DIMACS Workshop on

Algorithmic Mathematical Art: Special Cases and Their Applications

May 11 - 13, 2009

DIMACS Center, CoRE Building, Rutgers University

Jean-Marie Dendoncker

1. Projects : IT'S MATHEMAGIC - Van Maat tot Math

0. A picture of the context

A primary school with

- 100% of the children who don't speak Dutch at home
- 65 % of the children are underprivileged
- 45% are refugees

"How can we help these children?"

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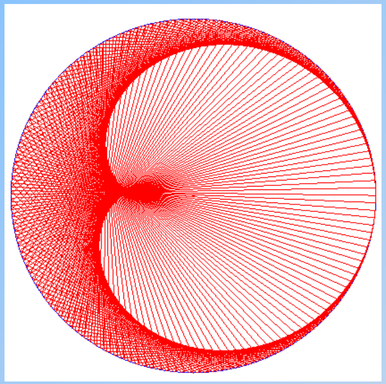
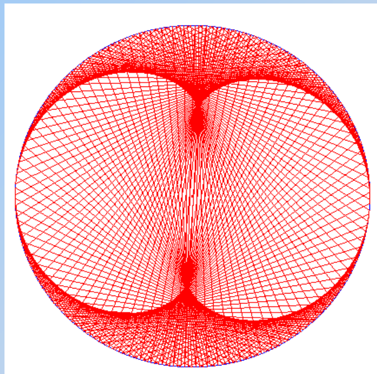
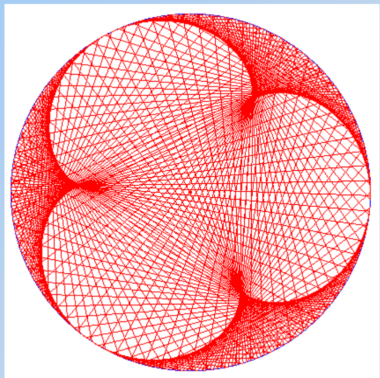
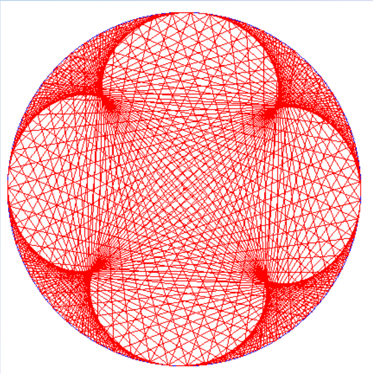
1.1 Arithmetic algorithm: tables of multiplication









1.2 Geometric algortihm: wavefronts in a two dimensional representation

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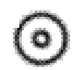

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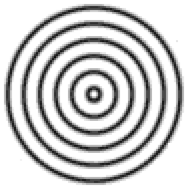

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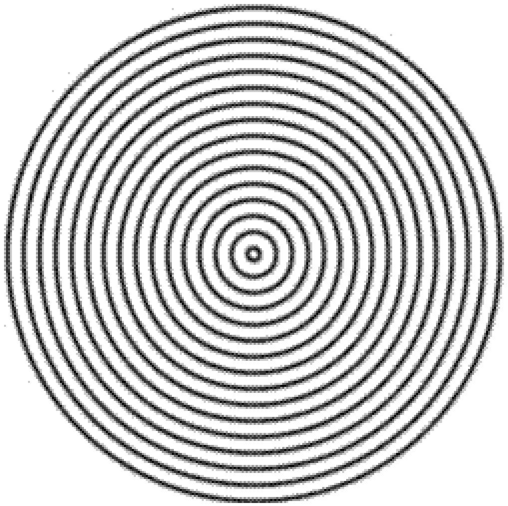

STOP !!

Play Back

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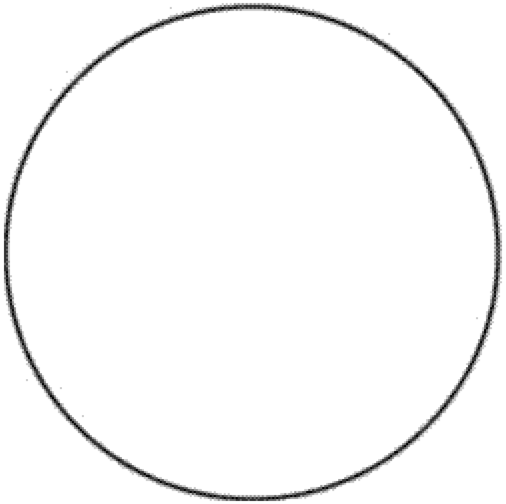

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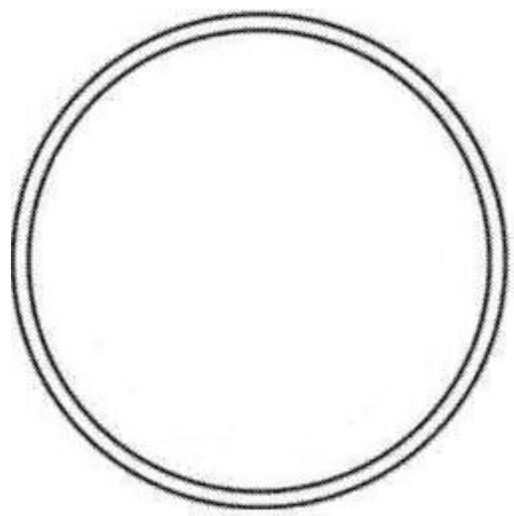

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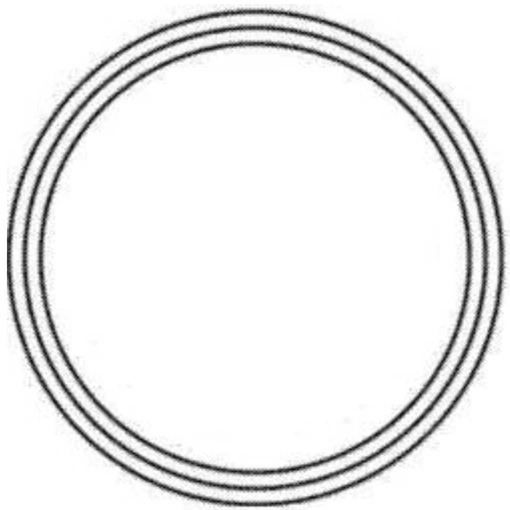

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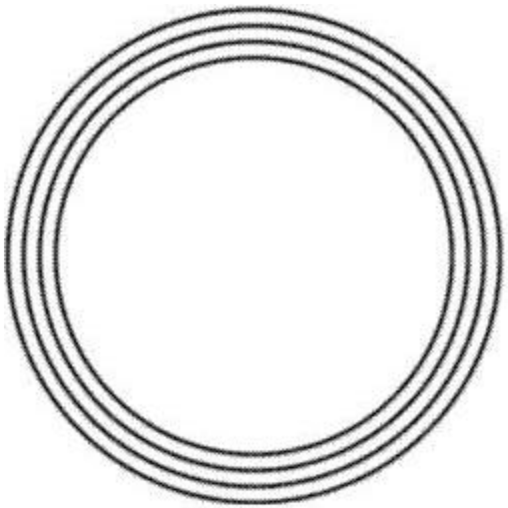

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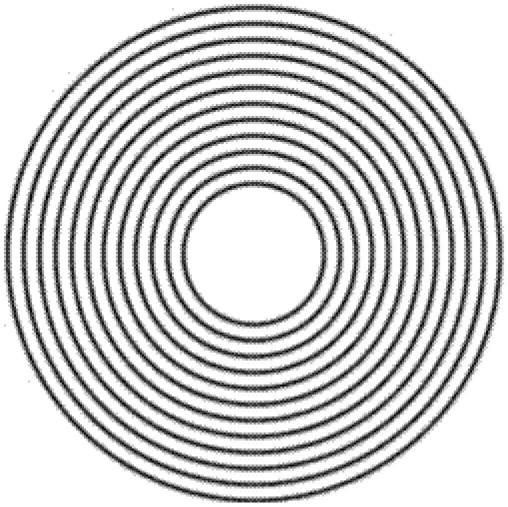

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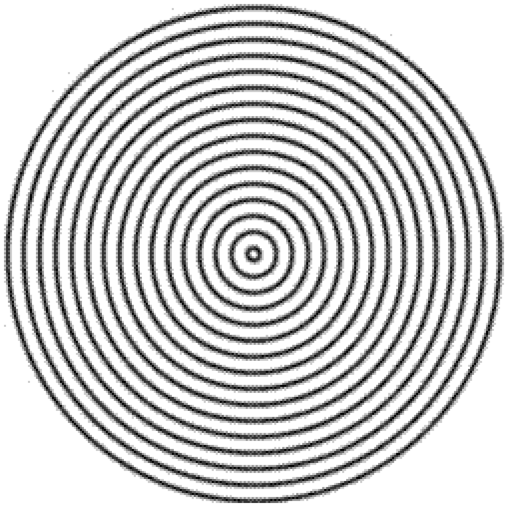

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1.2 Geometric algortihm: wavefronts in a two dimensional representation

What if the basic curve is an ellipse instead of a circle?

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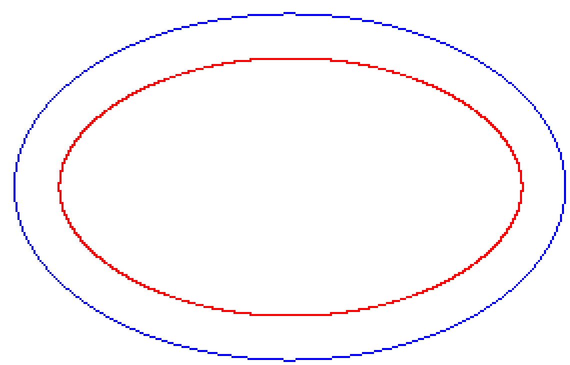
1.2 Geometric algortihm: wavefronts in a two dimensional representation



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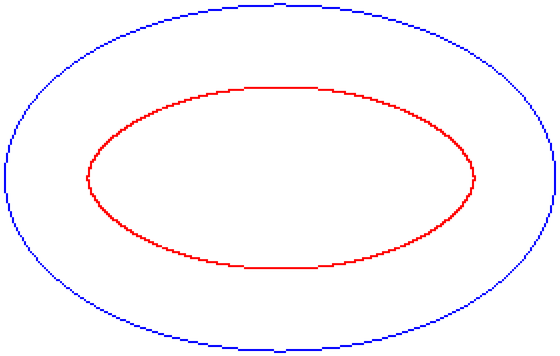

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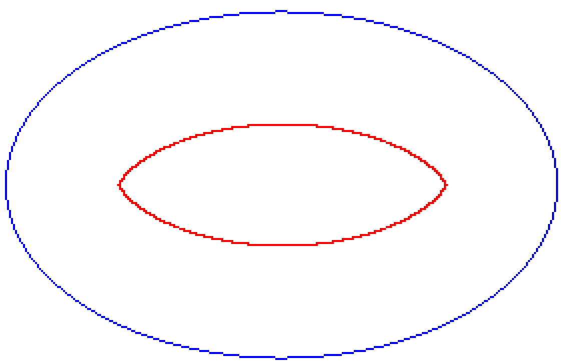

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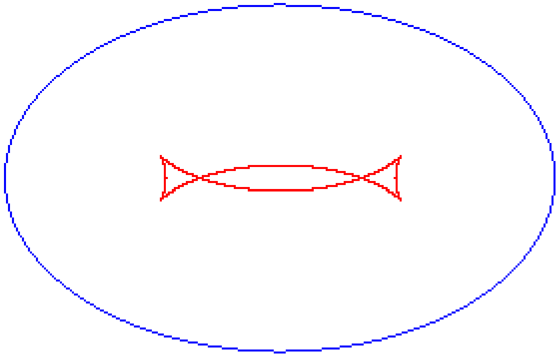

