A Price-Based Approach for Controlling Networked Distributed Energy Resources

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Piscataway, NJ
February 20, 2013
Outline

1. Introduction
2. Game-Theoretic Problem Formulation
3. Characterization of the DER Game
4. A Distributed Algorithm for Equilibrium Seeking
5. Numerical Examples
6. Concluding Remarks
Motivation

Distributed Energy Resources (DERs) can potentially be utilized to provide ancillary services

- Power electronics grid interfaces commonly used in DERs can provide reactive power support for voltage control
- Plug-in-hybrid vehicles (PHEVs) can provide active power for up and down regulation
Control and Coordination of DERs

- Effective control of DERs is key for enabling their utilization in providing ancillary services

**Potential solution:** centralized control (where each DER is commanded from a centralized decision maker)
- Requires a communication network connecting the central controller with each distributed resource
- Requires up-to-date knowledge of distributed resource availability on the distribution side

**Alternative approach:** utilize distributed strategies for control and coordination of DERs, which offer several potential advantages
- Easy and affordable deployment (no requirement for communication infrastructure between centralized controller and various devices)
- Ability to handle incomplete global knowledge of DER availability
- Potential resiliency to faults and/or unpredictable DER behavior
Consider a set of entities, referred to as aggregators, that through some market-clearing mechanism, are requested to provide certain amount of energy over some period of time.

Each aggregator can influence the energy provision/consumption of a group of DERs by offering them a pricing strategy.

**Objective:** to incentivize the DERs to provide or consume energy, as appropriate, so as to allocate among them the amount of energy that the aggregator has been asked to provide.
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Distributed Energy Resources

- Each DER is a decision maker and can freely choose to participate after receiving a request from its aggregator.

- DER actions include idle, provide, or consume energy.

- DER decisions depend on its own utility function, along with the pricing strategy (for provision/absorption) designed by the aggregator.

- DERs are price anticipating, i.e., they are aware that the aggregator designs the pricing as a function of the average energy available.

- DERs are able to collect information from “neighboring” DERs with which they can exchange information.
Problem Formulation

- Let $V = \{v_1, \ldots, v_n\}, \ n \in \mathbb{Z}_{\geq 1}$ denote the set of DERs.

- Let $x_i(t) \in [0, 1]$ denote the energy level of DER $v_i$ at time $t \in \mathbb{R}_{\geq 0}$.

- Let $\mathcal{X} \in \mathbb{R}$ be the amount of energy that the aggregator has contracted to provide/consume over some period of time:
  - when $\mathcal{X} \in \mathbb{R}_{< 0}$, the aggregator needs to encourage the DERs to provide energy.
  - when $\mathcal{X} \in \mathbb{R}_{> 0}$, the aggregator needs to encourage the DERs to consume energy.
Pricing Functions

- The aggregator incentivizes the DERs via pricing to provide or consume energy within a time interval \([0, T]\)

- Price per unit of energy provided/consumed is set for a time interval \([0, T]\) based on the DER average energy level, \(\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}\), at the end of the time interval
  - Each DER decides its \(x_i\) at the beginning of the time interval

- The quantities \(P_c(\bar{x})\) and \(P_p(\bar{x})\), which are obtained as the outputs of some mappings

\[
P_c : [0, 1] \rightarrow \mathbb{R}_{\geq 0} \quad \text{and} \quad P_p : [0, 1] \rightarrow \mathbb{R}_{\geq 0},
\]
gever, respectively, the price per unit of energy that DERs pay when consuming energy and receive when providing
DERs Payoff Functions

- Let $U_i : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$ denote the utility function of DER $v_i$.

