Distinguisher-Dependent Simulation

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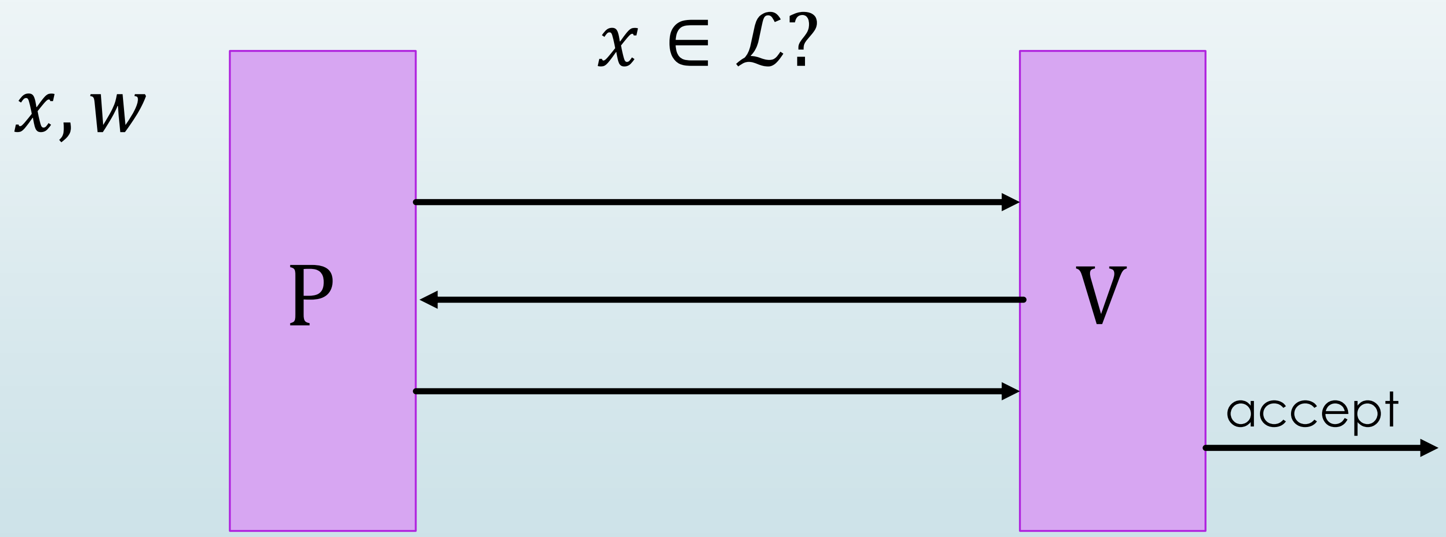
Joint work with Abhishek Jain, Yael Kalai and Ron Rothblum

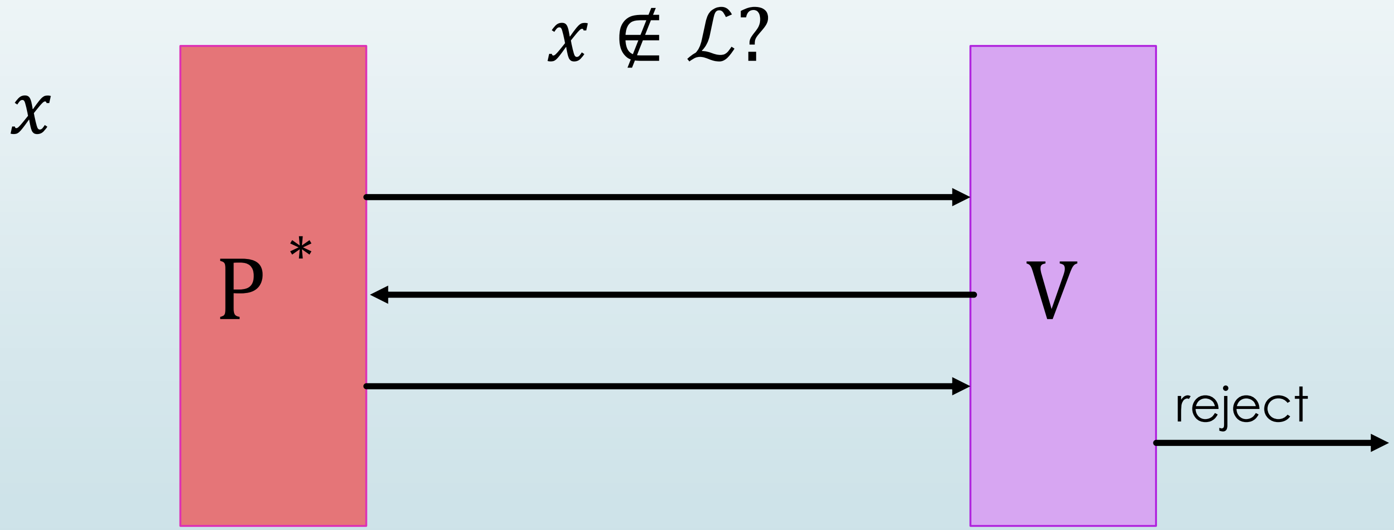
Interactive Proofs for NP

Interactive Proof (GMR85, Babai85)



Security Against Malicious Provers

Soundness



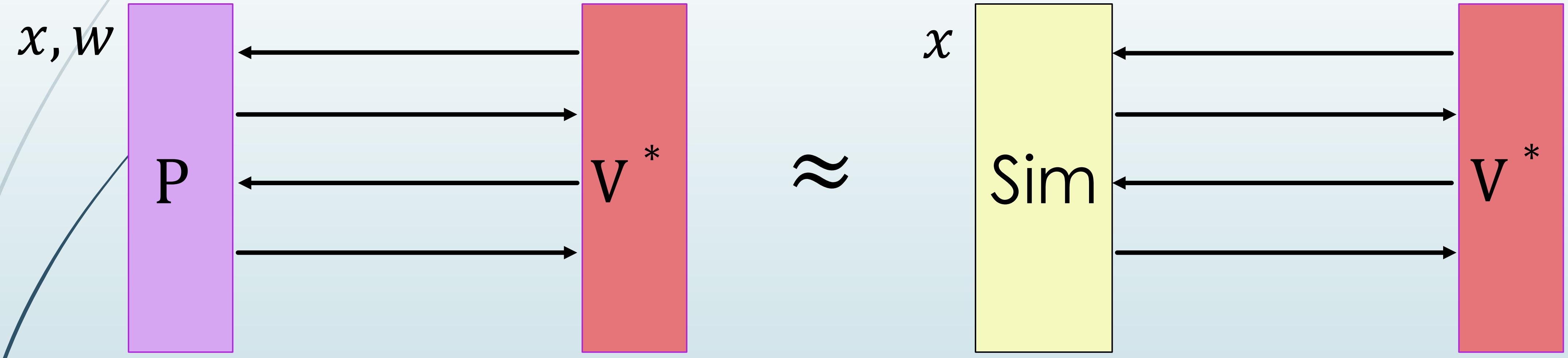
Security Against Malicious Verifiers

Shouldn't learn witness w

- Zero-Knowledge (GMR85)
- Distributional Zero-Knowledge (Goldreich93)
- Weak Zero-Knowledge (DNRS99)
- Witness Hiding (FS90)
- Witness Indistinguishability (FS90)
- Strong Witness Indistinguishability (Goldreich93)

