Throughput and Delay Optimal Resource Allocation in Multiple Access Fading Channels

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Multiple Access Communications

- Multiple access (many to one): multiple senders transmit to one receiver (possibly) over fading channels.
- Ex: cellular telephony, satellite networks, local area networks.

Central Problems

- Contention/interference - resource sharing.
- Bursty sources $\Rightarrow$ random number of active senders.
- Network/MAC layer QOS issues - throughput, delay.
- Physical layer issues - channel modelling, coding, detection.

Need for Cross-Layer Approach

- Multiple access network theory (ALOHA, CSMA) - concentrates on source burstiness and delay; poor modelling of noise and interference.
- Multiple access information theory - concentrates on channel modelling and coding; ignores random arrival of messages and delay.
- Need more unified **cross-layer** framework:
  - Random packet arrivals affect resource sharing.
  - Choice of modulation and coding affects QOS issues.
  - Random fading affects resource allocation.
  - Gallager (85), Ephremides and Hajek (98).

New Approach

- Goal:
  - Combine information-theoretic limits with QOS issues.
  - Establish fundamental bounds on throughput/delay performance.
- Implementation:
  - Random arrivals, information-theoretic optimal coding.
  - Power control and rate allocation as function of fading and queue states to optimize throughput and delay

Previous Work

- Telatar and Gallager (95)
  - Achievable multiple access scheme with feedback.
  - Poisson arrivals; no queueing; single-user decoding; processor sharing system.
- Telatar (95)
  - Analogy between MAC and multi-processor queue.
  - Each user has **fixed** pool of bits to send.
  - Optimal processor assignment to minimize average packet delay.
- Yeh (01)
  - Poisson arrivals; queueing.
  - Optimal rate allocation from $\mathcal{C}$ to min. average packet delay.
  - Longer Queue Higher Rate (LQHR) policy strongly delay optimal.

Multiple Access Fading Channel

- Continuous-time M-user Gaussian multiple access fading channel with bandwidth W:

$$Y(t) = \sum_{i=1}^{M} \sqrt{H_i(t)} X_i(t) + Z(t).$$

- $\{Z(t)\}$: white Gaussian noise, density $N_0/2$.
- Slowly-varying and flat-fading (under-spread) channel.

Multiple Access Fading Channel

- Block fading model, block length $= T$.
- T large enough for reliable communication at a fixed fade.
- $\{\boldsymbol{H}(t) = (H_1(t), \ldots, H_M(t))\}$ modulated by finite-state ergodic Markov chain.
- Transmitter i has (long-term) average power constraint $\overline{P}_i$, and (short-term) peak power constraint $\hat{P}_i$.

Information-theoretic Capacity Region $\mathcal{C}(\boldsymbol{h}, \boldsymbol{p})$

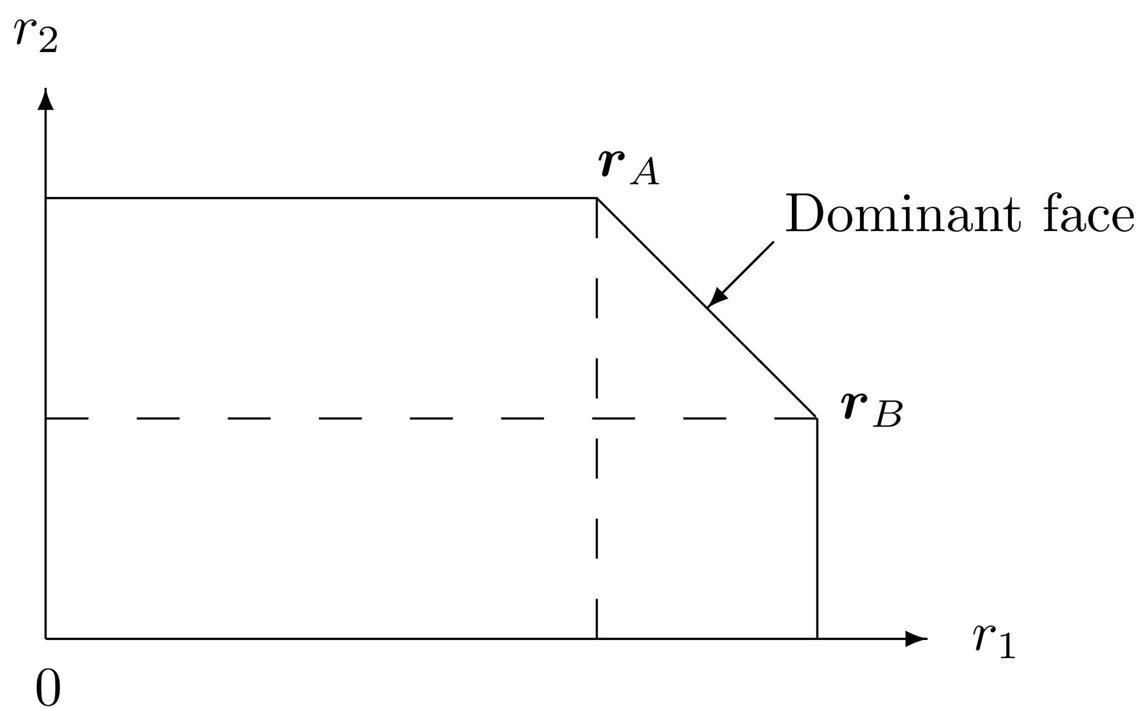
(Ahlswede, Liao, Cover, Wyner 1971-75)