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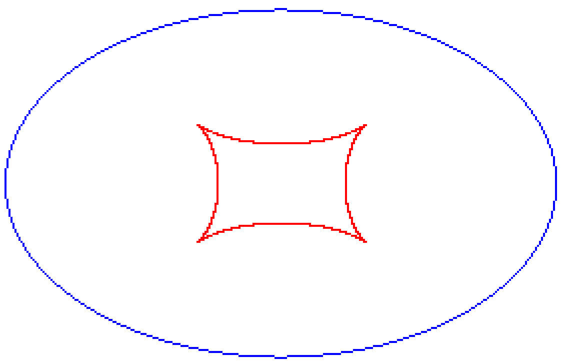

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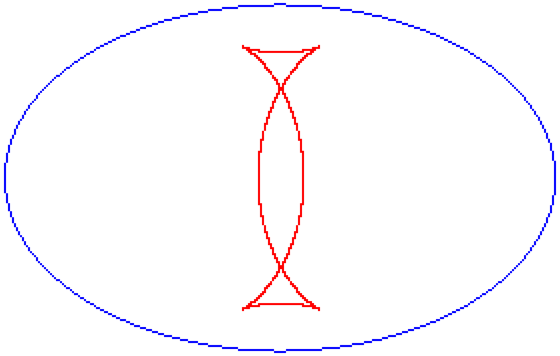

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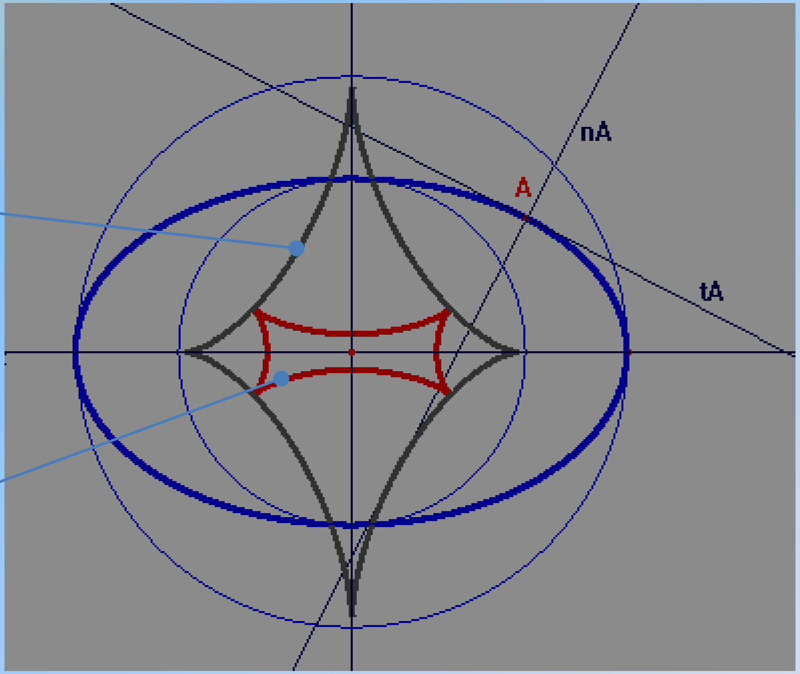
1.2 Geometric algortihm: wavefronts in a two dimensional representation

Solution:

Evolute ellipse



Tetracuspid curve

1.3 Geometric algortihm in a three dimensional representation

A further generalization is to visualize curves in space

1.3.1 Wavefront surface
1.3.2 Cardioid and nephroid
1.3.3 Hyperbolic paraboloid
1.3.4 Conoid
1.3.5 Surface of Scherk
1.3.6 Elliptic surface

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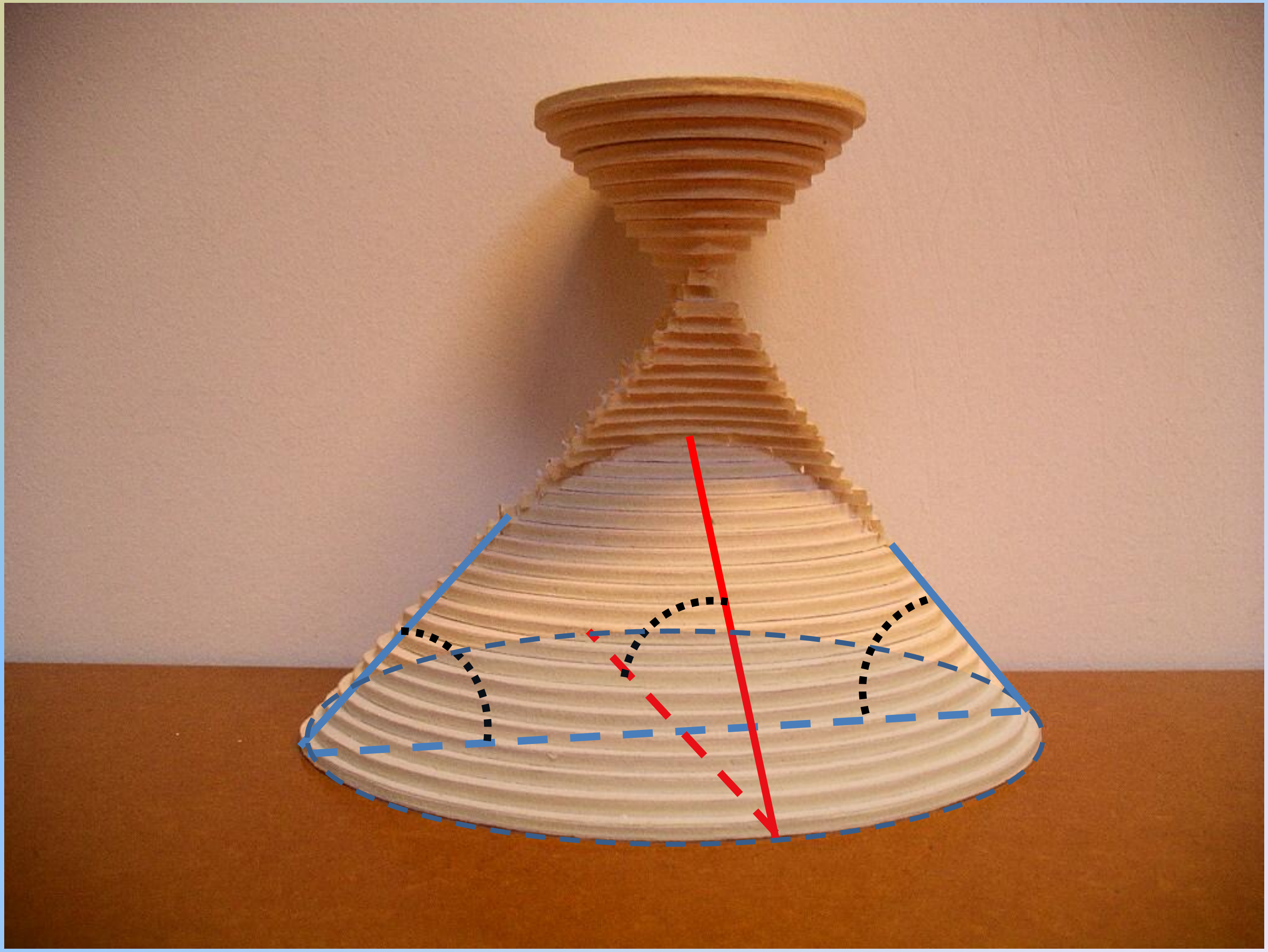
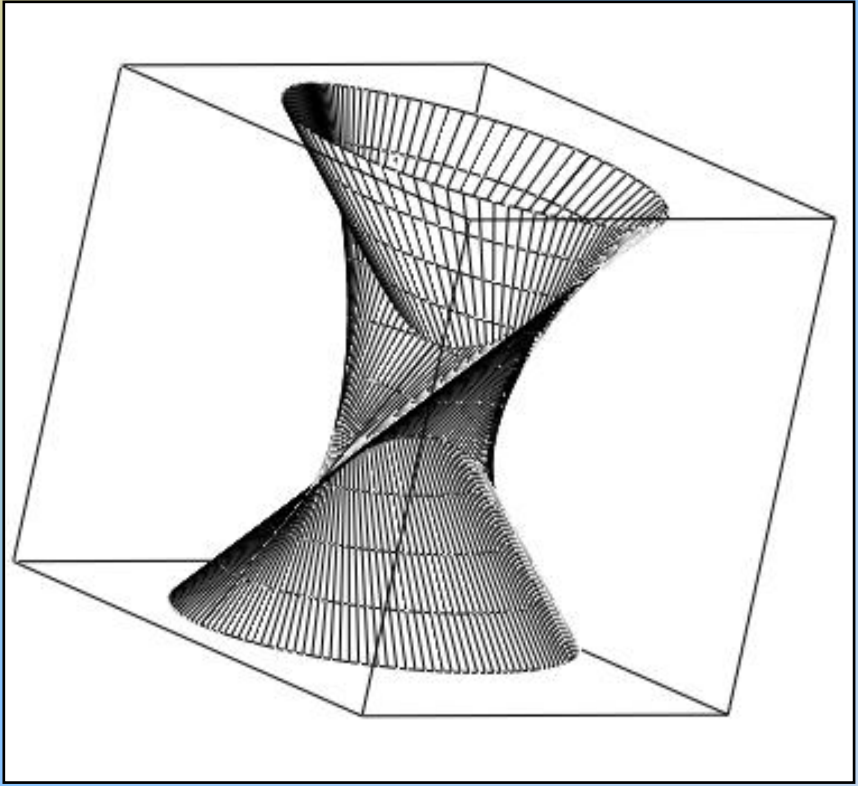
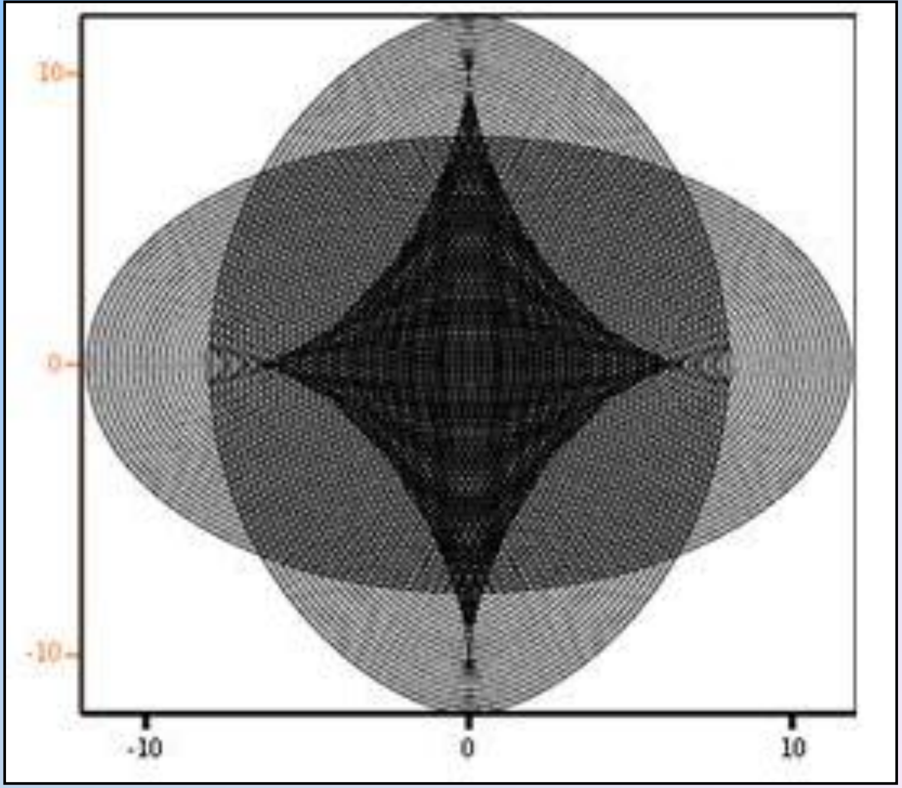
1.3.1 Wavefronts in a three dimensional representation