- Assume that each $U_i$ is an increasing function of the available energy:
  - at no cost, it is beneficial to keep as much energy as possible

- Similar to other resource allocation problems [Johari, Tsitsiklis, ’06], each DER wishes to maximize a payoff function of the form

$$ f_i(x_i, x_{-i}, P_c, P_p) = \begin{cases} U_i(x_i) - (x_i - x_i^0)P_c(x_i), & x_i > x_i^0, \\ U_i(x_i) - (x_i - x_i^0)P_p(x_i), & x_i \leq x_i^0, \end{cases} $$

where $(x_i^0, x_{-i}^0)$ denotes the initial energy profile of all DERs.
Objective of the Aggregator

- Ensure that the DERs collectively provide $X \in X_{agg}$ units of energy; thus it wishes to maximize

$$f_{agg}(x, P_c, P_p) = -|X - \sum_{i=1}^{n} \alpha_i (x_i - x_0^i)|,$$

where $\alpha_i \in \mathbb{R}_{>0}$, for all $i \in \{1, \ldots, n\}$

- Based on the description given thus far, the aggregator and the DERs define a game

$$G_{DERs-AGG} = (V \cup \{v^{agg}\}, [0, 1]^n \times X_{agg}, f_1 \times \ldots \times f_n \times f_{agg}),$$

where players wish to maximize their payoff functions
Some Problem Statements

(a) **[Existence of equilibria]** Given the pricing strategies of the aggregator $P_c$ and $P_p$, does there exist a Nash equilibrium solution to the DER game $G_{DERs}$ as defined below?

$$G_{DERs} = (V, [0, 1]^n, f_1 \times \ldots \times f_n)$$

If so, is the Nash equilibrium unique?

(b) **[Distributed equilibria seeking]** If the answers to both parts of (a) are positive, can the DERs use a (distributed) strategy to seek the Nash equilibrium, after the pricing strategy is fixed?

(c) **[Optimal pricing]** If the answer to the existence question is positive, does there exists pricing strategies $P_c$ and $P_p$ such that

$$x^* \in \{ z \in X \mid z = \arg\max_x f_{agg}(x, P_c, P_p) \},$$

where $x^*$ is the Nash equilibrium of the DER game $G_{DERs}$?
Focus of this talk: problems (a) and (b), i.e., for a given prices strategy, we study the DER game $G_{\text{DERs}}$ and propose a distributed algorithm that allows the DERs to seek for the Nash equilibrium when unique
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Claim: under proper assumptions on the DER payoff functions $G_{DERs}$ is a concave game
Casting $G_{\text{DERs}}$ as a Concave Game

- A group of $n$ players $\{v_1, \ldots, v_n\}$
  - In $G_{\text{DERs}}, v_i$ is the $i$th DER

- Player $v_i$ takes action from $S_i \subset \mathbb{R}^{d_1}$, nonempty, convex and compact
  - In $G_{\text{DERs}}, S_i = [0, 1], \forall i$

- Strategy set for all players is $S = S_1 \times \ldots \times S_n$
  - In $G_{\text{DERs}}, S = [0, 1] \times \ldots \times [0, 1] = [0, 1]^n$ [no shared constraints]

- When players take actions according to $\mathbf{x} = (x_1, \ldots, x_n), x_i \in \mathbb{R}^d$, the payoff function $f_i : S \to \mathbb{R}$ of the $i$th player is $f_i(\mathbf{x})$
  - In $G_{\text{DERs}}$:
    $$f_i(x_i, x_{-i}, P_c, P_p) = \begin{cases} U_i(x_i) - (x_i - x_i^0)P_c(\bar{x}), & x_i > x_i^0, \\ U_i(x_i) - (x_i - x_i^0)P_p(\bar{x}), & x_i \leq x_i^0, \end{cases}$$

- $f_i$ is a locally Lipschitz concave mapping in its $i$th argument
  - What assumptions do we need on the $f_i$'s of the $G_{\text{DERs}}$ so the conditions above are satisfied?
DER Game Payoff Functions

- **Assumptions:**
  - The utility function $U_i$ is concave, nondecreasing, and continuously differentiable, for all $i \in \{1, \ldots, n\}$
  - The function $P_c$ is convex, twice differentiable, and nondecreasing
  - The function $P_p$ is concave, twice differentiable, nondecreasing
  - $P_c(\bar{x}) \geq P_p(\bar{x})$, for all $\bar{x} \in [0, 1]$
  - The payoff function $f_i$ is **not necessarily** differentiable

**Proposition:** Under the assumptions above, the payoff function $f_i$ of each DER is concave in its first argument
Existence and Uniqueness of Equilibrium Points

Using a classical result on concave games [Rosen, ’65], we have the following result:

**Theorem:** Under the assumptions on the DER pay-off functions, \( G_{\text{DERs}} \) has a Nash equilibrium.

What about uniqueness?