- Fixed $\boldsymbol{h} = (h_1, \ldots, h_M)$ and $\boldsymbol{p} = (p_1, \ldots, p_M)$.
- $\mathcal{C}(\boldsymbol{h}, \boldsymbol{p})$ = set of $\boldsymbol{r} \in \mathbb{R}_+^M$ such that

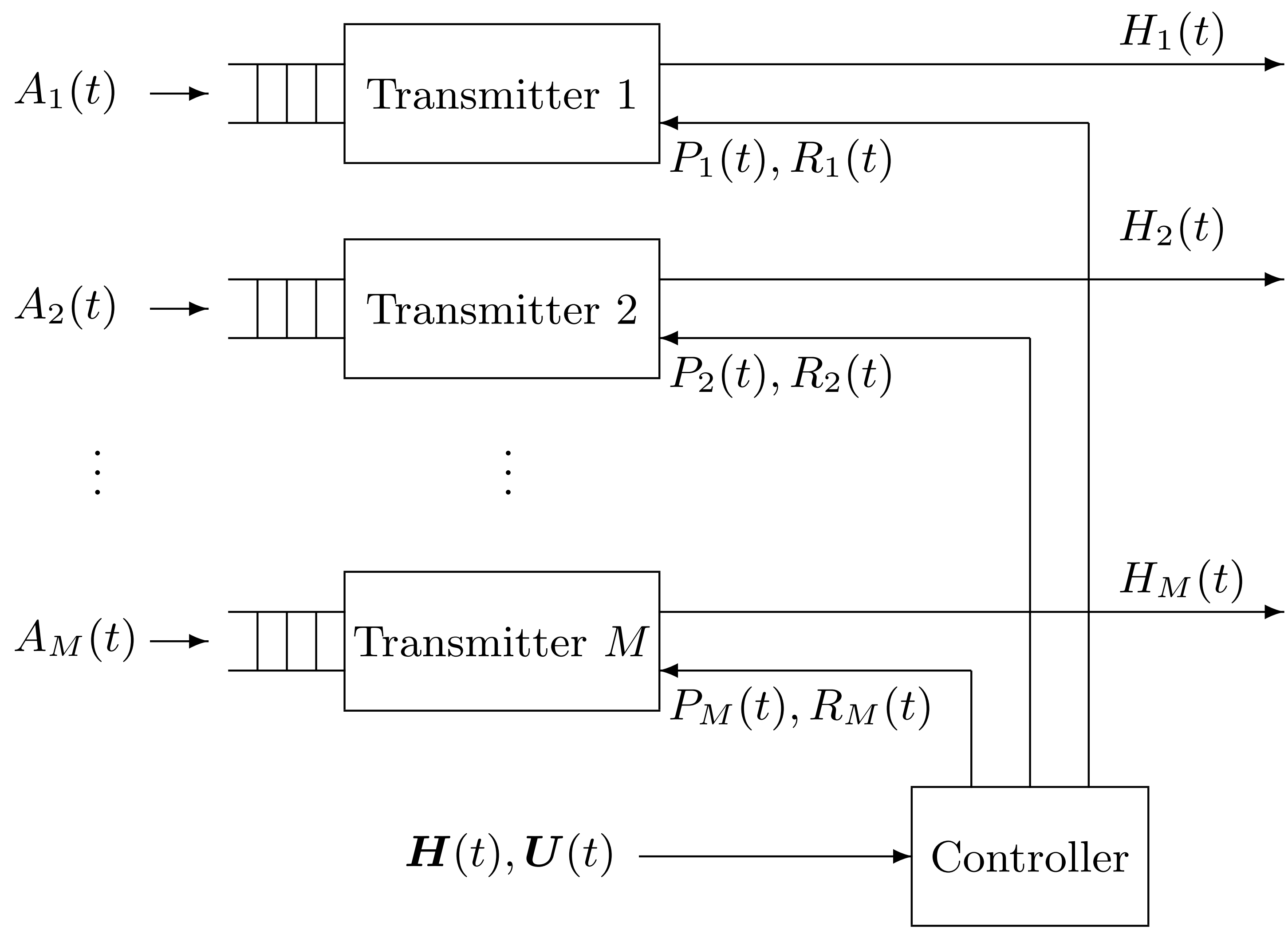
$$\sum_{i \in S} r_i \leq W \log \left(1 + \frac{\sum_{i \in S} h_i p_i}{N_0 W} \right), \ \ \forall S \subseteq \{1, \ldots, M\}.$$

