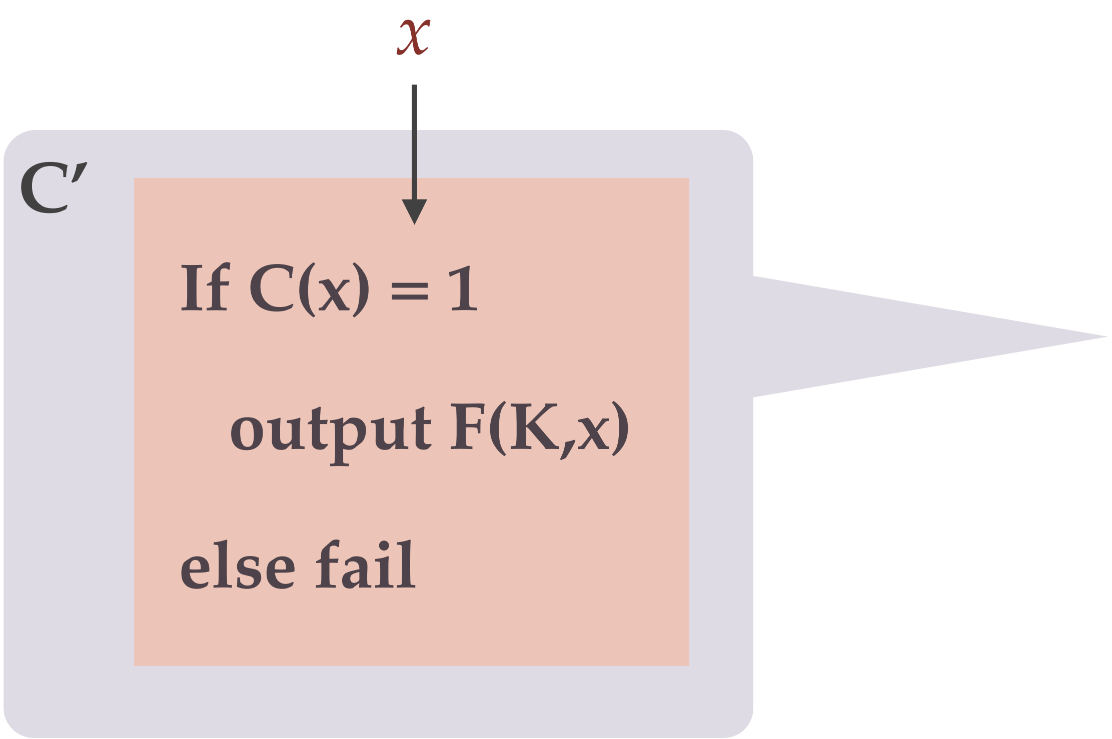
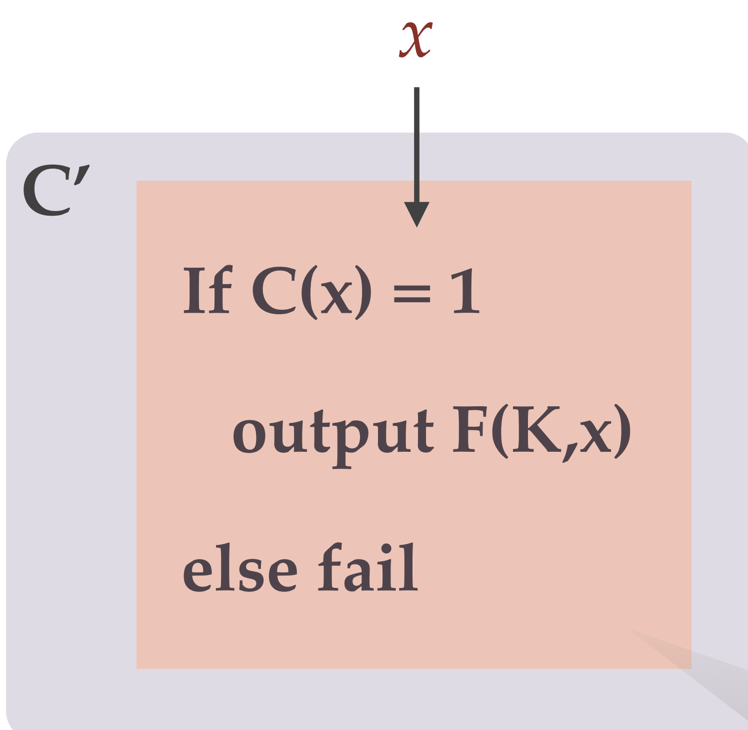


Obfuscate the circuit C′

Constrained key = O(C′)

CPRF Construction from iO



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Can prove this construction secure under iO

CPRF Construction: Proof Overview

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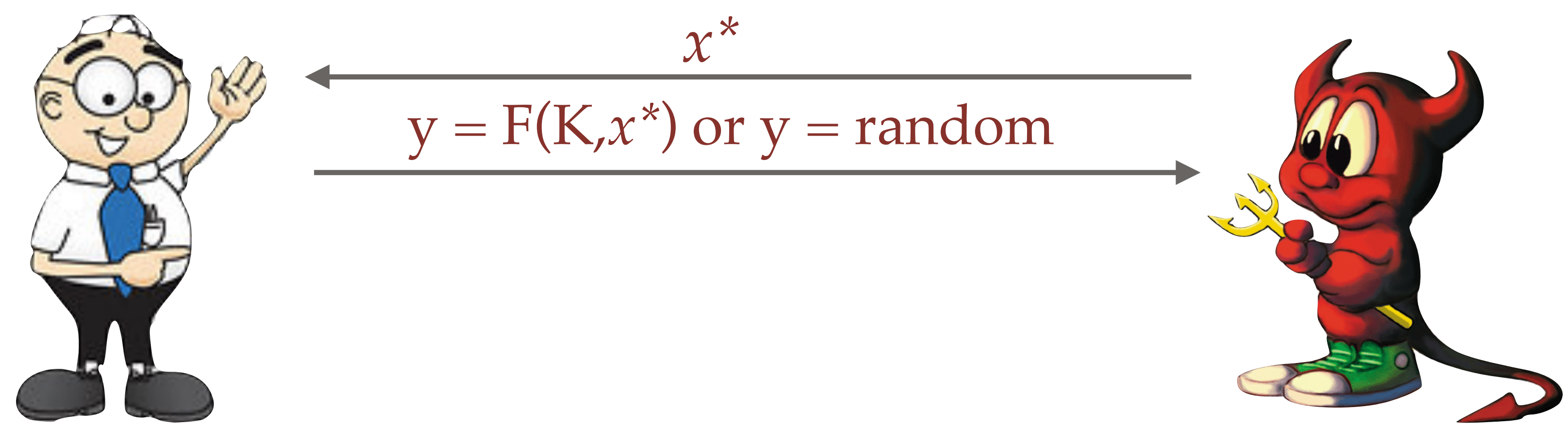

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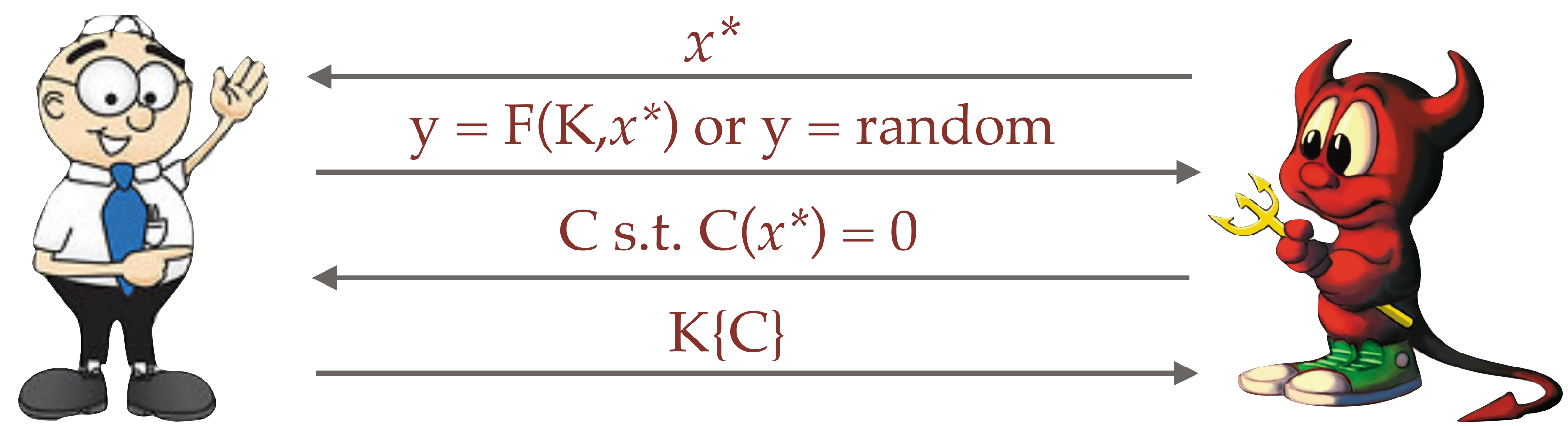

CPRF Construction: Proof Overview

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K

If C(x) = 1

output F(K,x)

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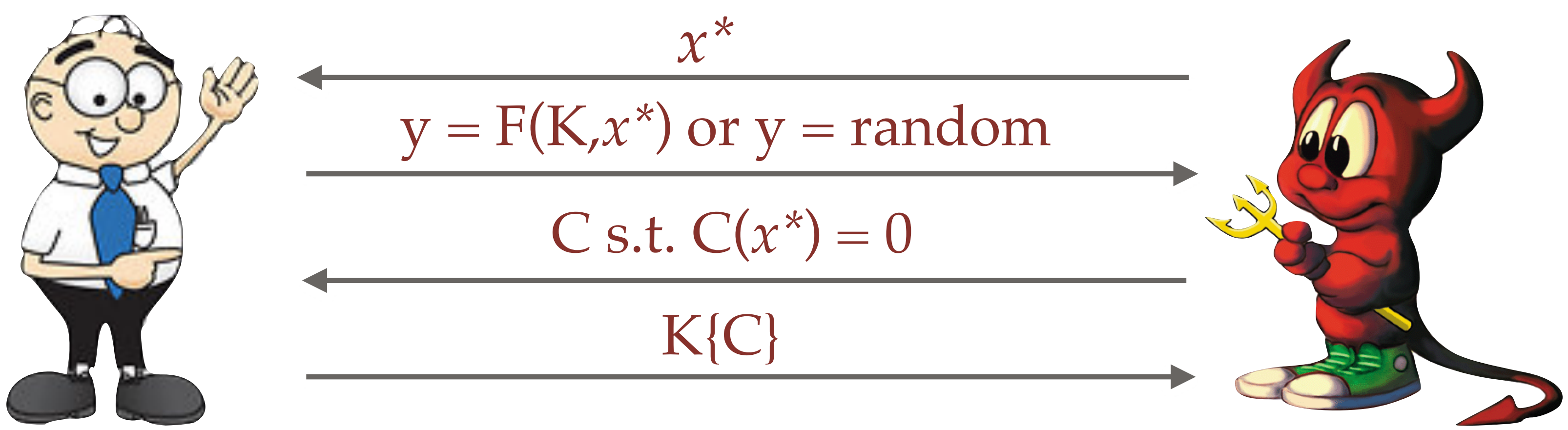

CPRF Construction: Proof Overview

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If C(x) = 1

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x

$K\{x^*\}$

$y^* = F(K,x^*)$

If C(x) = 1

if x = x*

output y*

else output F(K,x)

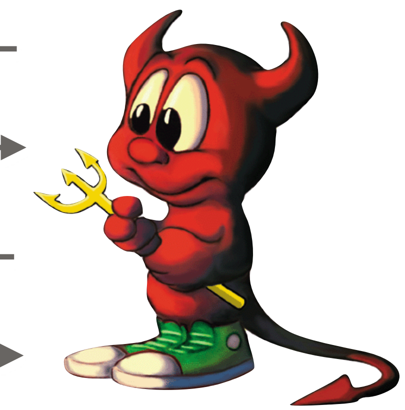
else fail

x^*

$y = F(K,x^*)$ or y = random

C s.t. $C(x^*) = 0$

K{C}



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⇓ iO

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$y^* = F(K,x^*)$

If C(x) = 1

  if x = x*

    output y*

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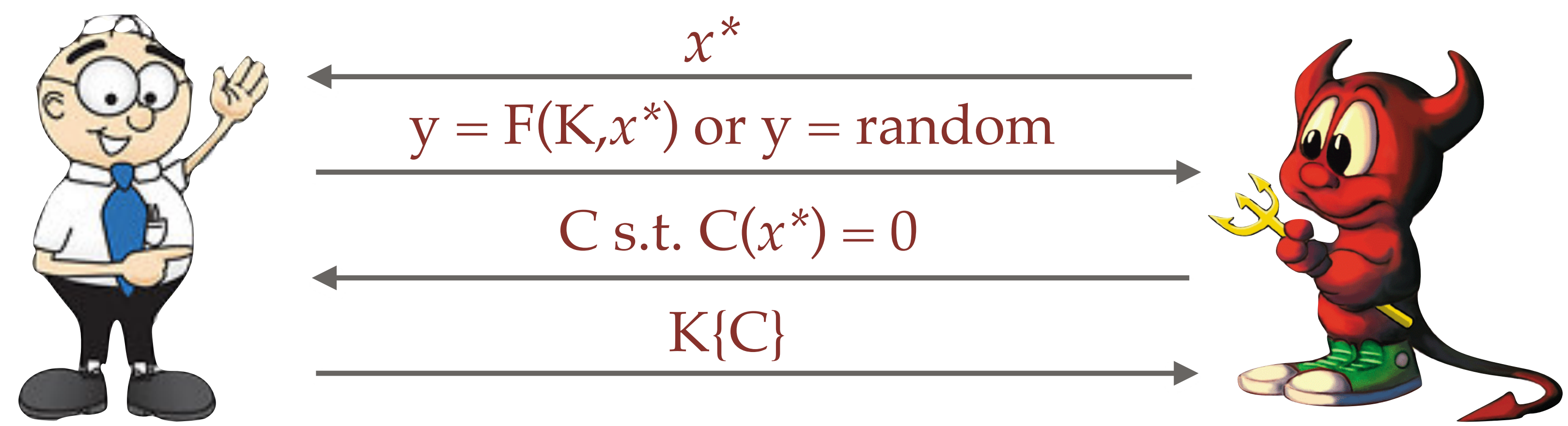

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iO ⇓

x ↓

$K\{x^*\}$

$y^* = F(K,x^*)$

If C(x) = 1

if x = x*

output y*

else output F(K,x)

