Tutorial: Algorithmic Issues in Network Resource Reservation Problems

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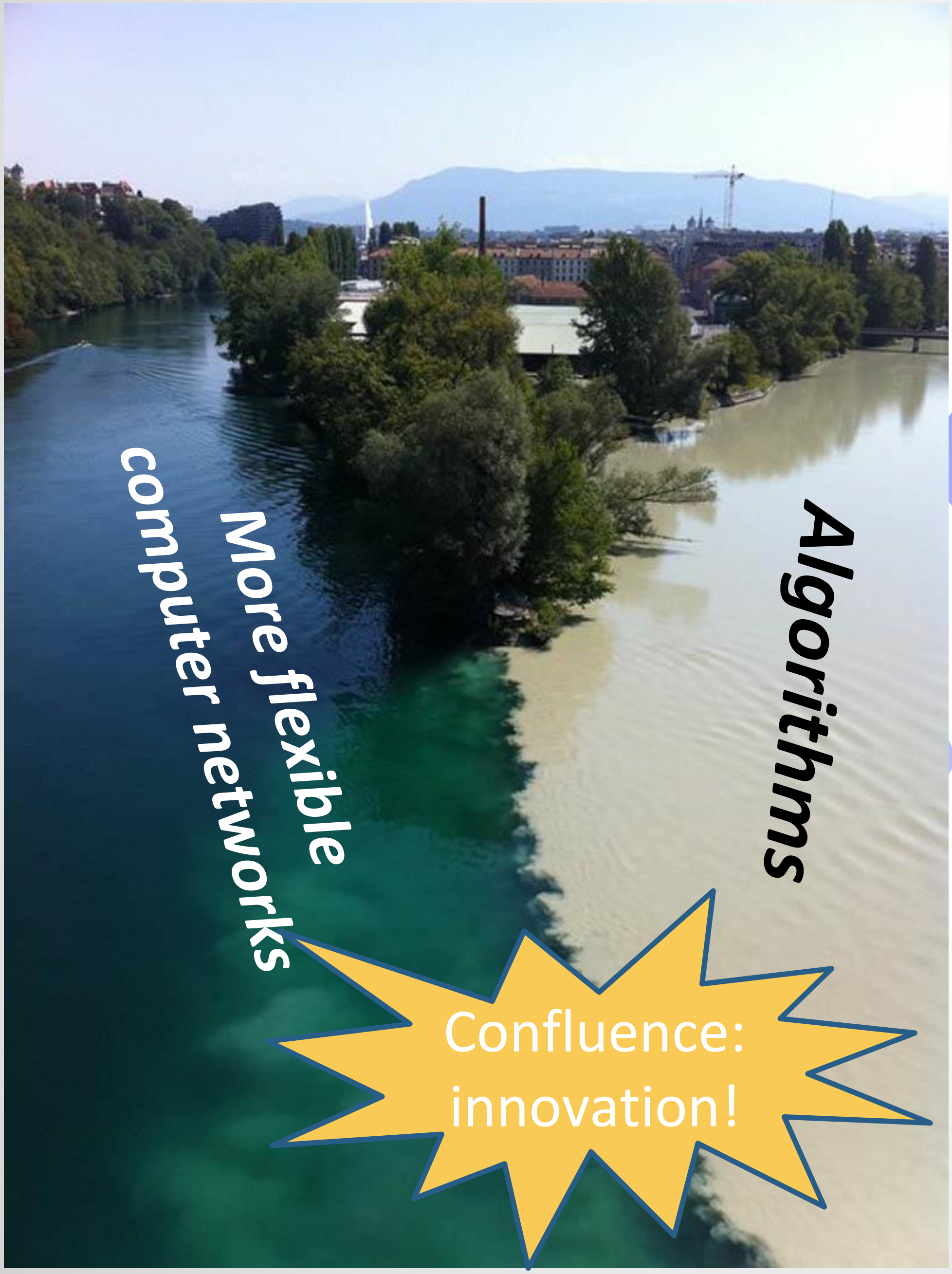
Aalborg University, Denmark & TU Berlin, Germany

DIMACS Workshop on Algorithms
for Data Center Networks

A rehash: It’s a great time to be a scientist!



More flexible computer networks

Algorithms

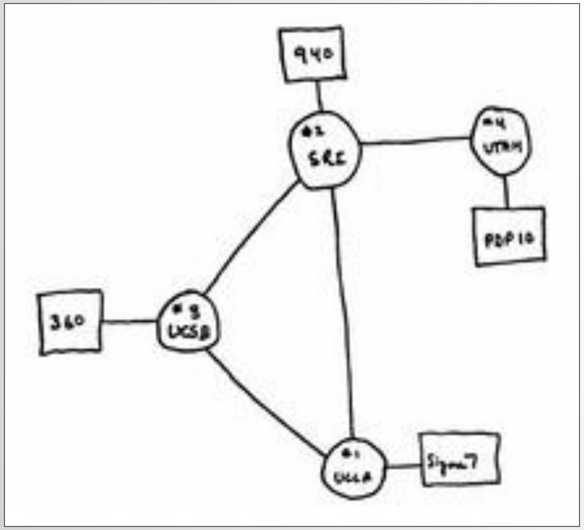
Confluence: innovation!

How to exploit these flexiblities? How not to shoot in our feet? Can be challenging!

”We are at an interesting inflection point!”
Keynote by George Varghese at SIGCOMM 2014

New Flexiblities: It’s About Time!

- Datacenter networks, enterprise networks, Internet: a critical infrastructure of the information society
- We have seen a huge shift in scale and applications…
- … but many Internet protocols hardly changed!



Applications: file transfer, email

Goal: connectivity between researchers

New requirements and challenges!

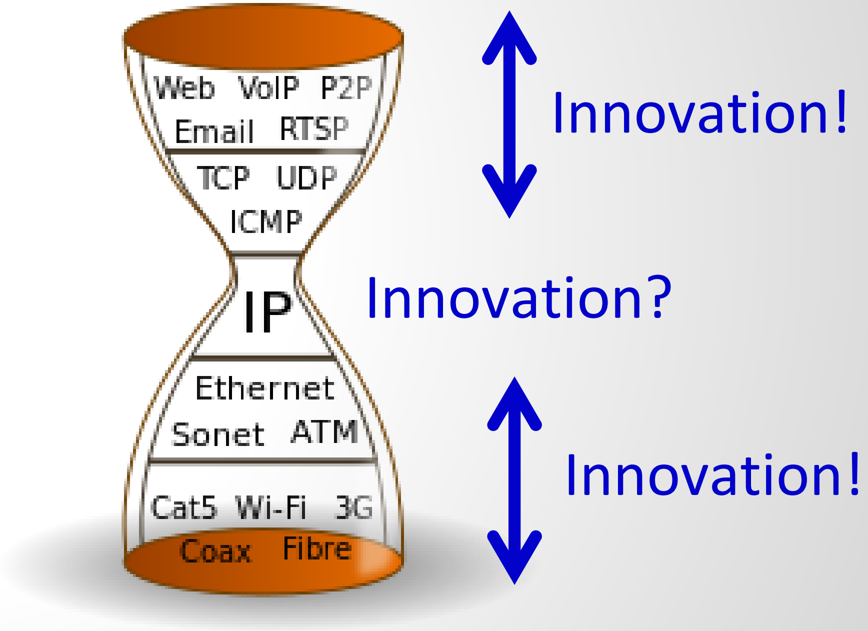
Applications: live streaming, IoT, etc.