- Reliable communication possible inside $\mathcal{C}(\boldsymbol{h}, \boldsymbol{p})$, impossible outside $\mathcal{C}(\boldsymbol{h}, \boldsymbol{p})$, for any coding and modulation scheme.
- **Polymatroid** structure (Tse and Hanly 98).

Two-User Capacity Region $\mathcal{C}(\boldsymbol{h}, \boldsymbol{p})$



Multiple Access Channel with Random Arrivals



Arrivals and Unfinished Work

- $\{A_i(t)\}$ = ergodic packet arrival process to transmitter i.
- User i packets i.i.d. $\sim F_{Z_i}(\cdot), \mathsf{E}[Z_i] < \infty$.
- $U_i(t)$ = number of untransmitted bits in queue i at time t.

Power Control and Rate Allocation

- Controller: $(\boldsymbol{H}(t), \boldsymbol{U}(t)) \mapsto (\boldsymbol{P}(t), \boldsymbol{R}(t))$.
- Two stages:

1. **Power control policy** $\mathcal{P}$:

$$\boldsymbol{p} = \mathcal{P}(\boldsymbol{h}, \boldsymbol{u})$$

   s.t. for all i, $\mathsf{E}[\mathcal{P}_i(\boldsymbol{H}, \boldsymbol{U})] \leq \bar{P}_i$, $\mathcal{P}_i(\boldsymbol{h}, \boldsymbol{u}) \leq \hat{P}_i$ for all $(\boldsymbol{h}, \boldsymbol{u})$.

2. **Rate allocation policy** $\mathcal{R}$:

$$\boldsymbol{r} = \mathcal{R}(\boldsymbol{h}, \boldsymbol{p}, \boldsymbol{u}) \in \mathcal{C}(\boldsymbol{h}, \boldsymbol{p}).$$

Main Results

- Stability region $\mathcal{S}$ of all bit arrival rates for which all queues can be kept finite.
- For given power control policy, find throughput optimal rate allocation policy.
- In symmetric scenario, find delay optimal rate allocation policy for any symmetric power control policy.

Stability Region $\mathcal{S}$

- $\lambda_i = \lim_{t\to\infty} A_i(t)/t$ = packet arrival rate to queue i.
- $\rho_i = \lambda_i \mathsf{E}[Z_i]$ = bit arrival rate to queue i.
- Define $f_i(\xi) = \limsup_{t\to\infty} \frac{1}{t}\int_0^t 1_{\{U_i(\tau)>\xi\}} d\tau$.
- System **stable** if $f_i(\xi) \to 0$ as $\xi \to \infty$ for all i.
- $\mathcal{S}$ = set of all $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_M)$ for which can stabilize system.

Stability Region $\mathcal{S}$

- Assume $\{A_i(t)\}$ modulated by finite-state ergodic Markov chain.

