

# Abstracts for Approximation Algorithms: The Last Decade and the Next

June 13-17, 2011

Room 104 (Large Auditorium)  
Department of Computer Science  
Princeton University

**Monday, June 13**

## **On the Unique Games Conjecture**

Subhash Khot [khot@cs.nyu.edu]

This will be a survey of recently discovered connections between the Unique Games Conjecture and computational complexity, algorithms, discrete Fourier analysis, and geometry.

## **Subexponential Algorithms for Unique Games and Related Problems**

David Steurer [dsteurer@cs.princeton.edu]

We give a subexponential-time approximation algorithm for the Unique Games problem: Given a Unique Games instance with optimal value  $1 - \epsilon^3$  and alphabet size  $k$ , our algorithm finds in time  $\exp(k \cdot n^\epsilon)$  a solution of constant value (say at least 0.99).

We also obtain subexponential algorithms with similar approximation guarantees for Small-Set Expansion and Multi Cut. For Max Cut, Sparsest Cut and Vertex Cover, our techniques lead to subexponential algorithms with improved approximation guarantees on interesting subclasses of instances.

Khot's Unique Games Conjecture (UGC) states that it is NP-hard to achieve approximation guarantees such as ours for Unique Games. While our results stop short of refuting the UGC, they do suggest that Unique Games is significantly easier than NP-hard problems such as Max 3-SAT, Label Cover and more, that are believed not to have subexponential algorithms achieving a non-trivial approximation guarantee.

The main component in our algorithms is a new kind of graph decomposition that may have other applications: We show that every graph with  $n$  vertices can be efficiently partitioned into disjoint induced subgraphs, each with at most  $n^\epsilon$  eigenvalues above  $1 - \epsilon^3$ , such that at most 0.01 of the edges do not respect the partition.

Joint work with Sanjeev Arora and Boaz Barak.

## **Vertex-Connectivity Survivable Network Design**

Sanjeev Khanna [sanjeev@cis.upenn.edu]

In the vertex-connectivity survivable network design problem (SNDP), we are given an undirected graph with costs on edges, along with a vertex-connectivity requirement for each pair of vertices. The goal is to find a minimum-cost subset of edges that satisfies the given set of pairwise connectivity requirements. In this talk, we will describe some recent progress on designing approximation algorithms for this problem.

## **Approximation Algorithms for Discrepancy Problems**

Nikhil Bansal [nikhil@us.ibm.com]

The minimum discrepancy problem is the following: Given a collection of sets  $S_1, \dots, S_m$ , color the elements red and blue such that each set is colored as evenly as possible.

I will describe an algorithm that finds low discrepancy colorings when the set system is guaranteed to have low hereditary discrepancy. Our algorithm is based on combining SDPs with previously known non-constructive techniques. Time permitting, we will also describe some more refined applications of this approach, such as a

constructive proof of Spencer's celebrated "six standard deviations suffice" result.

### **Why do we want a good ratio anyway? Approximation stability and proxy objectives**

Avrim Blum [avrim@cs.cmu.edu]

Often in a problem formulation, the objective to be optimized is a proxy for some other underlying goal. For example, a distance-based clustering objective (such as k-means or k-median) in the case of clustering protein sequences is really a proxy for a true goal of correctly identifying the functions of the proteins; computing an (approximate) equilibrium in a game may be a proxy for a true goal of predicting behavior. Making explicit the assumption (or hope) that a good solution to the objective being measured implies a good solution to the final goal provides structure that an algorithm can potentially use. As one example, for clustering we show that if a  $c$ -approximation to the k-means or k-median objective would be sufficient to yield solutions that are  $\epsilon$ -close to a desired target clustering, then we can produce clusterings that are  $O(\epsilon)$ -close to the target, even if obtaining a  $c$ -approximation is NP-hard. In particular, we achieve this guarantee for any constant  $c > 1$ . Furthermore if clusters are large, we can actually get  $\epsilon$ -close – as good as if we could approximate the objective to the NP-hard value. We also discuss initial steps in this direction for the problem of estimating Nash equilibria, and other problems where this approach may be fruitful.