- Under an additional condition [diagonally strict concavity], an extension of the result by Rosen to concave games with
  - non-smooth payoff functions, and
  - no shared constraints
  can be applied to guarantee uniqueness [Carlson, ’01]

- We assume uniqueness for the remainder of the talk.
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Distributed Nash Equilibrium Seeking

- At the Nash equilibrium \( x^* \in S \), for all \( i \),

\[
f_i(x^*) = \max_{y_i} \{ f_i(x_1^*, \ldots, x_{i-1}^*, y_i, x_{i+1}^*, \ldots, x_n^*) \mid y_i \in S_i \}
\]

i.e., no player can improve its payoff by unilaterally deviating from \( x^* \)

**Objective:** can an equilibrium be found, collaboratively, in spite of partial access to information?

**Assumptions:**

- Each DER can only communicate with its neighboring DERs
- Each DER has access to its own payoff function only
Main Idea for Achieving Distributed Nash Seeking

- For simplicity of exposition, consider the unconstrained version of $\mathbf{G}_{\text{DERs}}$, i.e., $x_i \in \mathbb{R}$, $\forall i$; then, the fix points of the function

\[ \Phi(x, y) = \sum_{i=1}^{n} f_i(y_1, \ldots, y_{i-1}, x_i, y_{i+1} \ldots, y_n), \]

restricted to the subset where $y = x$, correspond to the Nash equilibrium of $\mathbf{G}_{\text{DERs}}$

- Then, finding $x^*$ boils down to designing a distributed algorithm that allows the DERs to compute the fix points of $\Phi(x, y)$

- The distributed algorithm we propose is inspired by
  - Continuous-time distributed protocols for optimization problems [Wang and Elia ’10; Gharesifard and Cortés, ’12]
  - Nash-seeking strategies for noncooperative games [Frihauf, Krstic, and Başar, ‘12]
Consider a network of \( n \) DERs \( \{v_1, \ldots, v_n\} \)

The exchange of information between DERs is described by a connected graph, denoted by \( G \)

Let \( x^* \in X, X = [0, 1]^n \), denote the unique Nash equilibrium of the \( G_{\text{DERs}} = (V, [0, 1]^n, f_1 \times \ldots \times f_n) \)

Let \( x^i \in \mathbb{R}^n \) denote the estimate of DER \( v_i \) about \( x^* \)

Define \( \mathbf{x}^T = ((x^1)^T, \ldots, (x^n)^T) \in \mathbb{R}^{n^2} \)

Let \( L \in \mathbb{R}^{n \times n} \) denote the Laplacian of \( G \) and define \( L = L \otimes I_n \in \mathbb{R}^{n^2 \times n^2} \), where \( \otimes \) denotes the Kronecker product.
Discrete-Time Distributed Nash-Seeking Dynamics

- Due to the lack of differentiability of the payoff functions, we need to formulate the algorithm as a set-valued dynamical system.