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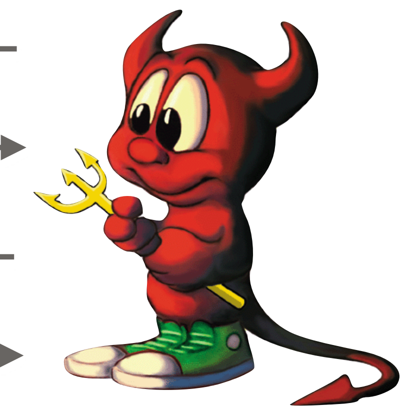
x^* ←

y = F(K,x^*) or y = random →

C s.t. C(x^*) = 0 ←

K{C} →



⇒

x ↓

$K\{x^*\}$

y^*= *random*

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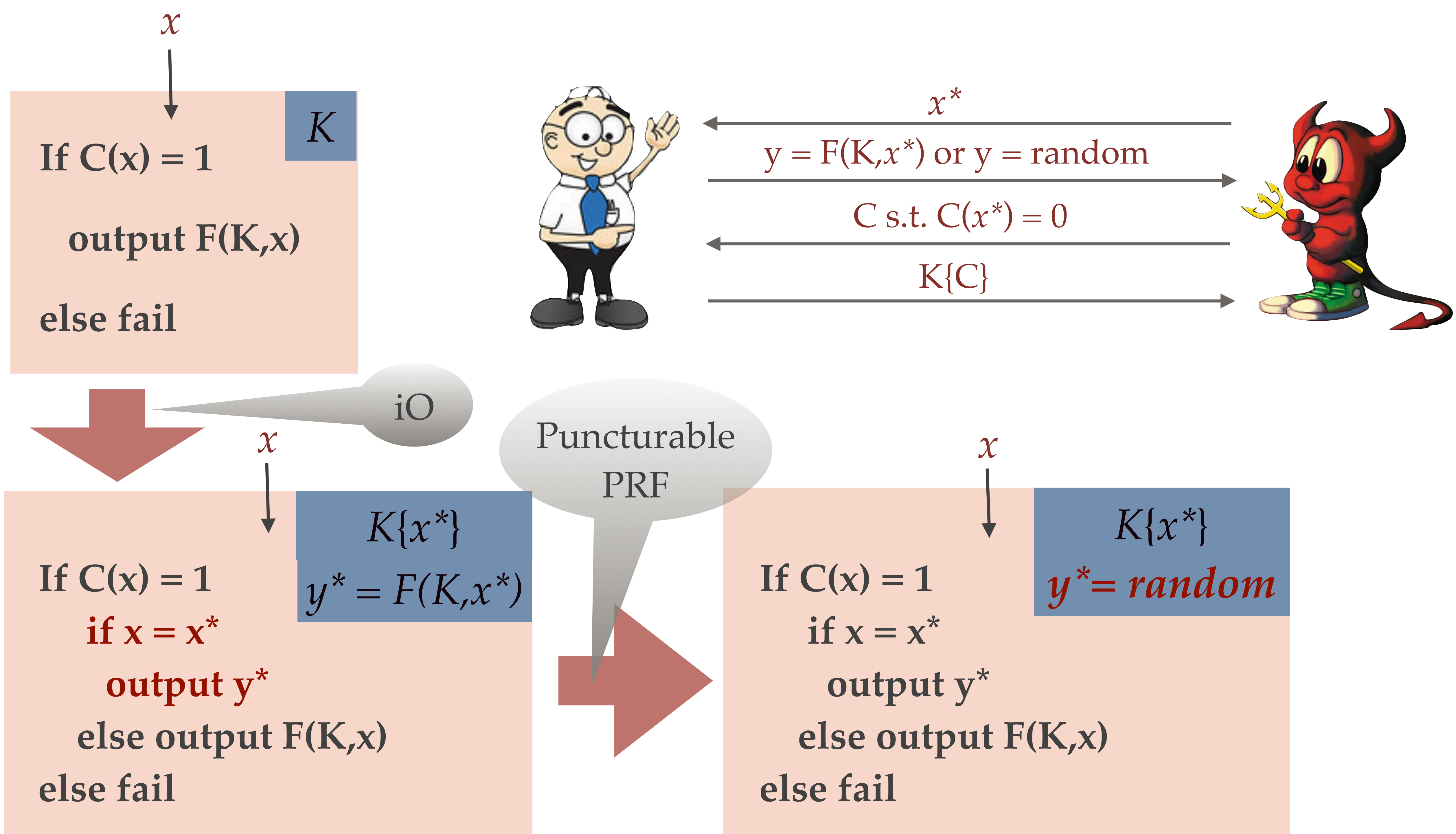

CPRFs for Unbounded Inputs

Motivation

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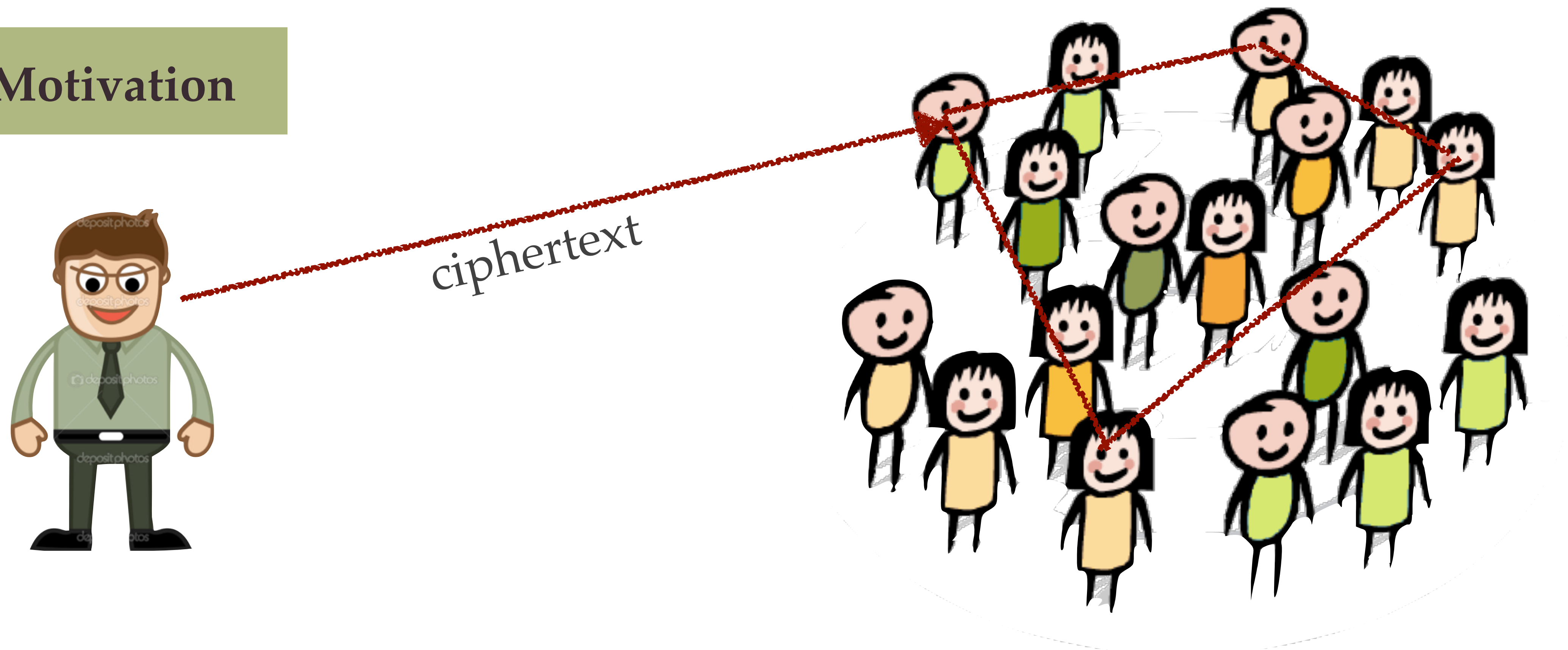

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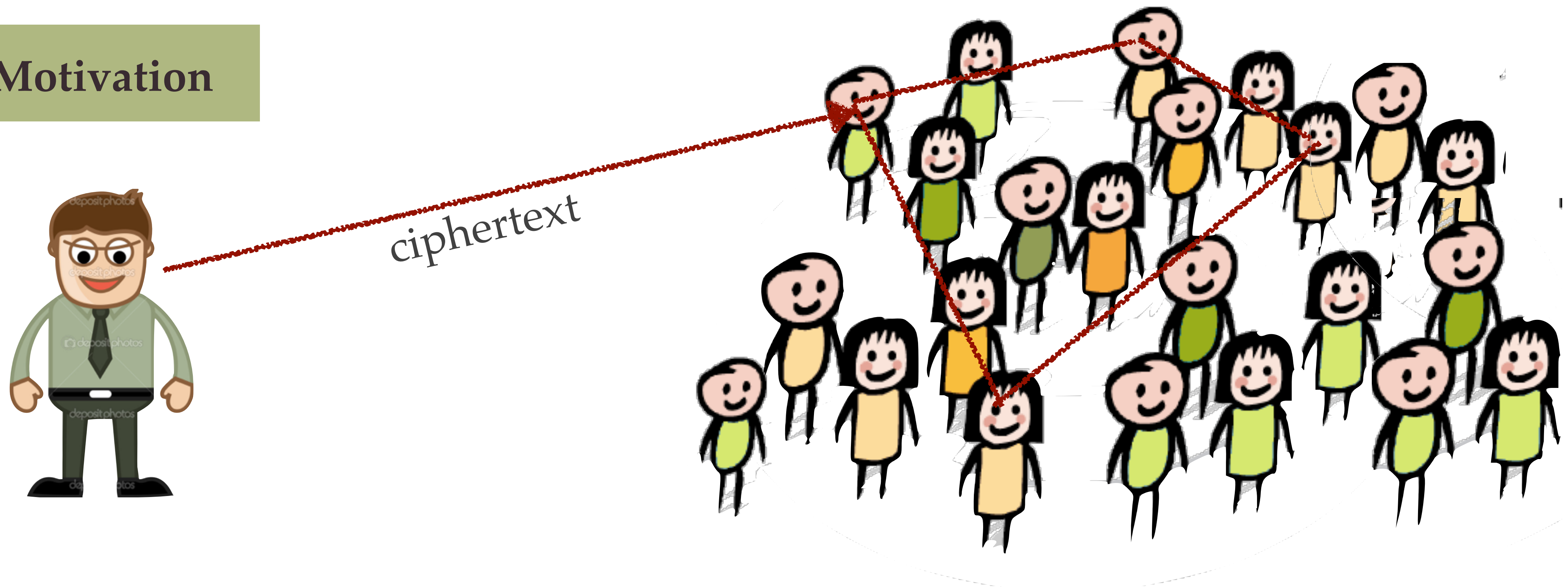
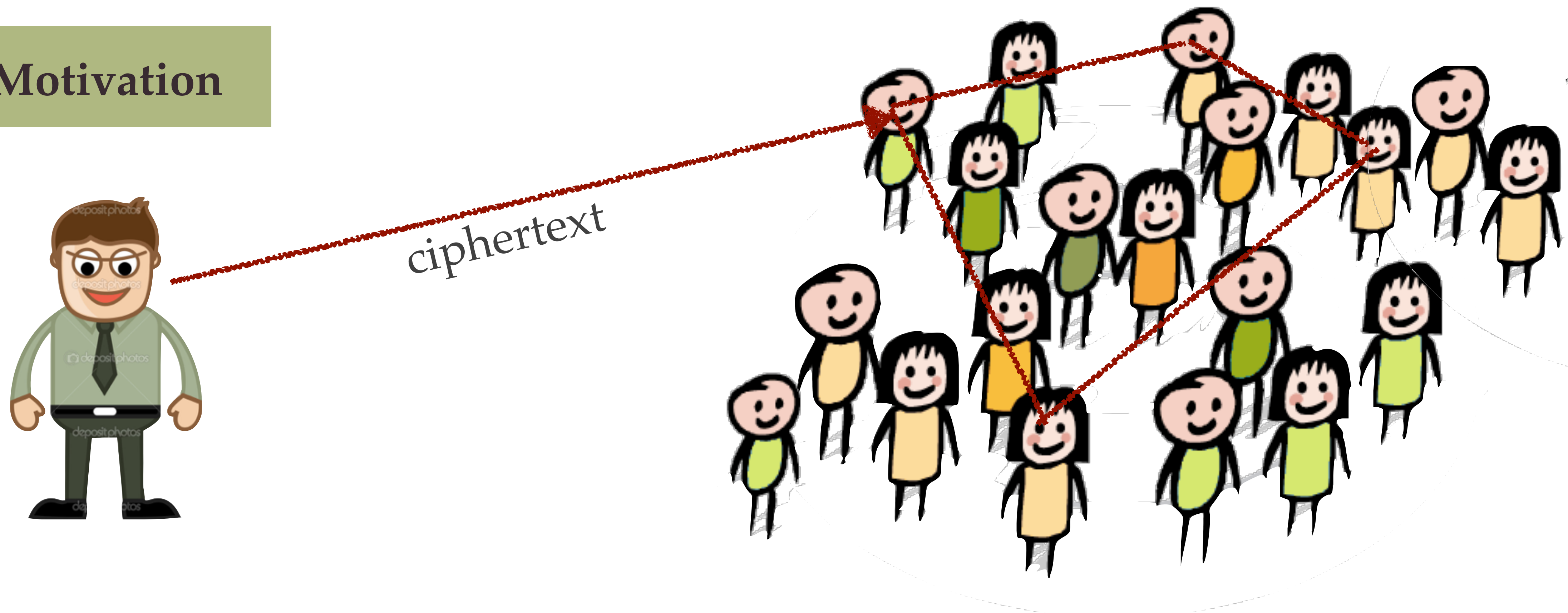


CPRFs for Unbounded Inputs

Motivation



CPRFs for Unbounded Inputs

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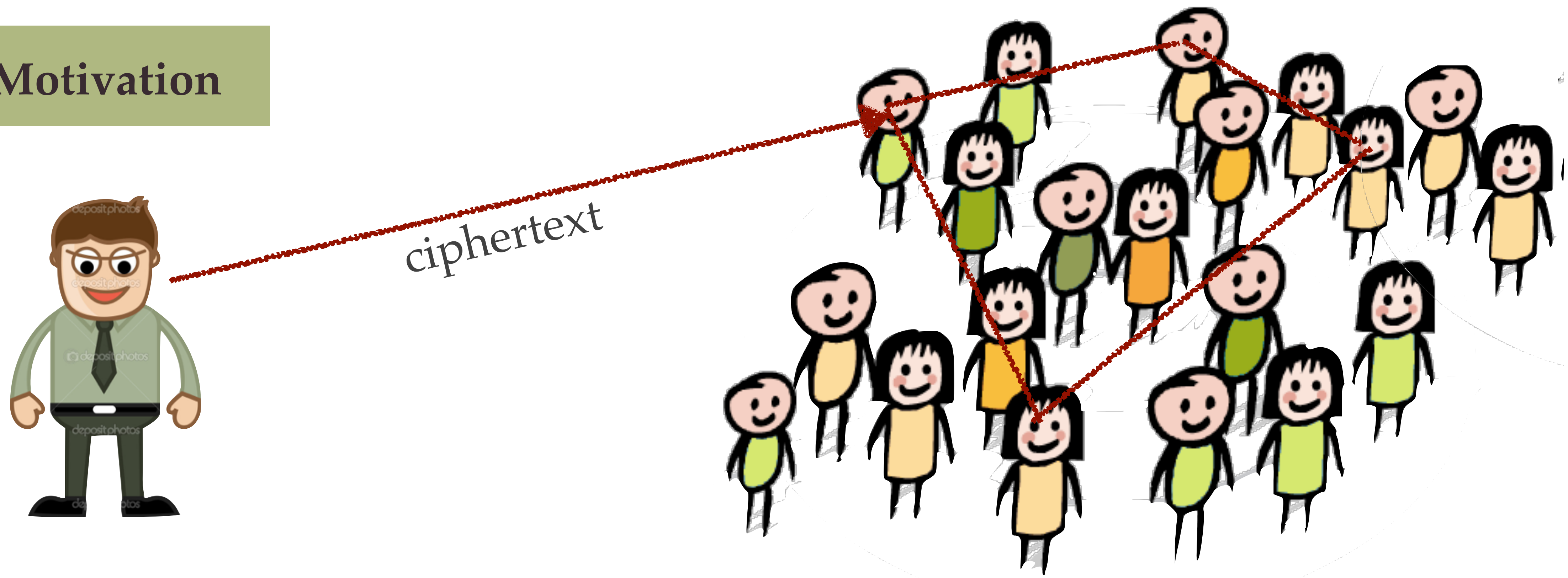