This talk includes work joint with Pranjal Awasthi, Nina Balcan, Anupam Gupta, Or Sheffet, and Santosh Vempala.

**Tuesday, June 14**

### **Online Matching and the Adwords Market**

Aranyak Mehta [aranyak@google.com]

The spectacular success of search and display advertising and its huge growth potential has attracted the attention of researchers from many aspects of computer science. A core problem in this area is that of Ad allocation, an inherently algorithmic and game theoretic question - that of matching ad slots to advertisers, under demand and supply constraints. Put very simply: the better the matching, the more efficient the market.

Interestingly, the seminal work on online matching, by Karp, Vazirani and Vazirani, was done over two decades ago, well before this motivation existed. In this talk, I will present an overview of several key algorithmic papers in this area, starting with its purely academic beginnings, to algorithms that address the actual Adwords problem. The theory behind these algorithms involves new combinatorial, probabilistic and linear programming techniques. Besides the analytical results, I will also present some approaches for using these algorithmic ideas in practice.

### **Understanding Karp-Vazirani-Vazirani's Online Matching (1990) via Randomized Primal-Dual**

Kamal Jain [kamalj@microsoft.com]

KVV online matching algorithm is one of the most beautiful online algorithms. The algorithm is simple though its analysis is not equally simple. Some simpler version of analysis are developed over the last few years. Though, a mathematical curiosity still remains of understanding what is happening behind the curtains, which has made extending the KVV algorithm hard to apply to other problems, or even applying to the more general versions of online matching itself.

In this talk I will present one possibility of lifting the curtains. We develop a randomized version of Primal-Dual schema and redevelop KVV algorithm within this framework. I will then show how this framework makes extending KVV algorithm to vertex weighted version essentially trivial, which is currently done through a lot of hard work in a brilliant paper of Aggarwal-Goel-Karande-Mehta (2010).

Randomized version of Primal-Dual schema was also a missing technique from our toolbox of algorithmic techniques. So this talk also fills that gap. This work is done just days ago. We are currently trying to extend it to other problems and this talk is an invitation to other colleagues to help.

Collaboration: This work is joint with Nikhil Devanur and Bobby Kleinberg.

### **Buy-at-bulk Network Design (with Protection)**

Chandra Chekuri [chekuri@cs.illinois.edu]

In the buy-at-bulk network design problem we are given a graph  $G=(V,E)$  and node pairs  $(s_1,t_1),\dots,(s_k,t_k)$ . Pair  $i$  has an associated demand  $d_i$  that needs to be routed between  $s_i$  and  $t_i$ . The goal is design a cheapest network to enable the pairs to route their demand where it is assumed that building/buying a capacity  $x$  on edge  $e$  of  $G$  costs  $f_e(x)$  where  $f_e$  is a sub-additive function. This problem, as the name suggests, is motivated by economies of scale present in many real-world network design problems, in particular those arising from telecommunication networks.

In addition to its practical importance, the problem has also been instrumental in the development of some new techniques in the design of approximation algorithms, and proving inapproximability results for routing problems. The talk will quickly survey the known results and techniques with a slight emphasis on the use of junction-trees which resulted in the first poly-logarithmic approximation for the non-uniform case (each edge has a different cost function  $f_e$ ). We will highlight the extension of the problem when the pairs need robustness to edge and node failures. This problem referred to as buy-at-bulk network design with protection is motivated by practical applications in high speed optical networks. Although it has been introduced a few years ago, its approximability is poorly understood other than in some special cases.

### **Online Algorithms, the Primal-Dual Method, and the k-Server Problem**

Seffi Naor [naor@cs.technion.ac.il]

The k-server problem is one of the most central and well studied problems in competitive analysis and is considered by many to be the "holy grail" problem in the field. In the k-server problem, there is a distance function defined over a metric space containing  $n$  points, and  $k$  servers located at the points of the metric space. At each time step, an online algorithm is given a request at one of the points of the metric space, and it is served by moving a server to the requested point. The goal of an online algorithm is to minimize the total sum of the distances traveled by the servers so as to serve a given sequence of requests. The k-server problem captures many online scenarios, and in particular the widely studied paging problem. I will present a new randomized algorithm for the k-server problem that achieves a polylogarithmic (in  $k$  and  $n$ ) competitive factor.

