Solving the economic power dispatch and related problems more efficiently (and reliably)

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Views expressed are not necessarily those of the Commission
Electricity fictions, frictions, paradigm changes and politics

19th century competition: Edison v. Westinghouse

1905 Chicago 47 electric franchises

20th century: Sam Insull’s deal
  - franchise ‘unnatural’ monopoly
  - cost-of-service rates
  - Incentives for physical asset solution

1927 PJM formed a ‘power pool’

1965 Blackout:
  - Edward Teller: “power systems need sensors, communications, computers, displays and controls”

2013 still working on it
Mild assumptions

- Over the next ten years
- Computers will be faster and cheaper
- Measurement will be faster and better
- Generic software will be faster and better

- The questions are how much?
- Research will determine how much!!
- How much does sub optimality cost?
World Gross Production (2009): 20,000 TWh
United States Gross Production (2009): 4,000 TWh
At $30/MWh: cost $600 billion/year (world)
  cost $120 billion/year (US)
At $100/MWh: cost $2,000 billion/year (world)
  cost $400 billion/year (US)
In US 1% savings is about than $1 to $4 billion/yr
FERC strategic goal: Promote efficiency through better market design and optimization software


_money can't buy me love_
Paradigm change
Smarter Markets
20??

What will be smarter?
Generators, transmission, buildings and appliances, communications, software and hardware, markets and incentives

What is the 21st century market design?
Locationally and stochastically challenged: Wind, solar, hydro
Fast response: batteries and demand
Harmonize wind, solar, batteries and demand
Greater flexibility: more options
new technologies need better markets

• Batteries, flexible generators, topology optimization and responsive demand
• optimally integrated
• off-peak
  – Generally wind is strongest
  – Prices as low as -$30/MWh
• Ideal for battery charging
<table>
<thead>
<tr>
<th>ISO</th>
<th>Generation megawatts</th>
<th>Transmission Lines (miles)</th>
<th>Population (millions)</th>
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<tbody>
<tr>
<td>CAISO</td>
<td>57,124</td>
<td>25,526</td>
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<td>ISO-NE</td>
<td>33,700</td>
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<td>Midwest</td>
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<td>NYISO</td>
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<td>SPP</td>
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<td>50,575</td>
<td>15</td>
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<tr>
<td>PJM</td>
<td>164,895</td>
<td>56,499</td>
<td>51</td>
</tr>
<tr>
<td>Total</td>
<td>506,711</td>
<td>206,713</td>
<td>172</td>
</tr>
</tbody>
</table>
ISO Markets and Planning

- Four main ISO Auctions
  - Real-time: for efficient dispatch
  - Day-ahead: for efficient unit scheduling
  - Generation Capacity: to ensure generation adequacy and cover efficient recovery
  - Transmission rights (FTRs): to hedge transmission congestion costs

- Planning and investment
  - Competition and cooperation
  - All use approximations due to software limitations
Real-time Market

- balancing market plus a look-ahead
- efficiently dispatch generation, load, transmission and ancillary services every 5 minutes
- Subject to N-1 reliability constraints
- Within the flexible limits of generators and transmission
Day-ahead Market

Woke up, got out of bed, ...

- scheduling and unit commitment market
- efficiently (from bids) schedule generation, load, transmission and ancillary services
- Subject to explicit reliability constraints
- Within the flexible limits of generators and transmission

Eight days a week is not enough to show I care
End-use consumers got to get you into my life

- Consumers receive very weak price signals
  - monthly meter; ‘see’ monthly average price
- On a hot summer day
  - wholesale price = $1000/MWh
  - Retail price < $100/MWh
  - results in market inefficiencies and
  - poor purchase decisions for electricity and electric appliances.

- Smart meter and real-time price are key
- Solution: smart appliances
  - real time pricing, interval meters and
  - Demand-side bidding
- Large two-sided market!!!!!!!!!!

He's as blind as he can be just sees what he wants to see
Open or close circuit breakers

Proof of concept savings using DCOPF

- provided 25% savings on an 118 bus test problem
- N-1 for IEEE 118 & RST 96 up to 16% savings
- ISO-NE network 15% savings or $.5 billion/yr

Potential

- all solutions have optimality gaps so higher savings may be found
- Currently takes too long to solve to optimality
- Better solutions are acceptable
Enhanced wide-area planning models

- More efficient planning and cost allocation through a mixed-integer nonlinear stochastic program.
- Integration into a single modeling framework
- Better models are required to:
  - Economically plan efficient transmission investments
  - Compute cost allocations
- In an environment of competitive markets with locationally-constrained variable resources and criteria for contingencies and reserve capacity.
Complete ISO market design
Not quite there yet