Broadcast encryption for unbounded set of users

CPRFs for Unbounded Inputs

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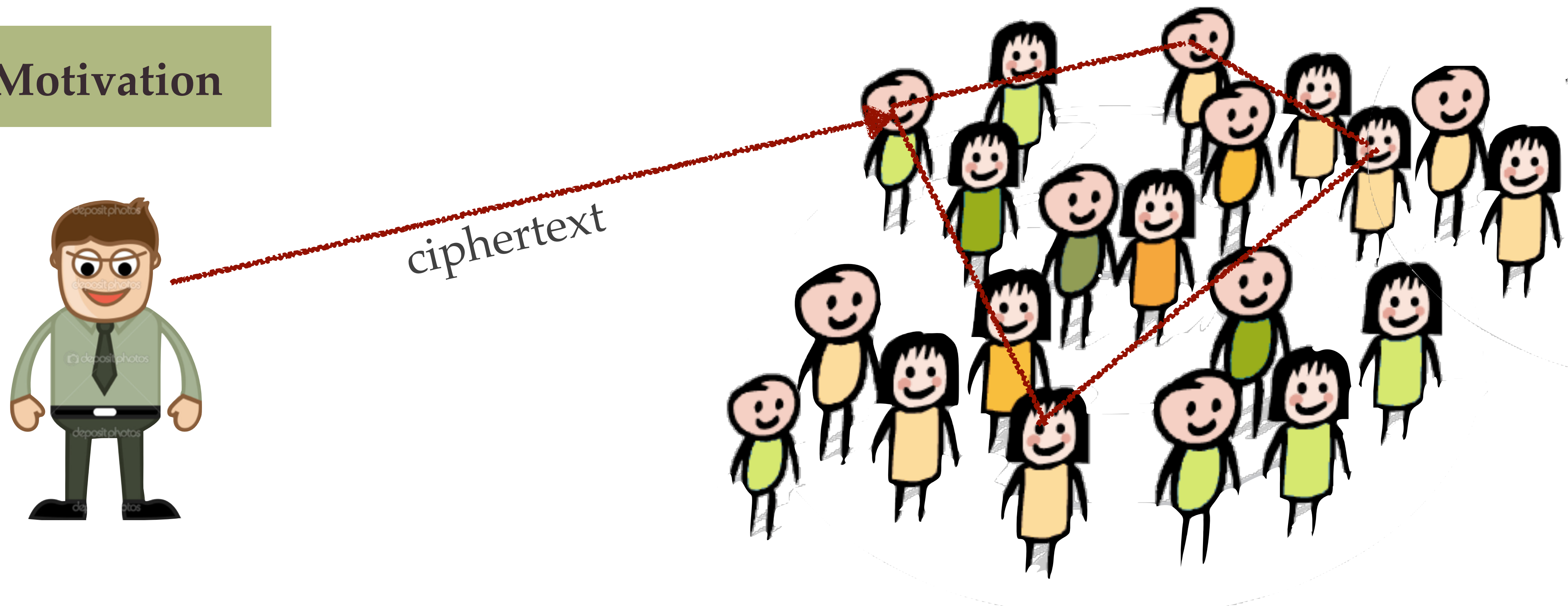


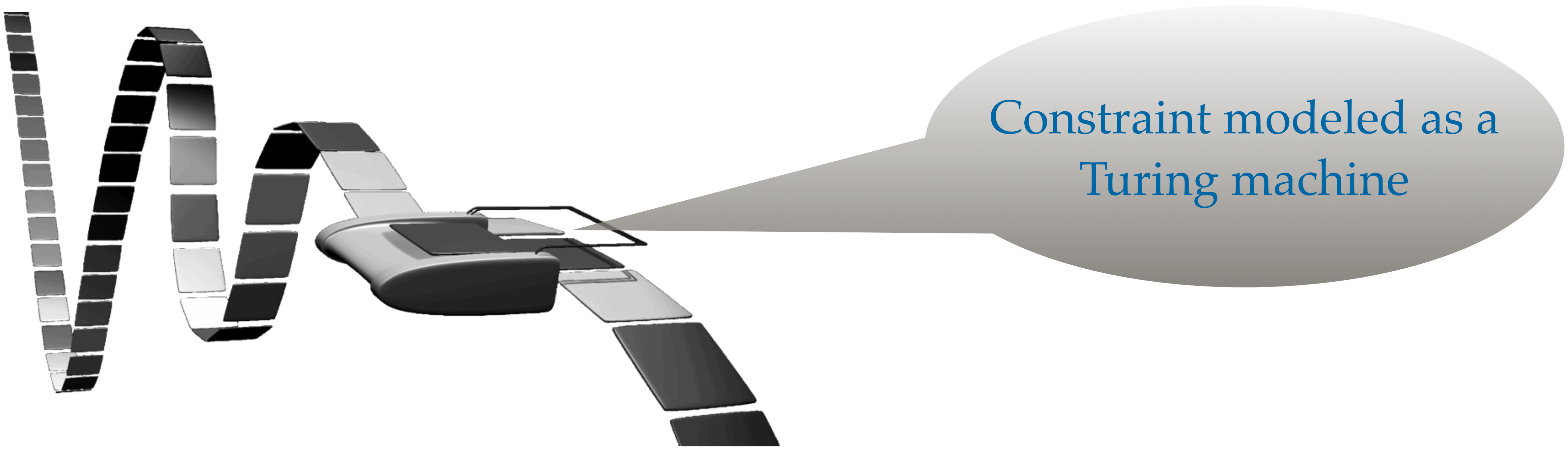
Broadcast encryption for unbounded set of users

- Inputs to the PRF and for the constraint can be of any size, not fixed a priori
- Cannot have constraint as a circuit: Model the constraint as a Turing machine

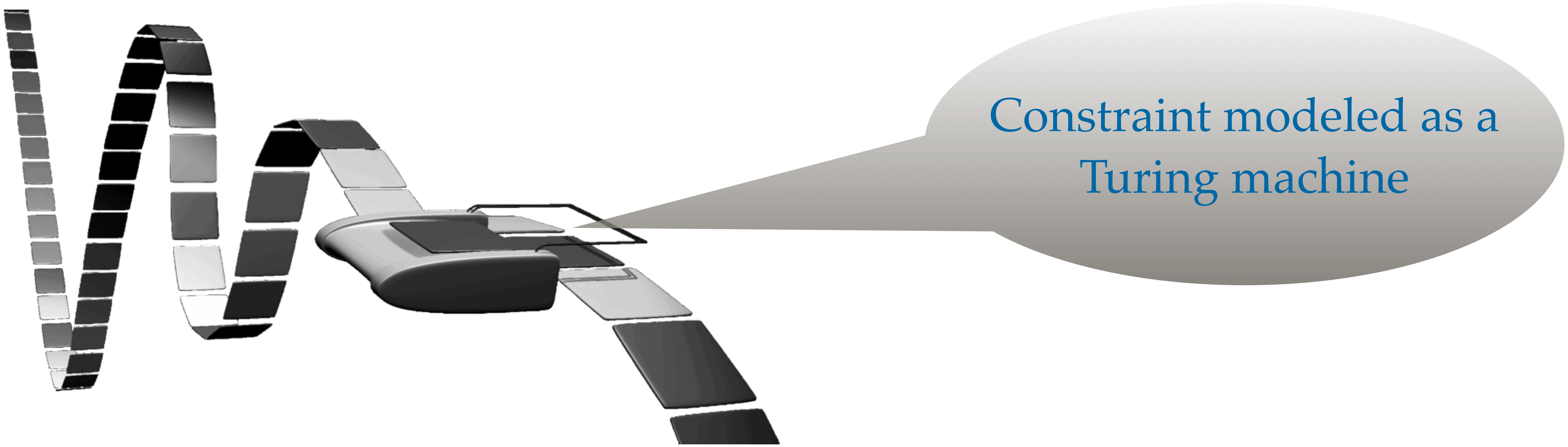
[Zhandry'14]

CPRFs for Unbounded Inputs

CPRFs for Unbounded Inputs



CPRFs for Unbounded Inputs



Results Prior to our Work

- CPRFs for unbounded inputs under the assumption that public-coin differing-inputs obfuscation exists [AFP'14]
- Evidence that differing-inputs obfuscation may not exist [GGHW'14] [BCP'14] [BSW'16]

Our Results

Our Results

Indistinguishability obfuscation for circuits exist

+

Injective Pseudorandom generators exist

Our Results

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+

Injective Pseudorandom generators exist

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Selectively secure constrained PRFs for unbounded inputs exist

Our Results

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Selectively secure constrained PRFs for unbounded inputs exist

Attribute-based Encryption for Turing Machines

CPRF Construction: Intuition

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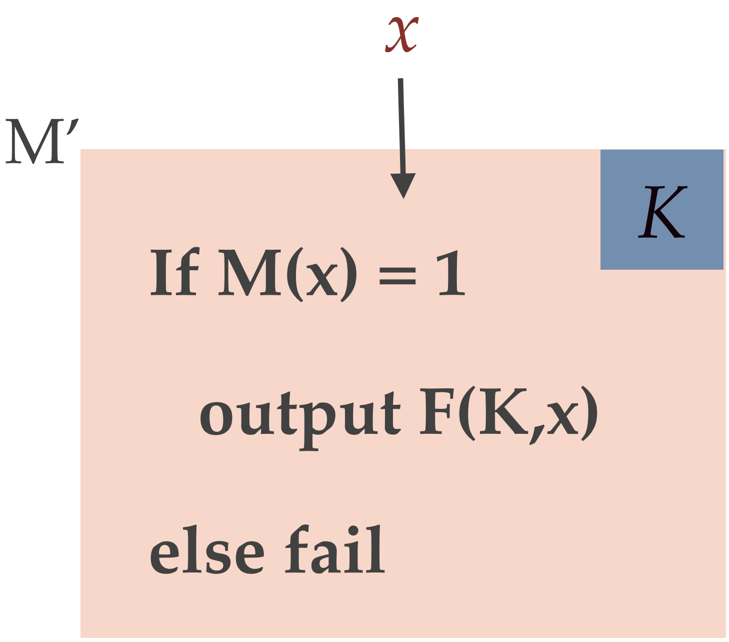

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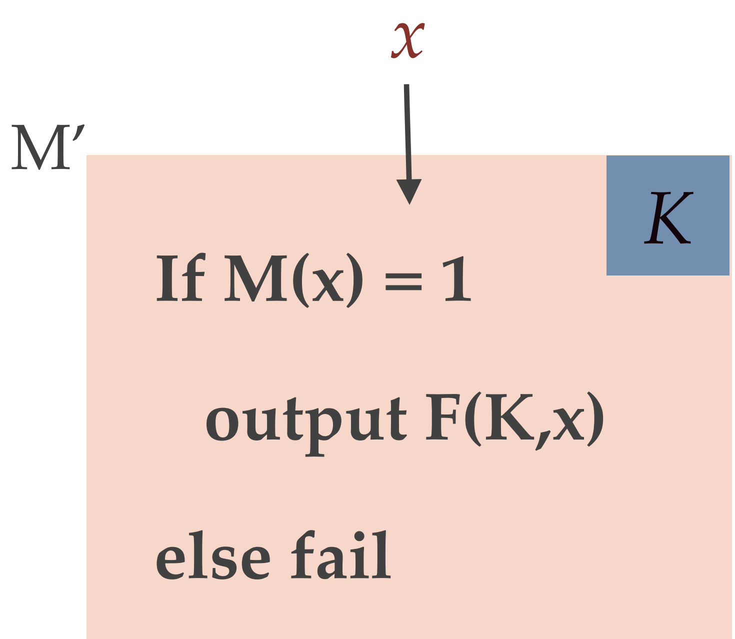
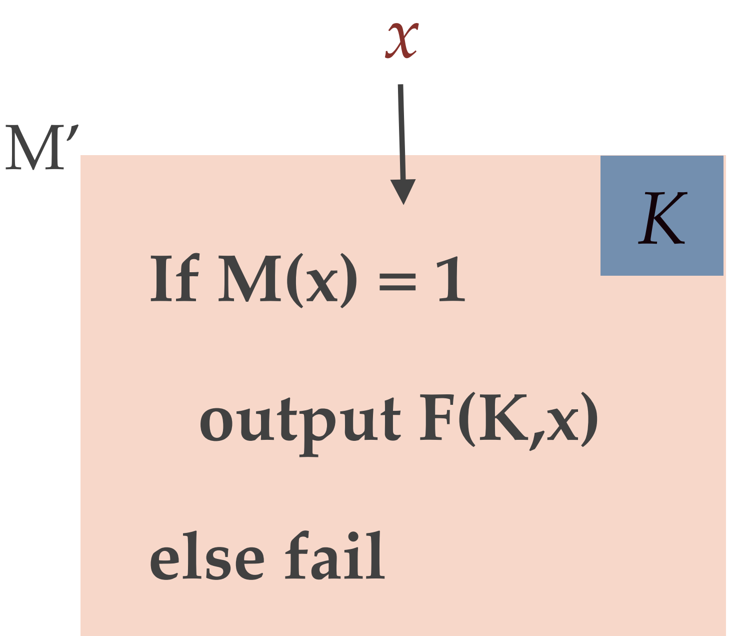


Constraint expressed as Turing machine:
Why not obfuscate this Turing machine ?