Goal: quality-of-service, predictable performance, low latency, …

Opportunity 1 of Network Virtualization: Overcoming Ossification

- ❑ Recent concern: Ossification in the network core
  - ❑ Are computer networks future-proof?
  - ❑ Meet the new requirements of new applications?

- ❑ Example Internet-of-Things:
  - ❑ IPv4: ~4.3 billion addresses, Gartner study: 20+ billion "smart things" by 2020
  - ❑ New security threats: recent DDoS attack based on IoT (almost 1TB/s, coming from webcames, babyphones, etc.)



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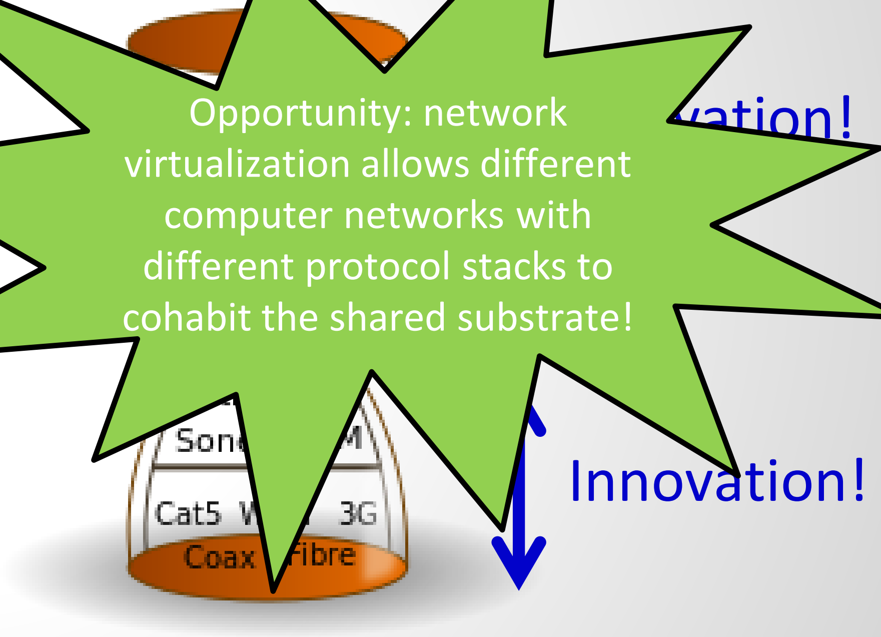
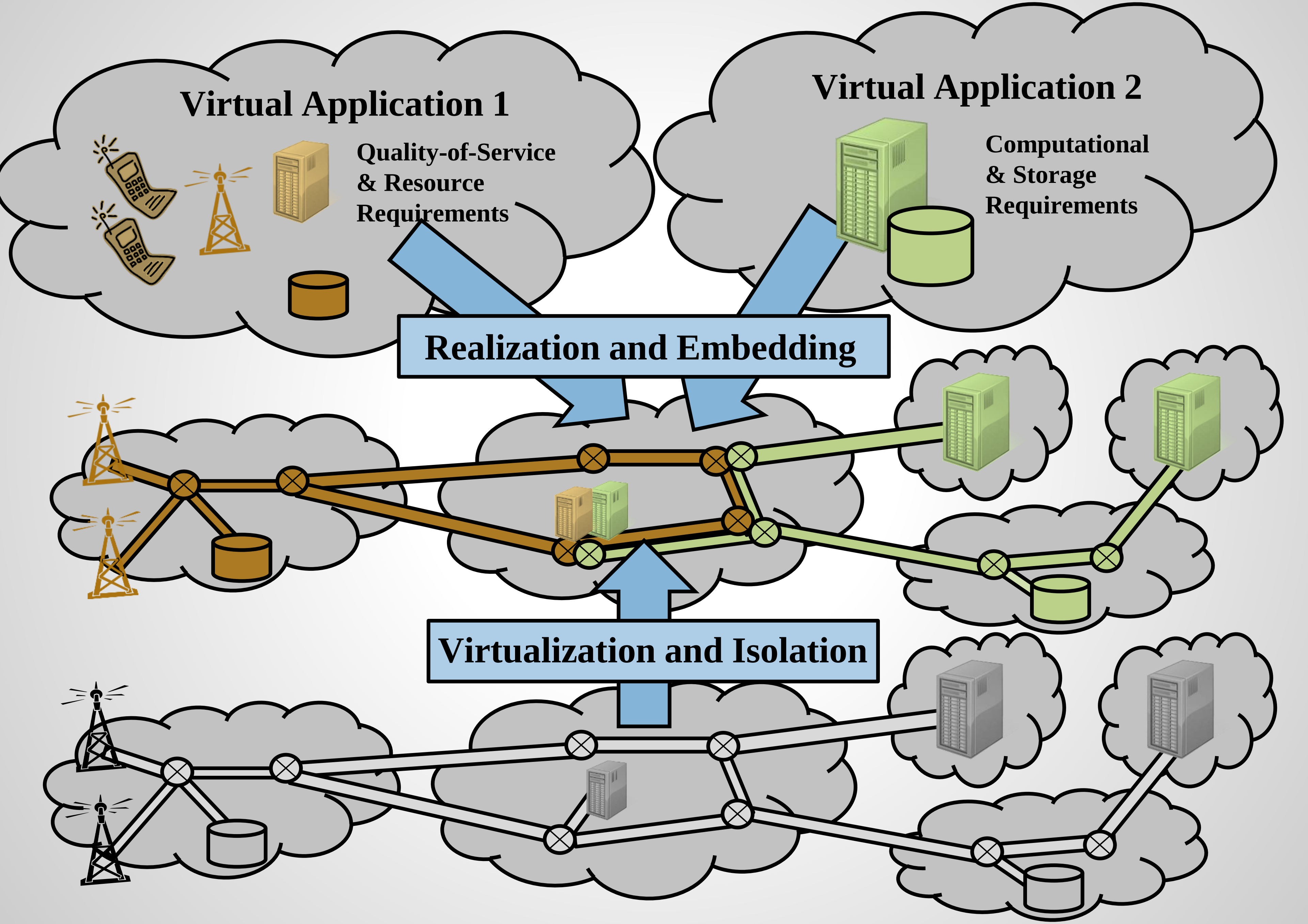


Opportunity: network virtualization allows different computer networks with different protocol stacks to cohabit the shared substrate!

