SOURCE CODING
AND
PARALLEL ROUTING

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CROSS-LAYER ISSUES

Compression (Layer 6) and Transmission (Layer 1)

- energy efficiency perspective.
- tradeoff between transmission (RF) and processing energy.
- in context of sensor networks, added feature of detection gives a special slant to compression

Compression (IT source coding) and Routing (Layer 3)

- coupling of information theory and networking.
- reveals novel trade-offs

MAIN IDEA

- **Multiple Description coding**
  - different (coupled) representations of source signals.
  - each description requires fewer bits than a single description.

- **Parallel Routing**
  - redundant transmission of packet copies over separate routes.
  - protects against long delays and/or errors

- **Joint Compression/Routing**
  - send each description over a separate route
  - “cancel” redundancy with compression

- **Trade-off study**

BACKGROUND

- Source emits i.i.d. Gaussian variables (0-mean, unit variance).
- D = mean squared error distortion
- R = representation rate (bits/symbol)
- $D = 2^{-2R}$
- Symbols are sent to a destination node; so modify distortion measure
- T: delay

$$D = \begin{cases} 2^{-2R}, & T \le \Delta \\ 1, & T > \Delta \end{cases}$$

- Think of each symbol as a separate "packet" of length R bits

BACKGROUND (Continued)

- Multiple (i.e. Double) Description Coding

  (Ozarow, ElCamal/Cover, Wyner etal circa ’80-’82)

$$R_1 + R_2 = R$$

$$D = \begin{cases} d_0 = \dfrac{2^{-2(R_1+R_2)}}{2^{-2R_1} + 2^{-2R_2} - 2^{-2(R_1+R_2)}} & , \ T_1 \leq \Delta \ \& \ T_2 \leq \Delta \\ d_1 = 2^{-2R_1} & , \ T_1 \leq \Delta \ \& \ T_2 > \Delta \\ d_2 = 2^{-2R_2} & , \ T_1 > \Delta \ \& \ T_2 \leq \Delta \\ 1 & , \ T_1 > \Delta \ \& \ T_2 > \Delta \end{cases}$$

- Each description is sent to destination over separate route
- ith description has rate R_i, individual mse distortion d_i, and delay T_i
- d_0 is joint distortion

BACKGROUND (Continued)

- Previous formula describes the boundary of the achievable rate-distortion region.
- "Inside" the region we have

$$d_i = 2^{-2R_i(1-\delta_i)}$$

$$d_0 = 2^{-2(R_1+R_2)} \cdot \frac{1}{1-(\sqrt{\Pi}-\sqrt{\Lambda})^2}$$

where $\Pi = (1\text{-}d_1)(1-d_2)$ & $\Lambda = d_1 d_2 - 2^{-2(R_1+R_2)}$

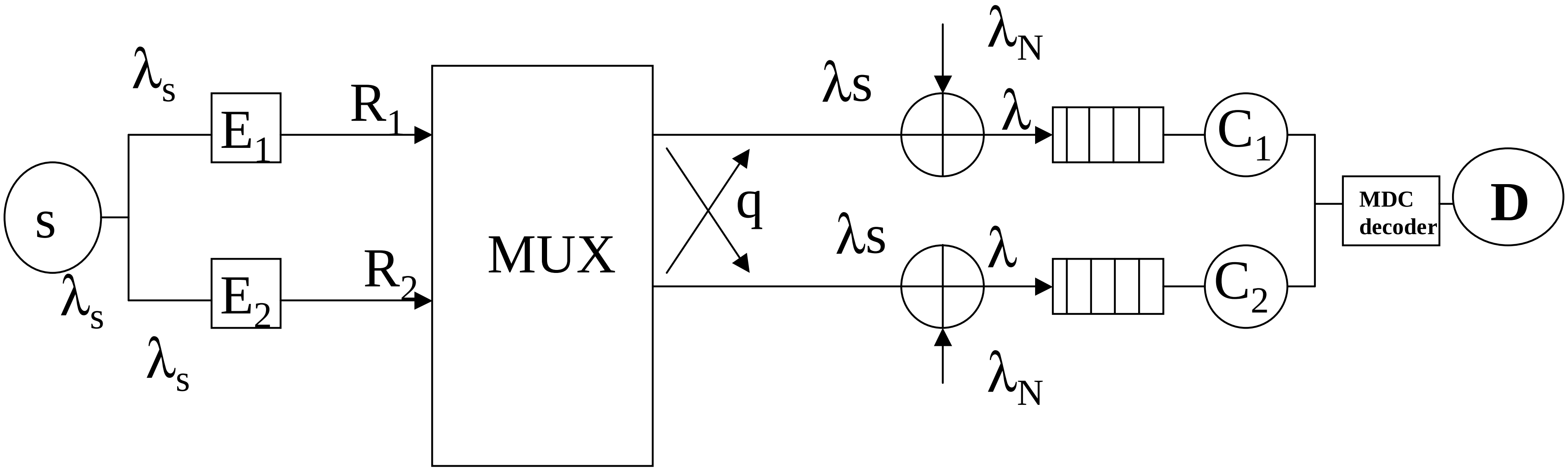
where $0 \le \delta_i \le 1$ represents the "redundancy" of the representations