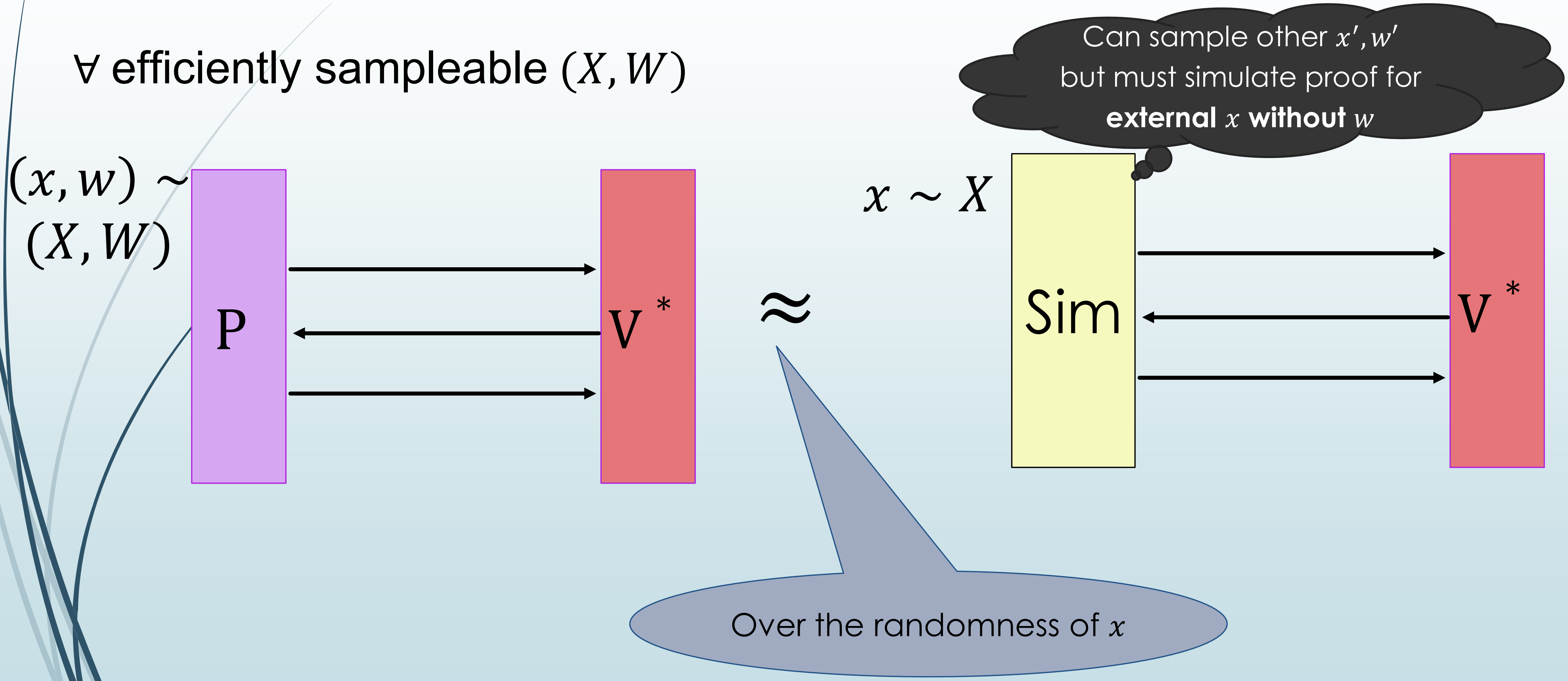
Zero-Knowledge

$\forall\, x,$

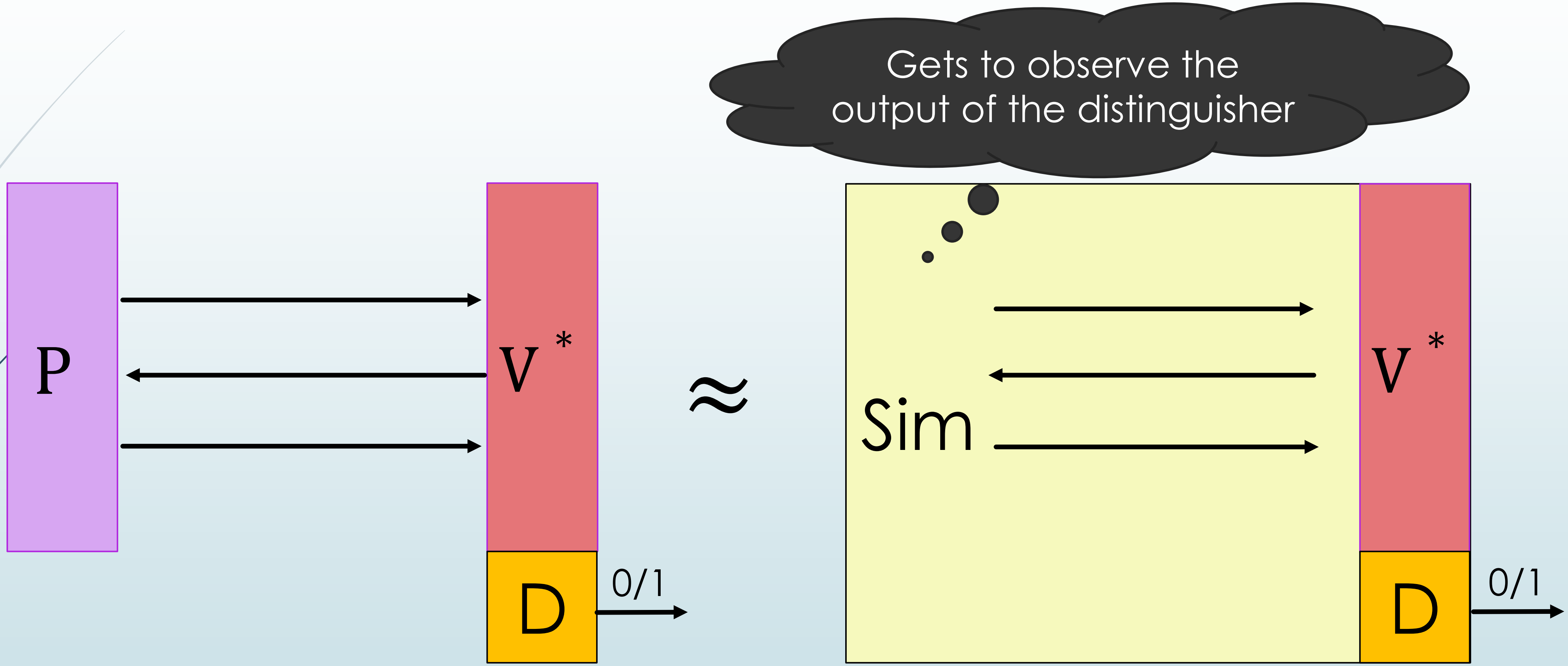


Distributional Zero-Knowledge

$\forall$ efficiently sampleable (X, W)