CPRF Construction: Intuition



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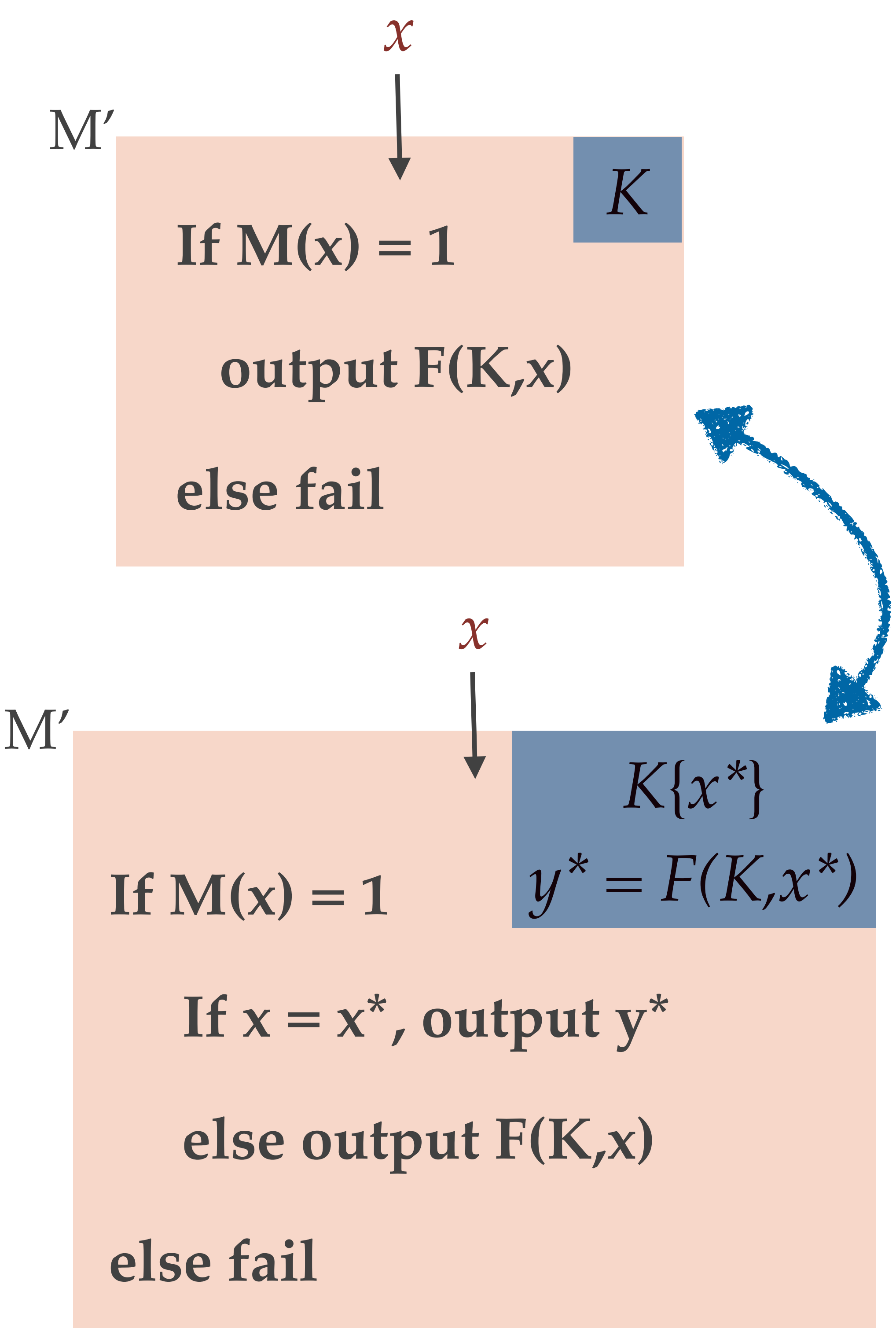
<u>iO for Turing machines:</u> Size of obfuscated TM depends on |M'|, requires a priori bound on |M'| and on input sizes

[KLW'14]

CPRF Construction: Intuition



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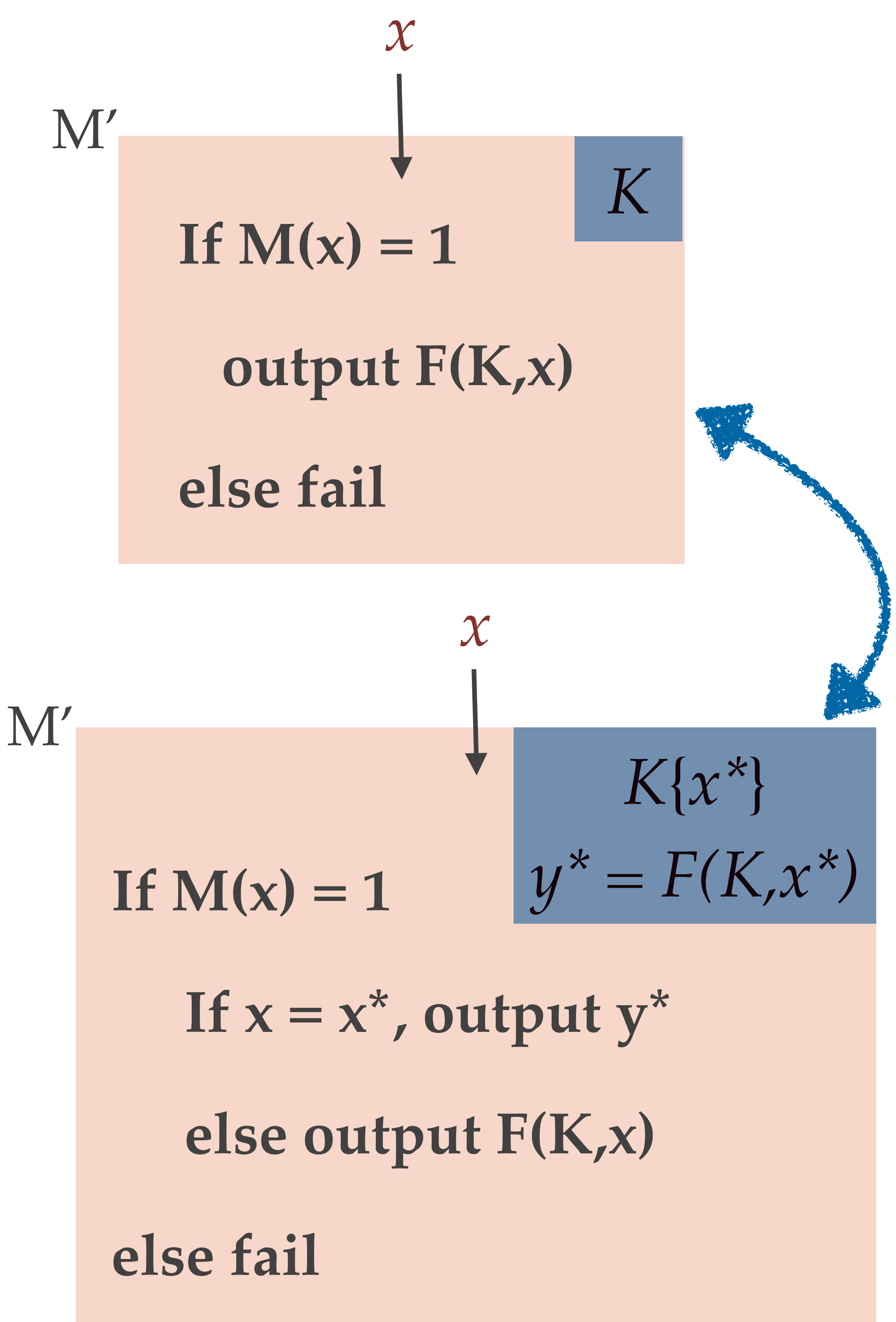

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[KLW'14]

Difficult to make this switch without knowing | x* |

CPRF Construction: Intuition

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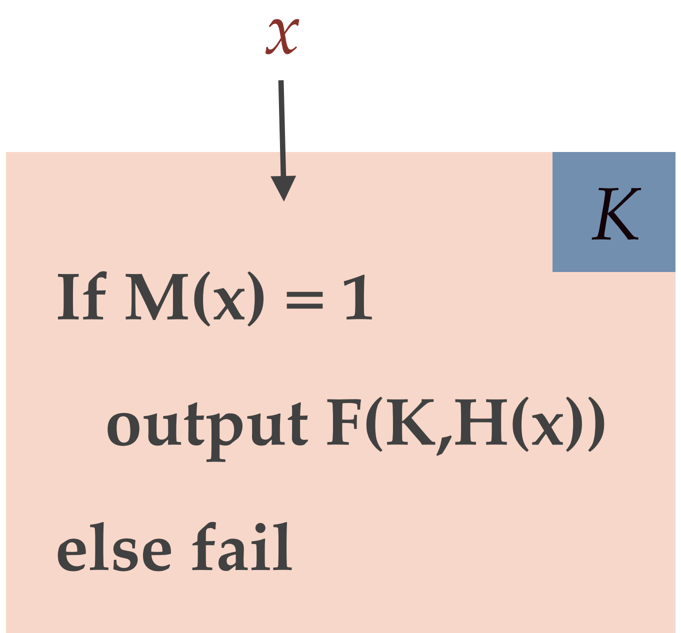

What if we map x to fixed sized strings?

Use F(K, H(x)) instead of F(K,x)?

CPRF Construction: Intuition

x

K

If M(x) = 1

output F(K,H(x))

else fail

What if we map x to fixed sized strings?

Use F(K, H(x)) instead of F(K,x)?

x

$K\{H(x^*) = h^*\}$

$y^* = F(K,h^*)$

If M(x) = 1
if H(x) = h*
output y*
else output F(K,H(x))
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x

$K\{H(x^*) = h^*\}$

$y^* = random$

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CPRF Construction: Intuition

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<u>iO fails:</u> u,v such that H(u) = H(v)
but M(u) = 1 and M(v) = 0

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output for x = u: F(K,h*)

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output for x = u: random

CPRF Construction: Intuition

<u>If we use some hash function:</u>

Crucial that no with input with $H(x) = h^*$ enters this region

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CPRF Construction: Intuition

<u>If we use some hash function:</u>

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Need "iO-friendly" hash-function

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Positional Accumulator

[KLW'14]

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Hash the value on tape

PA(x) = acc

<u>PRF Eval:</u> F(k,acc)

CPRF Construction: Intuition

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Positional Accumulator

[KLW'14]

- Succinct Verifiability
- Succinct Updatability
- Selective Enforcement

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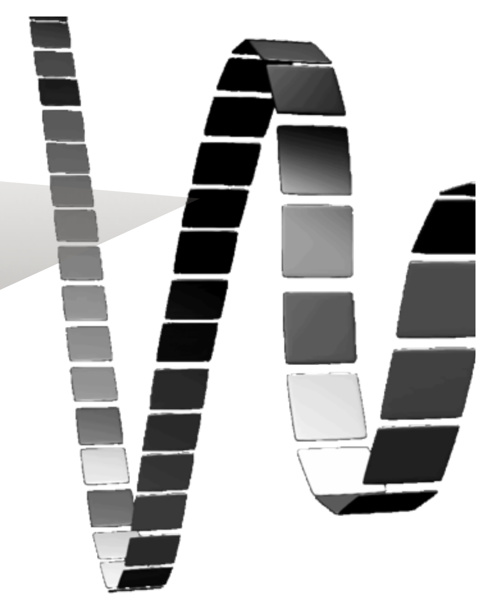

Construction Overview

Construction Overview

PRF Constrained Key

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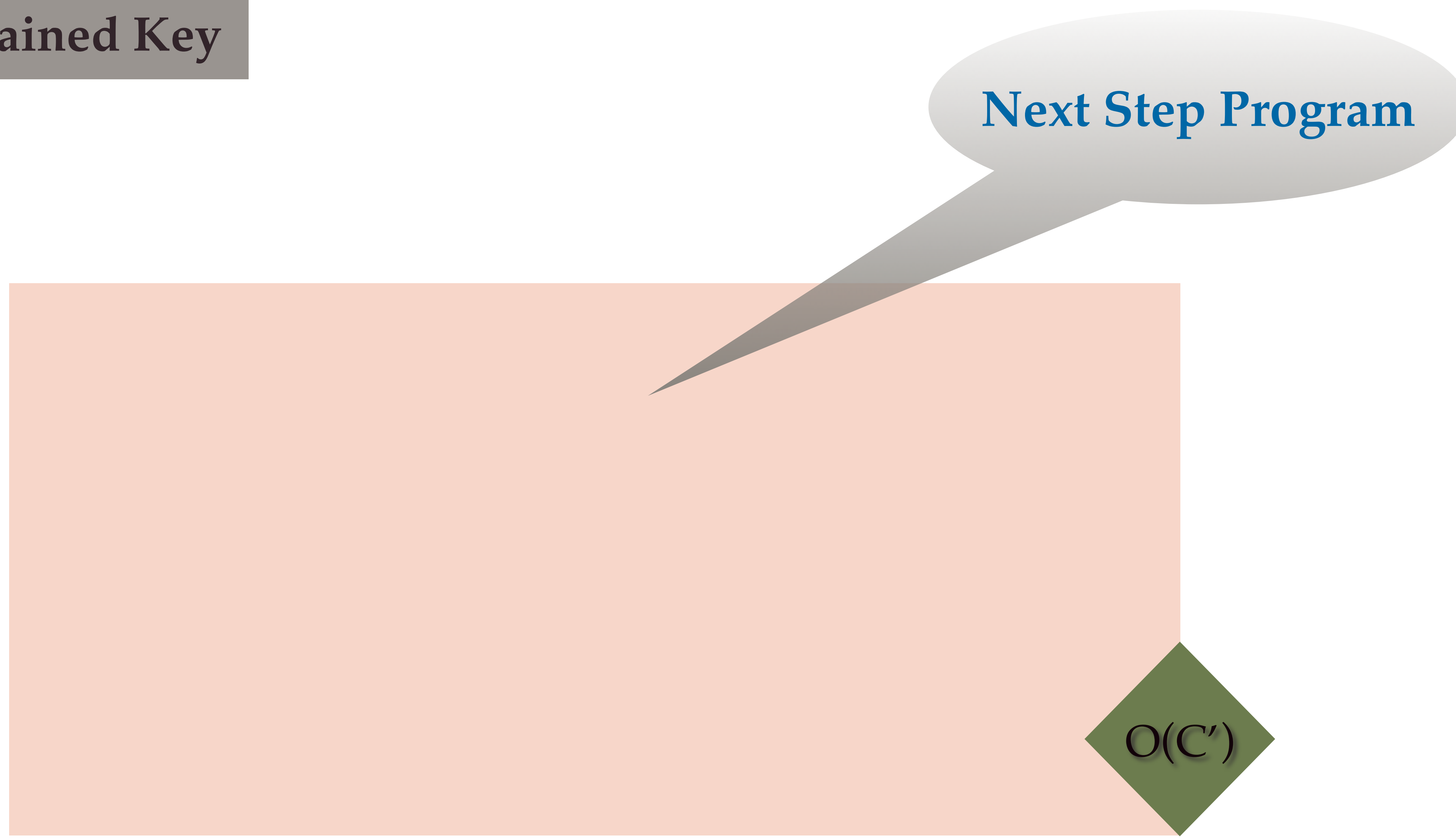

Construction Overview

PRF Constrained Key

Next Step Program

$(i, st_i, sym_i, pos_i, acc)$

k,*M*

- Compute $t(st_i, sym_i) = (st_{i+1}, sym_{i+1}, pos_{i+1})$
- If st_{i+1} is a reject state, output **fail**

  else if st_{i+1} is accept, output **F(*k*,acc)**

  else output **(st_{i+1}, sym_{i+1}, pos_{i+1})**

O(C')