- Note : $\delta_i \to 0$, no redundancy, "lean" compression, "effective" rate R_i, minimum distortion.

  $\delta_i \to 1$, maximum redundancy, ineffective compression, "effective" rate 0, maximum distortion

- Choice of δ affects distortion-rate values <u>and</u> representation complexity

SIMPLE NETWORK MODEL



Coding Parameters

$\alpha = \frac{R_1}{R_1 + R_2}$

$\delta_{1,}\ \delta_2$

λ_S = source symbol rate ("packets"/s)

λ_N = other traffic rate

$\lambda = \lambda_S + \lambda_N$

$R_1 + R_2 = R$ = bits/packet

q = network parameter

initially $C_1 = C_2$

- noiseless transmission

AVERAGE DISTORTION

$$E[D] = d_o \Pr[T_1 \le \Delta, T_2 \le \Delta] + \quad d_1 \Pr[T_1 \le \Delta, T_2 > \Delta] +$$

$$d_2 \Pr[T_1 >, T_2 \le \Delta] + \quad 1 \cdot \Pr[T_1 > \Delta, T_2 > \Delta]$$

<u>Objective</u>: Min E[D]

by choice of α, δ_1, δ_2, q

(for fixed R, $C_1 = C_2 = C$, λ, Δ)

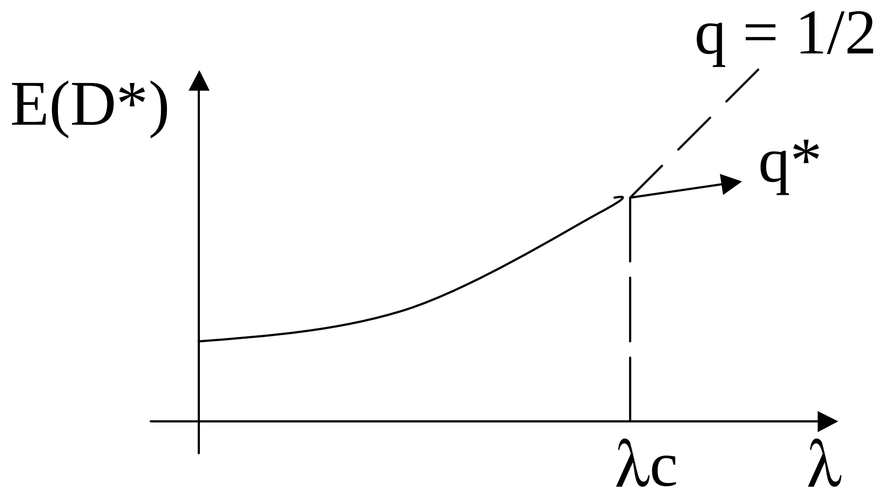
- Need queuing analysis to express the delay probability (use M/G/1 formulas)