Based on joint papers with Nikhil Bansal, Niv Buchbinder, and Aleksander Madry.

### **Exponential lower bounds for infinitary payoff game policy iteration and linear program simplex methods**

Oliver Friedmann [oliver.friedmann@googlemail.com]

Policy iteration is one of the most important algorithmic schemes for solving problems in the domain of determined game theory such as parity games, stochastic games and Markov decision processes, and many more. It is parameterized by an improvement rule that determines how to proceed in the iteration from one policy to the next. It is a major open problem whether there is an improvement rule that results in a polynomial time algorithm for solving one of the considered (two-player) game classes.

Simplex algorithms for solving linear programs are closely related to policy iteration algorithms. Like policy iteration, the simplex algorithm is parameterized by a so-called pivoting rule that describes how to proceed from one basic feasible solution in the linear program to the next. Also, it is a major open problem whether there is a pivoting rule that results in a (strongly) polynomial time algorithm for solving linear programs.

We describe our recent constructions for parity games that give rise to an exponential lower bounds for several improvement rules, and how to extend the lower bound to more expressive game classes like stochastic games. We show that our construction for parity games can be translated to Markov decision processes, transferring our lower bound to their domain, and finally show how the lower bounds for the MDPs can be transferred to the linear programming domain, solving problems that have been open for several decades.

### **Prize-collecting Frameworks**

Mohammad Taghi Hajiaghayi [hajiaghayi@MIT.EDU]

Prize-collecting problems are classic optimization problems and work on them has had a large impact, both in theory and practice. Prize-collecting problems involve situations where there are various demands that desire to be "served" by some structure and we must find the structure of lowest cost to accomplish this. However, if some of the demands are too expensive to serve, then we can refuse to serve them and instead pay a penalty. For example at AT&T, prize collecting programs have been incorporated in access network design tools and saved the company billions of dollars. This makes prize-collecting problems deserving of study in their own right. In particular, the prize-collecting Steiner

problems} are well-known network design problems with several applications in expanding telecommunications networks, cost sharing, and Lagrangian relaxation techniques. In the prize-collecting framework, recently a new clustering paradigm, namely prize-collecting clustering, has been introduced in which items are vertices of a graph. Vertices have their own budgets and the goal is to cluster them such that the cost of (connections in) each cluster can be paid by the budget of its participants. Furthermore, we want vertices in different clusters be in some sense far from each other.

In this talk after presenting the general prize-collecting framework and its practical applications, we describe prize collecting clustering and its use in improvement on the approximation ratios for prize-collecting Steiner tree, Steiner forest, Steiner networks, and TSP and even obtaining PTASs for some of these problems and others on planar graphs.

### **Approximation Algorithms for Stochastic Problems**

Anupam Gupta [anupamg@cs.cmu.edu]

There has been a considerable amount of work on approximation algorithms for stochastic combinatorial optimization, and also for online problems with stochastic arrivals. This talk will survey some results and techniques in this area.

### **Network Design with Diseconomies of Scale**

Lisa Zhang [ylz@research.bell-labs.com]

Power efficiency has emerged as an important concern in the design and operation of networks. In this talk we first model power-aware routing as a multi-commodity flow problem where the cost function displays diseconomies of scale. For this problem we present a polylogarithmic approximation. We also briefly present a simulation study which aims to identify as realistically as possible the opportunity of power savings in IP networks.

### **Iterative Methods in Combinatorial Optimization**

Mohit Singh [mohit@cs.mcgill.ca]

In this talk we will demonstrate *iterative methods* as a general technique to analyze linear programming formulations of many combinatorial optimization problems. The technique builds on the result of Jain who introduced the iterative rounding framework in the 90's. The talk will focus on the recent applications to degree bounded network design problems where the task is to minimize the cost of the network and also satisfy given degree bounds on nodes. The method enables to achieve additive approximation algorithms for these problems which add to a rather small list of combinatorial optimization problems which have an additive approximation algorithm.