Construction Overview

PRF Constrained Key

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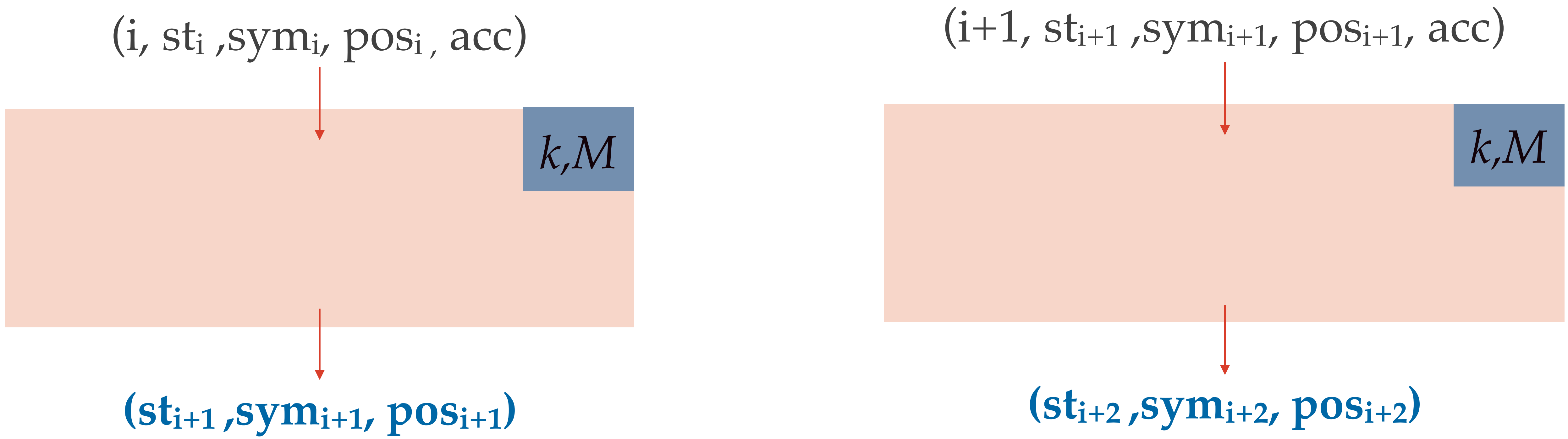
O(C')

Issue: User can input illegal states, symbols to this program

Construction Overview



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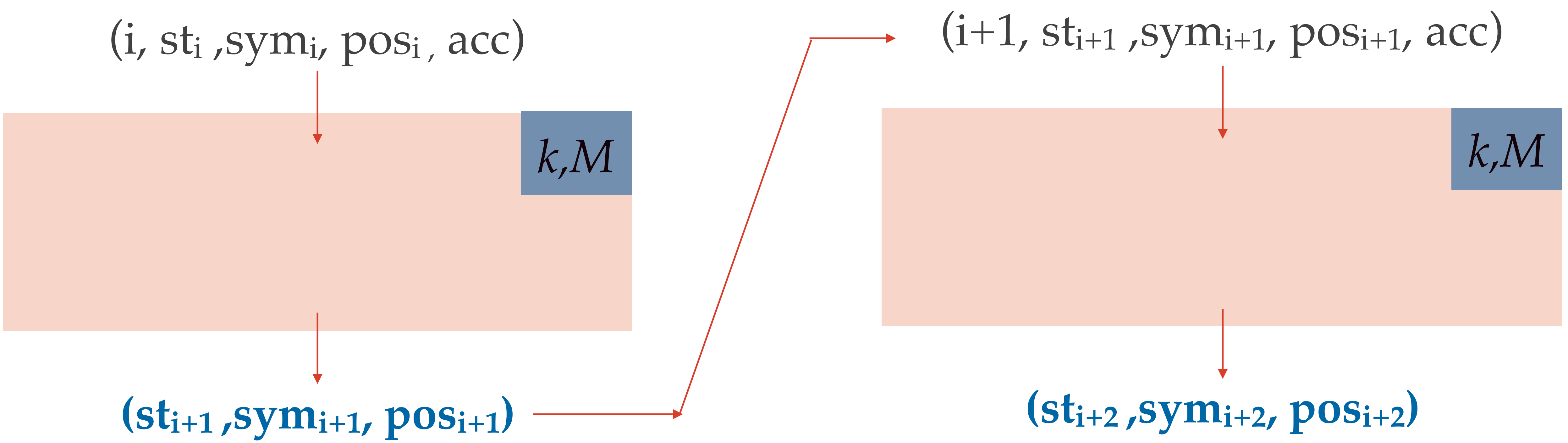

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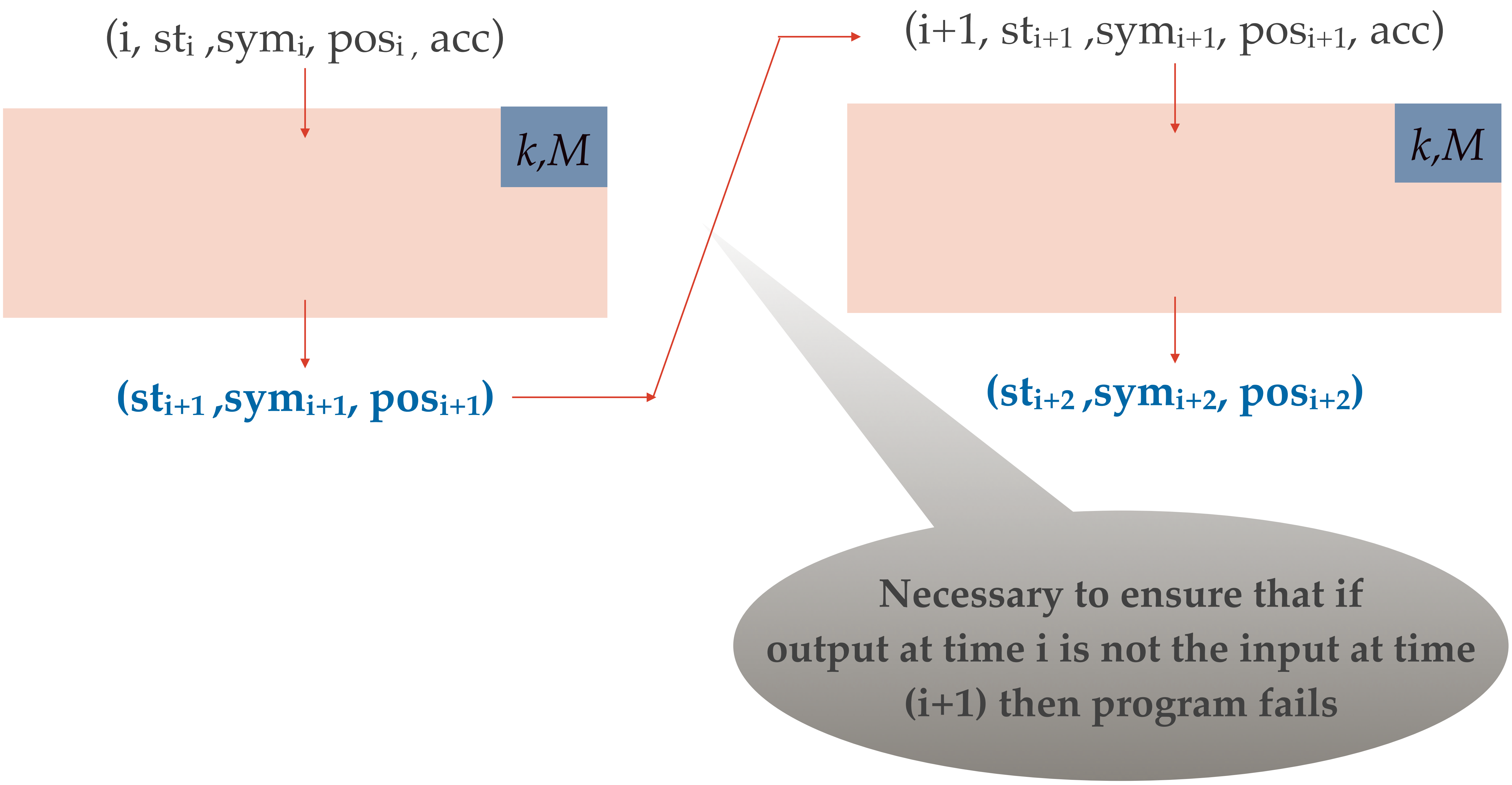

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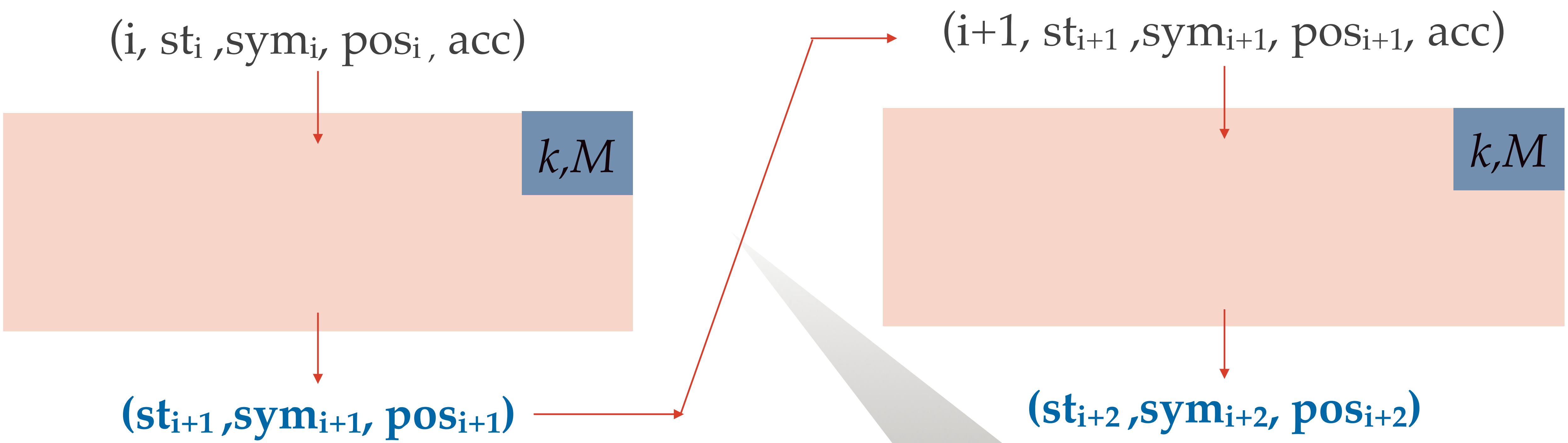


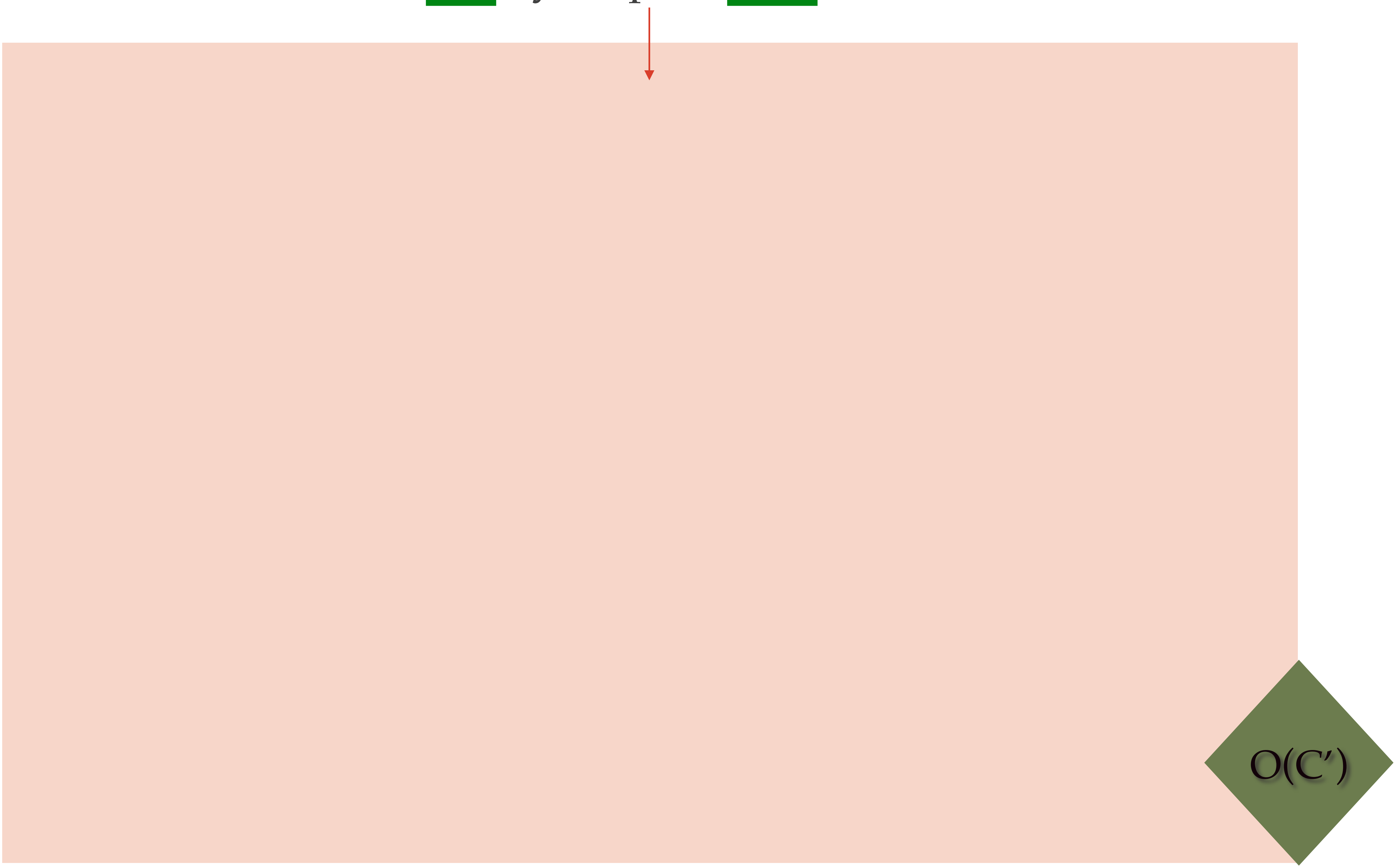
Necessary to ensure that if output at time i is not the input at time (i+1) then program fails

[KLW'14]