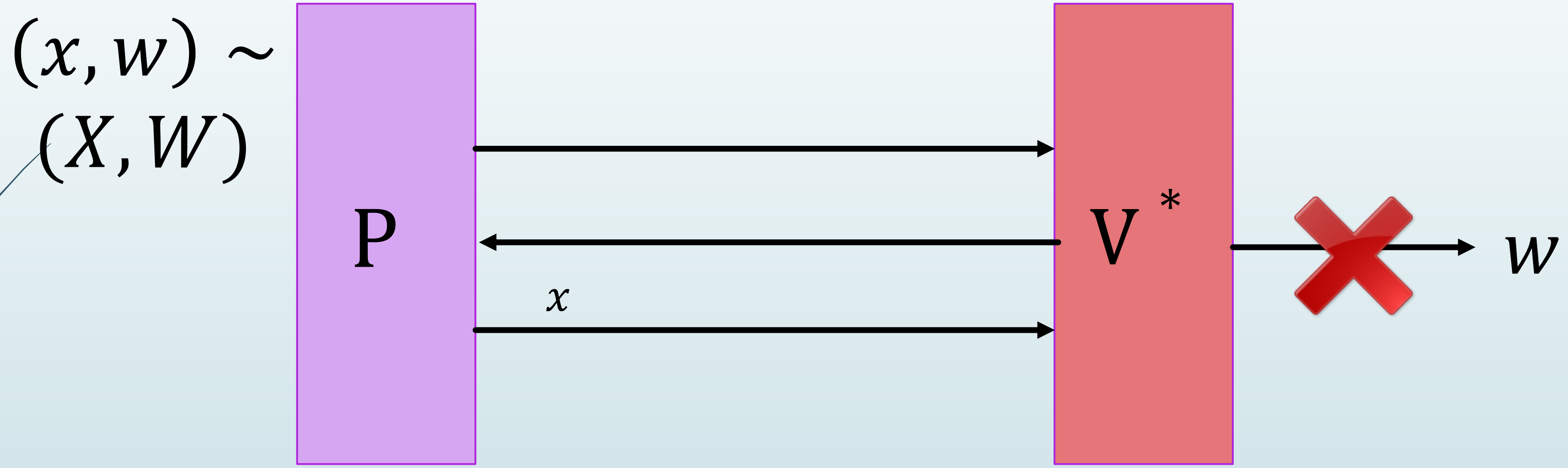
Weak Zero-Knowledge



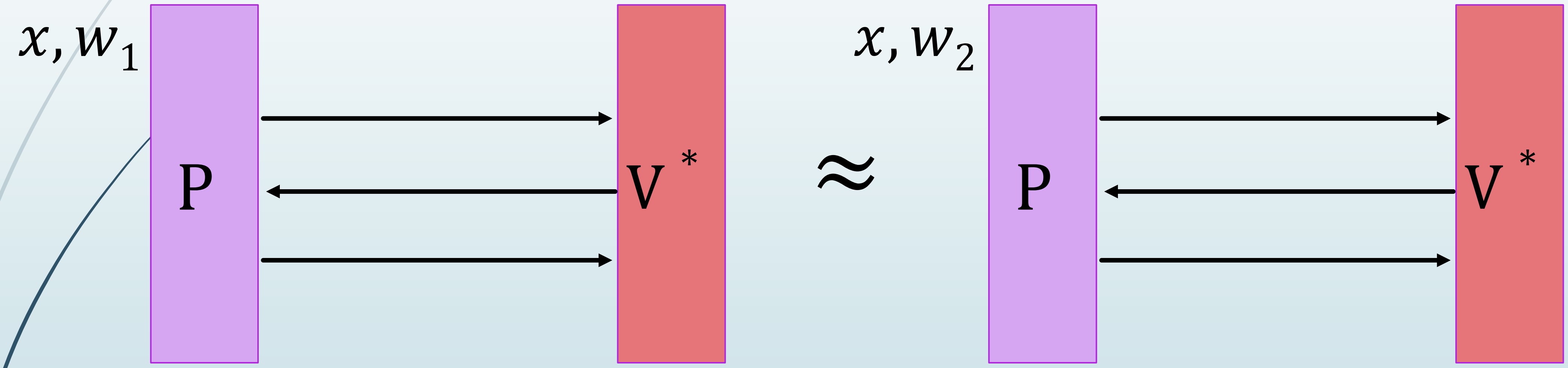
$$Pr[D = 1|real] \; - \; \Pr[D = 1|Sim] \leq negl$$

Witness Hiding

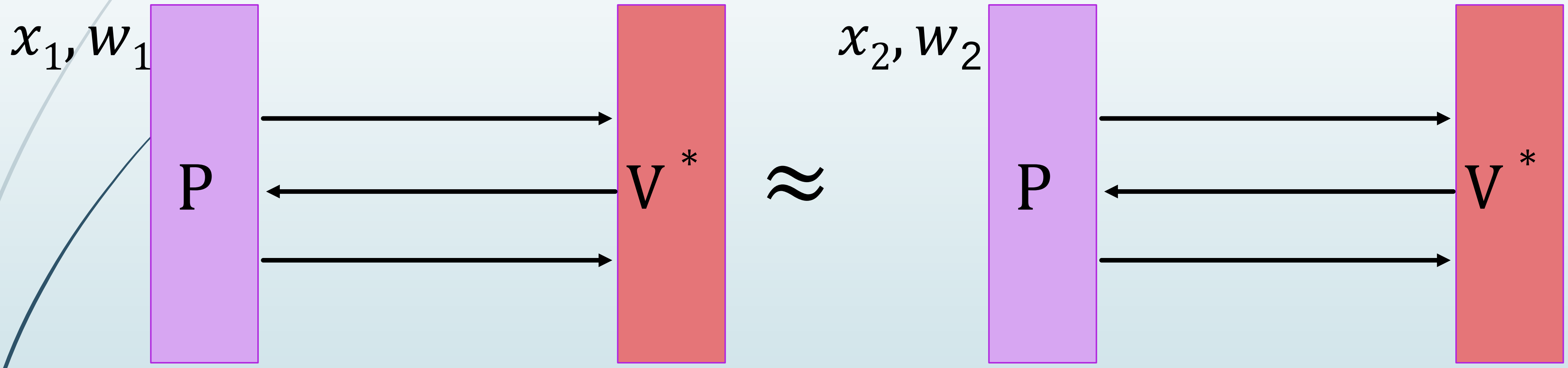
∀ efficiently sampleable (X, W) with hard to find witnesses,