Some properties of the wavefront surface

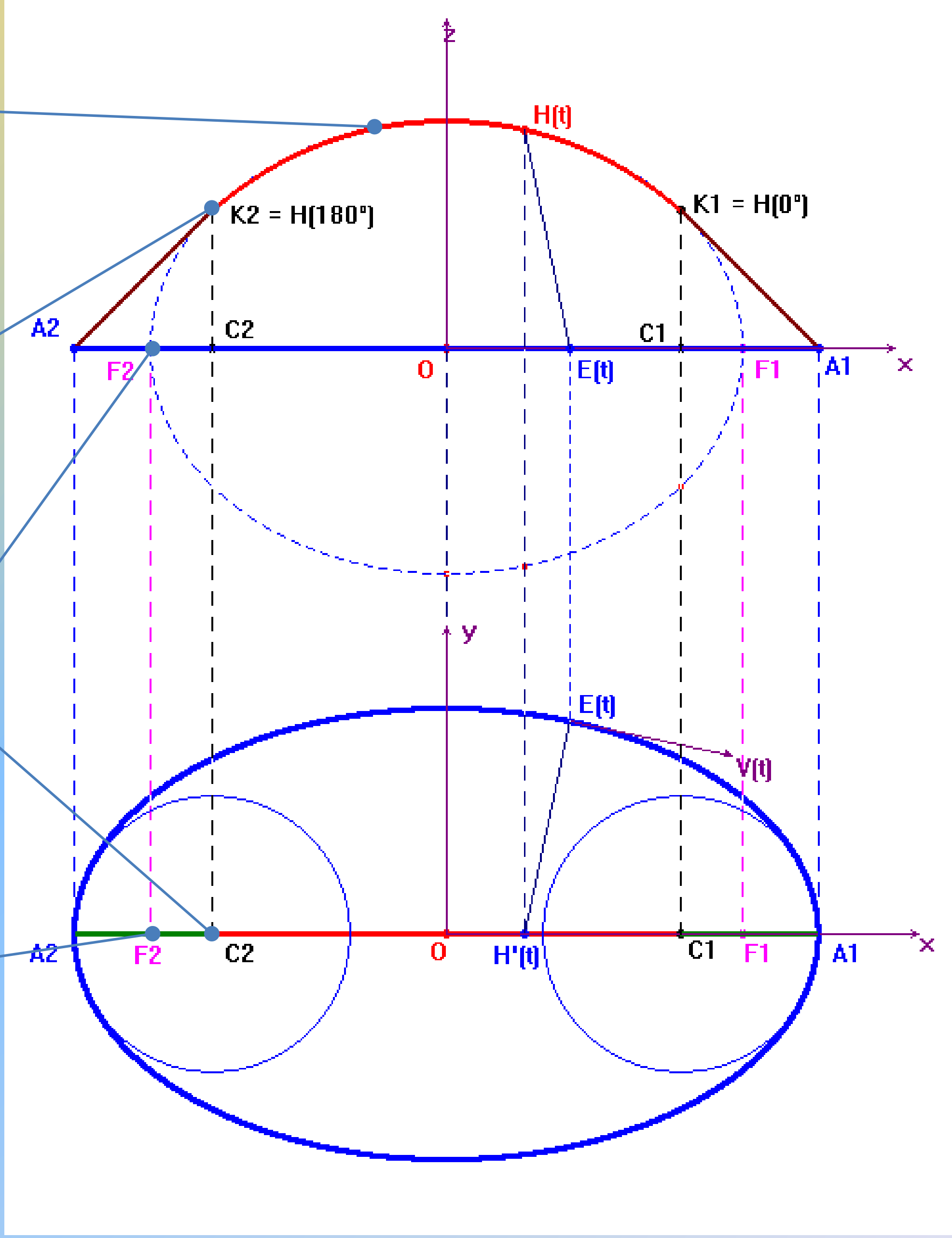




elliptic ridge

Point of curvature of the basic ellipse

focal point of the basic ellipse



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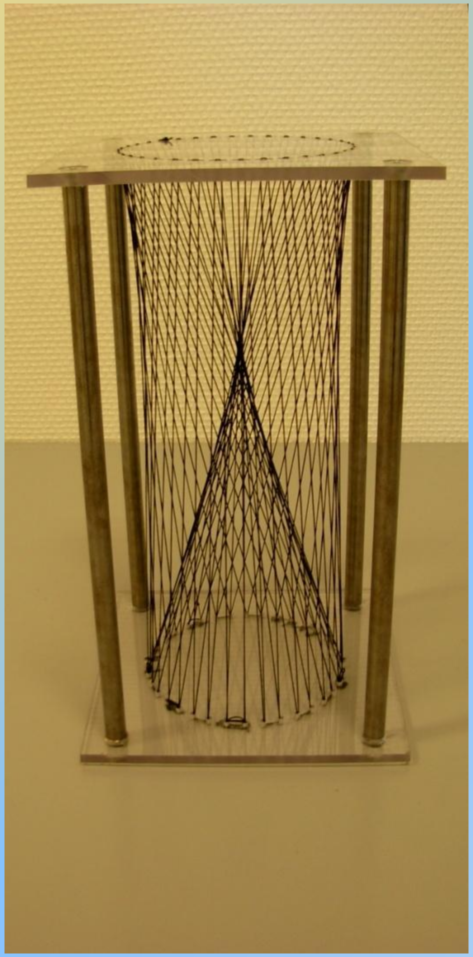
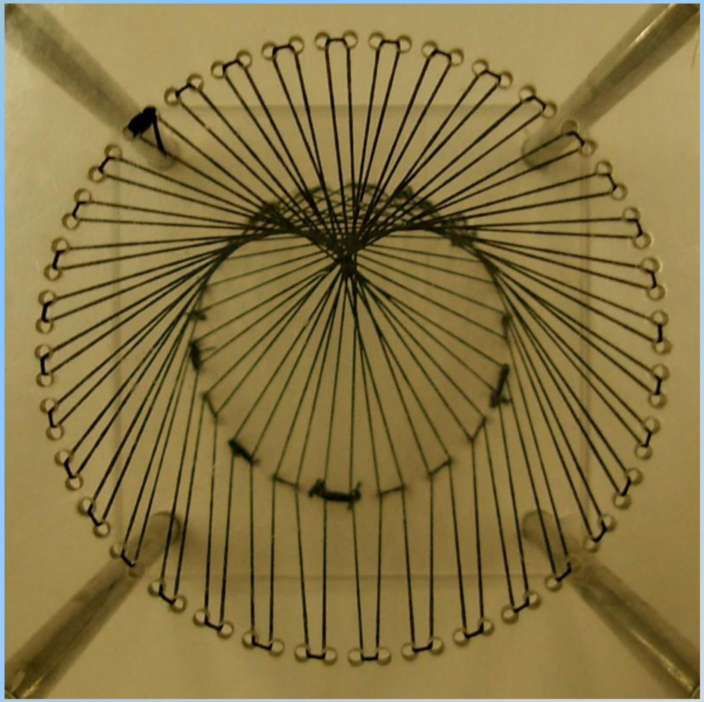
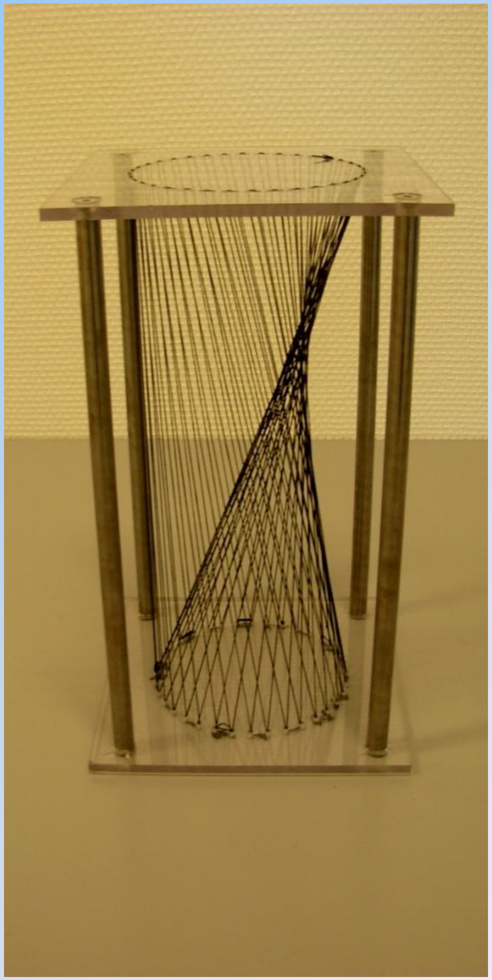
1.3.2 Cardioid and nephroid in a three dimensional representation







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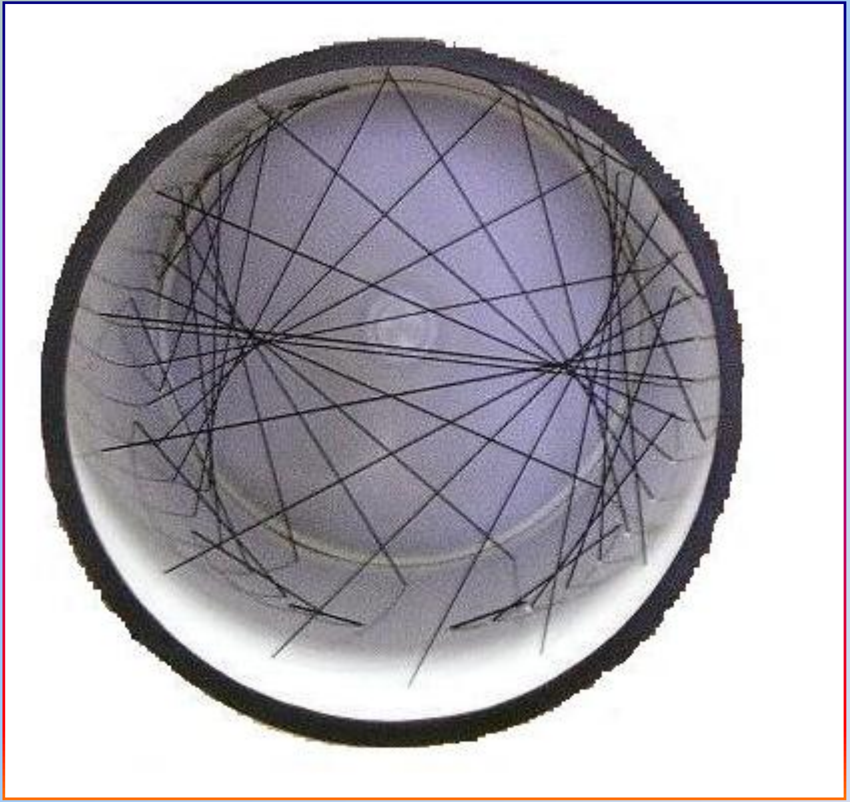
1.3.2 Cardioid and nephroid in a three dimensional representation



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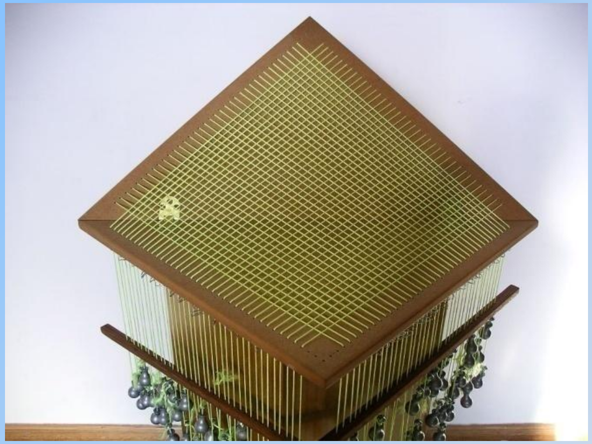
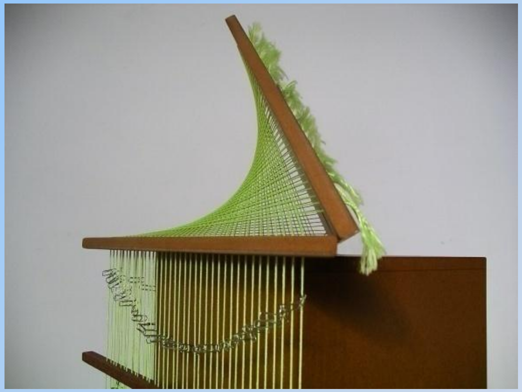
1.3.3 Hyperbolic paraboloid

Using the same way of curve stitching to visualise a parabola it's possible to do the same in space.