- Use ``Splittable'' Signatures !
- Use Accumulator at each step

CPRF Construction: Almost Final

$(t, st_i, \underline{sig}, sym_i, pos_i, \underline{acc_i}, acc)$



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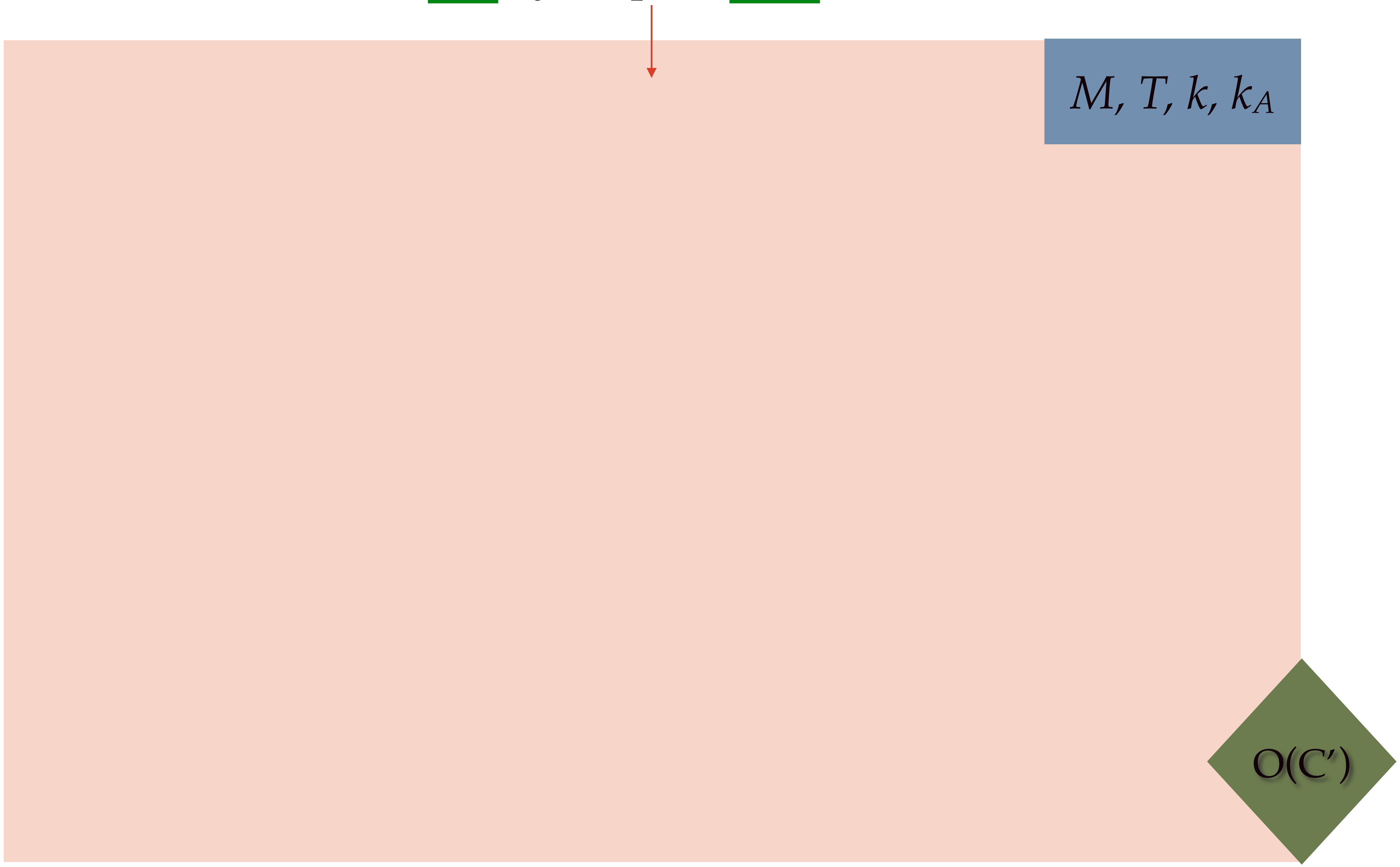

CPRF Construction: Almost Final

(t, st_i , sig, sym_i, pos_i , acc_i, acc)

M, T, k, k_A

- **Verify Current Accumulator Value** acc_i, else **fail**

O(C')

CPRF Construction: Almost Final

$(t, st_i, sig, sym_i, pos_i, acc_i, acc)$

M, T, k, k_A

- **Verify Current Accumulator Value** acc_i, else **fail**
- $r = F(k_A, (acc, t - 1))$; $(SK_A, VK_A) = Setup(1^k; r_A)$
  Verify Signature on (st_i, pos_i, acc_i), else **fail**

O(C')

CPRF Construction: Almost Final

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Next Step

O(C')

CPRF Construction: Almost Final

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  Next Step
- acc_{i+1} = **Update Accumulator**(pos_i , sym_{i+1}, acc_i)

O(C')

CPRF Construction: Almost Final

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  Next Step
- acc_{i+1} = **Update Accumulator**$(pos_i, sym_{i+1}, acc_i)$
- $r' = F(k_A, (acc, t))$; $(SK'_A, VK'_A) = Setup(1^k; r_A')$
  Output **$(st_{i+1}, sym_{i+1}, pos_{i+1}, acc_{i+1})$**
  Output **New Signature on $(st_{i+1}, pos_{i+1}, acc_{i+1})$**

O(C′)

CPRF Construction: Almost Final

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O(C')

Proof Intuition

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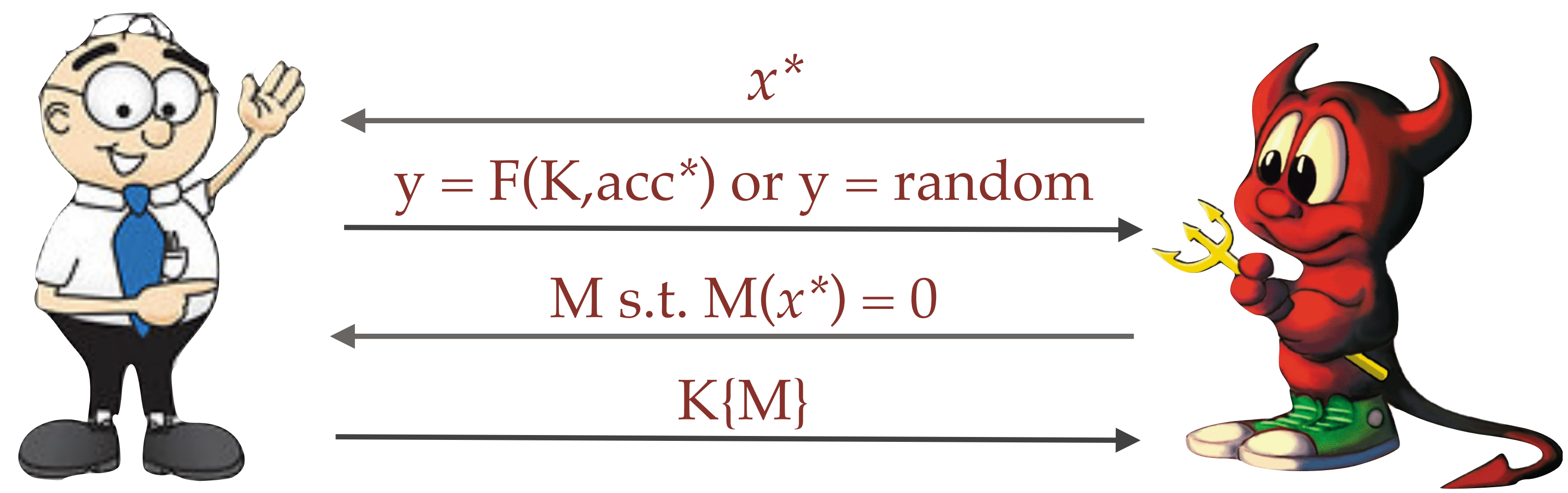
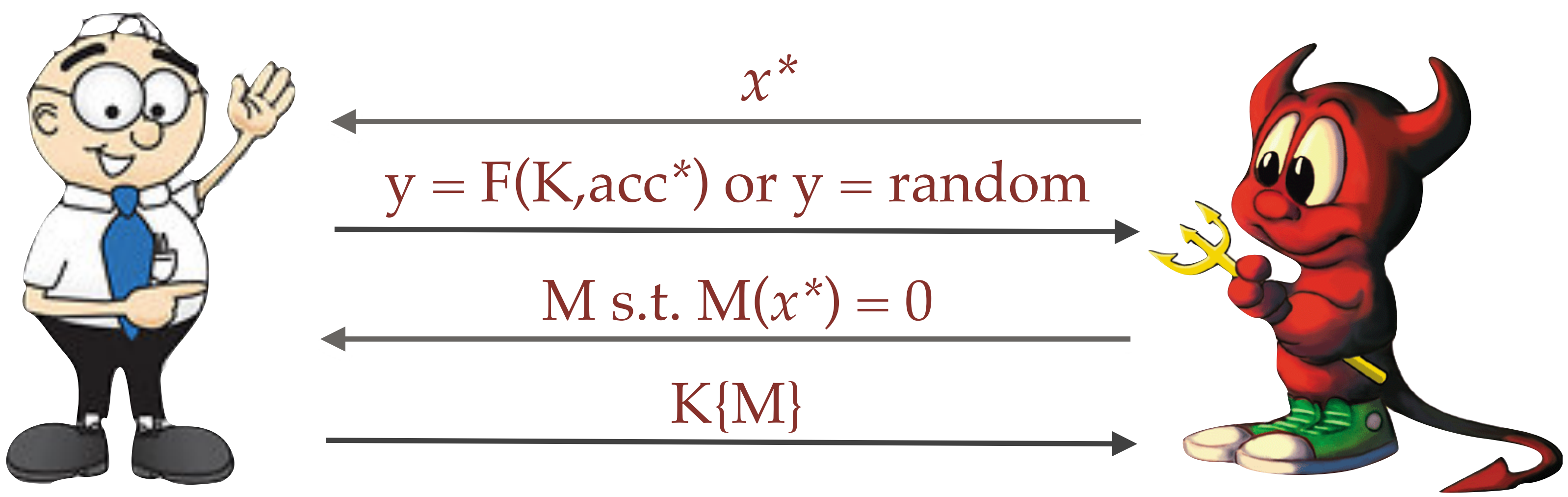


Proof Intuition

x

k, M

If M(x) = 1
output F(k,acc)
else fail



Proof Intuition

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x

k{acc*}, M
$y^* = F(K, acc^*)$

If M(x) = 1
  if acc = acc*
    output y*
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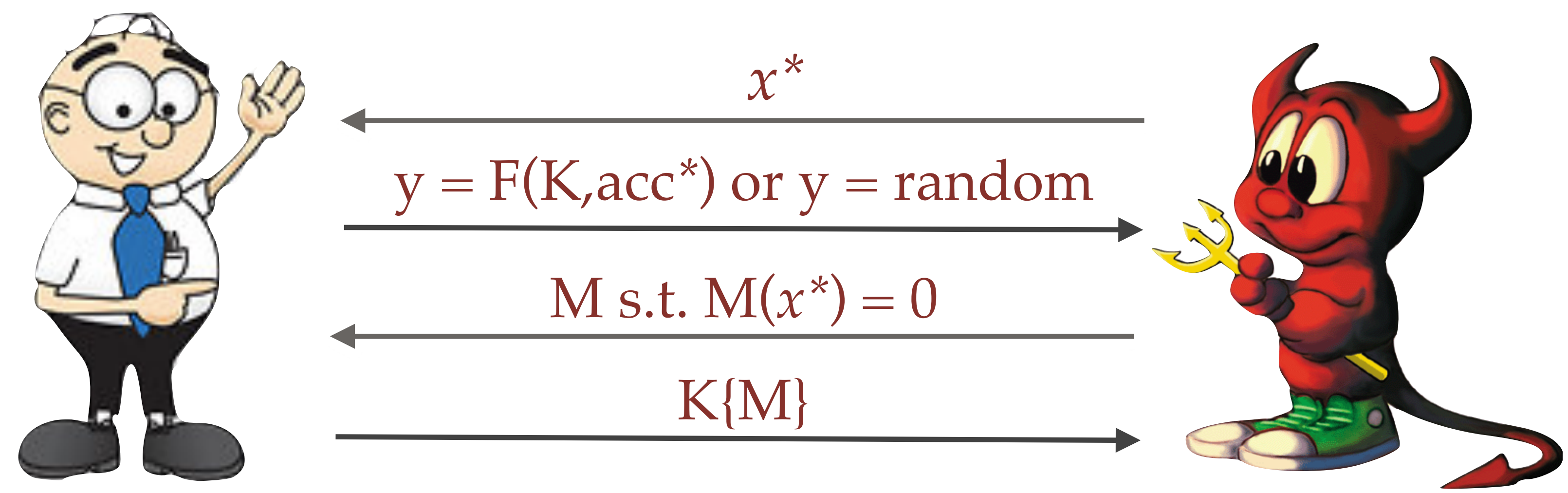
else fail



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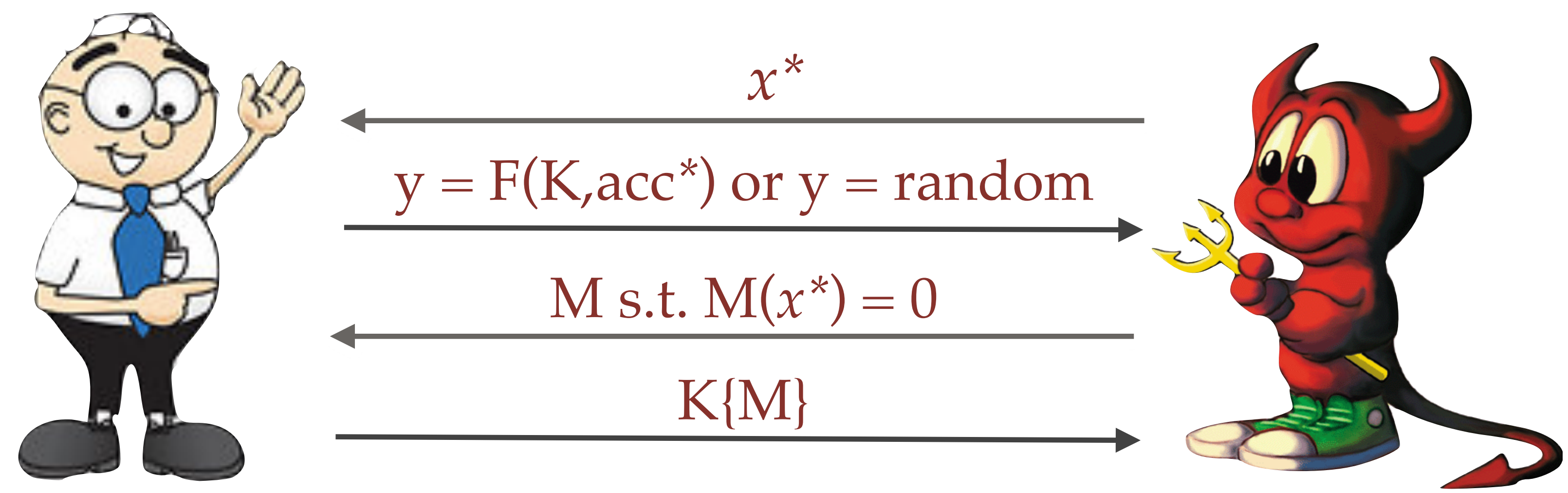