Theorem 1 $\mathcal{S} = \mathcal{C}(\overline{\boldsymbol{P}}, \hat{\boldsymbol{P}})$ = *information-theoretic capacity region under power control (Tse and Hanly 98).*

- $\mathcal{C}(\overline{\boldsymbol{P}}, \hat{\boldsymbol{P}}) = \bigcup_{\mathcal{P} \in \mathcal{F}} \mathcal{C}(\mathcal{P})$.
- $\mathcal{F} = \{\mathcal{P} : \mathsf{E}[\mathcal{P}_i(\boldsymbol{H})] \leq \overline{P}_i, \forall i; \mathcal{P}_i(\boldsymbol{h}) \leq \hat{P}_i, \forall \boldsymbol{h}, \forall i\}$.
- $\mathcal{C}(\mathcal{P}) = \mathsf{E}[\mathcal{C}(\boldsymbol{H}, \mathcal{P}(\boldsymbol{H}))]$.

Stability Theorem

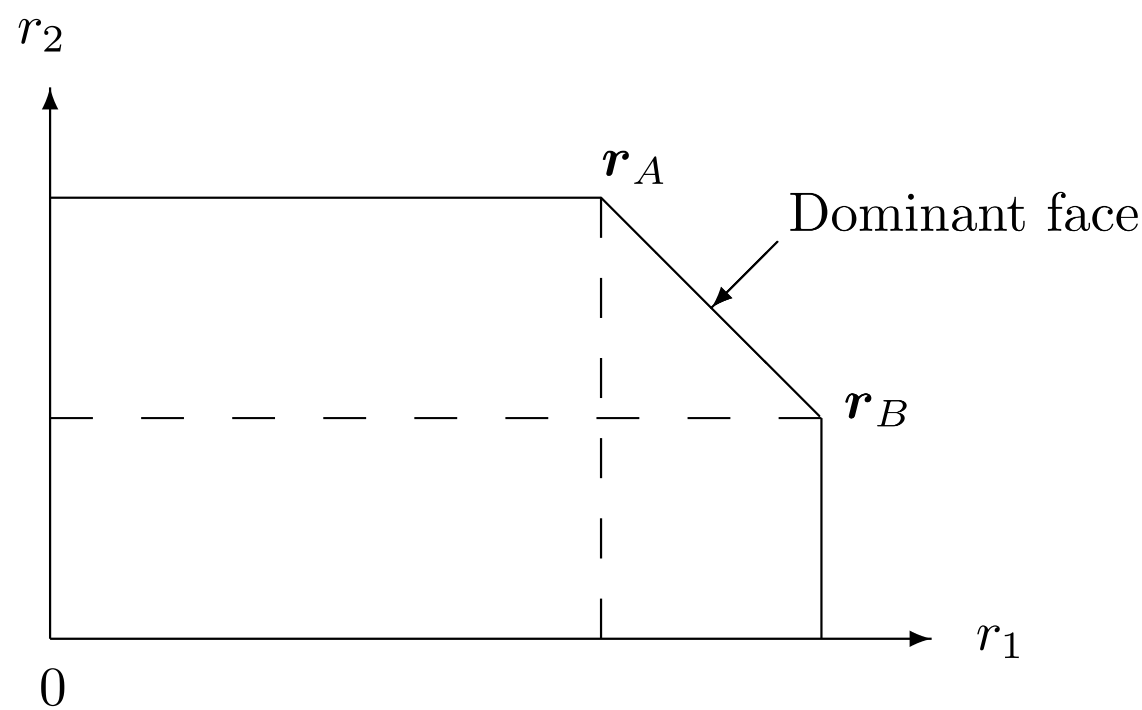
- **Achievability**: $\boldsymbol{\rho} \in \text{int}(\mathcal{S})$: knowing $\boldsymbol{\rho}$ and statistics of $\{\boldsymbol{H}(t)\}$, can stabilize system using stationary $\mathcal{P}, \mathcal{R}$ depending only on current channel state.
- **Converse:** $\boldsymbol{\rho} \notin \mathcal{S}$: cannot stabilize system, even with non-stationary policy with knowledge of queue state and/or knowledge of future events, so long as

$$\limsup_{t \to \infty} \frac{1}{t} \int_0^t p_i(\tau) d\tau \leq \overline{P}_i \ \forall i; \quad p_i(\tau) \leq \hat{P}_i, \forall \tau, \forall i.$$