⇒ Smarter markets
  ⇒ Full demand side participation with real-time prices
  ⇒ Smarter hardware, e.g., variable impedance
  ⇒ Better approximations, e.g., DC to AC
  ⇒ Flexible thermal constraints and transmission switching
  ⇒ Smarter software with high flop computers

⇒ Electric network optimization has roughly
  ⇒ $10^6$ nodes
  ⇒ $10^6$ transmission constraints
  ⇒ $10^5$ binary variables

⇒ Potential dispatch costs savings: 10 to 30%
Mixed Integer Nonconvex Program

maximize $c(x)$
subject to $g(x) \leq 0,$
      $Ax \leq b$
      $l \leq x \leq u,$
      some $x \in \{0,1\}$

$c(x), g(x)$ may be non-convex

I didn't know what I would find there
Mixed Integer Program

I didn't know what I would find there.

\[
\begin{align*}
\text{maximize} & \quad cx \\
\text{subject to} & \quad Ax = b, \\
& \quad l \leq x \leq u, \\
& \quad \text{some } x \in \{0,1\}
\end{align*}
\]

- Better modeling for
  - Start-up and shutdown
  - Transmission switching
  - Investment decisions

- Solution times improved by \( > 10^7 \) in last 30 years
  - 10 years becomes 10 minutes

And though the holes were rather small
They had to count them all

It was twenty years ago today
MIP Paradigm Shift
Let me tell you how it will be

⇒ Pre-1999
  ⇒ MIP can not solve in time window
  ⇒ Lagrangian Relaxation
    ⇒ solutions are usually infeasible
    ⇒ Simplifies generators
    ⇒ No optimal switching

⇒ 1999 unit commitment conference and book
  ⇒ MIP provides new modeling capabilities
  ⇒ New capabilities may present computational issues
  ⇒ Bixby demonstrates MIP improvements

⇒ 2011 MIP creates savings > $500 million annually
⇒ 2015 MIP savings of > $1 billion annually
Power Flow and Simplifications

\[ G = \frac{R}{R^2 + X^2} \]
\[ B = \frac{-X}{R^2 + X^2} \]

**AC Model (physics)**

\[ P_{ik} = G_k V_i^2 - G_k (V_i V_j) \cos(\Theta_i - \Theta_j) - B_k (V_i V_j) \sin(\Theta_i - \Theta_j) \]
\[ Q_{ik} = -B_k V_i^2 - G_k (V_i V_j) \sin(\Theta_i - \Theta_j) + B_k (V_i V_j) \cos(\Theta_i - \Theta_j) - B_{capik} V_i^2 \]

**DC Model (market model approximation. Can we do better?)**

\[ P_{ik} = -B_k (\Theta_i - \Theta_j) \]
"DC " Optimal Flow Problem

\[
\begin{align*}
\text{max} & \quad \sum_i b_i d_i - \sum_i c_i p_i \\
& \quad \sum_i d_i - \sum_i p_i = 0 \\
& \quad d_i \leq d_i^{\max} \\
& \quad p_i \leq p_i^{\max} \\
& \quad p_{ijk} = \sum_i df_{ki}(p_i - d_i) \leq p_{ijk}^{\max} \\
\text{max} & \quad \sum_i b_{in} d_{in} - \sum_i c_{in} p_{in} \\
& \quad \sum_i d_{in} - \sum_i p_{in} = \sum_{nk} p_{njk} \\
& \quad d_i \leq d_i^{\max} \\
& \quad p_i \leq p_i^{\max} \\
& \quad p_{ijk} = B_{ijk} \theta_{ij} \\
& \quad \theta_{\min i j} \leq \theta_{ij} \leq \theta_{\max i j}
\end{align*}
\]

\text{dual variables}\]

\[
\begin{align*}
& \lambda \\
& \lambda_n \\
& \alpha_i^{\max} \\
& \beta_i^{\max} \\
& \mu_k^{\max}
\end{align*}
\]
AC Optimal Flow Problem

⇒ “DCOPF” formulations
  ⇒ linearize the nonlinearities and
  ⇒ drop variables (voltage and reactive power)
  ⇒ simplify the problem
  ⇒ add binary variables

⇒ ‘ACOPF’ formulation
  ⇒ continuous nonconvex optimization problem
Power Flow Equations

Polar Power-Voltage: 2N nonlinear equality constraints

\[ P_n = \sum_{mk} V_n V_m (G_{nmk}\cos\theta_{nm} + B_{nmk}\sin\theta_{nm}) \]
\[ Q_n = \sum_{mk} V_n V_m (G_{nmk}\sin\theta_{nm} - B_{nmk}\cos\theta_{nm}) \]

Rectangular Power-Voltage: 2N quadratic equality constraints

\[ S = P + j\, Q = \text{diag}(V)I^* = \text{diag}(V)[YV]^* = \text{diag}(V)Y^*V^* \]

Rectangular Current-Voltage (IV) formulation.