Opportunity 2: Enable Resource Sharing for Improved Utilization



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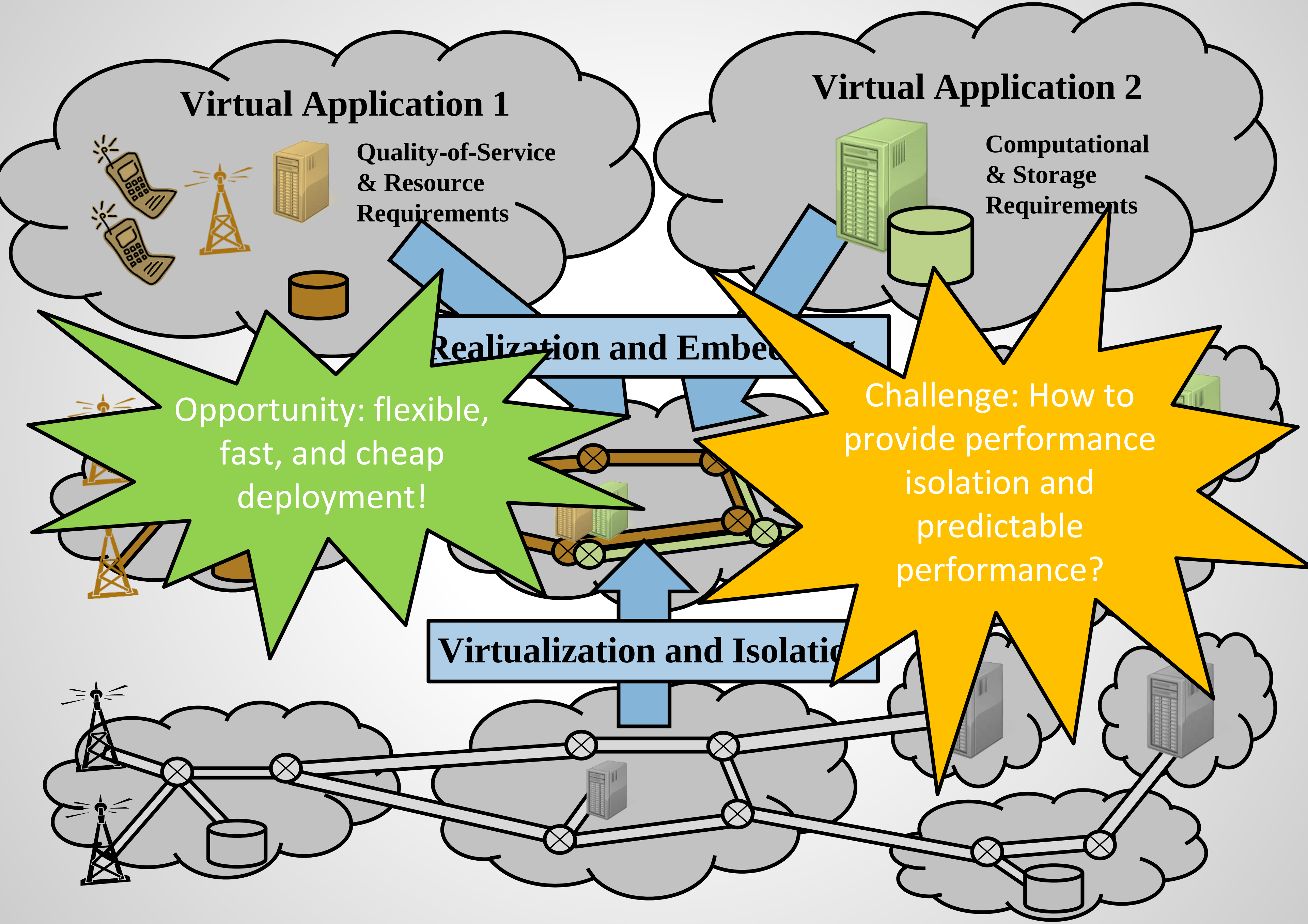

Opportunity: flexible, fast, and cheap deployment!

Challenge: How to provide performance isolation and predictable performance?

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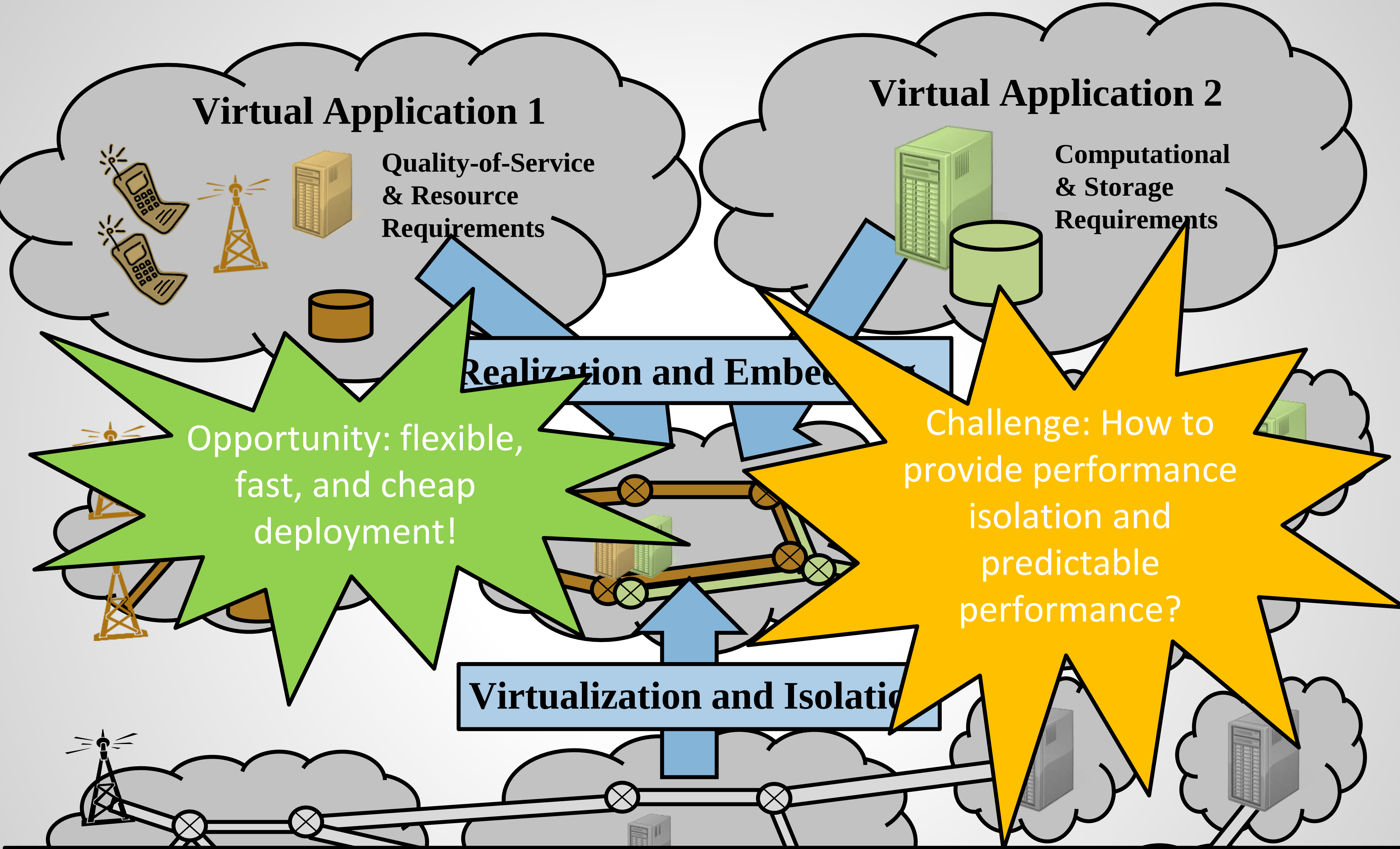


Opportunity: flexible, fast, and cheap deployment!

Challenge: How to provide performance isolation and predictable performance?