x

k\{acc*\}, M
$y^* = F(K,\text{acc}^*)$

If M(x) = 1
  if acc = acc*
    output y*
  else output F(k,acc)
else fail

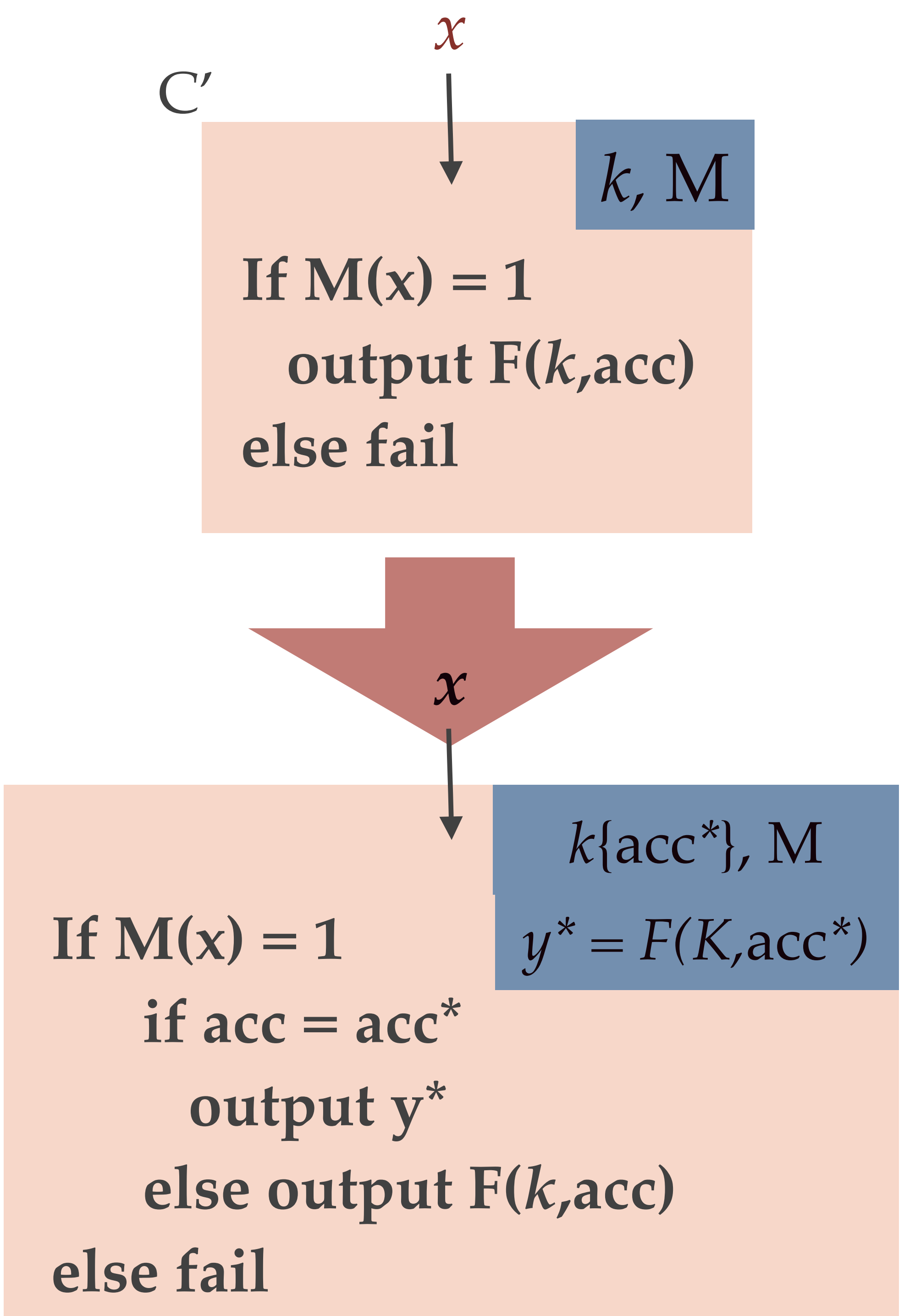
x

k\{acc*\}, M
y*= random

If M(x) = 1
  if x = x*
    output y*
  else output F(k,acc)
else fail

Proof Intuition

Proof Intuition



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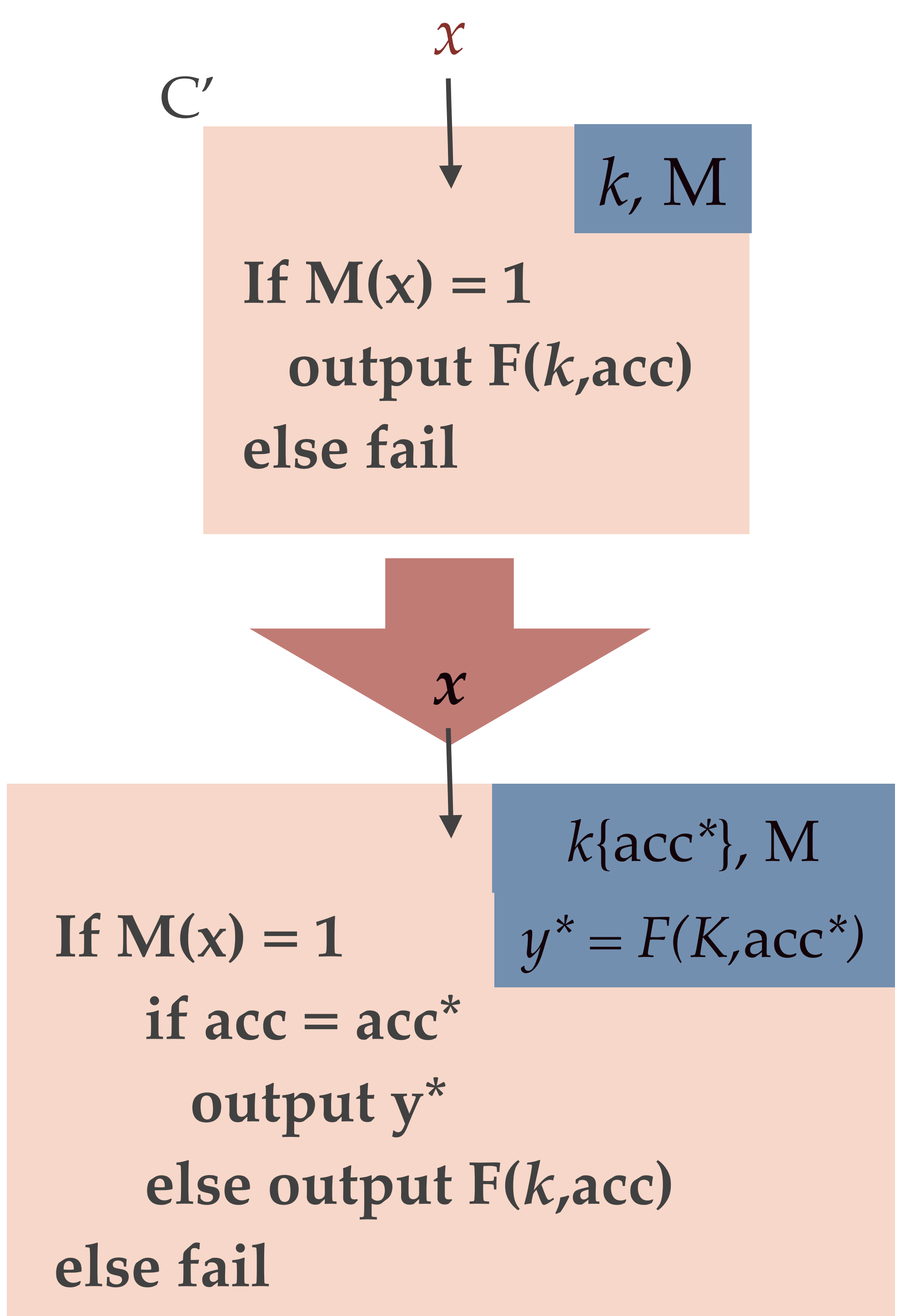
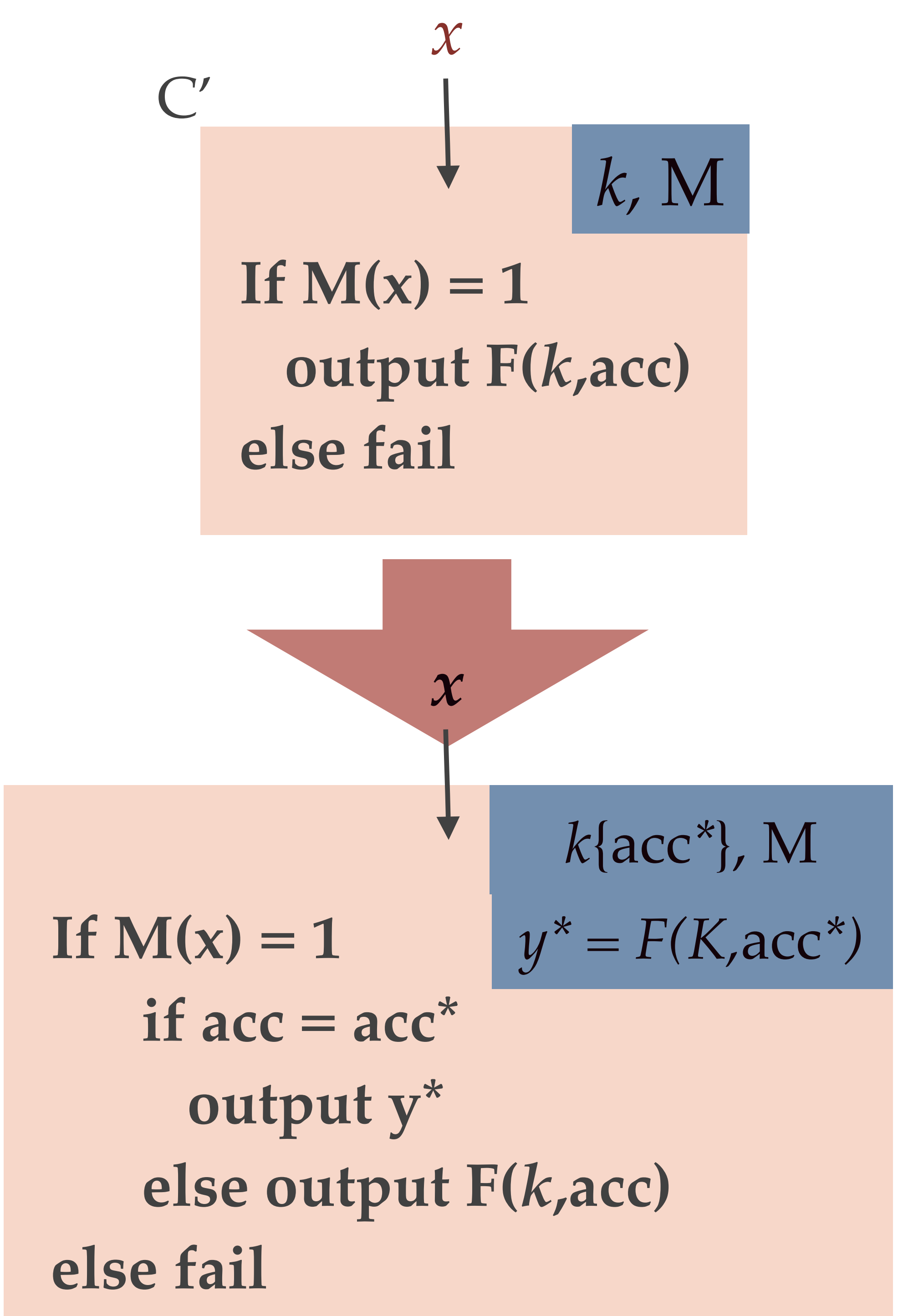


Ensure that O(C′) does not reach accept state for any input with acc = acc* :

Proof Intuition

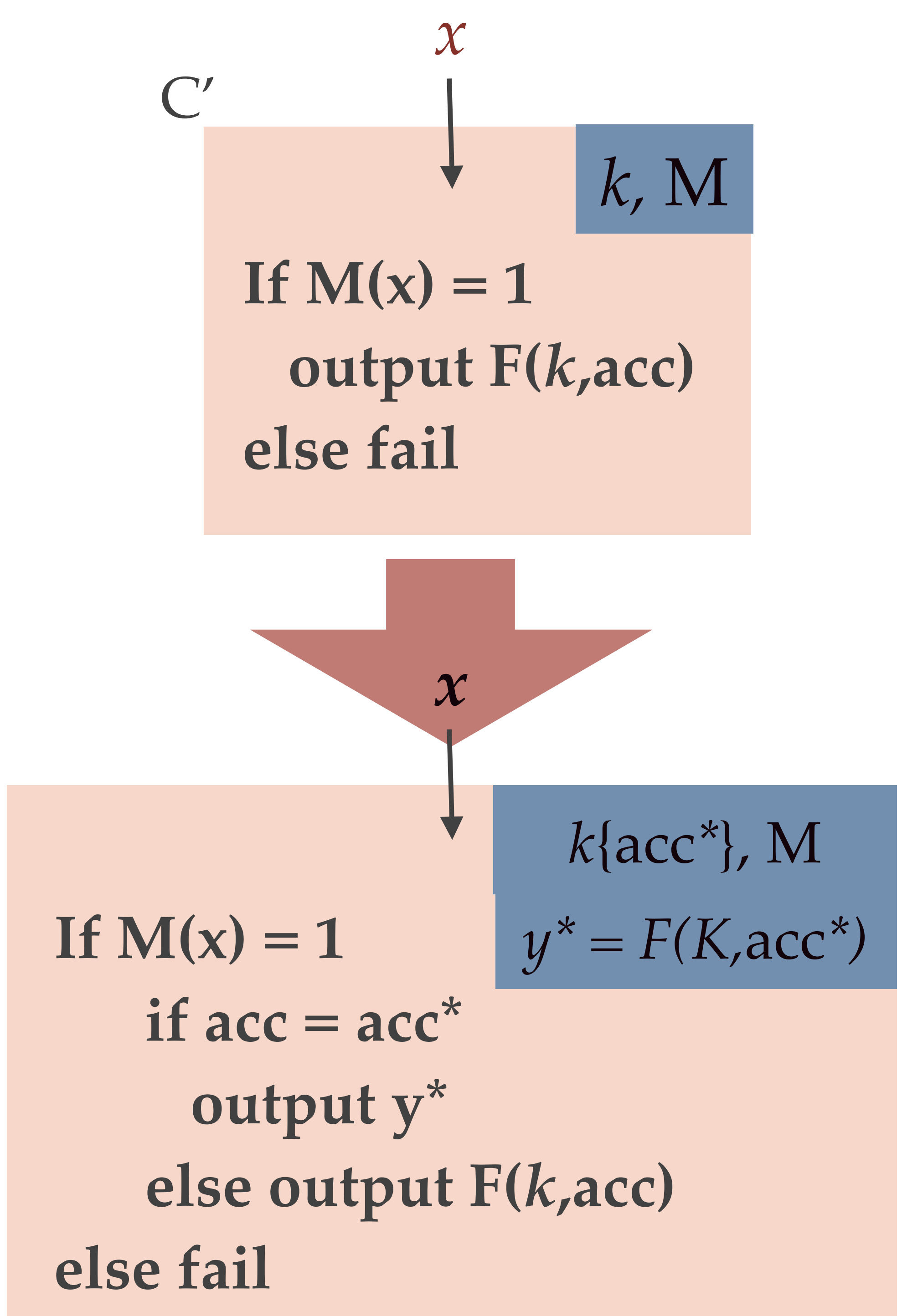


Ensure that O(C′) does not reach accept state for any input with acc = acc* :

Steps taken by M on x*

1. C′ does not reach accept state for any input with acc = acc* within first t* steps

Proof Intuition



Ensure that O(C′) does not reach accept state for any input with acc = acc* :

Steps taken by M on x*

1. C′ does not reach accept state for any input with acc = acc* within first t* steps

2. C′ does not reach accept state for any input with acc = acc* for t > t* steps

Proof Intuition

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1. C' does not reach accept state for any input with acc = acc* within first t* steps

2. C' does not reach accept state for any input with acc = acc* for $t > t^*$ steps

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1. C' does not reach accept state for any input with $acc = acc^*$ within first t^* steps

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$O(t^*)$ hybrids using properties of:

- positional accumulator
- splittable signatures

Proof Intuition

1. C′ does not reach accept state for any input with acc = acc* within first t* steps

2. C′ does not reach accept state for any input with acc = acc* for t > t* steps

Cannot use hybrids for each step i > t

Exponential hybrids

Proof Intuition

1. C′ does not reach accept state for any input with acc = acc* within first t* steps

2. C′ does not reach accept state for any input with acc = acc* for $t > t^*$ steps

Novel technique using PRG

Cannot use hybrids for each step $i > t$

Exponential hybrids

Final Construction

$(t, st_i, sig, sym_i, pos_i, acc_i, \underline{seed}, proof, acc_n)$

Final Construction

(t, st_i ,sig, sym_i, pos_i , acc_i, seed, proof, acc_n)