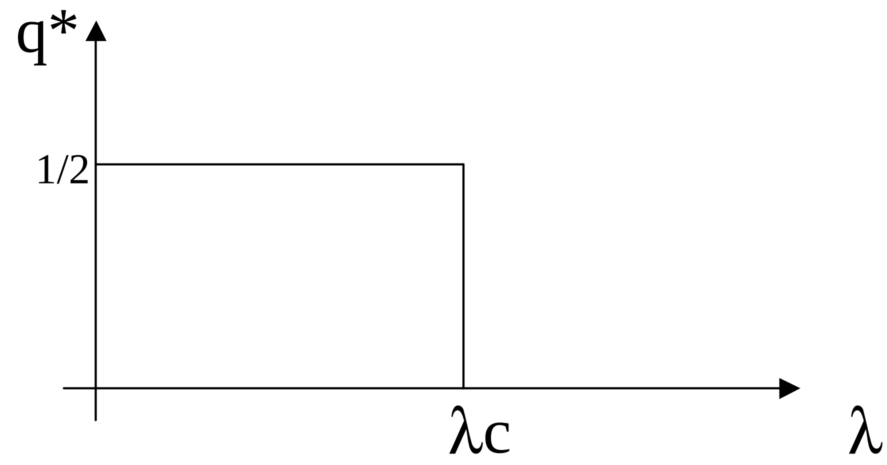
- Perform Numerical Minimization

- * = will denotes optimal values

FIRST RESULTS

- Phase Transition Behavior

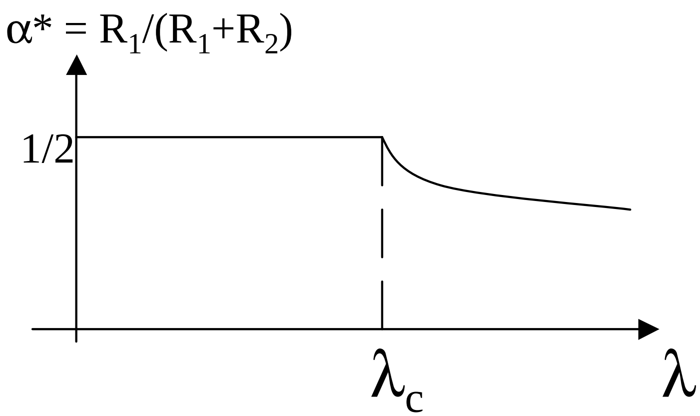




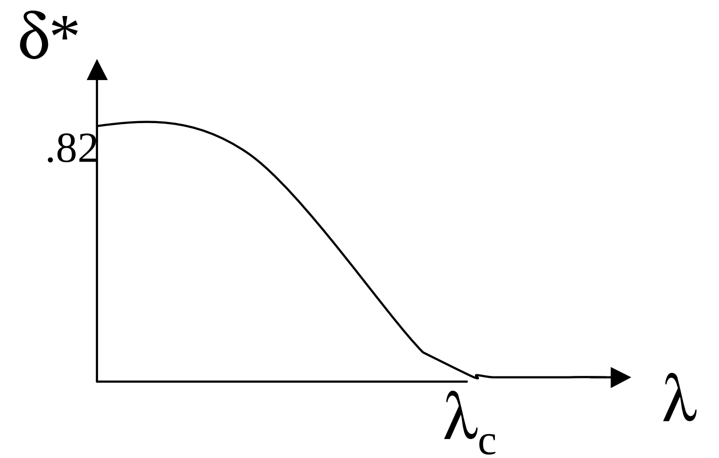
- Beyond a critical load value do not mix traffic
  (i.e. dedicate each description completely to its path)

- Below that value mix thoroughly (50-50)