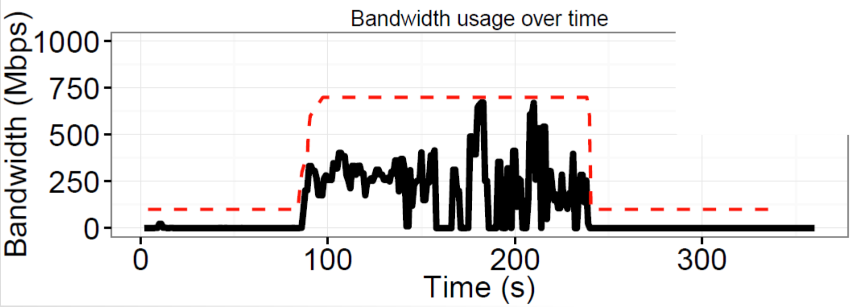
In general: For a predictable application performance, performance isolation needs to be provided along *all* involved resources.

Focus Today: *The Network*

The Network Matters

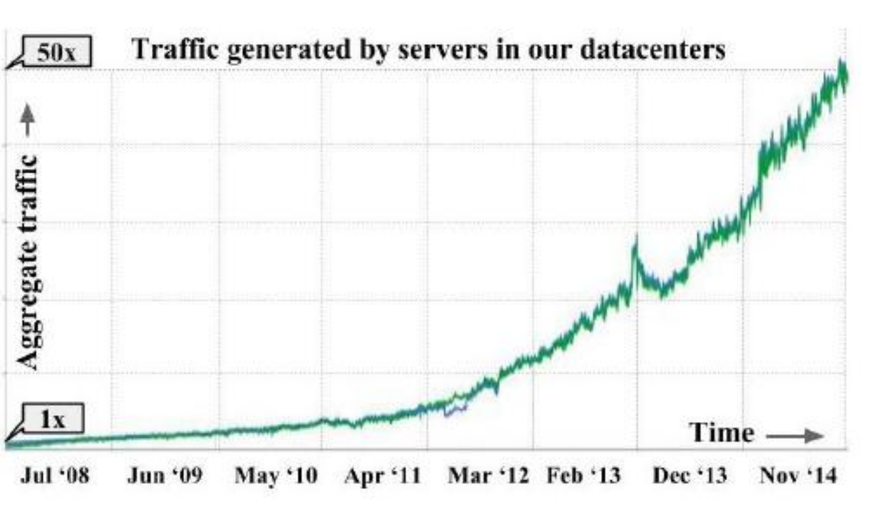
- Cloud-based applications generate significant network traffic
  - E.g., scale-out databases, streaming, batch processing applications

Example 1: Hadoop Terrasort job



shuffle phase

Example 2: Aggregate Server Traffic in Google datacenter



Jupiter rising @ SIGCOMM 2015

Example 3: More memory-based systems (network becoming bottleneck again)

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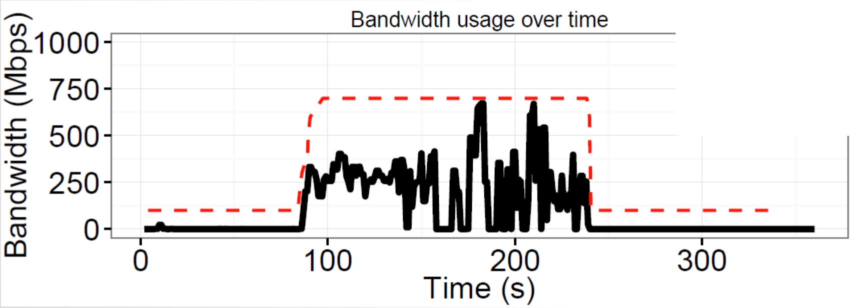

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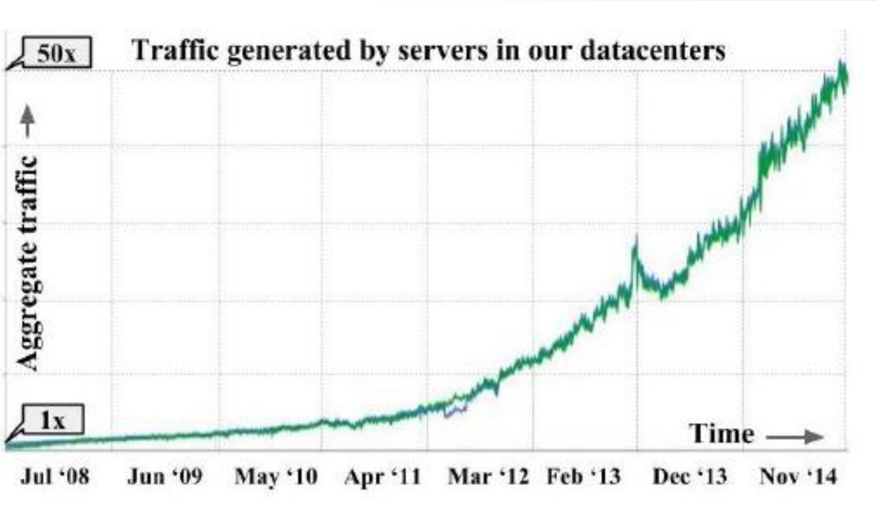

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As much time is spent on communication: For a predictable application performance, bandwidth resources need to be reserved explicitly.

The Network Matters

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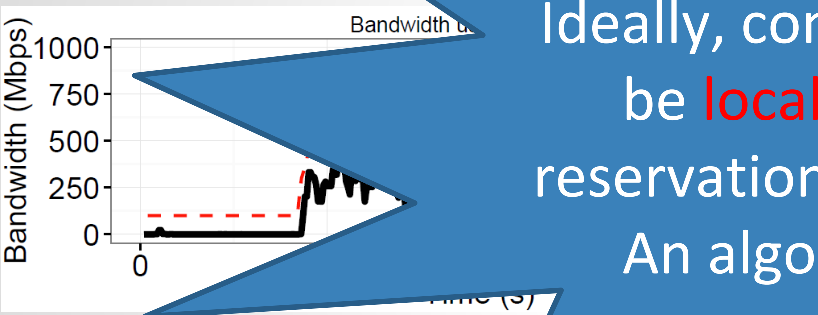
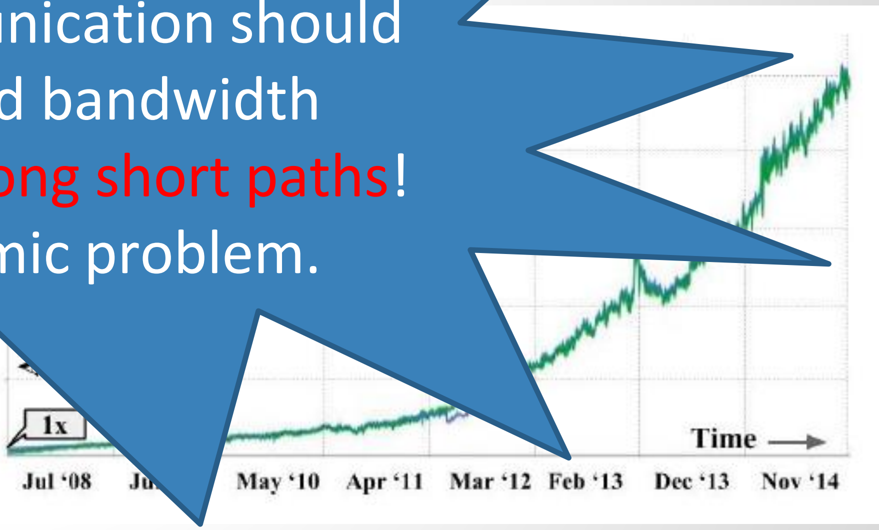


shuffle phase

Examp... Aggregate Server Traffic in ...