Witness Indistinguishability

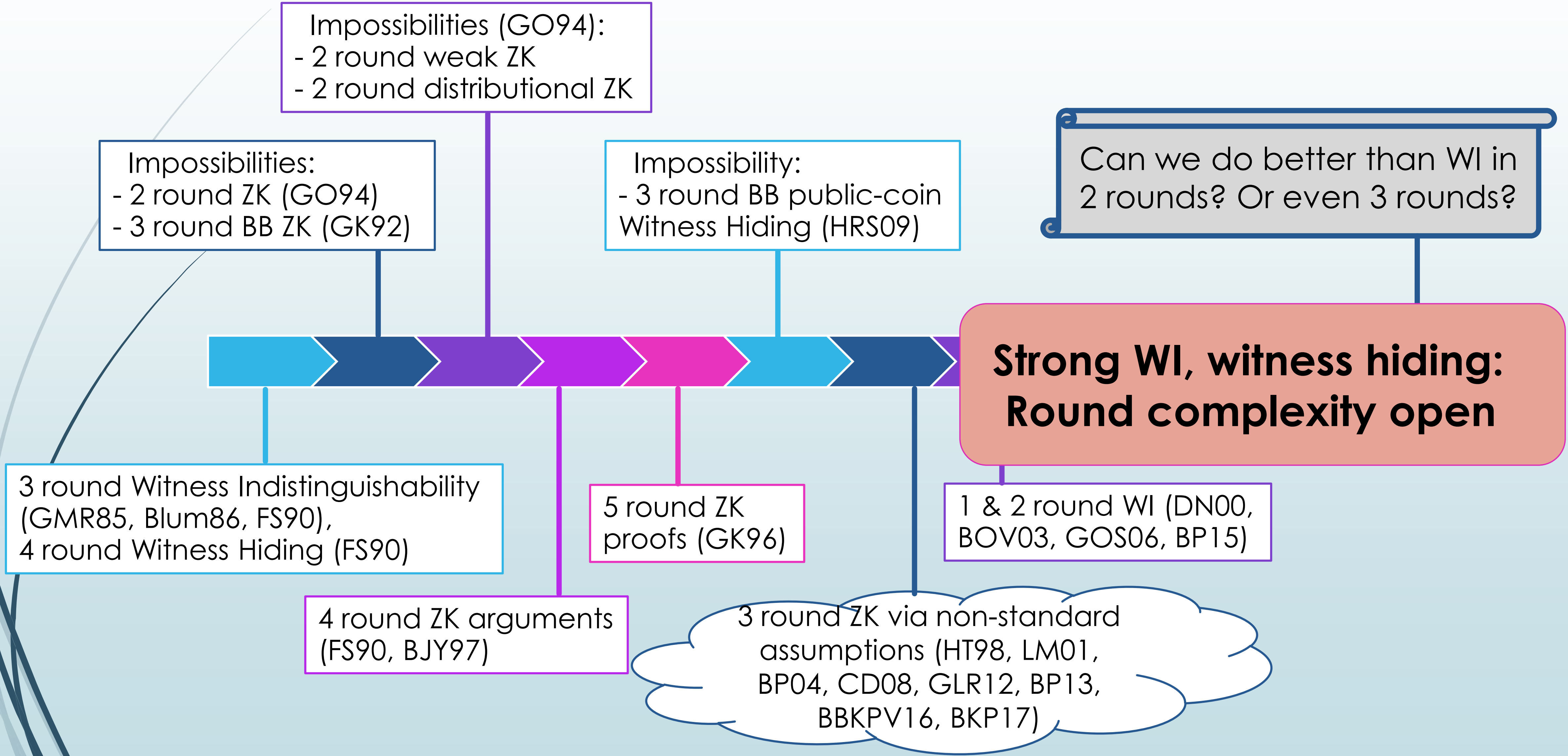


Strong Witness Indistinguishability



when $x_1 \approx x_2$

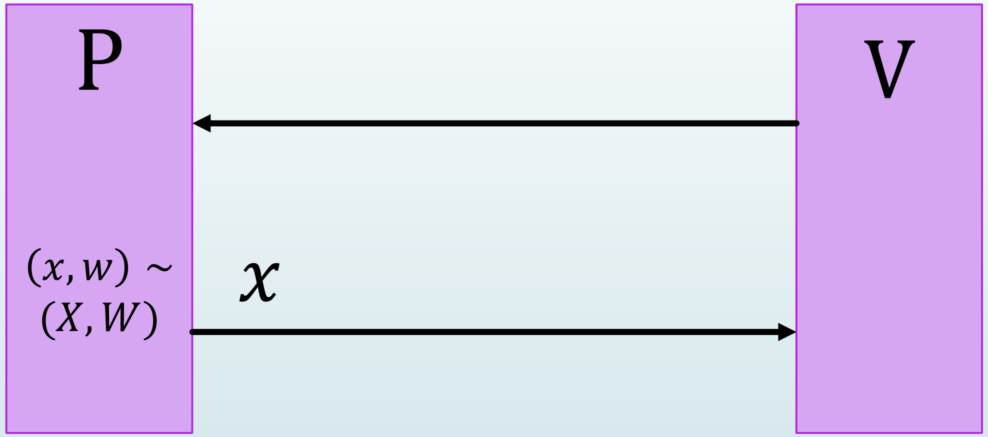
Round Complexity Timeline



Overcoming Barriers

Distributional Protocols

- Prover samples instance x from some distribution

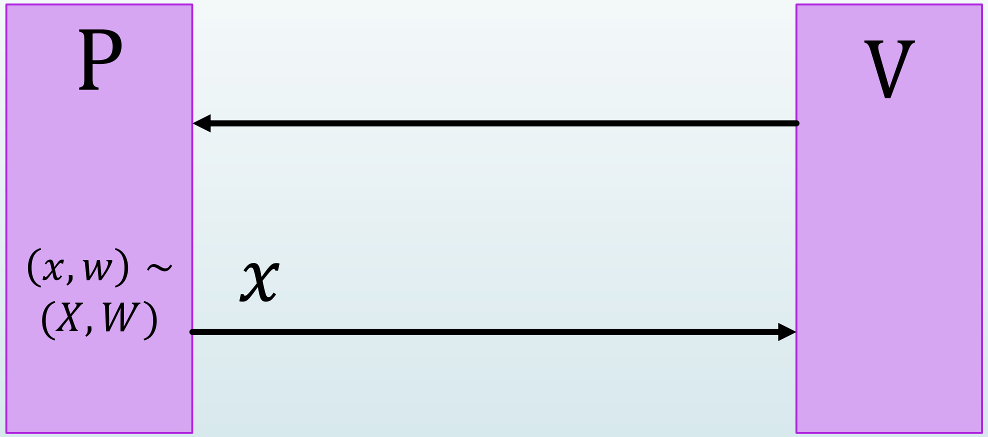


Why should we care?

- ZK proofs used to prove correctness of cryptographic computation
- Almost always, instances are chosen from some distribution
- Strong WI, WH by definition are distributional notions

Distributional Protocols

- Prover samples instance x from some distribution

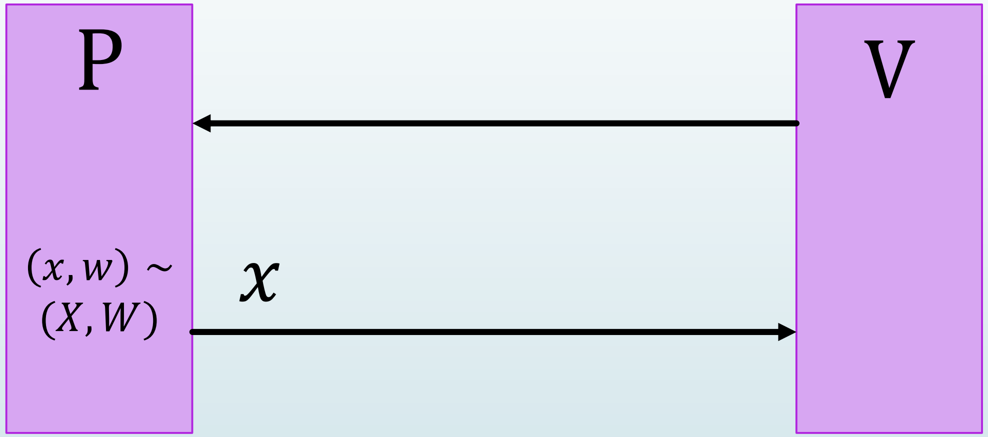


- Useful in secure computation: [KO05, GLOV14, COSV16]
- Our paper: extractable commitments, 3 round 2pc
- Specific 2 & 3 round protocols: [KS17, K17, ACJ17]

- In 2 round protocols, P sends x together with proof
- Adaptive soundness: P* samples x after V's message
- We will restrict to: **delayed-input** protocols
- Cheating verifier cannot choose first message depending on x

Distributional Protocols, Delayed-Input

- Prover samples instance x from some distribution



- Simulate the view of malicious V*, when V* is committed to 1st message, before P reveals instance x?
- **Distributional privacy for delayed-input statements.**
- Get around negative results!

Our Results

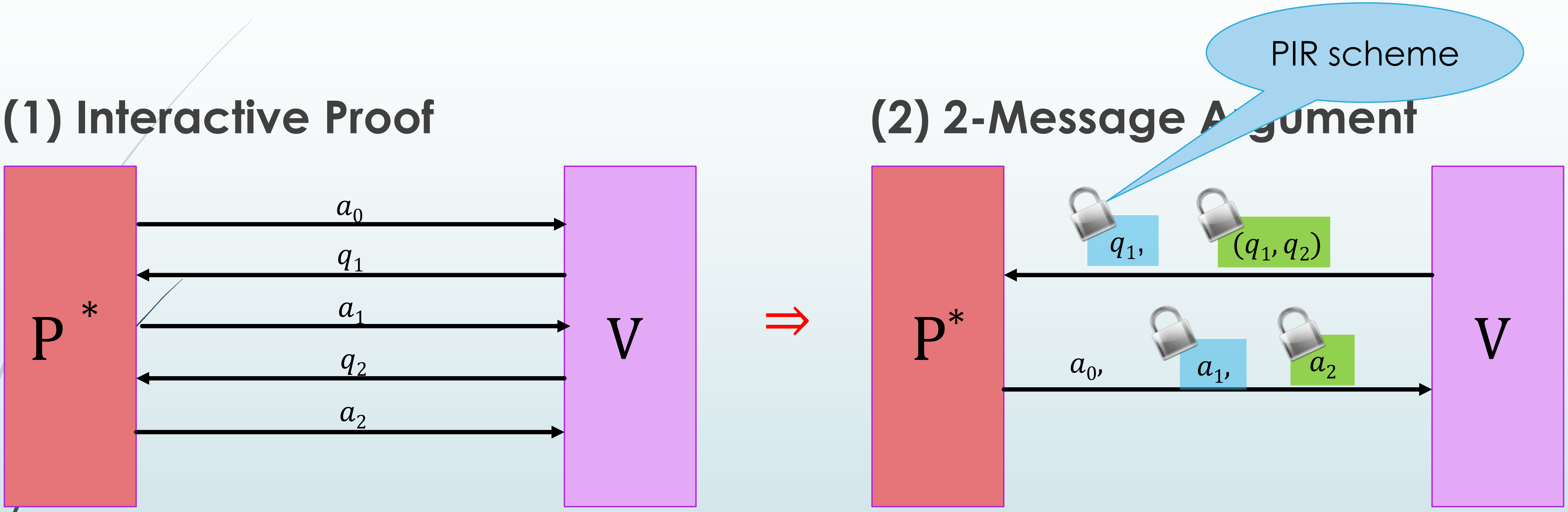
Assuming quasi-polynomial DDH, QR or N^{th} residuosity, we get

- **2 Round arguments in the delayed-input setting**
  - **Distributional weak ZK**
  - **Witness Hiding**
  - **Strong Witness Indistinguishability**
- **2 Round WI arguments** [concurrent work: BGISW17]
  - Previously, trapdoor perm (DN00), b-maps (GOS06), or iO (BP15)
- 3 Round protocols from polynomial hardness + applications