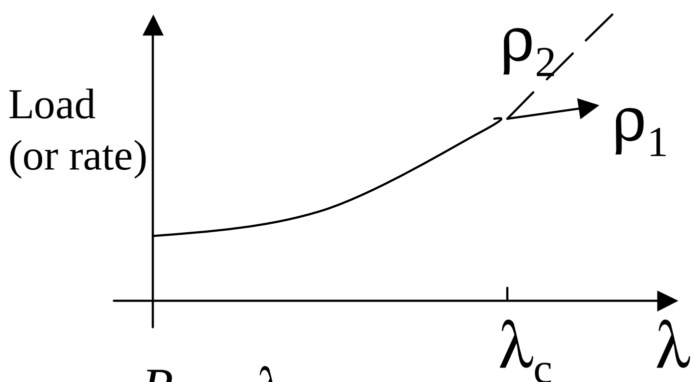
FIRST RESULTS (Continued)



Below λc encode symmetrically (no advantage to differentiate descriptions)



Gradually drop the redundancy Factor to zero (“lean” compression)

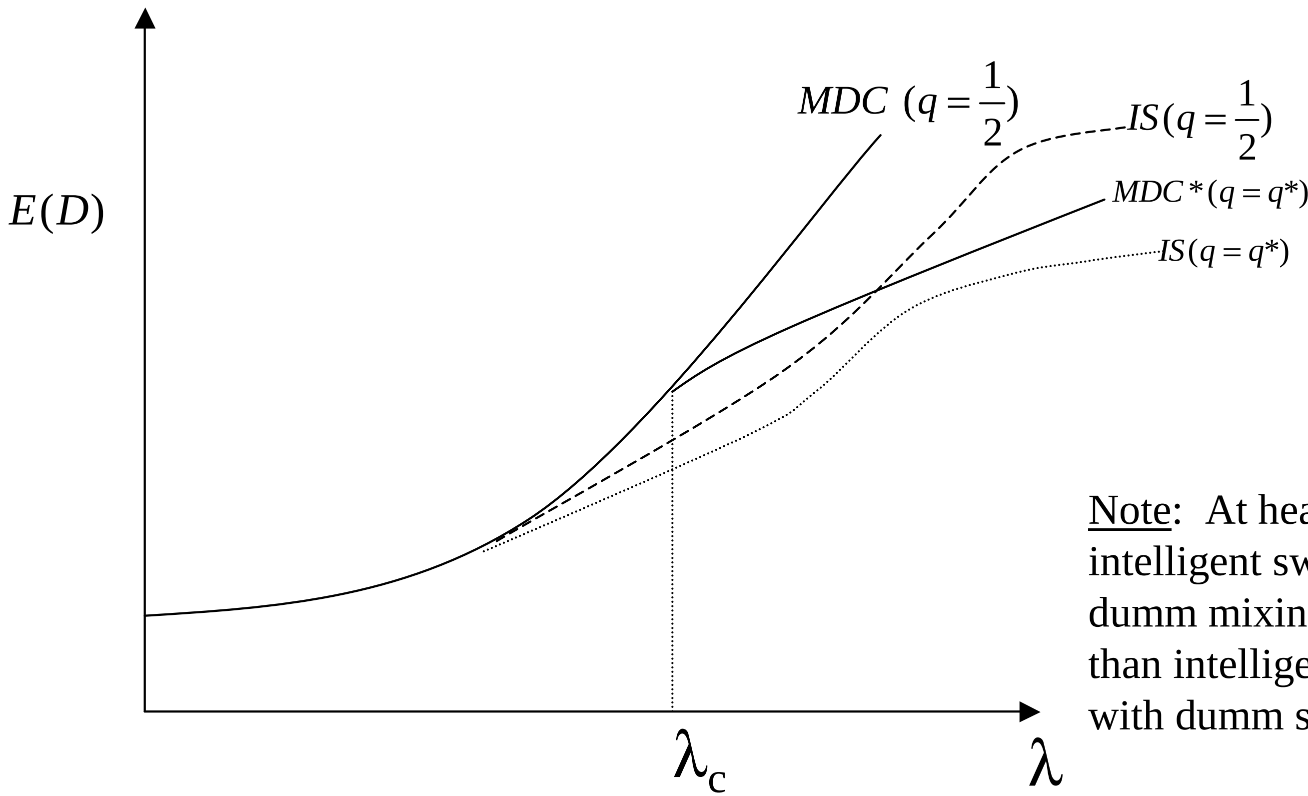


$$\rho_i = \lambda \frac{R_i}{C_i} = \frac{\lambda}{C} R_i$$

- Keep the load on one queue below saturation and send all the remaining traffic to the other queue