Jupiter rising @ SIGCOMM 2015

Ideally, communication should be local. And bandwidth reservations along short paths! An algorithmic problem.

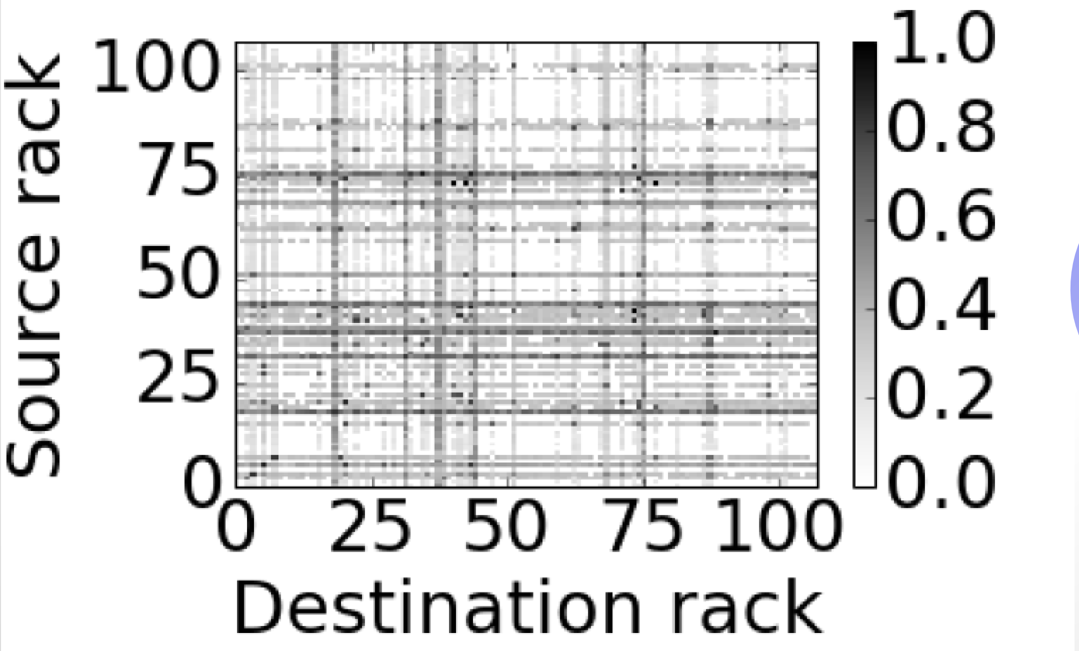
Example 3: More memory-based systems (network becoming bottleneck again)

As much time is spent on communication: For a predictable application performance, bandwidth resources need to be reserved explicitly.

Structure in Traffic Matrix = Optimization Opportunities

- At the same time, traffic matrices are often far from random and uniform, but have a lot of structure and are sparse

Example 1: Often little to no traffic between many racks



Without **taking this structure into account**, some links may be **overprovisioned** and others **underprovisioned**
.

Heatmap of rack-to-rack traffic
ProjecToR @ SIGCOMM 2016

Focus Today: *The Network*

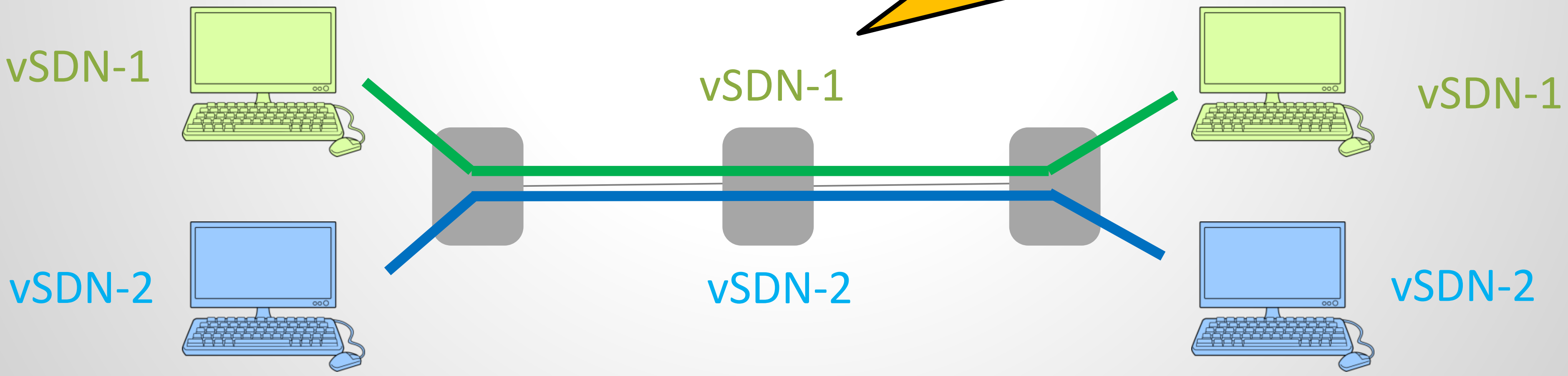
We will be talking a lot about bandwidth reservations.
But: Predictable network performance is about more, and interference can come in many flavors!

<remark>

The Many Faces of Performance Interference

Consider: 2 **SDN-based virtual networks (vSDNs)** sharing physical resources!

