Back to the old LYM (=YBLM) inequality

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Let $\mathcal{F}$ be a family of distinct subsets of $[n] = \{1, 2, \ldots, n\}$ and suppose that $\mathcal{F}$ is a Sperner family, that is, no member of $\mathcal{F}$ is a subset of another one. Let $f_i(\mathcal{F}) = f_i(0 \leq i \leq n)$ denote the number of i-element members of $\mathcal{F}$. The vector $(f_0, f_1, \ldots, f_n)$ is called the profile vector of $\mathcal{F}$. The well-known LYM inequality gives a linear bound on the profile vectors of Sperner families. This linear bound has been improved by Bey, giving a polynomial bound what is quadratic if only two of the f_is are non-zero. Our recent work with Jerry Griggs gives a further improvement replacing the quadratic curve with a convex broken line.