Sim depends on distinguisher

New Technique:
Black-box Simulation in 2 Rounds

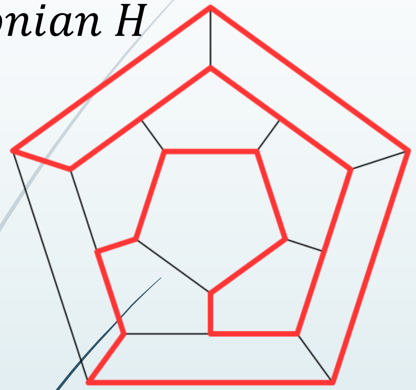
Kalai-Raz (KR09) Transform

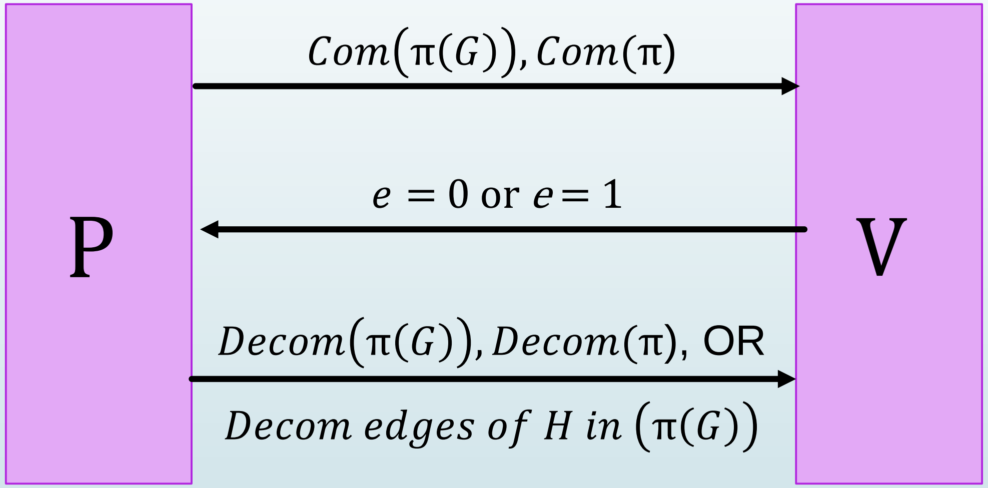


- KR09: Assuming quasi-polynomially secure PIR, (2) is sound against adaptive PPT P*.
- Our goal: 2 message arguments for NP with privacy.
- Apply KR09 transform to three round proof of Blum86.

Blum Protocol for Graph Hamiltonicity

Graph G,
Hamiltonian H

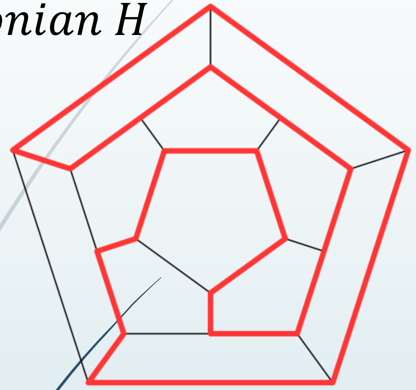


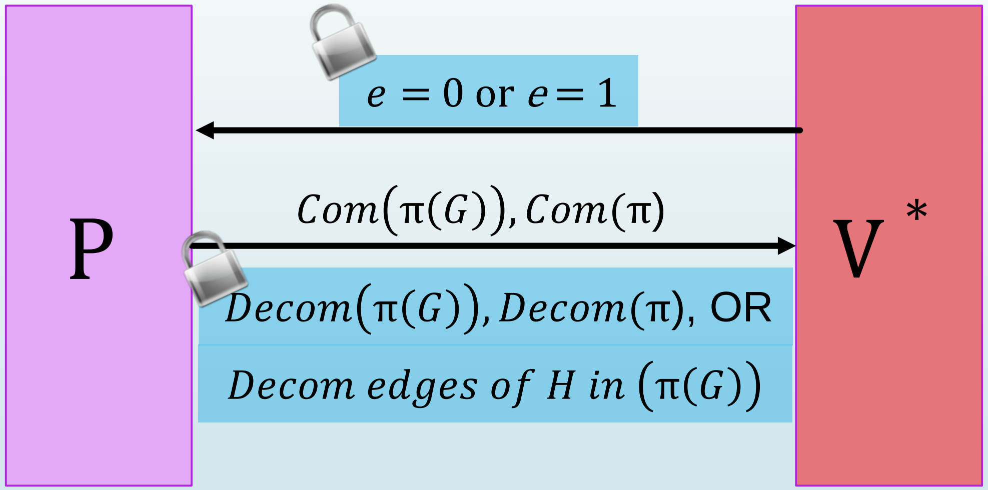


- Honest verifier zero-knowledge: Sim that knows e can simulate.
- Repeat in parallel to amplify soundness. Preserves honest verifier ZK.

KR09 transform on Blum

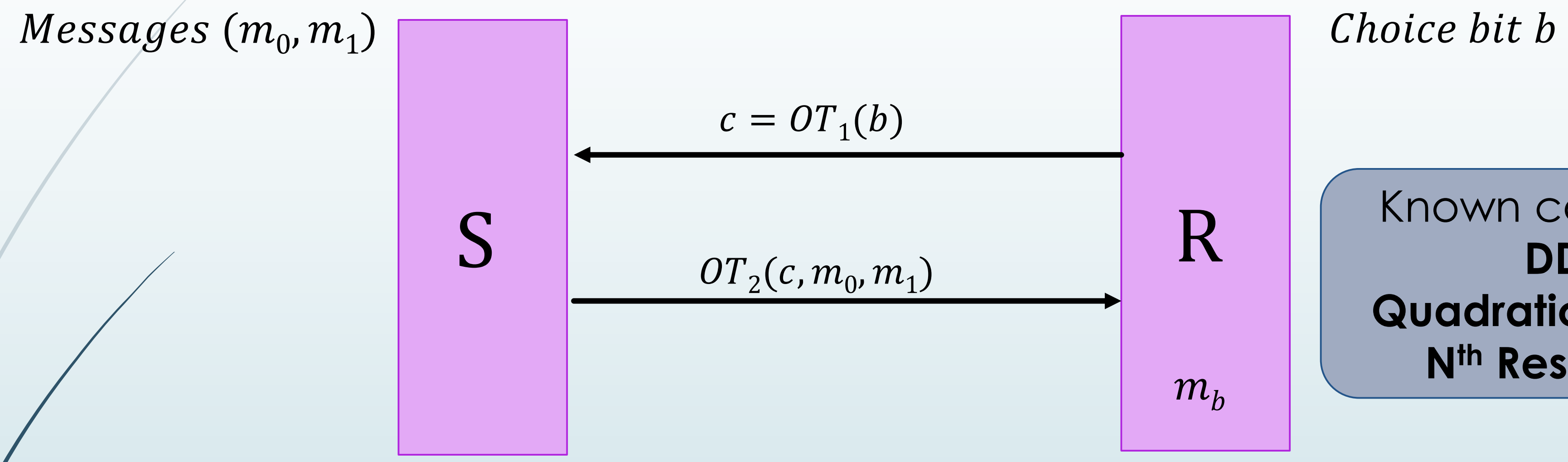
Graph G,
Hamiltonian H





- Remains honest verifier zero-knowledge.
- What if malicious V* sends malformed query that doesn't encode any bit?
- Prevent this by using a special PIR scheme.

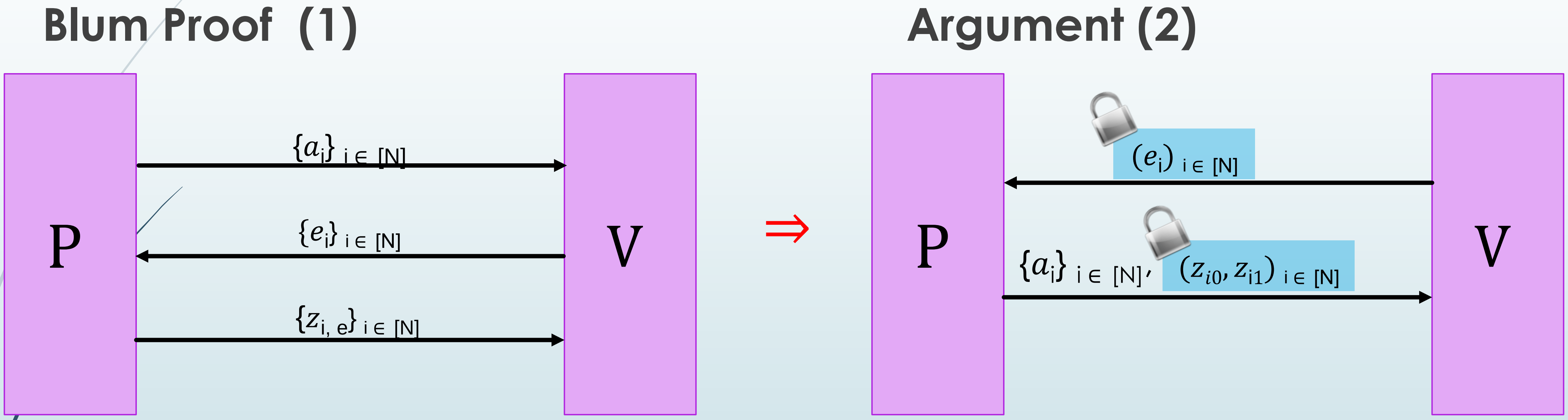
2-Message Oblivious Transfer



Known constructions from **DDH** (NP01), **Quadratic Residuosity** and **N^th Residuosity** (HK05)