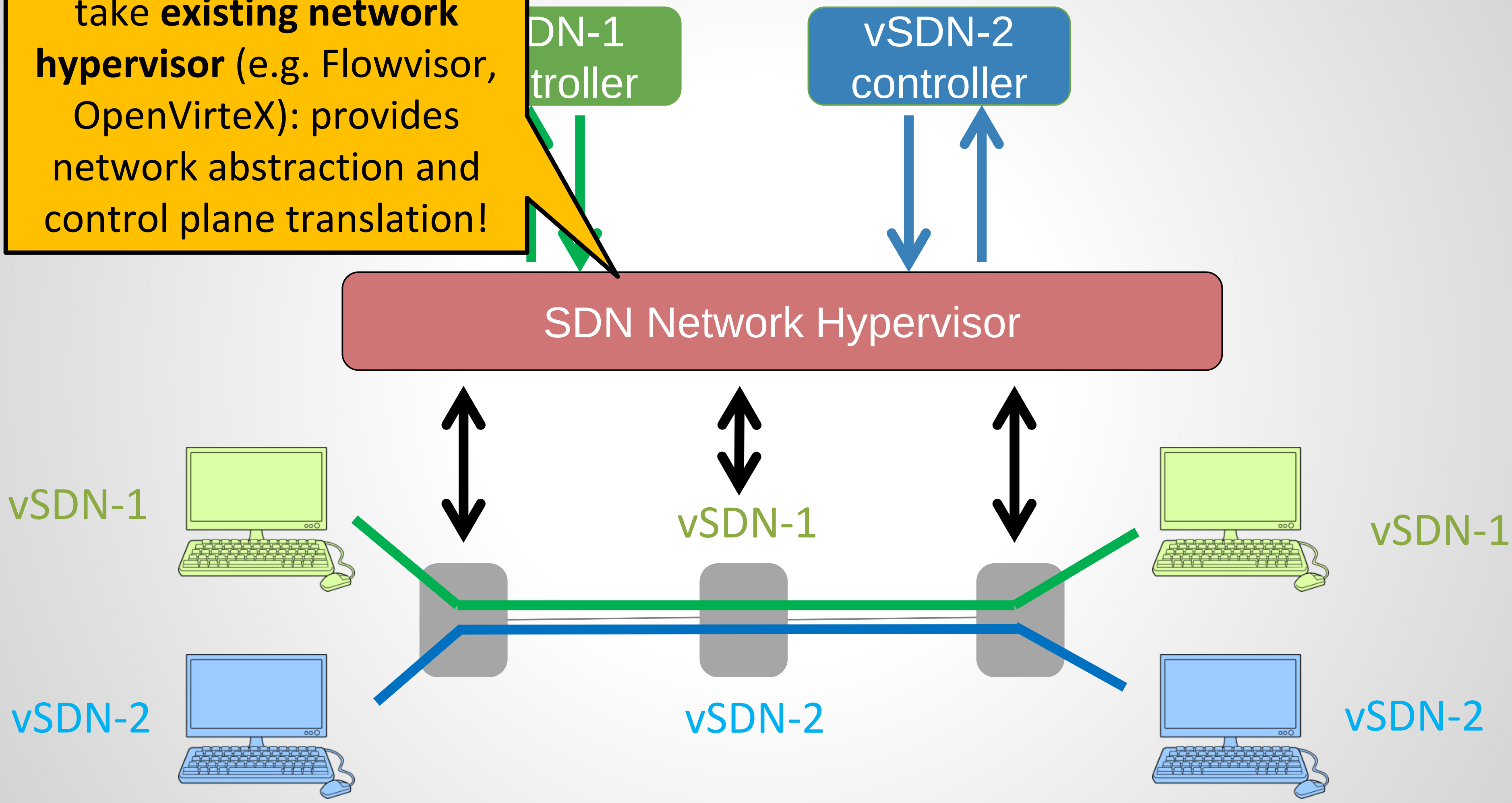
Assume: perfect performance isolation on the network!



An Experiment: 2 vSDNs with bw guarantee!

The Many Faces of Performance Interference

To enable **multi-tenancy**, take **existing network hypervisor** (e.g. Flowvisor, OpenVirteX): provides network abstraction and control plane translation!



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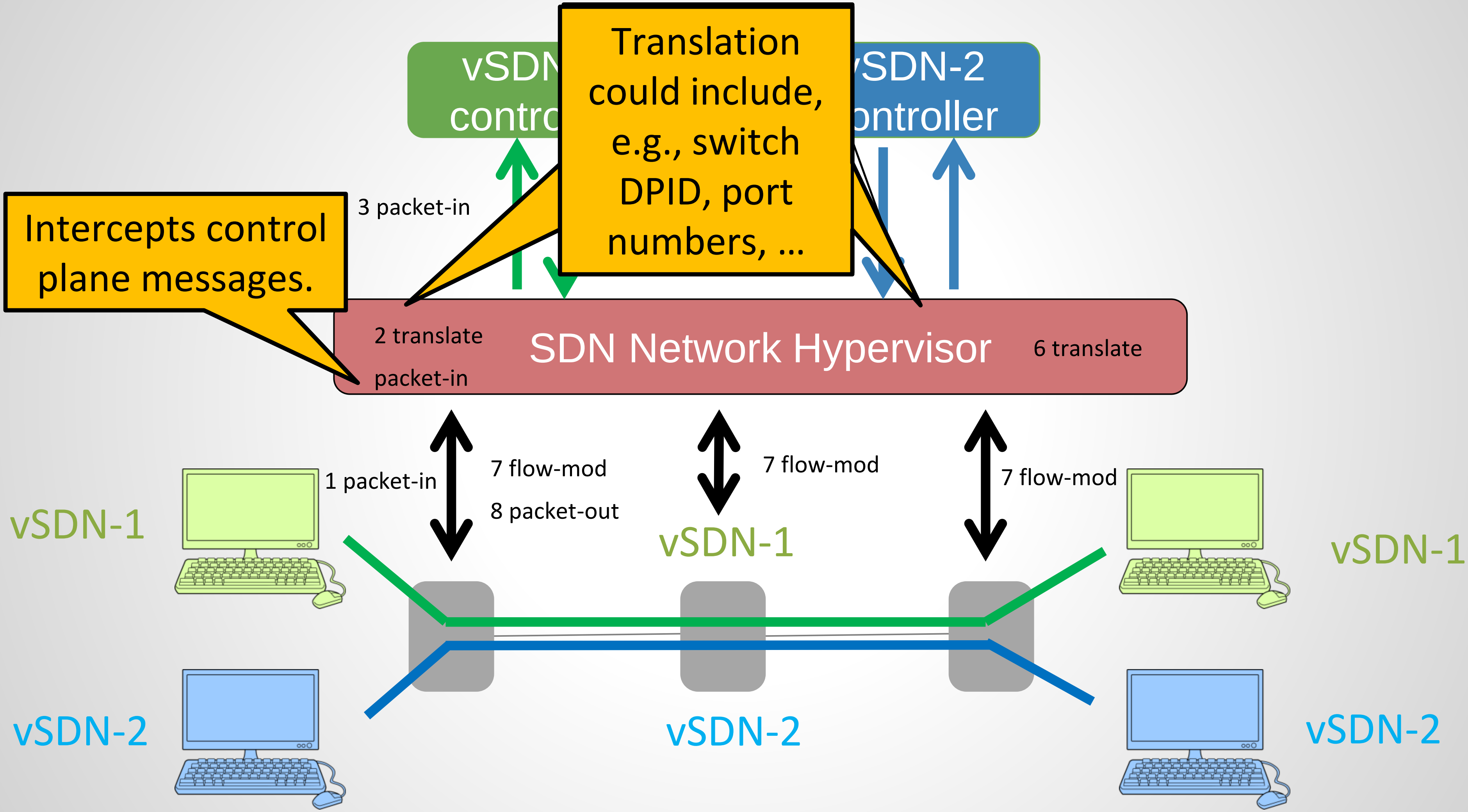

Translation could include, e.g., switch DPID, port numbers, …

Intercepts control plane messages.

An Experiment: 2 vSDNs with bw guarantee!