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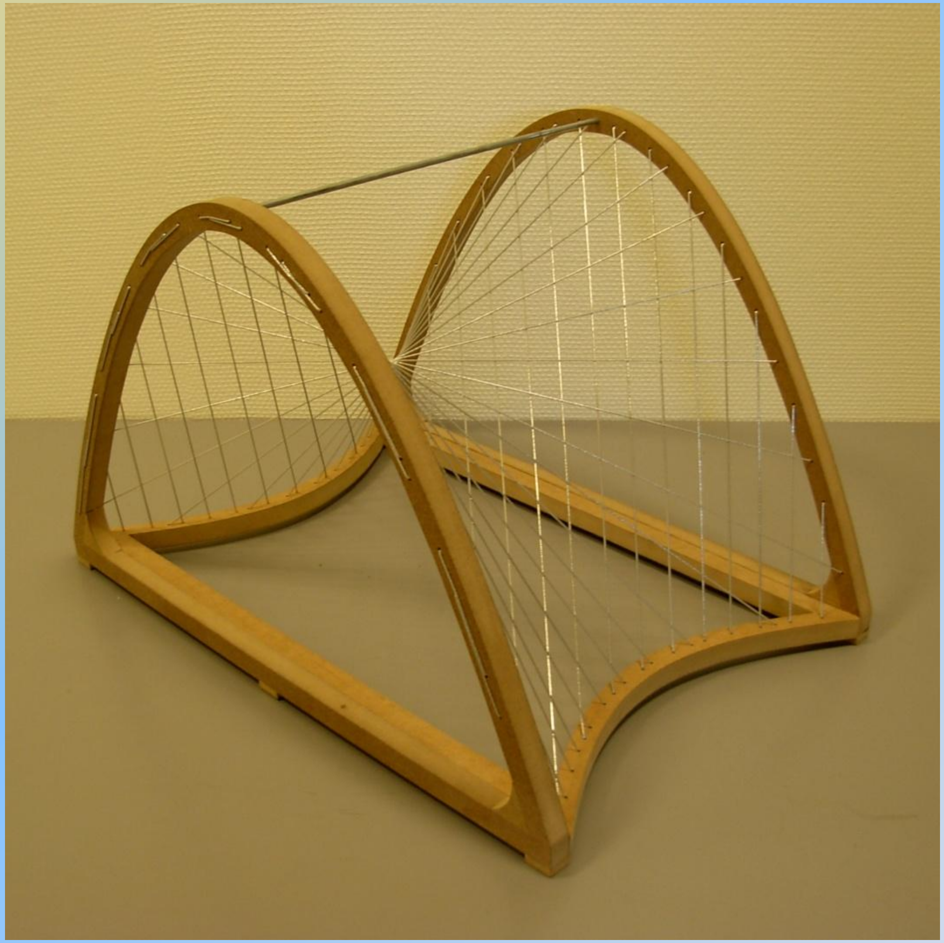
1.3.3 Hyperbolic paraboloid



1.3.3 Hyperbolic paraboloid ??

I have a little problem:

‘ There’s a hole in my bucket ‘

by Harry Belafonte

(and my mother)

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1.3.3 Hyperbolic paraboloid , NO

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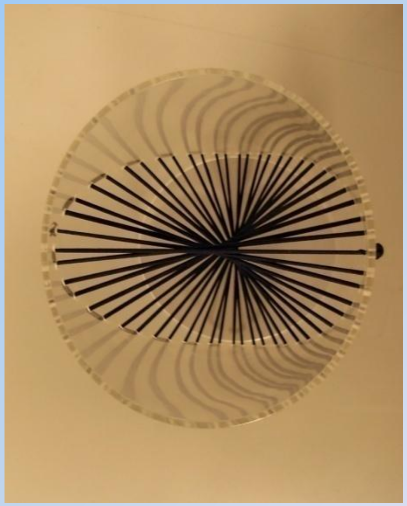
1.3.4 Conoid



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1.3.5 Surface of Scherk

The hyperbolic paraboloid should not be confused with the surface of Scherk (1798-1885). This surface is the only non trivial minimal translation surface. It can be given, with disregard of a translation and homothetic transformation, by the equation .

$$z = \ln\left|\frac{\cos y}{\cos x}\right|$$

It is formed by shifting in perpendicular planes without losing contact with each other the two curves ,

$$g(x) = -\frac{1}{c}\ln\left|\cos(cx + c_0)\right| + c_1 \qquad h(x) = \frac{1}{c}\ln\left|\cos(cx + d_0)\right| + d_1$$

with integration constants c_0, c_1, d_0, d_1

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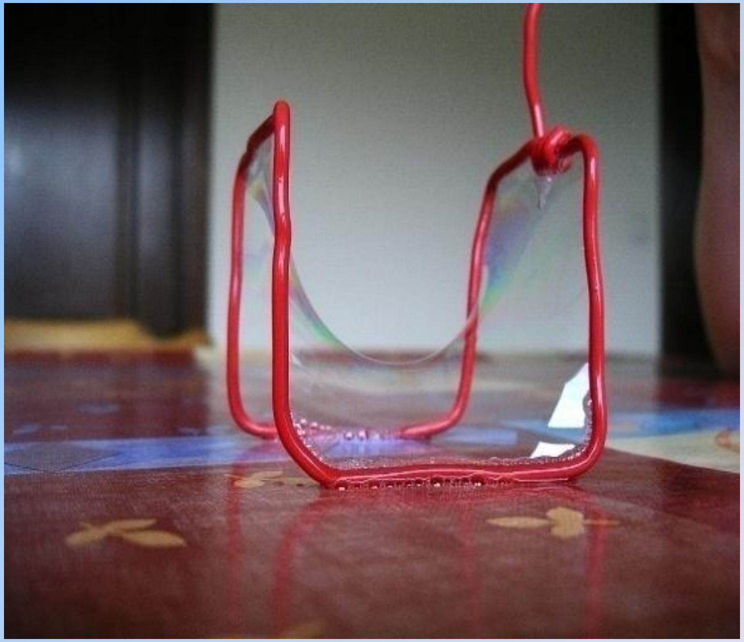
1.3.5 Surface of Scherk

as a translation surface



as a minimal surface

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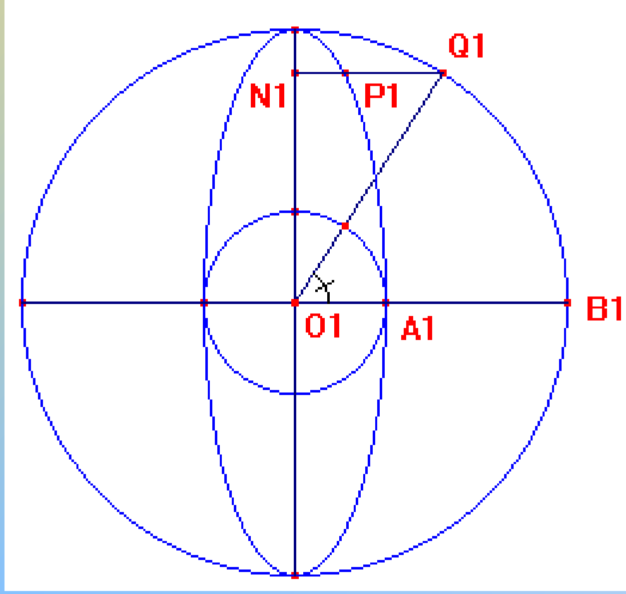
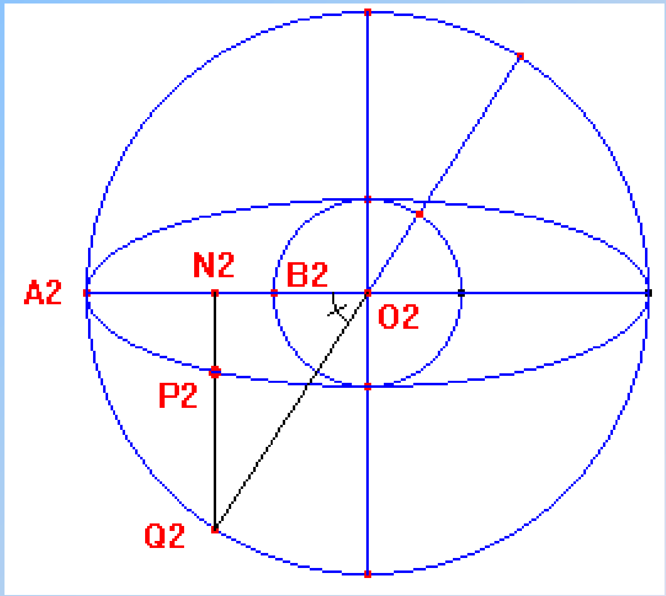
1.3.6 Elliptic surface

$$|O_1A_1| = \frac{1}{3}|O_1B_1|, \quad |O_2B_2| = \frac{1}{3}|O_2A_2| \qquad B_1\hat{O}_1Q_1 = A_2\hat{O}_2Q_2$$





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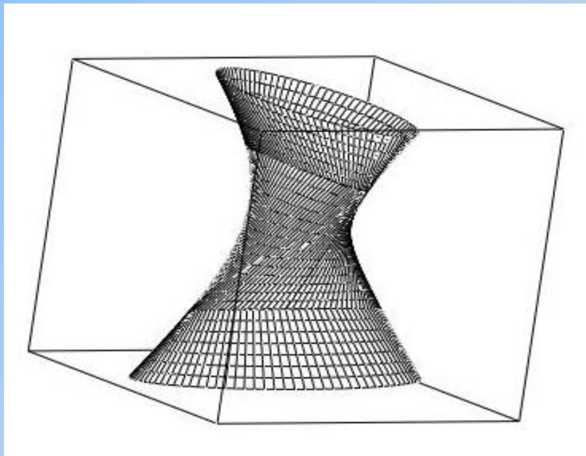
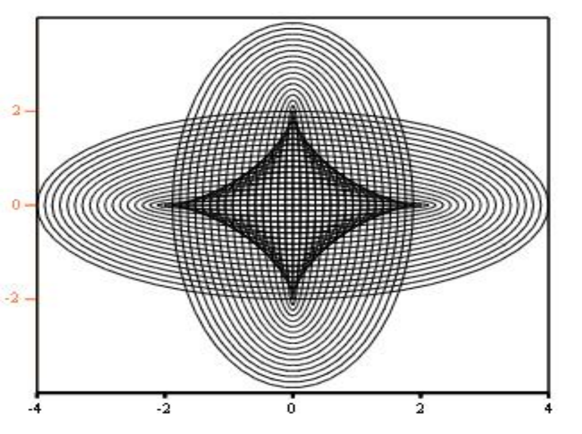
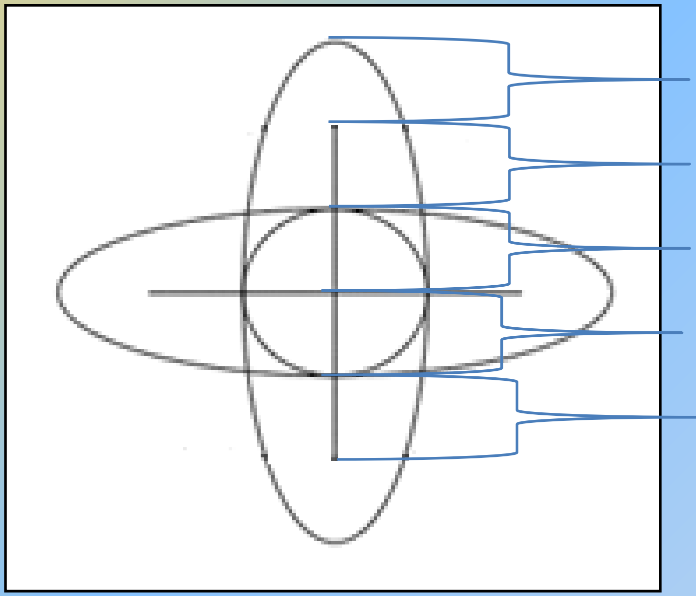
1.3.6 Elliptic surface