INTELLIGENT SWITCH

- drop packets whose sojourns times exceed Δ (while still in queue).
- only change: “impatient customer” queuing behavior



<u>Note</u>: At heavy loads intelligent switch with dumm mixing is worse than intelligent mixing with dumm switch

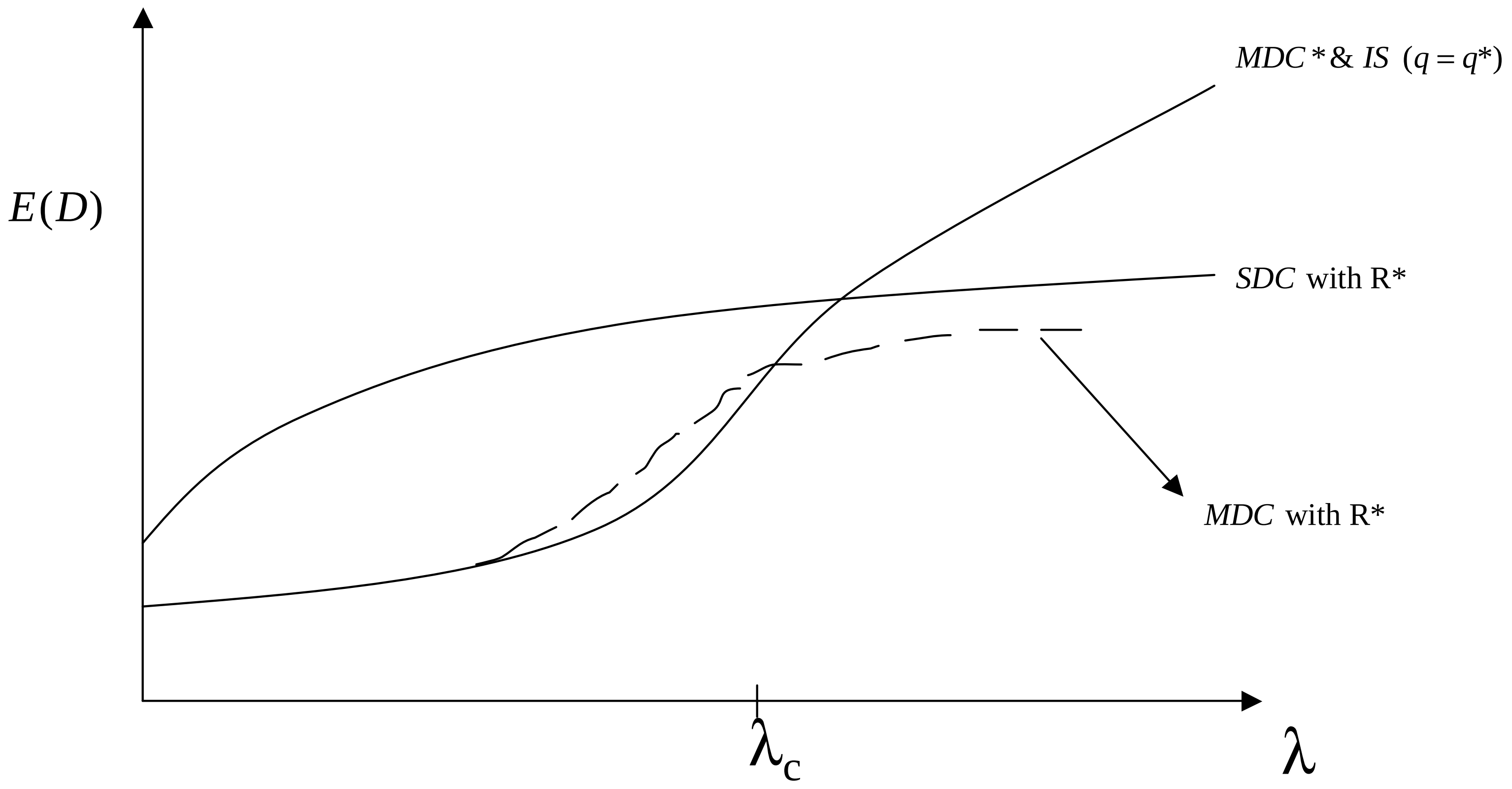
Explanation:
- IS drops packets “uniformly” at both queues
- Optimal mixing gives up on one queue totally (garbage bag) but keeps one queue maximally useful