Network-wide \textbf{LINEAR} constraints: 2N linear equality constraints

\[ I = YV = (G + jB)(V^r + jV^j) = GV^r - BV^j + j(BV^r + GV^j) \]
where \( I^r = GV^r - BV^j \) and \( I^j = BV^r + GV^j \)
Solving the ACOPF with Commercial Solvers

- CONOPT, KNITRO, MINOS, IPOPT, and SNOPT with default settings
- 7 test problems from 118 to 3000 bus problems
- $B\Theta$ and hot initialization methods outperformed the uniform random initialization
- ACOPF in rectangular coordinates compared to polar
  - Solves faster and is more robust
- IPOPT and SNOPT are faster and more robust
- Simulated parallel process using all solvers
  - is much faster and 100% robust
Rectangular IV-ACOPF formulation.

Network-wide objective function: Min \( c(P, Q) \)

Network-wide constraint: \( I = YV \)

Bus-specific constraints:

\[
P = V^r \cdot I^r + V^j \cdot I^j \leq P^{\text{max}}
\]

\[
P^{\text{min}} \leq P = V^r \cdot I^r + V^j \cdot I^j
\]

\[
Q = V^j \cdot I^r - V^r \cdot I^j \leq Q^{\text{max}}
\]

\[
Q^{\text{min}} \leq Q = V^j \cdot I^r - V^r \cdot I^j
\]

\[
V^r \cdot V^r + V^j \cdot V^j \leq (V^{\text{max}})^2
\]

\[
(V^{\text{min}})^2 \leq V^r \cdot V^r + V^j \cdot V^j
\]

\[
(i^r_{nmk})^2 + (i^j_{nmk})^2 \leq (i^{\text{max}}_{nmk})^2 \quad \text{for all } n, m, k
\]
The Linear Approximations to the IV Formulation

We take three approaches to constraint formulation.

If the constraint is nonconvex,
  - use the first order Taylor series approximation
  - restricted step size
  - updated at each LP iteration

If the constraint is convex,
  - preprocessed linear constraints (polygons)
  - add tight linear cutting planes that remove the current solution from the linear feasible region
  - kept for subsequent iterations

Active constraints for minimum voltage
Preprocessed Linear Voltage and Current Maximum Constraints

\[(v_r^m)^2 + (v_j^m)^2 \leq (v_{\text{max}}^m)^2\]

\[\cos(\theta^s)v_r^n + \sin(\theta^s)v_j^n \leq v_{\text{max}}^n\]

for \(s = 0, 1, \ldots, s_{\text{max}}; n\)
Iterative Linear Cuts.
Non-Convex Minimum Voltage Constraints.

\[(v_{\text{min}}^m)^2 \leq (v_r^m)^2 + (v_j^m)^2\]

⇒ the linear approximation is problematic.
⇒ approximation and eliminates parts of the feasible region
⇒ Since higher losses occur at lower voltages, the natural tendency of the optimization will be toward higher voltages
⇒ Use active set approach
Non-convex first order approximation at bus $n$ around $\mathbf{v}_n^r$, $\mathbf{i}_n^r$, $\mathbf{v}_n^j$, $\mathbf{i}_n^j$

$$p_n^\approx = \mathbf{v}_n^r \mathbf{i}_n^r + \mathbf{v}_n^j \mathbf{i}_n^j + \mathbf{v}_n^r \mathbf{i}_n^j + \mathbf{v}_n^j \mathbf{i}_n^j - (\mathbf{v}_n^r \mathbf{i}_n^r + \mathbf{v}_n^j \mathbf{i}_n^j)$$

The Hessian has eigenvalues: 2 are 1 and 2 are -1

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$q_n^\approx = \mathbf{v}_n^j \mathbf{i}_n^r - \mathbf{v}_n^r \mathbf{i}_n^j - \mathbf{v}_n^r \mathbf{i}_n^j + \mathbf{v}_n^j \mathbf{i}_n^r - (\mathbf{v}_n^j \mathbf{i}_n^r - \mathbf{v}_n^r \mathbf{i}_n^j)$$

The Hessian has eigenvalues: 2 are 1 and 2 are -1

$$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$
Computational experience

- MINOS, CONOPT, IPOPT, KNITRO, SNOPT with default setting
- naïve implementation of iterative LP IV-ACOPF

- Problems: 14, 30, 57, 118, 300 bus; no line limits
- Ten random starting points