Throughput Optimal Resource Allocation

- Find "universal" power/rate policy to stabilize system even if $\boldsymbol{\rho}$ not known, as long as $\boldsymbol{\rho} \in \mathrm{int}(\mathcal{S})$.
- Must use both $\boldsymbol{H}(t)$ and $\boldsymbol{U}(t)$.
- Suppose know $\boldsymbol{\rho} \in \mathcal{C}(\mathcal{P}) = \mathsf{E}[\mathcal{C}(\boldsymbol{H}, \mathcal{P}(\boldsymbol{H}))]$.
- Assume $\{H_i(kT)\}$ i.i.d. for each i, $\{A_i((k+1)T) - A_i(kT)\}$ i.i.d. for each i.
- Assume $\mathsf{E}[(A_i((k+1)T) - A_i(kT))^2] < \infty$.

No Work Conservation



Throughput Optimal Rate Allocation

Theorem 2 *Given $\mathcal{P} \in \mathcal{F}$, throughput optimal rate allocation policy is*

$$\boldsymbol{r}^* = \mathcal{R}^*(\boldsymbol{h}, \mathcal{P}(\boldsymbol{h}, \boldsymbol{u}), \boldsymbol{u}) = \arg \max_{\boldsymbol{r} \in \mathcal{C}(\boldsymbol{h}, \mathcal{P}(\boldsymbol{h}, \boldsymbol{u}))} \sum_{i=1}^{M} u_i r_i \qquad (1)$$

- Idea appeared in Tassiulas and Ephremides '92; McKeown, et al. '96; Tassiulas '97; Neely et al. '02.
- Here, motivated by delay optimality results.

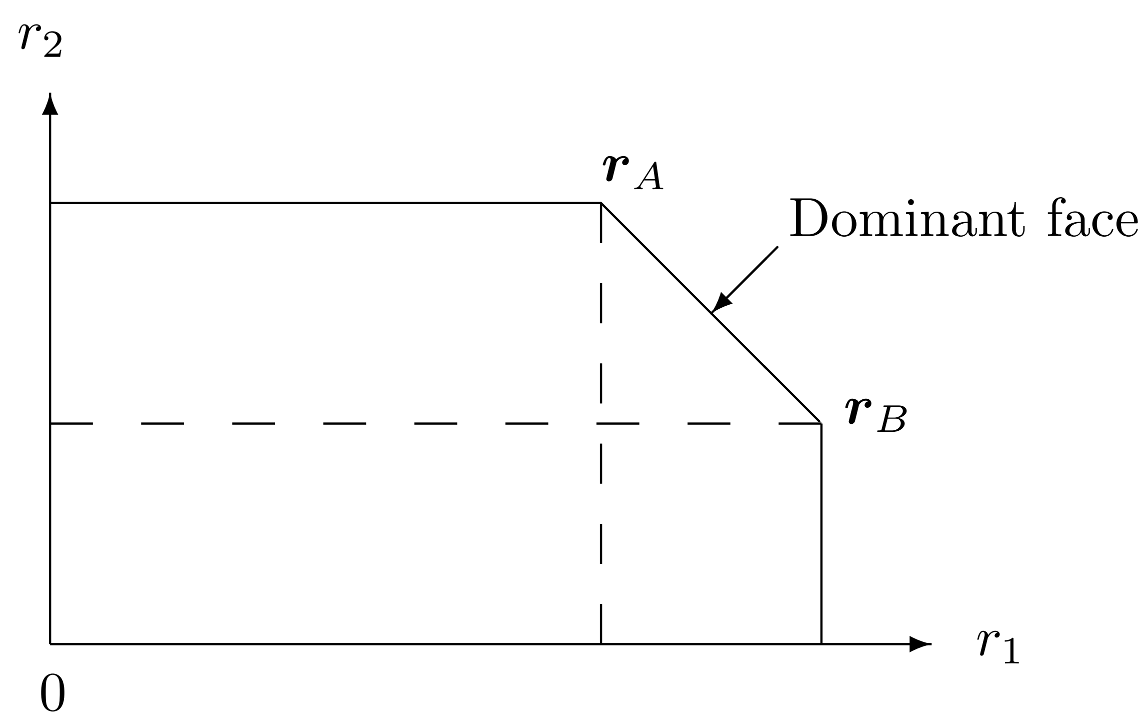
Longest Queue receives Highest Possible Rate (LQHPR)