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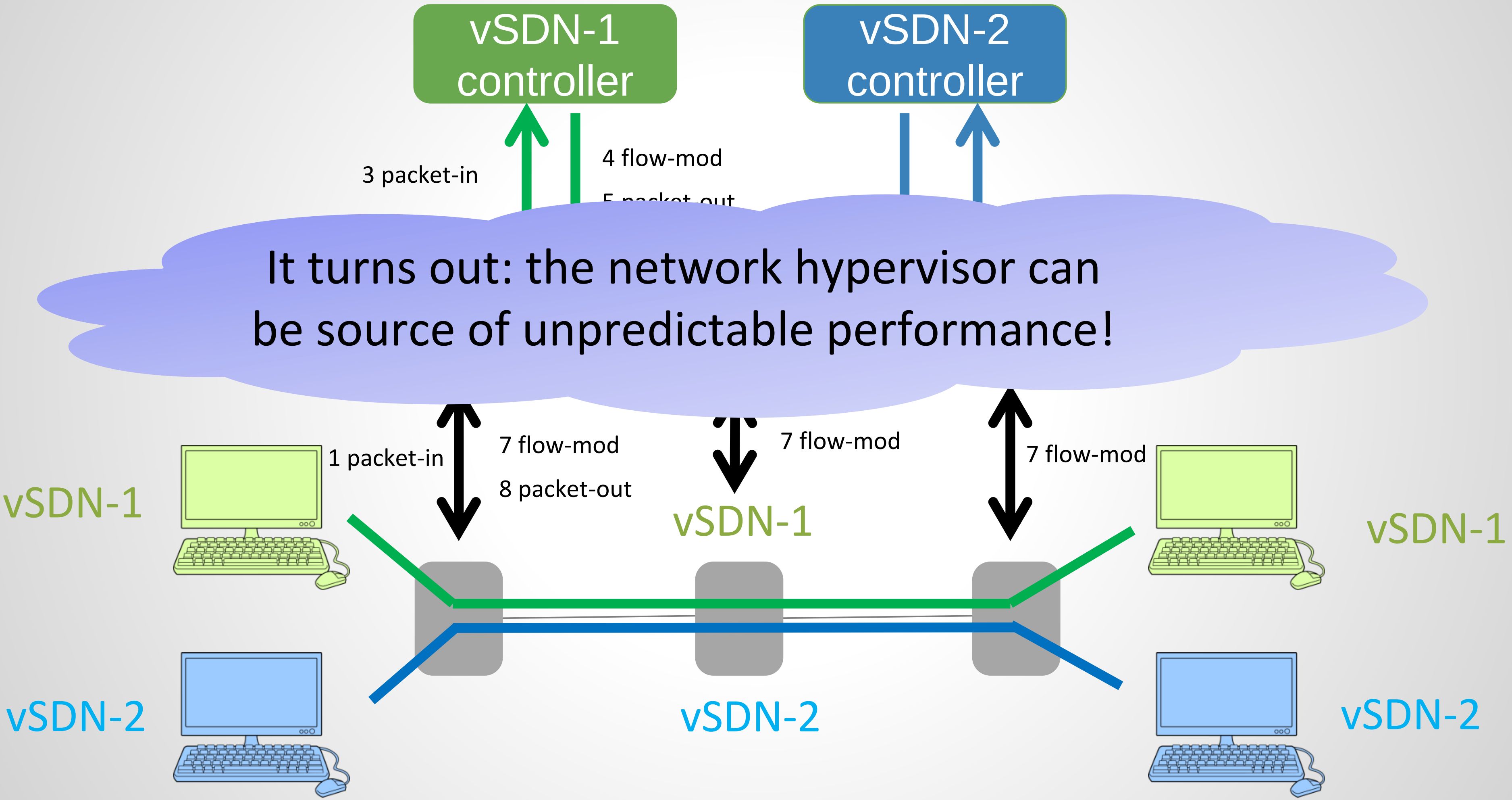
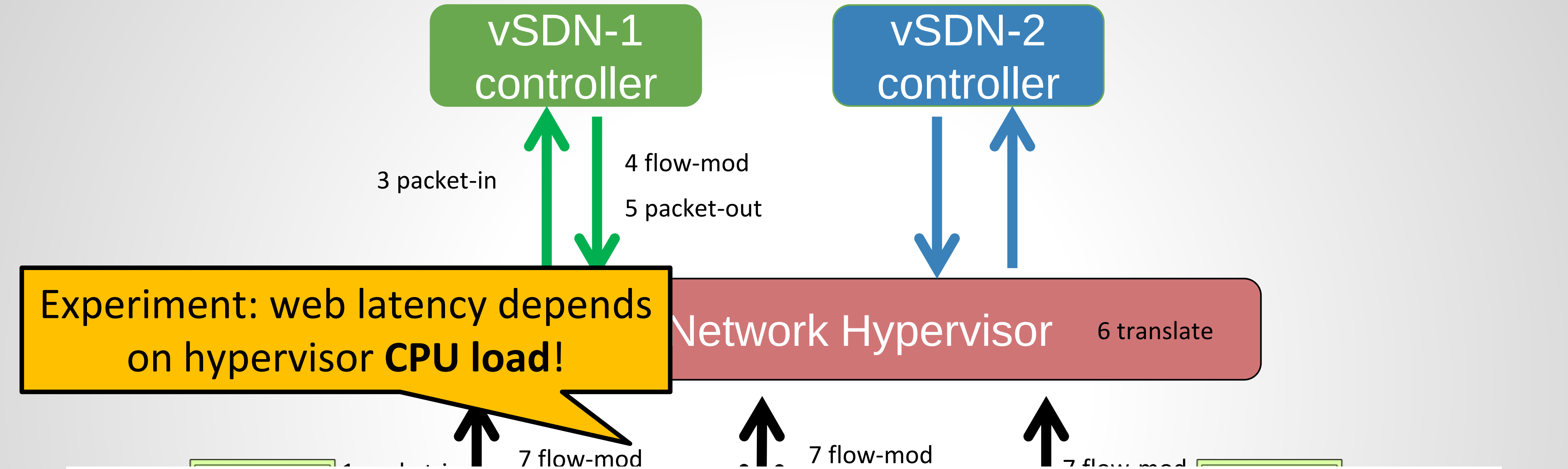


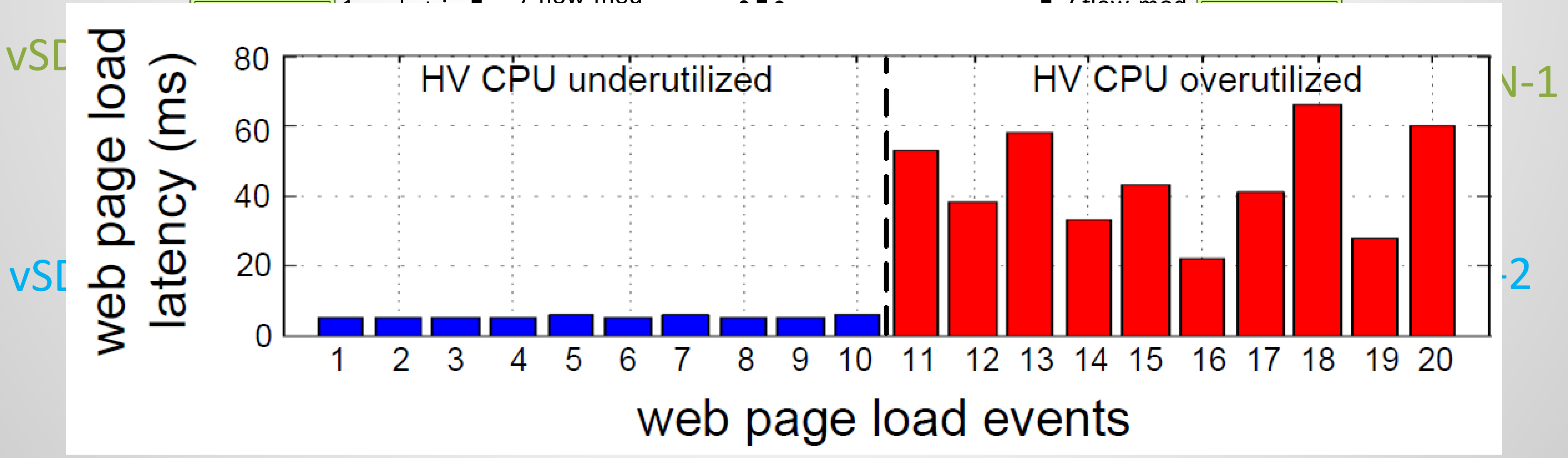
It turns out: the network hypervisor can be source of unpredictable performance!