- Results: iterative LP approach is faster or competitive with nonlinear solvers
Add binding line constraints
LIV-ACOPF: Minimize $\Sigma_n c_{pl}^l(p_n)+c_{ql}^l(q_n)$

\[i_{nmk}' = g_{nmk}(v_{rn} - v_{rm}) - b_{nmk}(v_{jn} - v_{jm}) \quad \text{for all } n, m, k\]

\[\dot{j}_{nmk}' = b_{nmk}(v_{rn} - v_{rm}) + g_{nmk}(v_{jn} - v_{jm}) \quad \text{for all } n, m, k\]

\[i_n' = \sum_{mk} i_{nmk}' \quad ; \quad \dot{j}_n' = \sum_{mk} \dot{j}_{nmk}' \quad \text{for all } n\]

\[p_n = v_{rn}i_n' + v_{jn}\dot{j}_n' + v_{rn}\dot{r}_n + v_{jn}\dot{j}_n - (v_{rn}\dot{r}_n + v_{jn}\dot{j}_n) \quad \text{for all } n\]

\[q_n = v_{jn}i_n' - v_{rn}\dot{j}_n - v_{rn}\dot{r}_n + v_{jn}\dot{r}_n - (v_{jn}\dot{r}_n - v_{rn}\dot{j}_n) \quad \text{for all } n\]

\[q_{min} \leq q_n \leq q_{max} ; \quad p_{min} \leq p_n \leq p_{max} \quad \text{for all } n\]

\[\cos(\theta_s)v_{rn} + \sin(\theta_s)v_{jn} \leq v_{max} \quad \text{for } s = 0, 1, \ldots, s^{max} ; \ n\]

\[(v_{rn_{nd}}/v_{nd})v_{rn} + (v_{jn_{nd}}/v_{nd})v_{jn} \leq v_{max} \quad \text{for } d = 0, \ldots, h-1; \ n\]

\[\cos(\theta_s)i_{nmk}' + \sin(\theta_s)\dot{j}_{nmk}' \leq i_{max}^l_{nmk} \quad \text{for } s = 1, \ldots, s^{max} ; \ k\]

\[(i_{nmk_{nd}}/i_{nmk_{nd}})i_{nmk}' + (j_{nmk_{nd}}/j_{nmk_{nd}})\dot{j}_{nmk}' \leq i_{max}^l_{nmk} \quad \text{for } d = 0, \ldots, h-1; \ k\]
Preprocessed Polygons

- 4, 8, 16, 32 and 128 sided polygons

**Results**

- 16 or 32 sided polygons best in a tradeoff between accuracy and solution time.

- With tight iterative cuts, the solution is always
  - within 2.5% of the best-known nonlinear solution and
  - usually less than 1%.

- With 16 preprocessed constraints, the iterative linear model 2 to 5x faster nonlinear solver (IPOPT).
Step-size limits for non-convex linearizations

⇒ improved performance of the iterative linear procedure
⇒ faster and more robust
⇒ up to six times to 10x faster than
  ⇒ the nonlinear solver and
  ⇒ without a step-size constraint.
⇒ best parameters are problem-dependent
Next steps for the ILIV-ACOPF

- Call back testing
  - Start from previous major iteration
- IV cost functions
  - Replace $\text{Min } c(P, I)$ with $\text{Min } c(I, V)$
  - Eliminate non-convex $P, Q$ constraints
  - Lower limit voltage constraints remain
- ILIV-AC OPF with binary variables
  - Unit commitment models
  - Optimal topology models
    - Preprocessed linear cut sets
    - Heuristics
Computational Research Questions

• Decomposition and Grid (parallel) computing
  – Real/reactive
  – Time
• Good approximations
  – Linearizations
  – convex
• Avoiding local optima
• Nonlinear prices
• Better tree trimming
• Better cuts
• Advance starting points

If you really like it you can have the rights
It could make a million for you overnight
Future ISO Software

⇒ **Real-time:**
  ⇒ *AC Optimal Power Flow* with <5 min dispatch, look ahead and explicit N-1 reliability

⇒ **Day-ahead:**
  ⇒ explicit N-1 *ACOPF* with unit commitment and transmission switching with <15 min scheduling

⇒ **Investment/Planning:**
  ⇒ Binary investment variables
  ⇒ *Greater detail and topology*
  ⇒ more time to solve
Acceptance of Paradigm Shifts

“A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.” Max Planck

The magical mystery tour is waiting to take you away, waiting to take you away.

“Ohhhhhhh . . . Look at that, Schuster . . . Dogs are so cute when they try to comprehend quantum mechanics.”
Market Design

"Everything should be made as simple as possible ... but not simpler." Einstein

The magical mystery tour is waiting to take you away, waiting to take you away.