1.3.6 Elliptic surface

Some properties of the elliptic surface



1.
Contour lines on ¼- ½- ¾ of the distence between the two ellipses .

2.
The angle between the rulings and the plane of the ellipses is constant.

3.
The length between two connected points P_1 and P_2 is constant.

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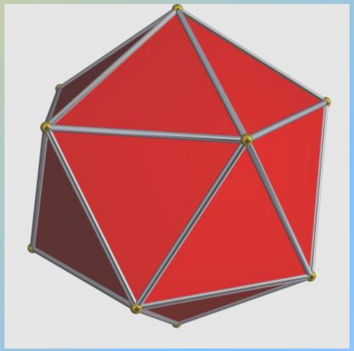
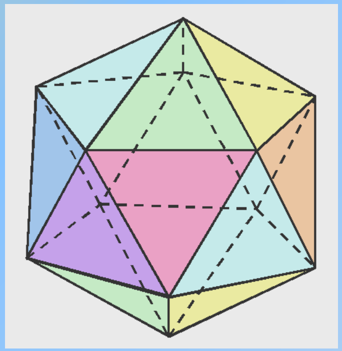
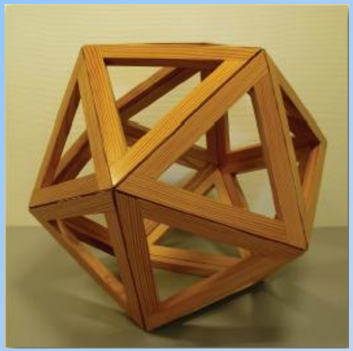
1.3.7 Two experiments : the Euler characteristic

Find the mystery of Euler

Exercise 1 Exercise 2 Exercise 3







Only after making the real models, more children understood the general calculating method.

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1.3.7 Two experiments : the mobile hyperboloid

Let 's try

1.3.7 Two experiments : conclusion

I can conclude as Prof. Eisenberg of the University of Colorado describes in his paper 'Mathematical Crafts for children: Beyond Scissors and Glue', *that mathematical crafts have to be seen as a strong element of mathematical education*.

It's clear that the use of algorithms gives to young underprivileged children a better structure not only in the use of mathematics but also in their lives.

2. Higher degree mathematics IT'S MATHEMAGIC

2.1 Finite geometry

A second type of examples of a practice of algorithms in mathematics is the use of graph theory.

The goal of this project was to make some representations of models that occur in a finite space.

2. Higher degree mathematics IT'S MATHEMAGIC

2.1 Fano configuration (7,7,3)

This problem has no concrete solutions.
Instead of using lines as real lines, we represented them as points. Lines are then nothing else then a sequence of three points.
After numbering the points respectively 1, 2, 3, … 7, it is possible to find the lines as

{1, 2, 3}, {1, 4, 5}, {1, 6, 7}, {2, 4, 6}, {2, 5, 7}, {3, 4, 7}, {3, 5, 6}

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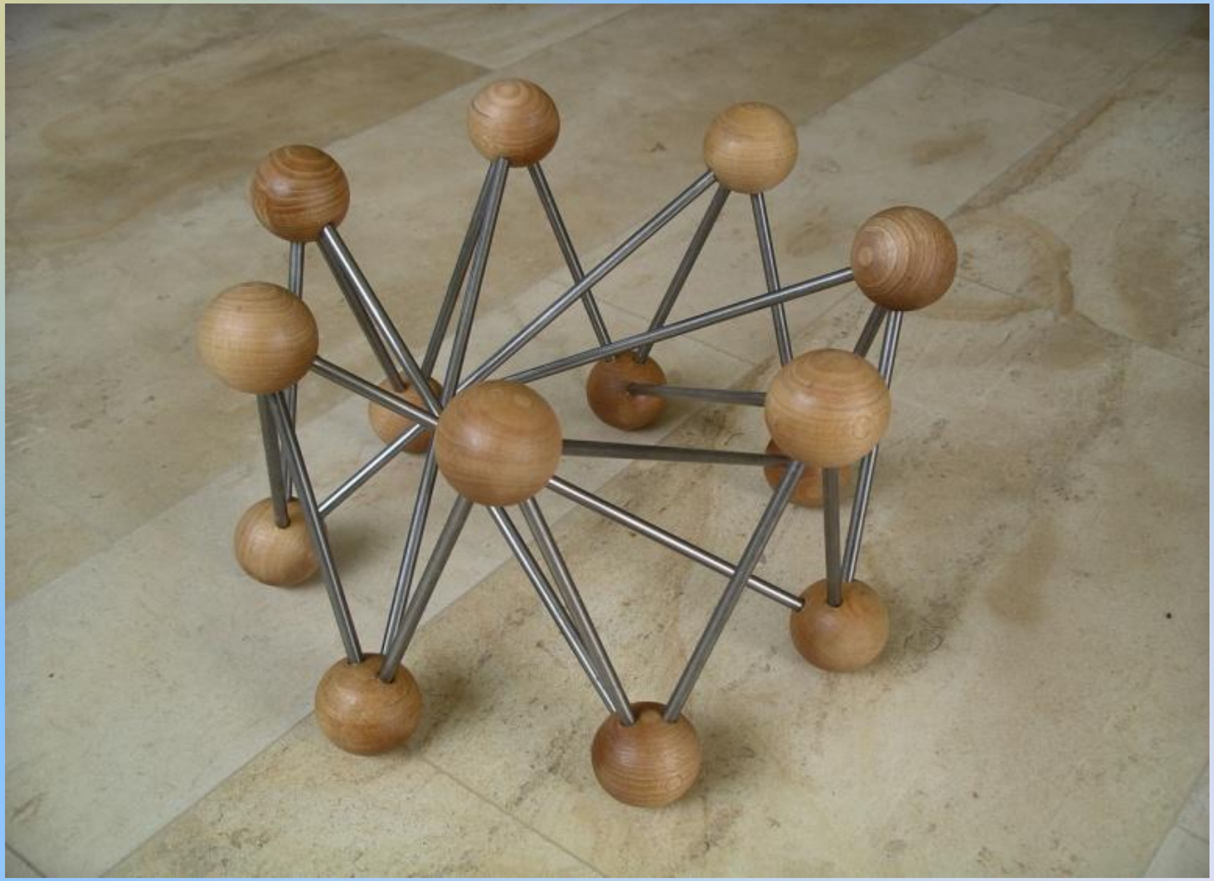
2.1 Fano configuration



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2.2 Desargues configuration (10,10,3)

To make a realizable spatial configuration, you number the points as 12, 13, 14, 15 | 23, 24, 25 | 34, 35 and 45

Solution:

The lines are formed by three points with only three different digits. Doing this you obtain:

{12, 13, 23}, {12, 14, 24}, {12, 15, 25}, {13, 14, 34}, {13, 15, 35}, {14, 15, 45}, {23, 24, 34}, {23, 25, 35}, {24, 25, 45} and {34, 35, 45}

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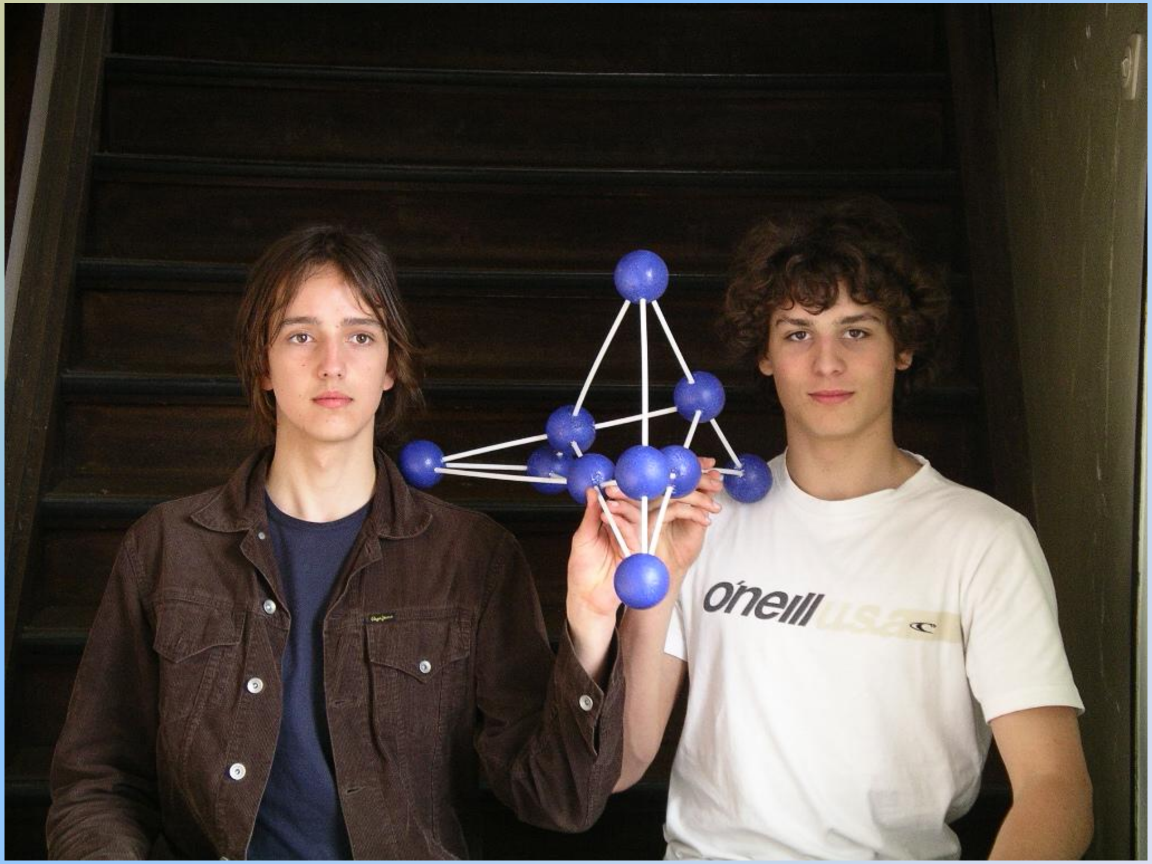
2.2 Desargues configuration



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2.3 (15,15,3) Tutte configuration (15,15,3)

For this purpose the points are numbered as

12, 13, 14, 15, 16	23, 24, 25, 26	34, 35, 36	45, 46	56

.

Solution:

Three points form a line if their numbers contain all the digits from 1 to 6

{12, 34, 56}, {12, 35, 46}, {12, 36, 45}, {13, 24, 56}, {13, 25, 46}, {13, 26, 45}, {14, 23, 56}, {14, 25, 36}, {14, 26, 35}, {15, 23, 46}, {15, 24, 36}, {15, 26, 34}, {16, 23, 45}, {16, 24, 35} {16, 25, 34}.