- Due to **polymatroidal** nature of $\mathcal{C}(\boldsymbol{h}, \mathcal{P}(\boldsymbol{h}, \boldsymbol{u}))$, solution to (1) has special form.
- Order queues $u_{[1]} \geq u_{[2]} \geq \cdots \geq u_{[M]}$.

$$r^*_{[i]} = W \log \left(1 + \frac{h_{[i]} \mathcal{P}_{[i]}(\boldsymbol{h}, \boldsymbol{u})}{\sum_{j<i} h_{[j]}(t) \mathcal{P}_{[j]}(\boldsymbol{h}, \boldsymbol{u}) + N_0 W} \right)$$

- Longest Queue receives Highest Possible Rate (LQHPR).
- LQHPR $\Leftrightarrow$ **adaptive successive decoding**: $u_{[M]}$ decoded first, $u_{[1]}$ decoded last.

Two-User Rate Allocation



- $u_1 \geq u_2 : \boldsymbol{r}_B$ $\qquad$ $u_1 < u_2 : \boldsymbol{r}_A$

Proof of Stability Theorem

- Stability of Markov chains based on negative Lyapunov drift.
- $V(\boldsymbol{U}) = \sum_i U_i^2$.
- Show there exists compact set $\Gamma \subset \mathbb{R}^M$ s.t. for some $\epsilon > 0$,

$$\mathsf{E}[V(\boldsymbol{U}(t+T)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t)] \leq -\epsilon$$

whenever $\boldsymbol{U} \notin \Gamma$.

Delay Optimal Resource Allocation

- Beyond stabilization, keep queues as short as possible.
- Find feasible $\mathcal{P}$ and $\mathcal{R}$ to minimize $\lim_{t\to\infty} \mathsf{E}[\sum_{i=1}^{M} U_i(t)]$ (average bit delay) for $\boldsymbol{\rho} \in \mathrm{int}(\mathcal{S})$.

Delay Optimal Rate Allocation

- Focus on symmetric Poisson/exponential case.
- $\{A_i(t)\} = \text{Poisson}(\lambda)$ for each i.
- All packets i.i.d. $\sim \exp(\mu)$.
- Queue state $\boldsymbol{Q}(t) = (Q_1(t), \ldots, Q_M(t))$ - number of packets.
- For fixed $\mathcal{P}$, find $\mathcal{R}$ to minimize $\lim_{t\to\infty} \mathsf{E}\left[\sum_{i=1}^{M} Q_i(t)\right]$ (average packet delay).
- Yeh '01: non-faded symmetric MAC.

Delay Optimal Rate Allocation

- Symmetric fading process $\boldsymbol{H}(t)$:
  For any $\boldsymbol{a} = (a_1, \ldots, a_M)$, $\Pr(H_1(t) = a_1, \ldots, H_M(t) = a_M) = \Pr\left(H_1(t) = a_{\pi(1)}, \ldots, H_M(t) = a_{\pi(M)}\right)$ for any permutation π.
  e.g. for every t, $H_1(t), \ldots, H_M(t)$ i.i.d.
- Symmetric power control $\mathcal{P}(\boldsymbol{h}, \boldsymbol{q}) = \mathcal{P}(\boldsymbol{h})$:
  $\mathcal{P}_i(a_1, \ldots, a_M) = \mathcal{P}_{\pi^{-1}(i)}\left(a_{\pi(1)}, \ldots, a_{\pi(M)}\right)$ for all π.
  e.g. $M = 2$ and $a_1 > a_2$: $\mathcal{P}_1(a_1, a_2) = \mathcal{P}_2(a_2, a_1)$.
  e.g. Knopp and Humblet ('95).
- For this case, $\max \sum u_i r_i$ (LQHPR) policy is **delay optimal**.