## **Wednesday, June 15**

### **Constructive Aspects of the Lovasz Local Lemma and their Applications**

Aravind Srinivasan [srin@cs.umd.edu]

Recent years have seen significant progress on the algorithmic aspects of the Lovasz Local Lemma: e.g., one can now handle super-polynomially many events that need to be avoided. I will survey this general area, as well as my joint work with Bernhard Haeupler and Barna Saha.

### **Generalized Online Matching with Concave Utilities**

Kamal Jain [kamalj@microsoft.com]

In this talk we consider a search engine's ad matching problem with soft budget. In this problem, there are two sides. One side is ad slots and the other is advertisers. Currently advertisers are modeled to have hard budget, i.e., they have full utility for ad slots until they reach their budget, and at that point they can't be assigned any more ad-slots. Mehta Saberi-Vazirani-Vazirani and Buchbinder-J-Naor gave a  $1-1/e$  approximation algorithm for this problem, the latter had a traditional primal-dual analysis of the algorithm.

In this talk, we consider a situation when the budgets are soft. This is a natural situation if one models that the cost of capital is convex or the amount of risk is convex. Having soft budget makes the linear programming relaxation as a more general convex programming relaxation. We still adapt the primal-dual schema to this convex program using an

elementary notion of convex duality. The approximation factor is then described as a first order non-linear differential equation, which has at least  $1-1/e$  as its solution. In many cases one can solve these differential equations analytically and in some cases numerically to get algorithms with factor better than  $1-1/e$ .

Credits: This work is based on many collaborations. Collaborators include: Mark Braverman, Niv Buchbinder, Nikhil Devanur, and Seffi Naor.

### **Steiner Tree Approximation via Iterative Randomized Rounding**

Fabrizio Grandoni [fabrizio.grandoni@gmail.com]

The Steiner tree problem is one of the most fundamental NP-hard problems: given a weighted undirected graph and a subset of terminal nodes, find a minimum-cost tree spanning the terminals. In a sequence of papers, the approximation ratio for this problem was improved from 2 to 1.55 [Robins, Zelikovsky-'05]. All these algorithms are purely combinatorial. A long-standing open problem is whether there is an LP relaxation of Steiner tree with integrality gap smaller than 2 [Vazirani, Rajagopalan-'99].

In this work we present an LP-based approximation algorithm for Steiner tree with an improved approximation factor. Our algorithm is based on a, seemingly novel, iterative randomized rounding technique. We consider an LP relaxation of the problem, which is based on the notion of directed components. We sample one component with probability proportional to the value of the associated variable in a fractional solution: the sampled component is contracted and the LP is updated consequently. We iterate this process until all terminals are connected. Our algorithm delivers a solution of cost at most  $\ln(4)+\epsilon < 1.39$  times the cost of an optimal Steiner tree. The algorithm can be derandomized using the method of limited independence.

As a byproduct of our analysis, we show that the integrality gap of our LP is at most 1.55, hence answering to the mentioned open question. This might have consequences for a number of related problems.

Joint work with J. Byrka, T. Rothvoss, and L. Sanità.

### **Allocating Goods to Maximize Fairness**

Julia Chuzhoy [cjulia@ttic.edu]

We consider the Max-Min Allocation problem, in which we are given a set of  $m$  agents and a set of  $n$  items, together with utilities  $u(A,i)$  of agent  $A$  for item  $i$ . Our goal is to allocate items to agents to maximize fairness. Specifically, the utility of an agent is the sum of its utilities for the items it receives, and we seek to maximize the minimum utility of any agent. While this problem has received much attention, its approximability has not been well-understood thus far: the best previously known approximation algorithm achieved a roughly  $O(\sqrt{m})$ -approximation, and in contrast, the best known hardness of approximation stands at factor  $2$ .

We present an approximation algorithm that achieves a  $\tilde{O}(n^{\epsilon})$  approximation in time  $n^{O(1/\epsilon)}$ , for any  $\epsilon = \Omega(\log \log n / \log n)$ . In particular, we obtain poly-logarithmic approximation in quasi-polynomial time, and for every constant  $\epsilon > 0$ , we obtain an  $O(n^{\epsilon})$ -approximation in polynomial time.