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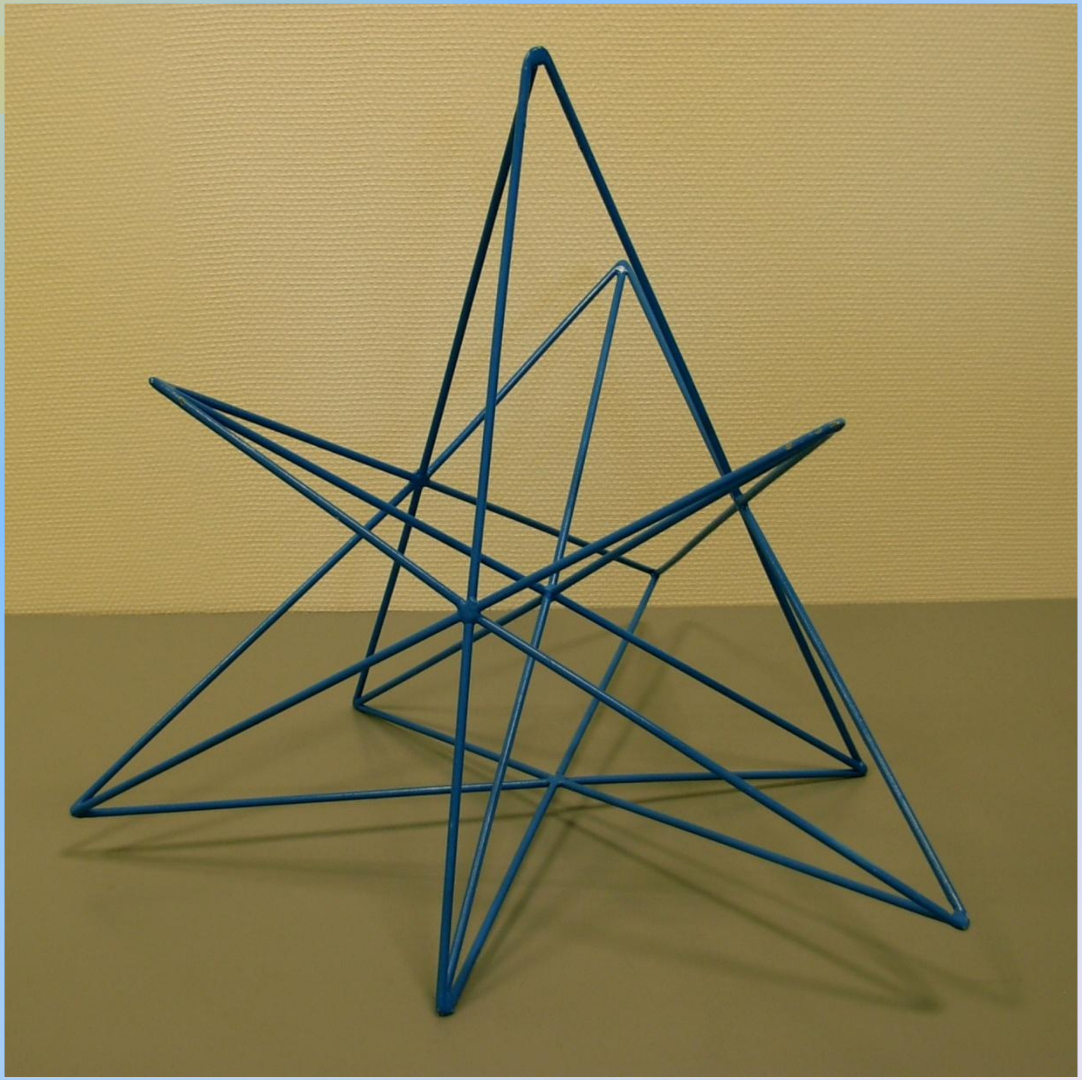
2.3 (15,15,3): GQ(2,2)



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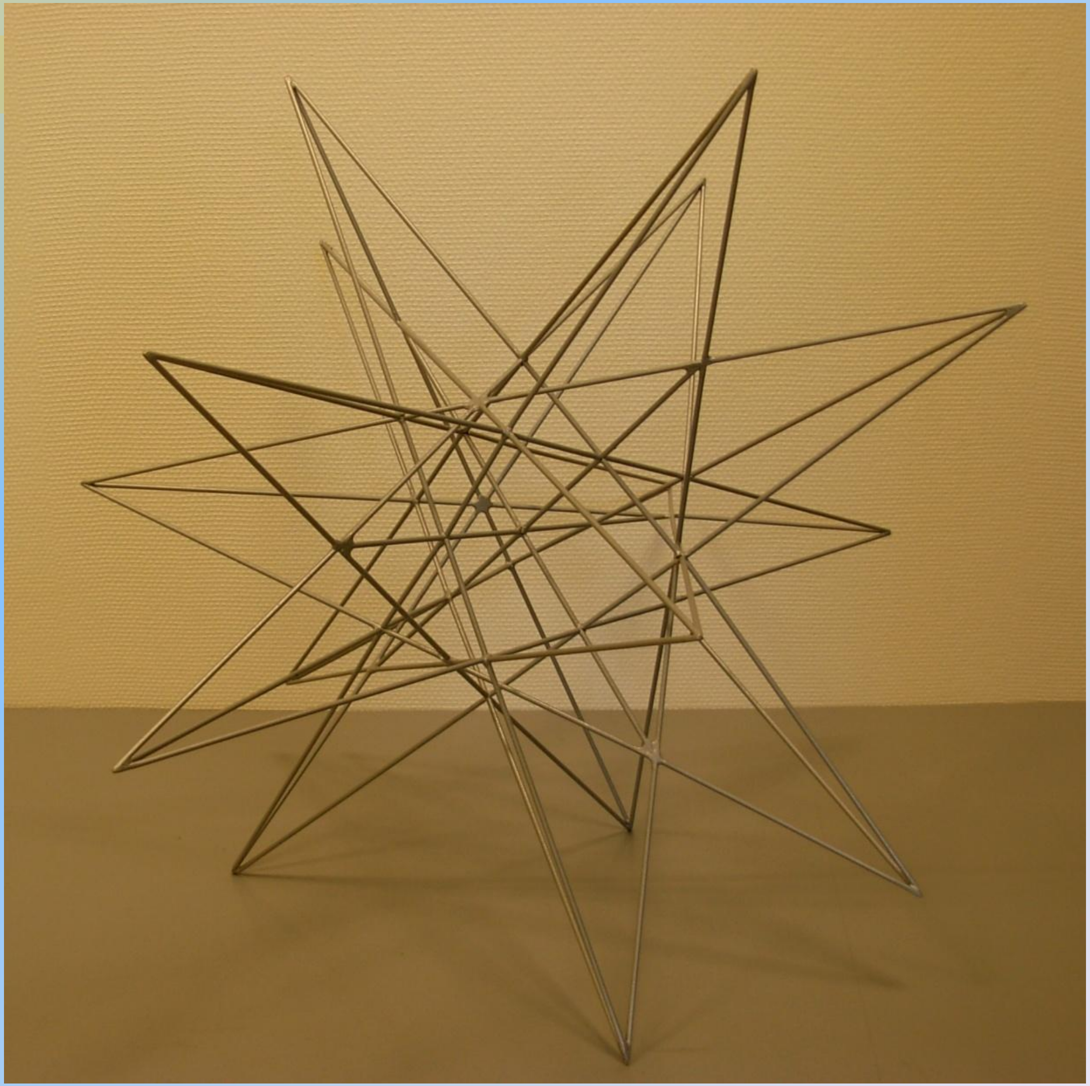
2.4 (45,27,3): GQ(4,2)



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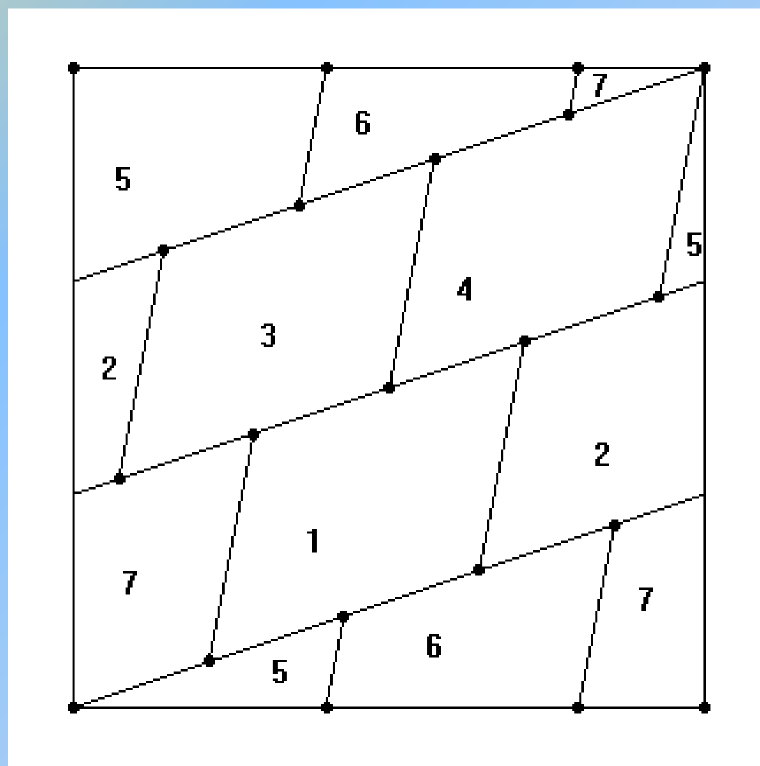
2.5 Exercise 1: a colorproblem on a torus

How many colors are at least needed to color an arbitrary map on a torus so that each adjacent 'country' has another color?

Solution:



{1, 3, 4}

{2, 4, 5}

{3, 5, 6}

{4, 6, 7}

{5, 7, 1}

{6, 1, 2}

{7, 2, 3}

i.e. the Fano configuration

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2.5 Exercise 1: model

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2.5 Exercise 2: two new olympic disciplines