- S cannot guess b
- R cannot distinguish $\mathrm{OT}_2(m_0, m_1)$ from :
  - $\mathrm{OT}_2(m_0, m_0)$ when $b = 0$, OR
  - $\mathrm{OT}_2(m_1, m_1)$ when $b = 1$.
- Every string c corresponds to $OT_1(b)$ for some bit b

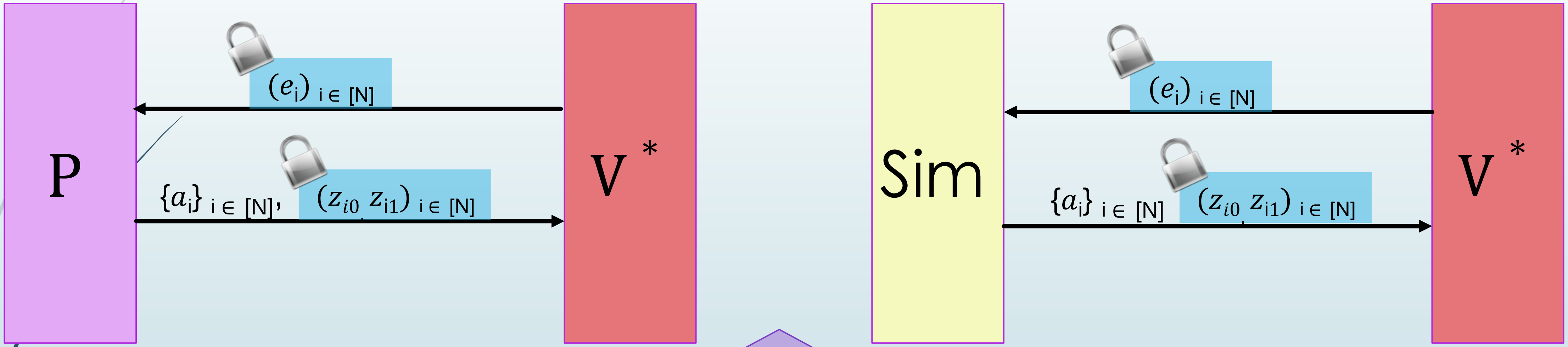
Kalai-Raz Transform on Blum using OT



- KR09: (2) remains sound against PPT provers, even if they choose x adaptively
- What about privacy?

Kalai-Raz Transform on Blum

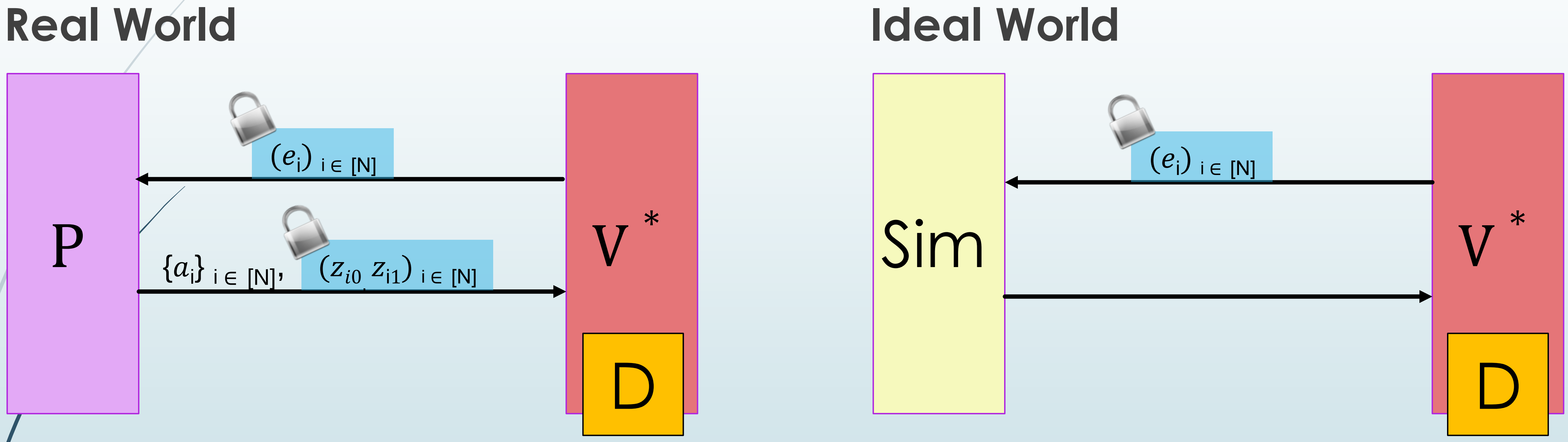
Real World



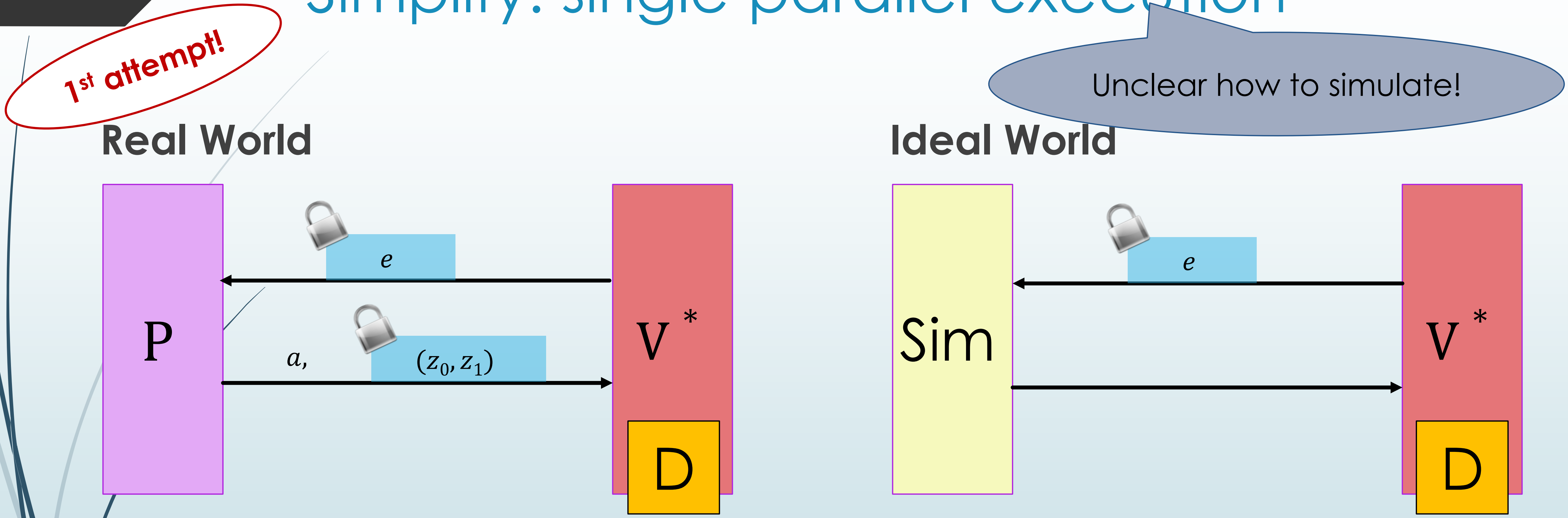
- Every message sent by ... cryption of some $\{e_i\}_{i \in [N]}$
- If Sim knew $\{e_i\}_{i \in [N]}$, the ... ZK).
- Privacy via super-poly sim ... encryption to find e_i [BGISW17]

Polynomial Simulation??