Joint work with Deeparnab Chakrabarty and Sanjeev Khanna.

### **The Edge-Disjoint Paths with Congestion problem: Algorithms, Lower Bounds and Open Problems**

Matthew Andrews [andrews@research.bell-labs.com]

In the Edge-Disjoint Paths problem (EDP) we are given a set of demand pairs in a graph and we wish to integrally connect as many of them as possible using disjoint paths. In the Edge-Disjoint Paths with Congestion problem (EDPwC) we wish to approximate the optimal solution to EDP but we allow a small amount of congestion on each edge. The last decade has seen a number of negative results for EDPwC based on high-girth instances and a number of positive results based on embedding an expander on the terminals. However, for the case of constant congestion in general graphs there is still an exponential gap between the best upper and lower bounds on the approximation ratio. In this talk we will attempt to put recent positive and negative results into a common framework and identify some hurdles that would need to be overcome in order to shrink the gap (hopefully within the next decade).

**Thursday, June 16**

### **Generic techniques to round SDP relaxations**

Prasad Raghavendra [nrprasad@gmail.com]

This talk will survey two general approaches to rounding solutions to SDP relaxations.

#### **Rounding by Miniatures:**

This technique of rounding SDPs for constraint satisfaction problems generalizes and unifies a large body of SDP based algorithms for CSPs. More specifically, it yields a generic algorithm that for every CSP, achieves an approximation at least as good as the best known algorithm in literature. The generic algorithm is guaranteed to achieve an approximation ratio equal to the integrality gap of an SDP relaxation known to be optimal under Unique Games Conjecture.

This is based on joint work with David Steurer.

#### **Rounding Using Correlations:**

Despite the numerous applications of semidefinite programming, in all but very few cases, the algorithms rely on arguably the simplest SDP relaxation. Hence the power of stronger semidefinite programming relaxations is not yet harnessed, leaving open the possibility that fundamental optimization problems like MaxCut and Vertex Cover could be approximated better using SDPs. The dearth of algorithms based on stronger SDP relaxations stems from the lack of general techniques to round these relaxations.

In this work, we present a technique to round SDP hierarchies using the underlying correlations between variables. To demonstrate the technique, we present a subexponential time algorithm for Unique Games based on the Lasserre SDP hierarchy. A subexponential time algorithm for Unique Games was previously designed in the work of Arora Barak-Steurer using a combination of subspace-enumeration (brute-force search) and spectral techniques.

This is joint work with David Steurer and Boaz Barak.

### **A Combinatorial, Primal-Dual Approach to Semidefinite Programs**

Satyen Kale [skale@yahoo-inc.com]

Algorithms based on convex optimization, especially linear and semidefinite programming, are ubiquitous in Computer Science. While there are polynomial time algorithms known to solve such problems, quite often the running time of these algorithms is very high. Designing simpler and more efficient algorithms is important for practical impact.

In my talk, I will describe applications of a Lagrangian relaxation technique, the Multiplicative Weights Update method in the design of efficient algorithms for various optimization problems. We generalize the method to the setting of symmetric matrices rather than real numbers. The new algorithm yields the first truly general, combinatorial, primal dual method for designing efficient algorithms for semidefinite programming. Using these techniques, we obtain significantly faster algorithms for approximating the Sparsest Cut and Balanced Separator in both directed and undirected weighted graphs to factors of  $O(\log n)$  and  $O(\sqrt{\log n})$ , and also for the Min UnCut and Min 2CNF Deletion problems. The algorithm also has applications in quantum computing and derandomization.

This is joint work with Sanjeev Arora.

### **Discrete extension and selection problems**

Rabani Yuval [yrabani@cs.huji.ac.il]

We will present the current state-of-the-art concerning the approximation of various graph labeling problems. Graph decompositions are the primary tool employed in solving such problems, and we will also present some of the unintended consequences of the work on labeling. We will explain the connection to Lipschitz extension and selection problems. Finally, we will discuss the limitations on approximation, the limitations of the current methods, and some open problems.