Shot put

Discus throw

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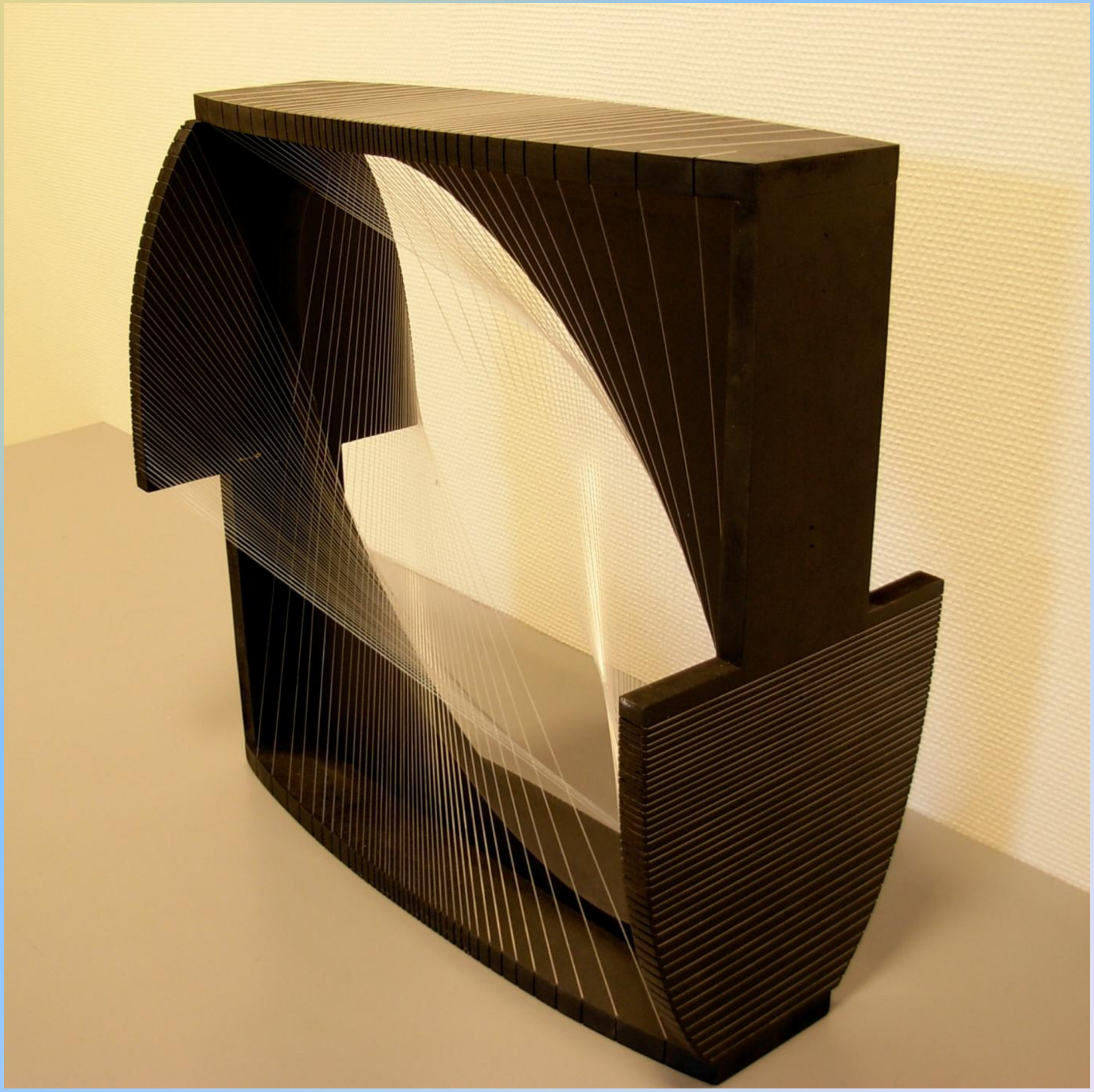
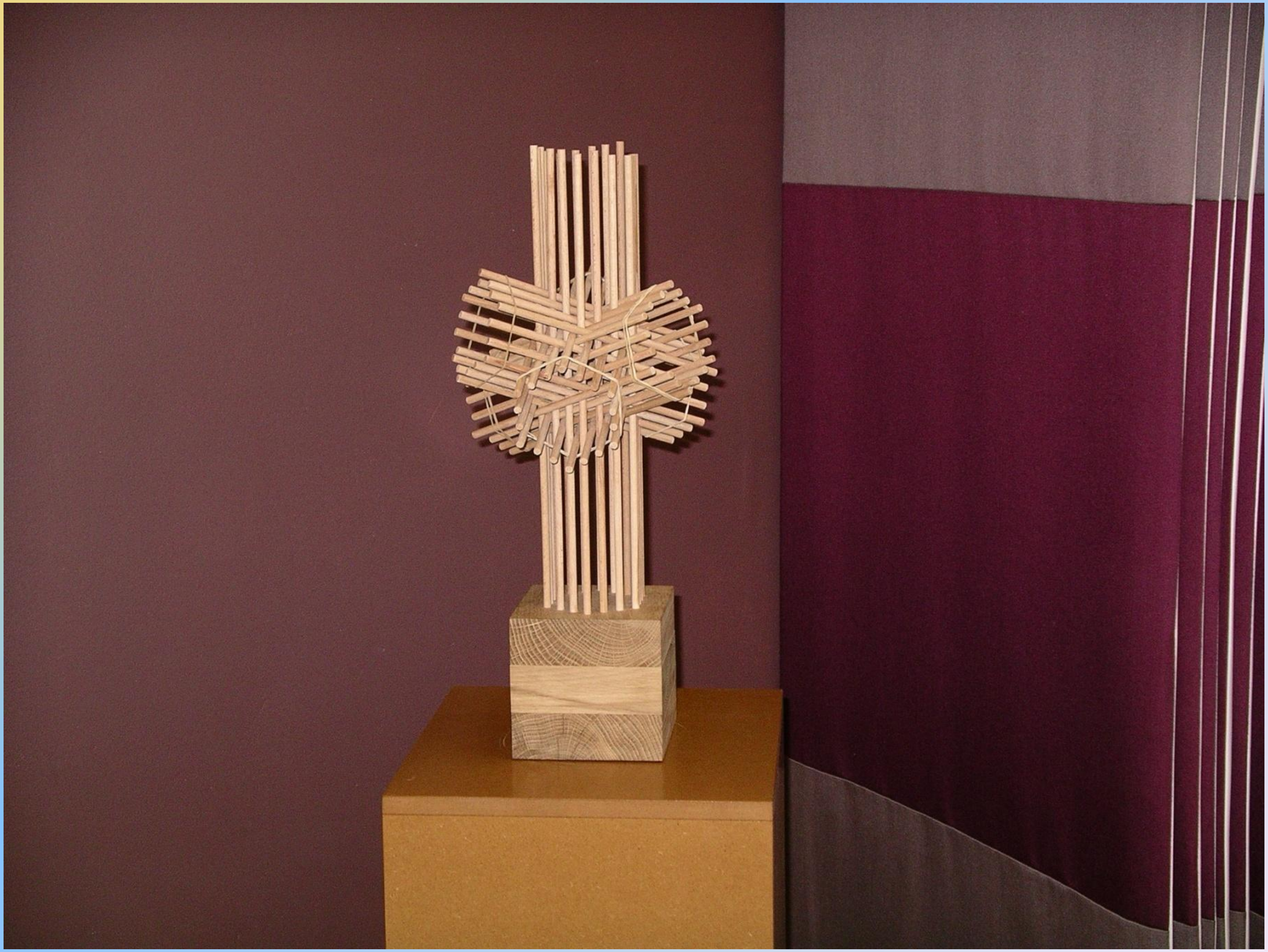
2.6 Other models





3. The architecture of Van der Laan and the use of the plastic number

- He formulated his fundamental ideas in

  Le nombre plastique
  Quinze leçons sur l’ordonnance architectonique
  Brill Leiden 1960

  De Architectonische ruimte
  Vijftien lessen over de dispositie van het menselijk verblijf
  Brill Leiden 1983

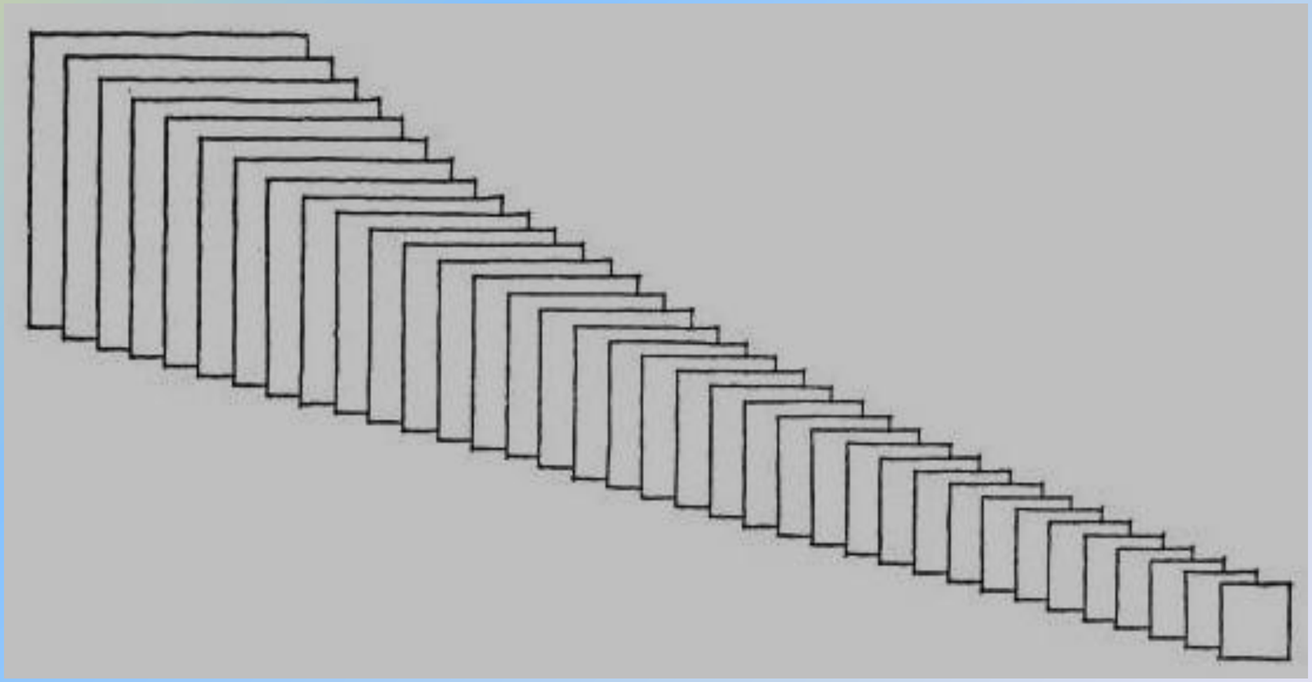
3. The architecture of Van der Laan and the use of the plastic number

STEP 1: first experiment

A sheet of paper of 50 cm torn in two equal parts produced in his experiment with 50 people lengths between 24.5 cm or 25.5 cm so that they differ 1/25 to each other.

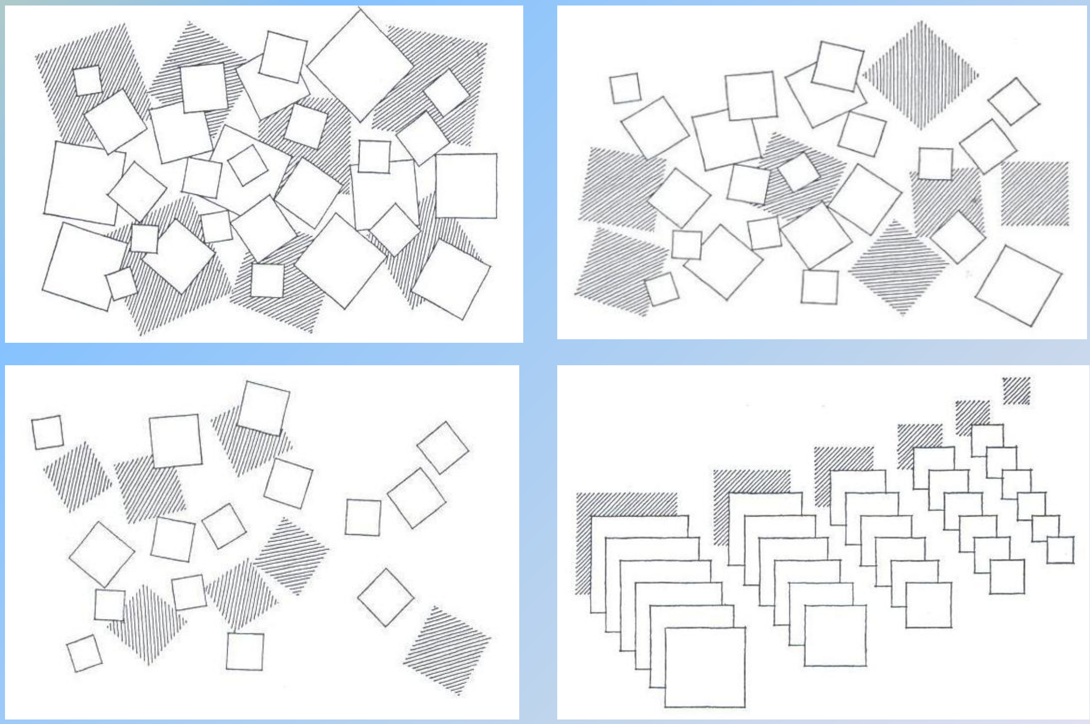
3. The architecture of Van der Laan and the use of the plastic number

STEP 2: second experiment



3. The architecture of Van der Laan and the use of the plastic number

STEP 3: classification of 36 squares



STEP 4: margin, type and order of size

ORDER OF SIZE						
40	42	56	75	98	130	MARGIN
	44	58	78	102	135	
	46	60	81	106	141	
	48	63	84	110	147	
	50	66	87	115	153	
	52	69	90	120	159	
	54	72	94	125	165	
Type I	Type II	Type III	Type IV	Type V	Type VI	

3. The architecture of Van der Laan and the use of the plastic number

STEP 5: the common ratio of the geometric sequence

To establish the basic proportion of the real three dimensional quantity it's necessary to know the size of the smallest different size.