Majorization and Weak Majorization

- Need to quantify load balancing.
- For $\boldsymbol{u} \in \mathbb{R}^M$, let

$$u_{[1]} \geq \cdots \geq u_{[M]}.$$

- For $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^M$,

$$\boldsymbol{u} \prec_w \boldsymbol{v} \quad \text{if} \quad \sum_{i=1}^{k} u_{[i]} \leq \sum_{i=1}^{k} v_{[i]}, \; k = 1, \ldots, M.$$

  Say $\boldsymbol{u}$ **weakly majorized** by $\boldsymbol{v}$. If equality holds for $k = M$, say $\boldsymbol{u}$ **majorized** by $\boldsymbol{v}$: $\boldsymbol{u} \prec \boldsymbol{v}$.

- Ex: $(1\ 1) \prec_w (3\ 0)$, $(1\ 1) \prec (2\ 0)$.
- See Marshall and Olkin (79).

Stochastic Weak Majorization

- Use **stochastic coupling** to show weak majorization on queue vectors, in a **stochastic** sense.
- $\boldsymbol{U} = (U_1, \ldots, U_M)$, $\boldsymbol{V} = (V_1, \ldots, V_M)$ random vectors. $\boldsymbol{U}$ is said to be **stochastically weak-majorized** by $\boldsymbol{V}$, $\boldsymbol{U} \prec_w^{st} \boldsymbol{V}$, if there exist random vectors $\tilde{\boldsymbol{U}}$ and $\tilde{\boldsymbol{V}}$ such that

(a) $\boldsymbol{U}$ and $\tilde{\boldsymbol{U}}$ are identically distributed.

(b) $\boldsymbol{V}$ and $\tilde{\boldsymbol{V}}$ are identically distributed.

(c) $\tilde{\boldsymbol{U}} \prec_w \tilde{\boldsymbol{V}}$ a.s.

Strong Delay Optimality of LQHPR

Theorem 3 *Let* $\boldsymbol{q}_0$ *be initial queue state. Let* $\boldsymbol{Q}(t)$ *be queue evolution under* g_{LQHPR} *for* $t \geq 0$*. Let* $\boldsymbol{Q}'(t)$ *be corresponding quantity under any policy* $g \in G_{\mathcal{D}}$*. Then under all symmetric* $\mathcal{P}$*,*

$$\boldsymbol{Q}(t) \prec_w^{st} \boldsymbol{Q}'(t) \;\; \forall t \geq 0.$$

- Proof: generalize stochastic coupling argument for non-faded symmetric MAC.

Consequences

Corollary 1

$$\mathsf{E}\left[\varphi(\boldsymbol{Q}(t))\right] \leq \mathsf{E}\left[\varphi(\boldsymbol{Q}'(t))\right] \quad \forall t \geq 0$$

for all $\prec_w$-preserving $\varphi : \mathbb{R}^M \mapsto \mathbb{R}$ for which expectations exist.

- φ is $\prec_w$-**preserving** if $\boldsymbol{x} \prec_w \boldsymbol{y} \Rightarrow \varphi(\boldsymbol{x}) \leq \varphi(\boldsymbol{y})$ for $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^M$.
- $\prec_w$-preserving $\Leftrightarrow$ Schur-convex, increasing.
- Includes all symmetric, convex and increasing real functions on $\mathbb{R}^M$.
- Examples:
  $\varphi(\boldsymbol{x}) = \max_{i_1 < i_2 < \cdots < i_k}(|x_{i_1}| + \cdots + |x_{i_k}|)$, $1 \leq k \leq M$;
  $\varphi(\boldsymbol{x}) = \sum_{i=1}^{M} |x_i|^r$ for $r \geq 1$ or $r \leq 0$;
  $\varphi(\boldsymbol{x}) = (\sum_{i=1}^{M} |x_i|^r)^{1/r}$ for $r \geq 1$.

Summary and Conclusions

- General framework for resource allocation in fading MAC with random arrivals.
- Stability region $\mathcal{S} = \mathcal{C}(\overline{\boldsymbol{P}}, \hat{\boldsymbol{P}})$.
- $\max \sum_i u_i r_i$ (LQPHR) policy throughput optimal for given $\mathcal{P}$.
- LQHPR minimizes average packet delay for any symmetric $\mathcal{P}$ in symmetric scenario.
- LQHPR implements adaptive successive decoding at physical layer.

Summary and Conclusions

- "Converse": LQHPR establishes **fundamental throughput/delay performance limit** for any multiple access coding scheme which meets any given required P_e (Fano).
- "Achievability": To approach rates in $\mathcal{D}$, need sufficiently large T and code over large number of bits.