Define

\[ \Psi(x, z) = \left\{ \left( -Lx - Lz + s_x, Lx \right) \mid s_x \in D_x \right\}, \]

with

\[ D_x = \{ u \mid u = (\eta_1, 0, \ldots, 0, \ldots, 0, \ldots, 0, \eta_n)^T, \eta_i \in \partial x_i f_i(x_i) \}, \]

computed by \( v_1 \) \quad \text{and} \quad \text{computed by } v_n

The distributed Nash-seeking dynamics is given by

\[ x(k + 1) \in P \left( x(k) - \delta (Lx(k) + Lz(k) - D_x(k)) \right), \]

\[ z(k + 1) = z(k) + \delta Lx(k), \]

with \( \delta > 0 \), and \( P = \prod_{i=1}^{n^2} P_i \), where \( P_i : \mathbb{R} \to [0, 1], \ i \in \{1, \ldots, n^2\} \), is the projection onto \( [0, 1] \).
Convergence Results [Gharesifard, D-G, and Bașar, ’13]

**Lemma:** When the graph $G$ is connected, the distributed Nash-seeking dynamics has at least one fixed point. Moreover, $(x^*, z^*)$ is a fixed point if and only if $x^* = 1_n \otimes x^*$, where $x^* \in X$ is the Nash equilibrium of the DER game $G_{DERs}$

**Theorem:** When the graph $G$ is connected, the distributed Nash-seeking dynamics is asymptotically convergent. Moreover, the projection onto the first component of its trajectory converges to $x^* = 1_n \otimes x^*$, where $x^* \in \mathbb{R}^n$ is the Nash equilibrium of $G_{DERs}$
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Consider a set of DERs \( \{v_1, \ldots, v_6\} \) with the adjacency matrix of the communication graph \( G \) given by

\[
A = \begin{pmatrix}
0 & 1 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}.
\]

The utility function \( U_i : [0, 1] \to \mathbb{R}_{\geq 0} \), \( i \in \{1, \ldots, 6\} \) of each DER is

\[
U_i(x_i) = u^1_i \log(1 + x_i) + u^2_i x_i,
\]

where \( U_i \) is normalized so that \( u^1_i, u^2_i \in (0, 1] \).

Assume linear pricings:

\[
P_c(\bar{x}) = c_1 \bar{x} + c_2, \\
P_p(\bar{x}) = d_1 \bar{x} + d_2,
\]

with \( P_c \) and \( P_p \) normalized so that \( c_1, d_1 \in (0, 1] \) and \( c_2, d_2 \in [0, 1] \).
High Price for Consumption; Provision is Encouraged

- Consider a scenario in which the aggregators need to encourage the DERs to provide energy.
- The aggregator chooses the pricing parameters as $c_1 = 0.9$, $c_2 = 0.9$, $d_1 = 0.8$, and $d_2 = 0.8$.
- Consider two cases:
  - **Case-1:** all DERs have low initial available energy and no incentive for consuming energy.
  - **Case-2:** all DERs have low initial energy available; the only DER with incentive for consuming energy is $v_5$. 

![Graphs showing energy consumption over time]
High Price for Consumption; Provision is not Encouraged

- Consider a scenario in which the aggregator increases the price for consuming energy when the average energy available is high.
- The aggregator chooses the pricing parameters as $c_1 = 0.7$, $c_2 = 0.1$, $d_1 = 0.1$, $d_2 = 0.1$.
- Consider two cases:
  - **Case-3:** all DERs have low initial energy available in the DERs and no incentive for providing energy.
  - **Case-4:** all DERs have low initial energy available in the DERs; the only DER with incentive for consuming energy is $v_5$.
Outline

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5. Numerical Examples
6. Concluding Remarks
Summary

- We proposed a framework for controlling DER energy provision and consumption via pricing strategies
  - A group of aggregators is responsible for providing a certain amount of energy predetermined by some market-clearing mechanism
  - The DERs are assumed to be price anticipating and also have their own individual utility functions

- We formulated the problem as a two-layer game in which the aggregator sets prices for energy consumption/provision

- For fixed pricing, we give conditions under which the DER-layer game is concave and conditions under which the equilibrium is unique

- We propose a discrete-time algorithm which allows the DERs to compute the Nash equilibrium when unique
Future Work

- Characterization of optimal pricing strategies for the aggregator in the context of mechanism design

- Extension of the convergence results to communication networks described by directed graphs

- Study groups of aggregators and their interconnections with the retail market layer

- Robustness and resilience in pricing strategies