Rely on the Distinguisher to find e



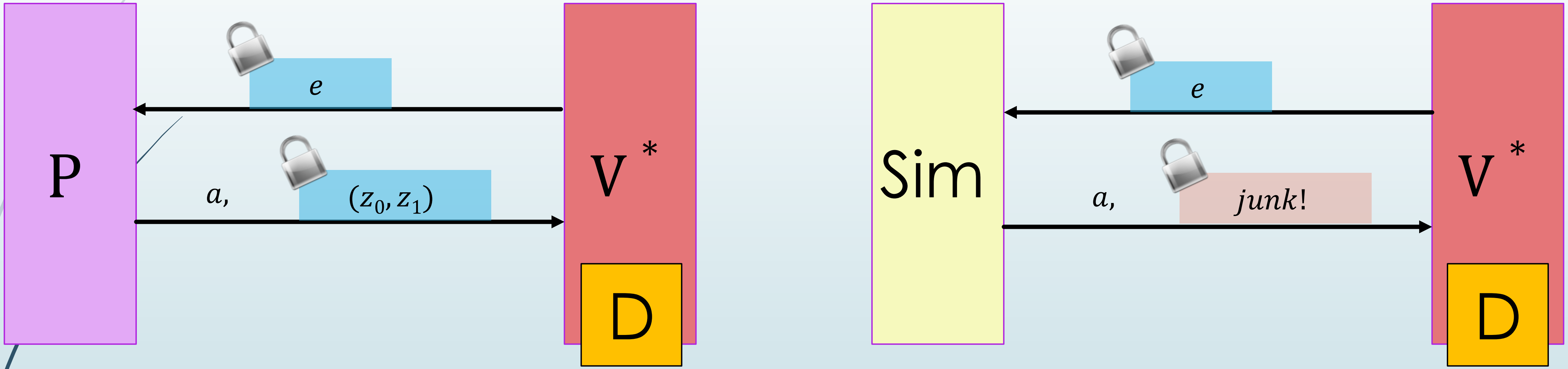
Simplify: single parallel execution



Simplify: single parallel execution

1st attempt!

Real World **Ideal World**

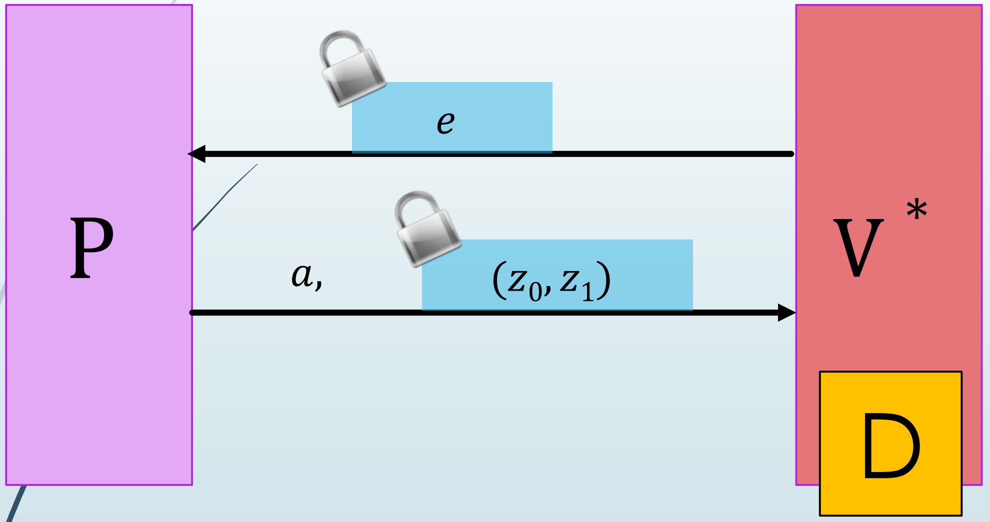


Can D tell the difference?

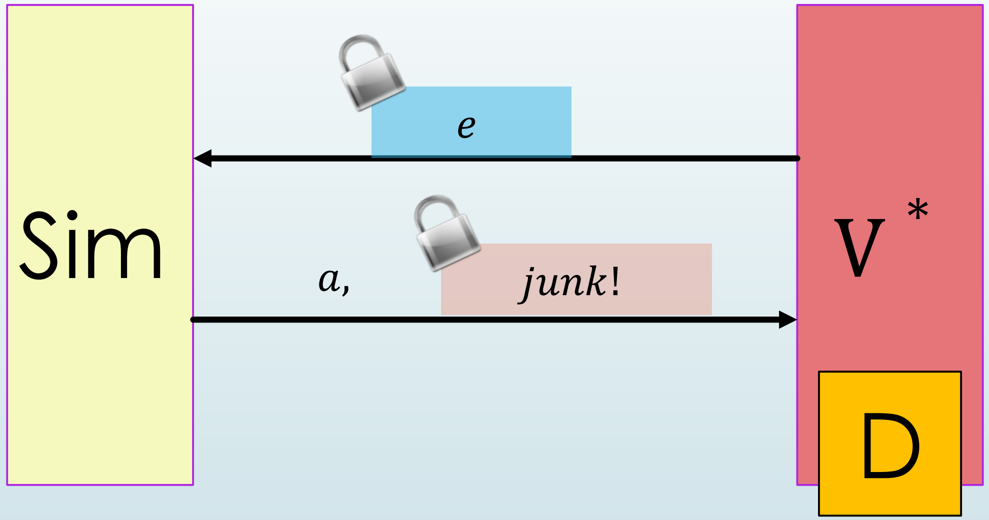
- Suppose **NOT**: eg, D doesn't know randomness for e
- a is already computationally hiding, Sim can easily sample $a,$ $junk!$

Simplify: Single parallel execution

Real World



Ideal World

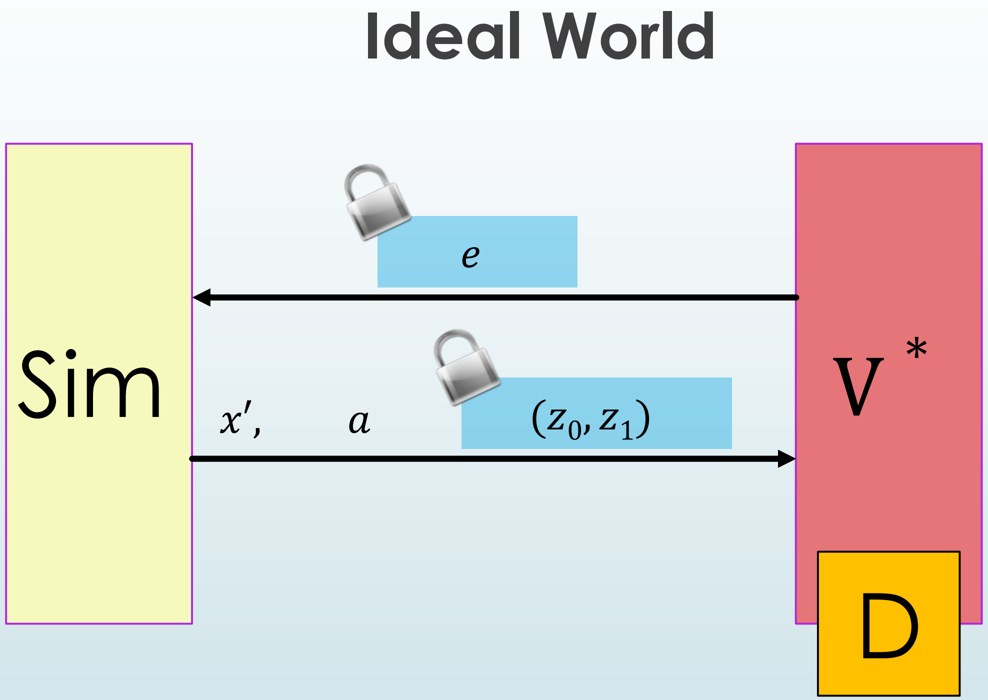


Can D tell the difference?

- Suppose **YES**: eg, D knows randomness for e
- Sim can't just sample $a,$ $junk!$: will be distinguishable!