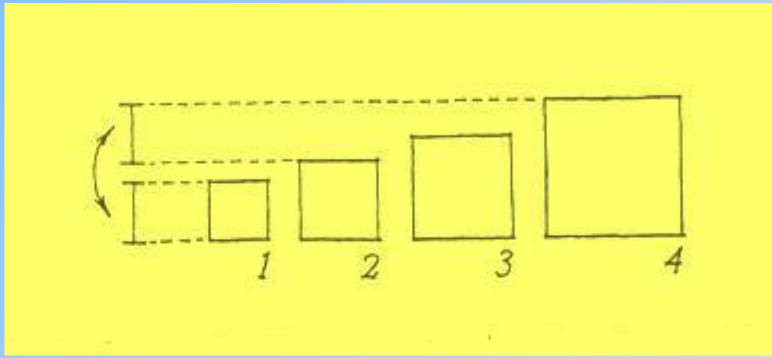
Dimension 1: I + I = II
Dimension 2: I + II = III → Golden number
Dimension 3: I + II =IV → Plastic number

Due to this last requirement, there exists a fixed common ratio between the threshold measures.

3. The architecture of Van der Laan and the use of the plastic number

STEP 5: the common ratio of the geometric sequence

So Van der Laan determined exactly the common ratio of the geometric sequence of the different threshold values of the different types of sizes.



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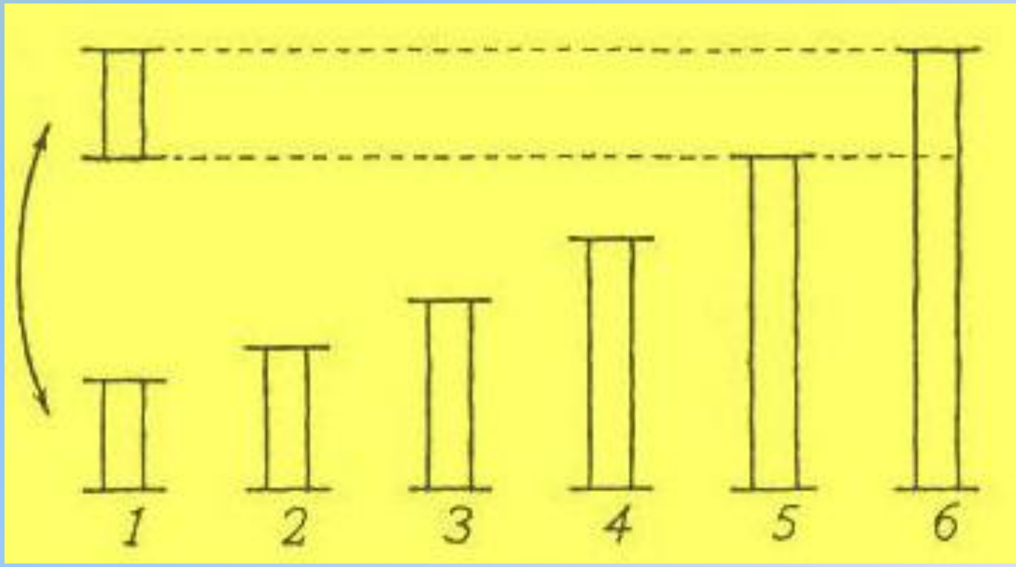
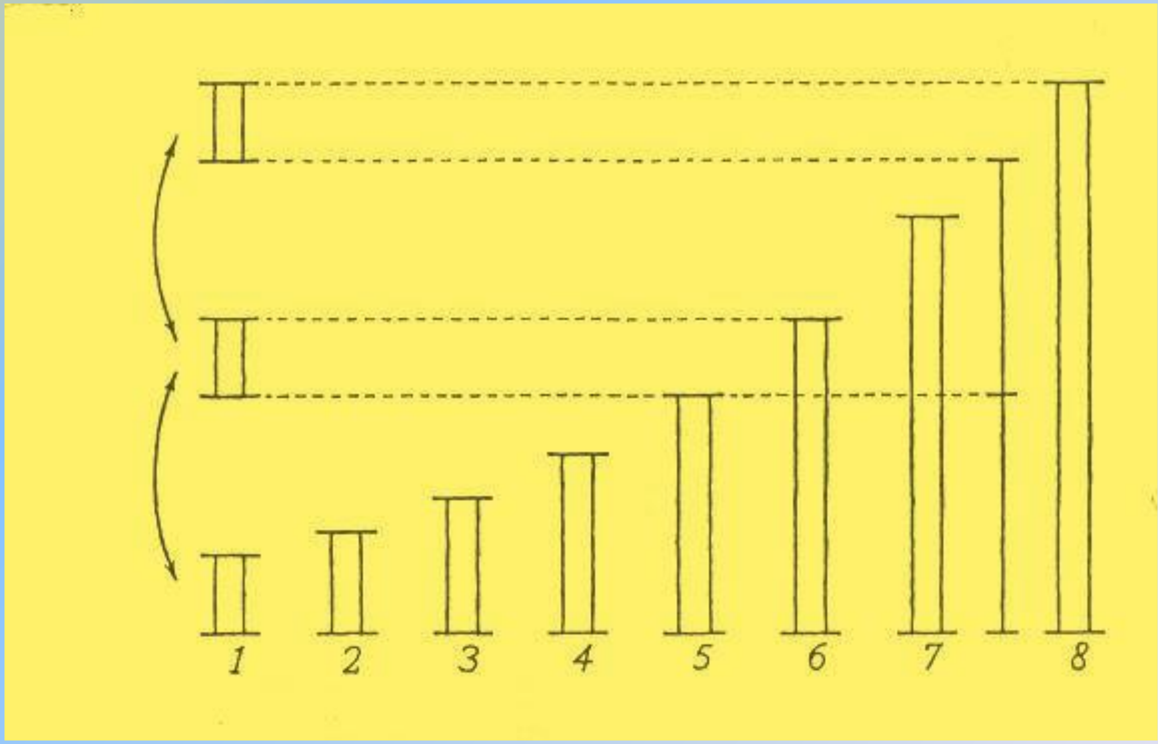
STEP 6: the extent of the order of size

As I +II= IV than VI-V = (III+IV) - (II+III) = IV – II = I



3. The architecture of Van der Laan and the use of the plastic number

STEP 7: a total system of eight measures



3. The architecture of Van der Laan and the use of the plastic number

CONCLUSION: a system of eight measures

Type	Name	Ratio	Plastic ratio
I	Small element	1	1
II	Great element	4/3	1,3247… = Ψ
III	Small piece	7/4	1,7548… = Ψ^2
IV	Great piece	7/3	2,3247… = Ψ^3
V	Small part	3	3,0795… = Ψ^4
VI	Great part	4	4,0795… = Ψ^5
VII	Small ensemble	5 1/3	5,4043… = Ψ^6
VIII	Great ensemble	7	7,1591… = Ψ^7

I + II = IV

Δ = 1

H(VIII, VII) = 6,1591

VIII – H(VIII,VII) = 1

STEP 8: morphic numbers

A real number *a* is a morphic number if there exist two natural numbers *k* and *l* so that

$$a+1=a^{k} \quad \text{and} \quad a-1=a^{-l}$$

Ψ is a morphic number as I + II = IV and VI – I = V

$$\downarrow \qquad \qquad \downarrow$$

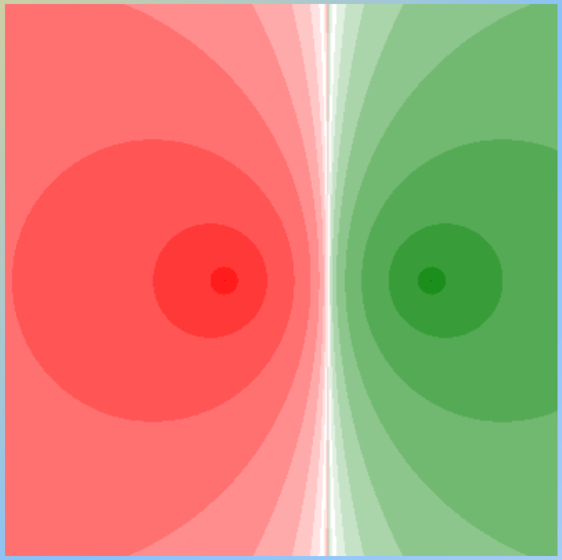
$$k=3 \qquad l=4$$

Only Φ and Ψ are morphic numbers. Kruijtzer, Aarts and Fokkink 2002

3. The architecture of Van der Laan and the use of the plastic number

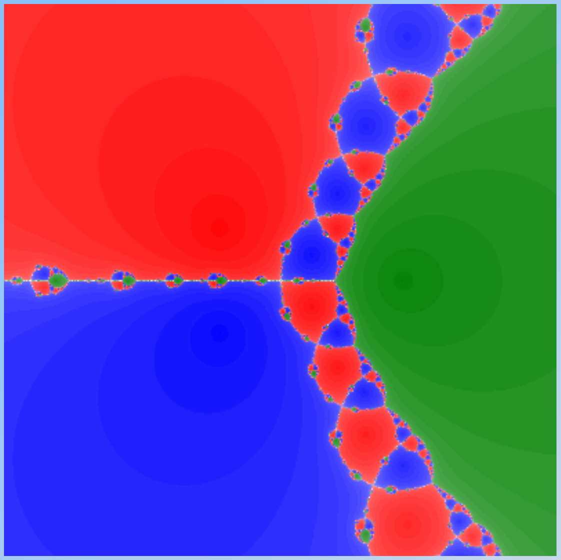
STEP 9: polynomiography

$z^2 = z + 1$



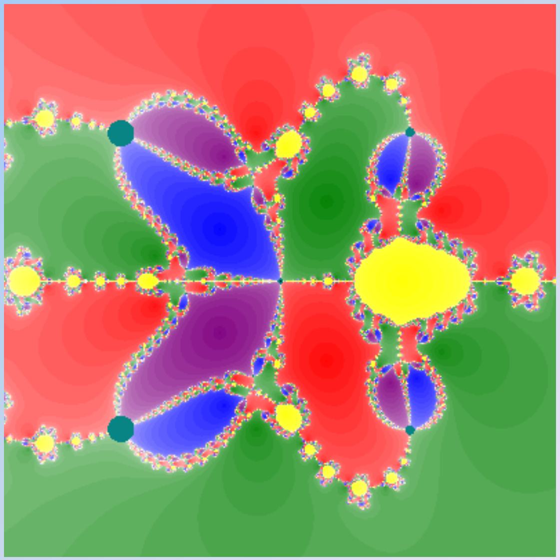
Golden number

$z^3 = z + 1$



Plastic number: I + II = IV

$z^5 = z^4 + 1$



Plastic number: VI – I = V

3. The architecture of Van der Laan and the use of the plastic number

Nunnery Waasmunster

Colors :

Floor: tint 5
Wall: 1
Ceiling: 3

3. The architecture of Van der Laan and the use of the plastic number

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