### **Approximating Metrics by Tree Metrics**

Kunal Talwar [kunal@microsoft.com]

Several optimization problems are NP hard to solve in general metric spaces. However, often these problems turn out to be easy on simple metrics such as tree metrics. A general approach to solving such problems is to approximate the input metric by a tree metric in such a way that solving the problem on the tree suffices to give a good solution for the original metric. I will survey these probabilistic tree embeddings, show some applications to approximation algorithms, and sketch a proof of an embedding with distortion  $O(\log n)$  for any  $n$  point metric.

### **Finding Dense Subgraphs**

Moses S. Charikar [moses@CS.Princeton.EDU]

The problem of finding dense subgraphs in a graph arises in a number of settings. This problem has frustrated the development of good algorithms and yet resisted attempts to prove complexity theoretic lower bounds (although some versions are known to be efficiently solvable). The assumption that the problem is hard has been used to construct cryptosystems and establish the computational complexity of evaluating financial derivatives.

Variants of this problem have been shown to be closely connected to the Unique Games Conjecture. In this talk, I will survey some of the results that are known for the densest subgraph problem, focusing on recent algorithmic results. These results (for worst case analysis) are inspired by studying a planted version of the problem and suggest a concrete distribution on inputs that seems to be barrier for further progress.

Based on joint work with Aditya Bhaskara, Eden Chlamtac, Uriel Feige and Aravindan Vijayaraghavan.

### **Approximability of Submodular Combinatorial Optimization Problems**

Pushkar Tripathi [pushkar.tripathi@gatech.edu]

Applications in complex systems such as the Internet have spawned recent interest in studying situations involving multiple agents with their individual cost or utility functions. In this paper, I will introduce an algorithmic framework for studying combinatorial problems in the presence of multiple agents with submodular cost functions. We study several fundamental covering problems (Vertex Cover, Shortest Path, Perfect Matching, and Spanning Tree) in this setting and establish tight upper and lower bounds for the approximability of these problems. We also study a special subclass of submodular functions that is succinctly representable namely, discounted submodular functions, and establish improved upper and lower bounds for these problems.

This is based on joint work with : Gagan Goel, Chinmay Karande and Lei Wang.

### **Scheduling to minimize flow time**

Naveen Garg [naveen@cse.iitd.ac.in]

We will consider the problem of scheduling jobs so as to minimize total flow time. The flow time of a job is the difference in its completion and release times. The talk will survey recent work on this problem for different machine models and will emphasize the role of linear programming in designing and analyzing the algorithms.

### **Introduction to LP and SDP hierarchies**

Madhur Tulsiani [tulsiani@cs.princeton.edu]

I will give overview of various hierarchies which strengthen linear and semidefinite programs by adding increasingly larger local constraints. I will describe few techniques for arguing about them and survey some results proved over the past few years.

## **Friday, June 17**

### **New Approximation Schemes for Optimization Problems in Planar Graphs**

Philip Klein [klein@cs.brown.edu]

Research on polynomial-time approximation schemes for optimization problems in planar graphs goes back to the pioneering work of Lipton and Tarjan (1977) and Baker (1983). Beginning in 2005, however, a flurry of results were obtained, greatly broadening the range of problems for which fast approximation schemes for planar graphs are

known. In this talk, we describe one approach to obtaining such approximation schemes, and sketch how this approach leads to fast approximation schemes for, e.g., Traveling Salesman, Two-Edge-Connected Subgraph, Steiner Tree, Steiner Forest, and Multiterminal Cut in planar graphs.

### **Universal Approximations in Network Design**

Rajmohan Rajaraman [rraj@ccs.neu.edu]

Research in network design over the past decade has revealed that it is often possible to derive network structures (e.g., routes, multicast trees) that yield good approximations simultaneously for a range of input instances. Examples include single-sink buy-at-bulk, oblivious routing, universal Steiner Tree, universal and a priori approximations for TSP.

This talk will focus on one class of results in this area -- universal approximations, which seek the design of a single structure that simultaneously approximates an optimal solution for every possible input. We will present the main results in this area, draw connections with related problems and techniques including graph decompositions, and conclude with some intriguing open problems.