M, T, PP, k, k_1, k_2, ……, k_l

Final Construction

(t, st_i ,sig, sym_i, pos_i , acc_i, seed, proof, acc_n)

M, T, PP, k, k_1, k_2, ……, k_l

- **Verify PRG(seed) =** PRG(s_m) for $2^m <= t < 2^{m+1}$, else **fail**

m^{th} interval

Final Construction

$(t, st_i, sig, sym_i, pos_i, acc_i, \underline{seed}, proof, acc_n)$

M, T, PP, k, k₁, k₂, ……, kₗ

- **Verify PRG(seed) =** PRG(s_m) for $2^m <= t < 2^{m+1}$, else **fail**

m^{th} interval

m^{th} landmark

- If $t+1 = 2^{m+1}$, then **new seed** = s_{m+1} ; else **new seed** = `` ''

Final Construction

(t, st_i ,sig, sym_i, pos_i , acc_i, seed, proof, acc_n)

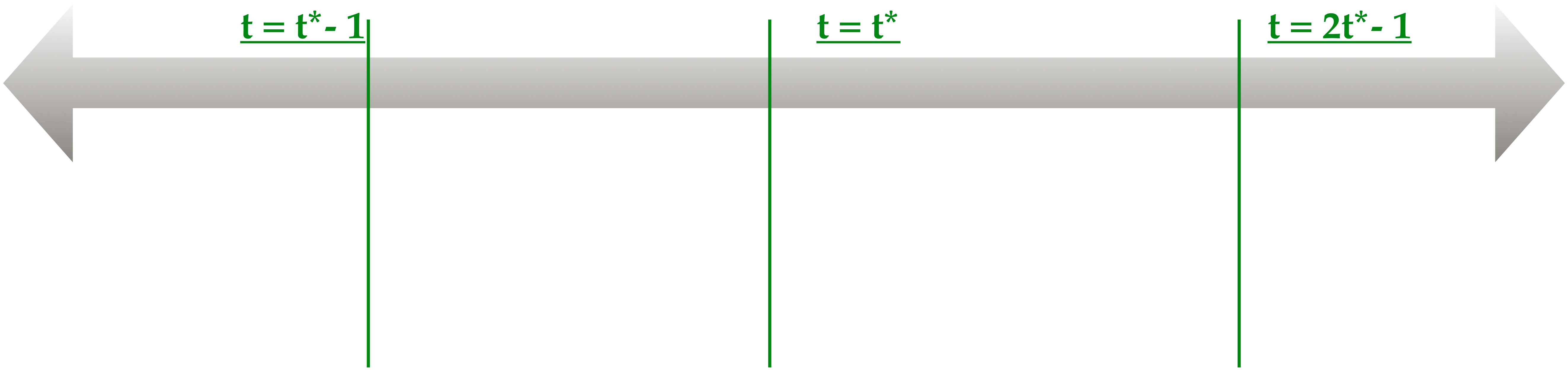
M, T, PP, k, k₁, k₂, ……, k_l

- **Verify PRG(seed) =** PRG(s_m) for $2^m <= t < 2^{m+1}$, else **fail**
- **Verify Current Accumulator Value** acc_i, else **fail**
- **Verify Signature on** (st_i, pos_i , acc_i), else **fail**
- Compute next step
- acc_{i+1} = **Update Accumulator**()
- Output **(st_{i+1} ,sym_{i+1}, pos_{i+1} ,acc_{i+1})**
  Output **New Signature on (st_{i+1} , pos_{i+1} , acc_{i+1})**
- If $t+1 = 2^{m+1}$, then **new seed** = s_{m+1} ; else **new seed** = `` ''

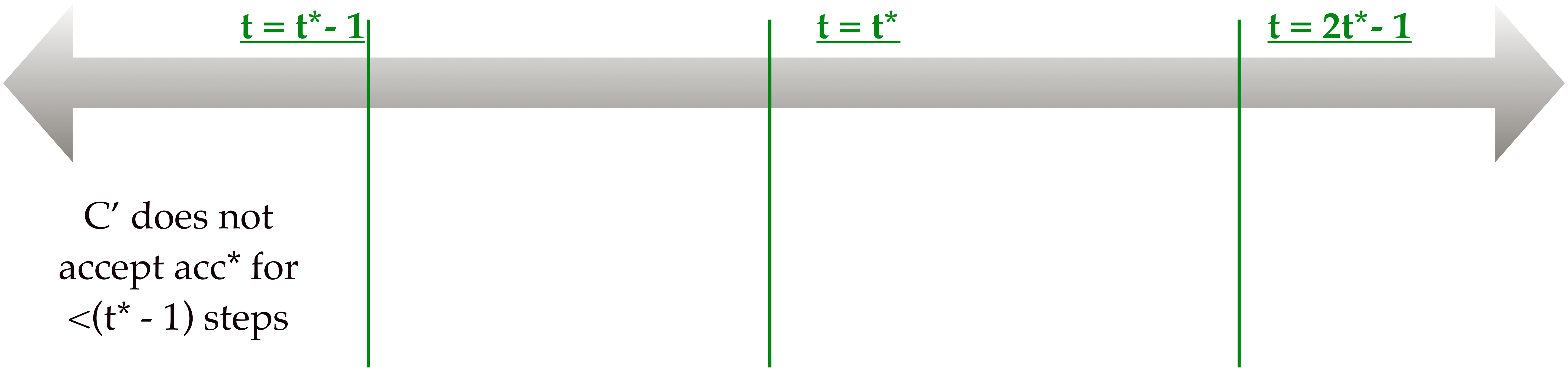
O(C')

Tail Hybrids

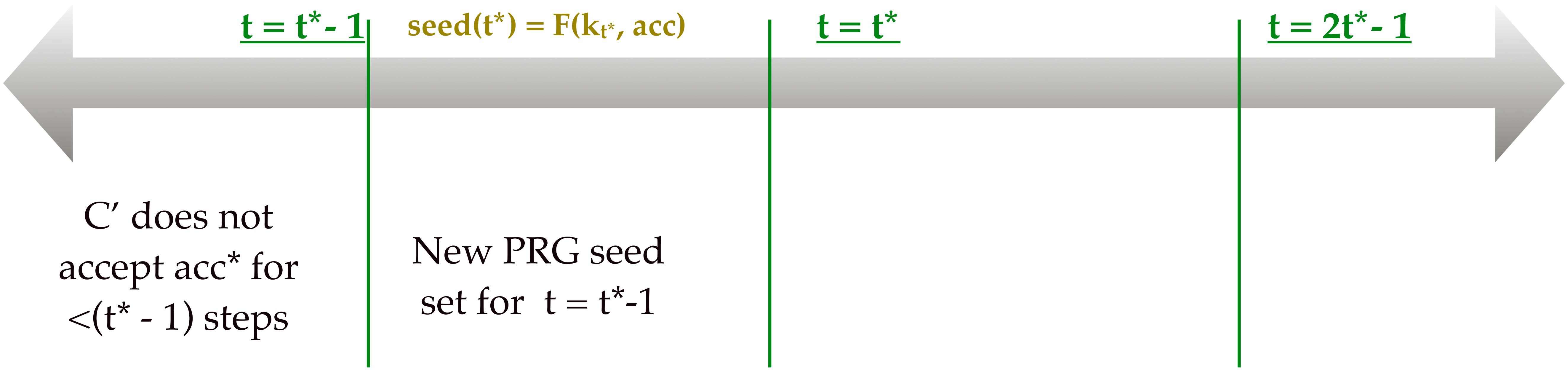
Tail Hybrids



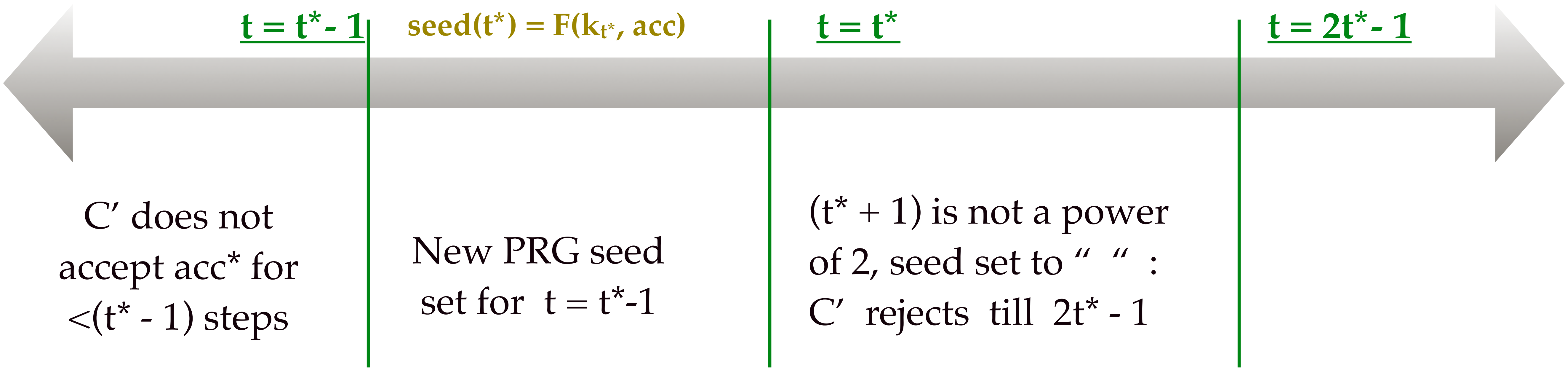
Tail Hybrids



Tail Hybrids



Tail Hybrids



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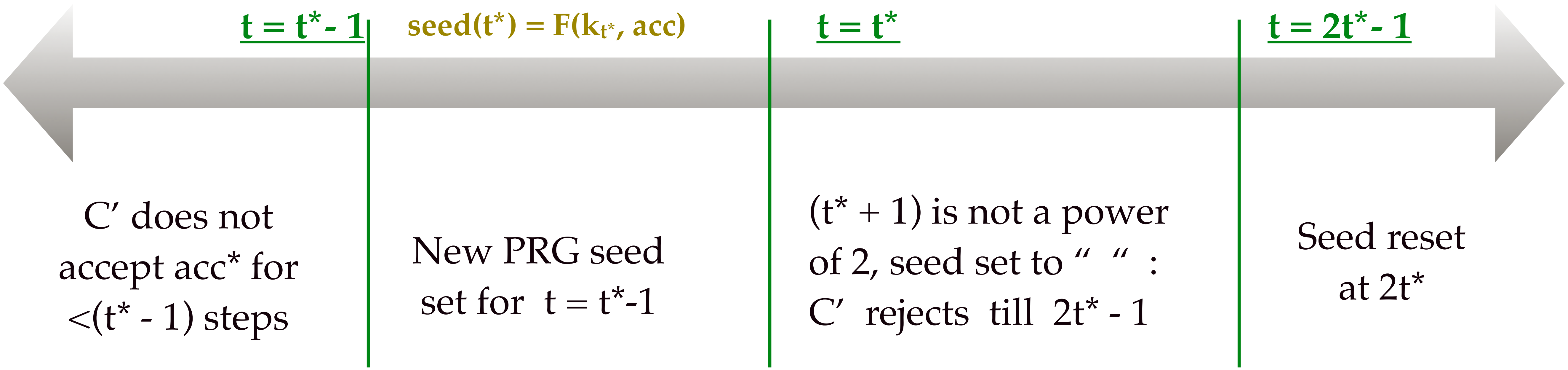
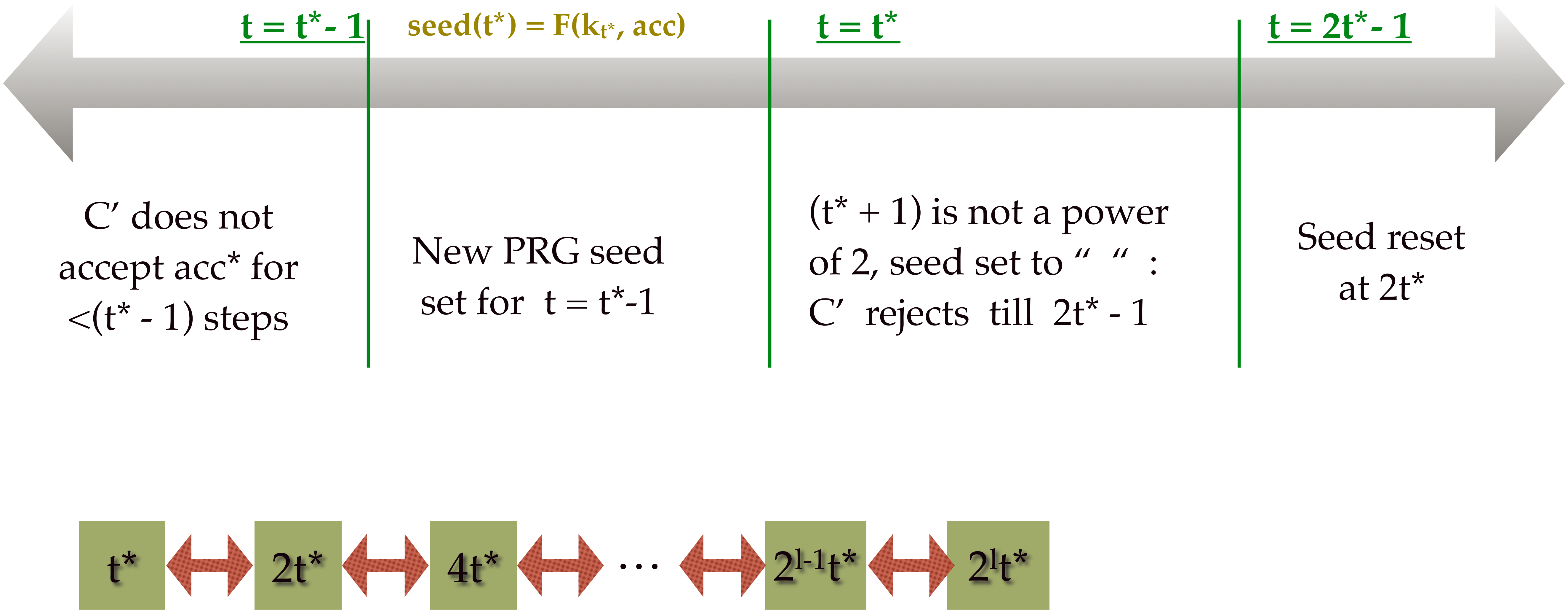


Tail Hybrids



Tail Hybrids

$t = t^* - 1$ | $seed(t^*) = F(k_{t^*}, acc)$ | $t = t^*$ | $t = 2t^* - 1$

C′ does not accept acc* for <(t* - 1) steps

New PRG seed set for t = t*-1

(t* + 1) is not a power of 2, seed set to “ “ : C′ rejects till 2t* - 1

Seed reset at 2t*

$t^* \leftrightarrow 2t^* \leftrightarrow 4t^* \leftrightarrow \cdots \leftrightarrow 2^{l-1}t^* \leftrightarrow 2^{l}t^*$

Possible to have O(k) hybrids for $O(2^k)$ steps !

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