PROBING FURTHER

- So far *R* was fixed (total rate)
- As λ increases, we may be able to control the load by manipulating packet lengths without the constraint that
  $R_1 + R_2 = \textit{fixed}$
- If there is an optimal R*, by symmetry we should have
  $$R_1^* = R_2^* = \frac{R^*}{2}$$
- Also, since both queues would be equally loaded, packets would be lost with low probability at both as we decrease *R*; hence we should choose δ_1^*, δ_2^* to minimize d_0
- In fact, then, $\delta_1^* = \delta_2^* = \frac{1}{R^*} \log \frac{2^{R^*} + 2^{-R^*}}{2}$

  & $d_0^* = 2^{-2R^*}$
- Not optimal

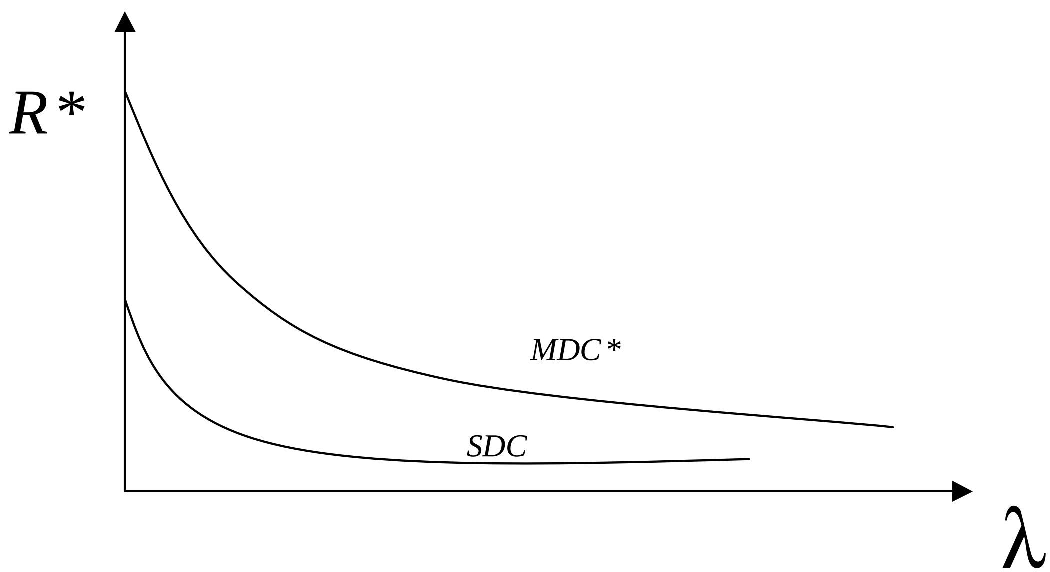
CONFIRMATION



Even SDC with optimal R* outperforms MDC* with IS at high loads

CONFIRMATION (CONTINUED)

- If instead of minimizing d_0 we minimize E[D] we find that both R^* and E(D) are indistinguishably close (hence, intuition was good)
- At very low loads ($\lambda \to 0$), one might expect that the optimum R^* might increase without bound.
- This is not the case (very long packets increase the delay sufficiently to wipe out distortion gains)



FURTHER THOUGHTS

- Are these trade-offs extendable to non-Gaussian symbols and non-trivial networks paths?
- Can we translate the results to practical compression schemes?
- What are the energy implications of the trade-off? Do we spend more or less energy when we use parallel paths with multiple descriptions?
- What happens if noise is added in the system?
- What happens in a wireless environment where inadvertent multicasting occurs?