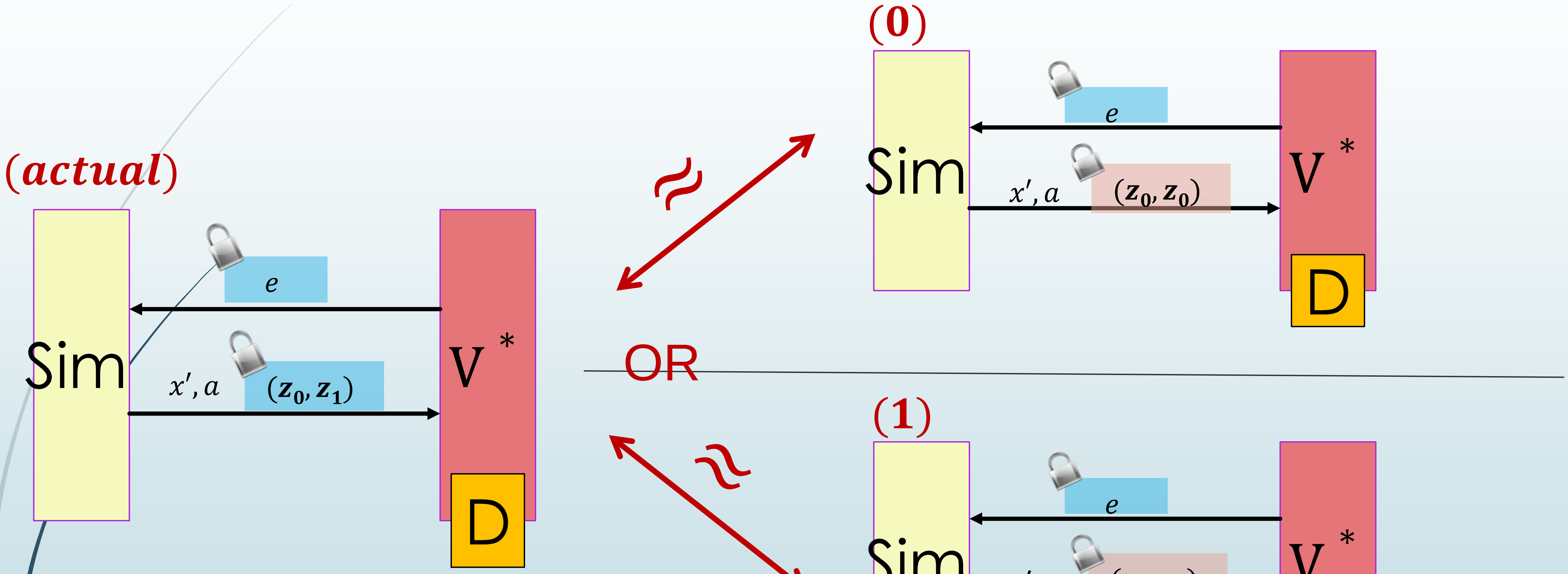
Sim will use D to extract *e* !

Recall: Distributional Simulation



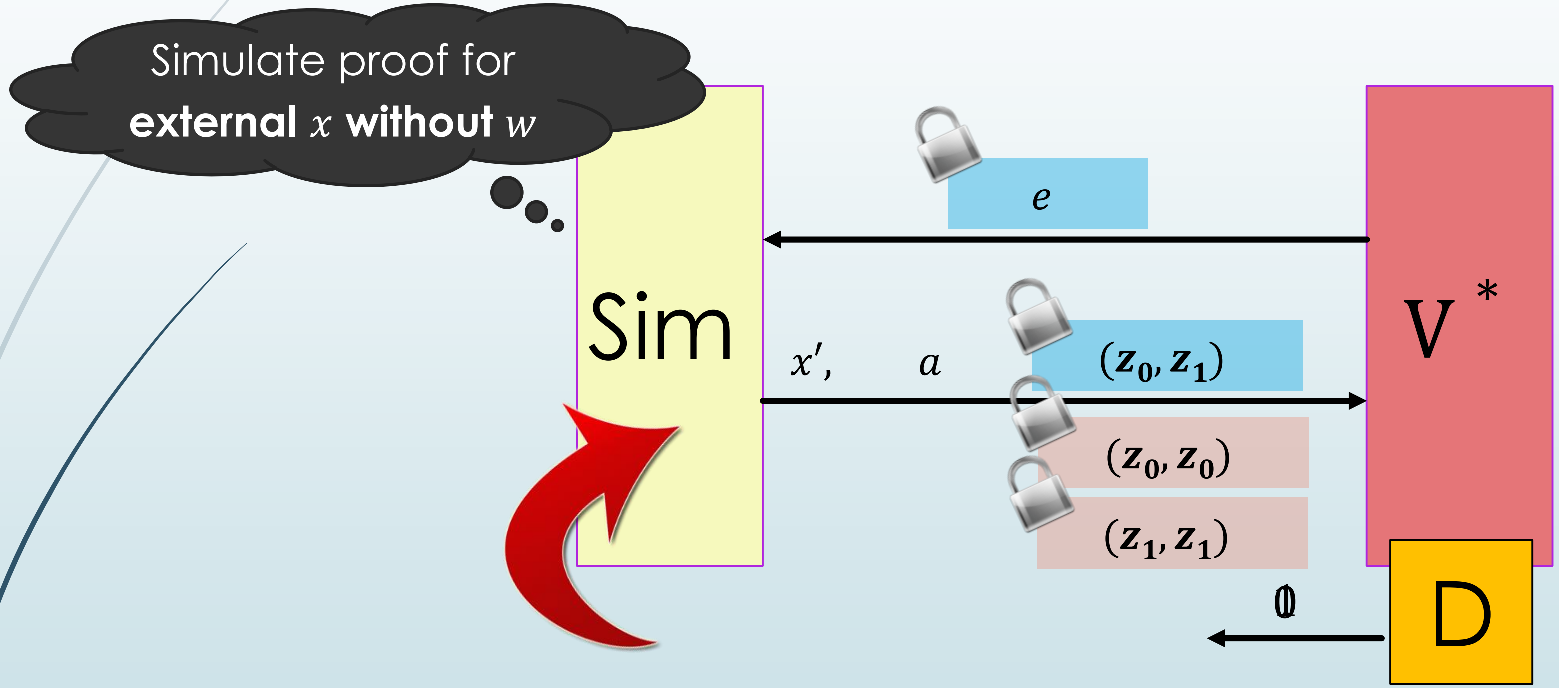
- Recall: want a simulator for $x \sim X$, which generates a proof without witness.
- However, Sim can sample other $(x', w') \sim (X, W)$ from the same distribution.
- Sim can also sample proofs for these other $(x', w') \sim (X, W)$.

Main Simulation Technique



Checks if $(actual) \approx (0)$
Or, if $(actual) \approx (1)$
Use this to extract e.

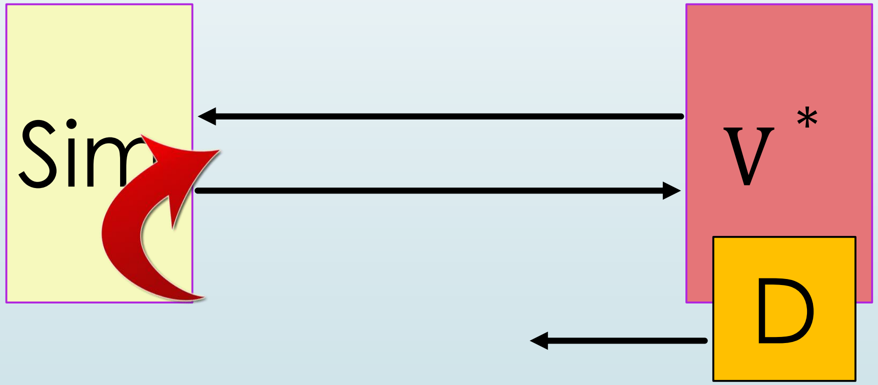
Polynomial Simulation



- Simulator *rewinds the distinguisher* to learn the OT challenge e.
- Technique extends to extracting $\{e_i\}_{i \in [N]}$ from parallel repetition.

Perspective: Extraction in Cryptography

- Black-box polynomial simulation strategy that requires only 2 messages.
- Previously, rewinding took more rounds



- Towards resolving open problems on round complexity of WH, strong WI.
- Applications to multiple 2-round, 3-round protocols, beyond proofs.

Conclusion & Open Problems

Round Complexity Timeline

3 round Witness Indistinguishability (FS90),
4 round Witness Hiding (FS90)

Impossibilities:
- 2 round ZK (GO94)
- 3 round BB ZK (GK92)

Impossibilities (GO94):
- 2 round weak ZK
- 2 round distributional ZK

4 round ZK arguments (FS90, BJY97)

5 round ZK proofs (GK96)

Impossibility:
- 3 round Witness Hiding (HRS09)

3 round ZK from non-std assumptions (HT98, LM01, BP04, CD08, GLR12, BP13, BBKPV16, BKP17)

1 & 2 round WI From TDPs / iO (DN00, BOV03, BP15)

Delayed-input setting:
- **Distributional weak ZK**
- **Witness Hiding, Strong WI**

2 rounds from quasi-poly &,
3 rounds from poly assumptions

2 round WI from quasi-poly DDH, QR, N^{th} residuosity

Open Questions

- 2 round protocols from *polynomial hardness*?
- Low round *public-coin* protocols with strong privacy?
- New applications of distinguisher-dependent simulation
- Other black-box/non-black-box techniques for 2 round protocols
  - A 2-round rewinding technique from superpoly DDH in [KS17, BKS17]

Thank you!