An Experiment: 2 vSDNs with bw guarantee!

The Many Faces of Performance Interference



Experiment: web latency depends on hypervisor **CPU load**!



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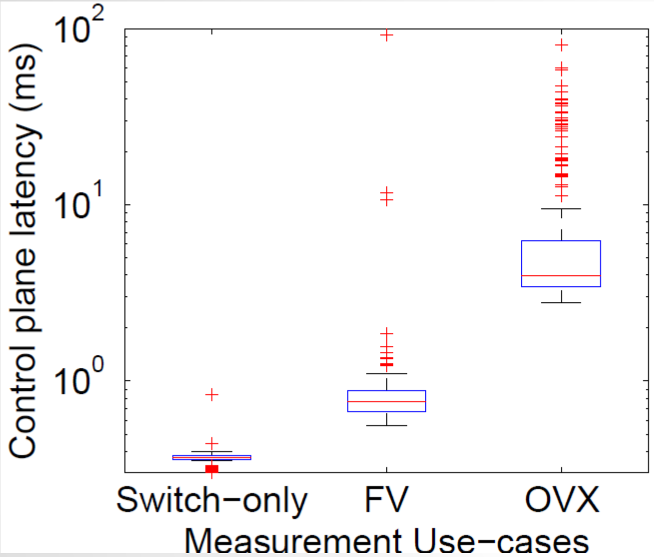
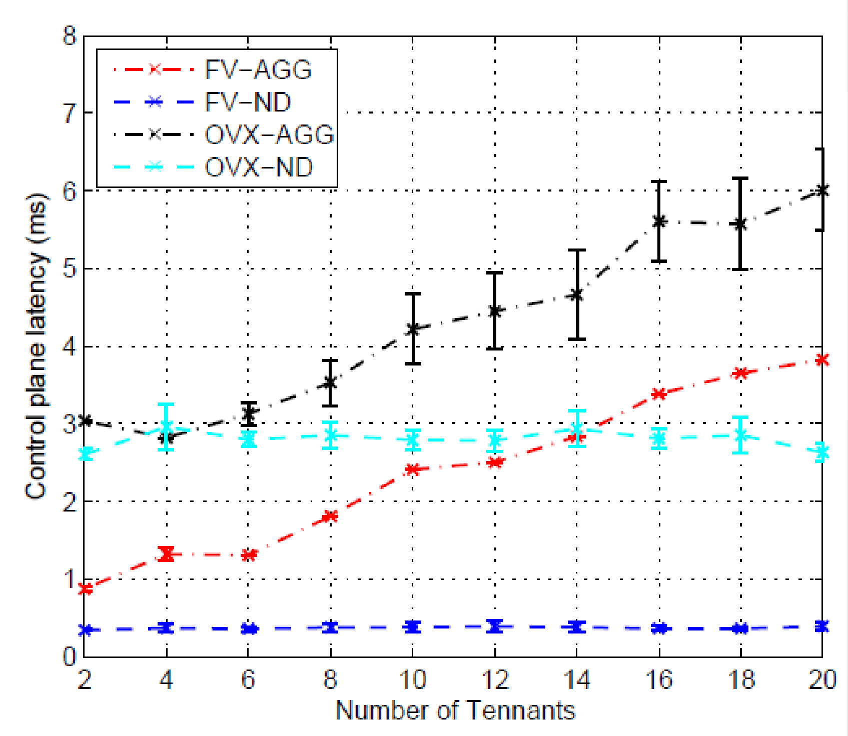


Performance also depends on hypervisor type…

(multithreaded or not, which version of Nagle's algorithm, etc.)

… number of tenants…



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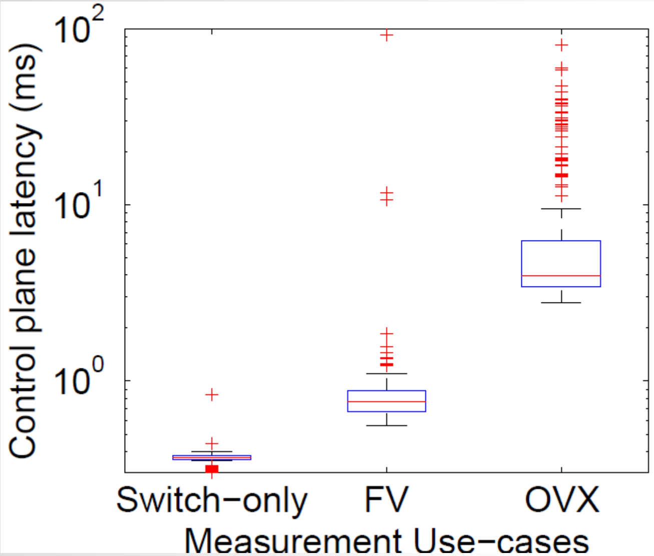

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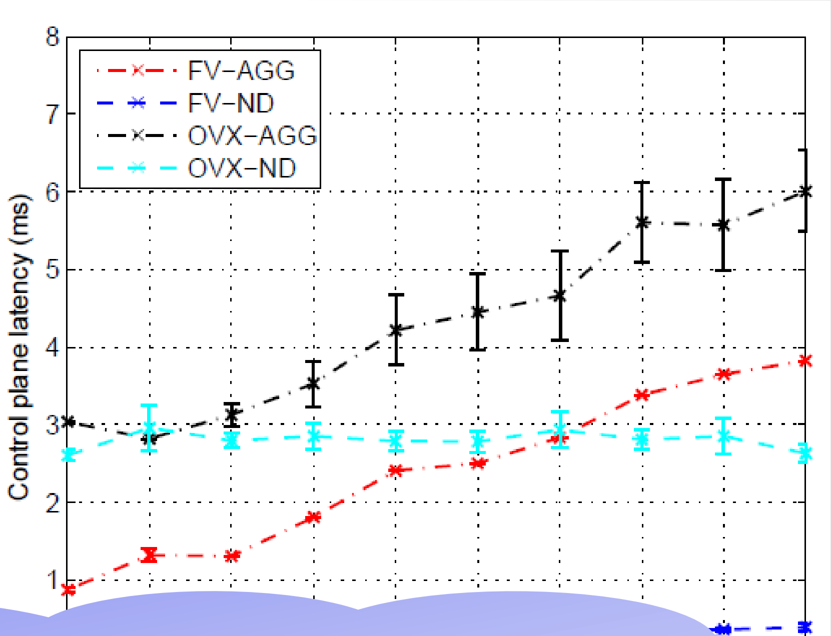

Performance also depends on hypervisor type

(multith

Conclusion: for predictable performance, need to account for all resources! But let us now focus on the network itself.

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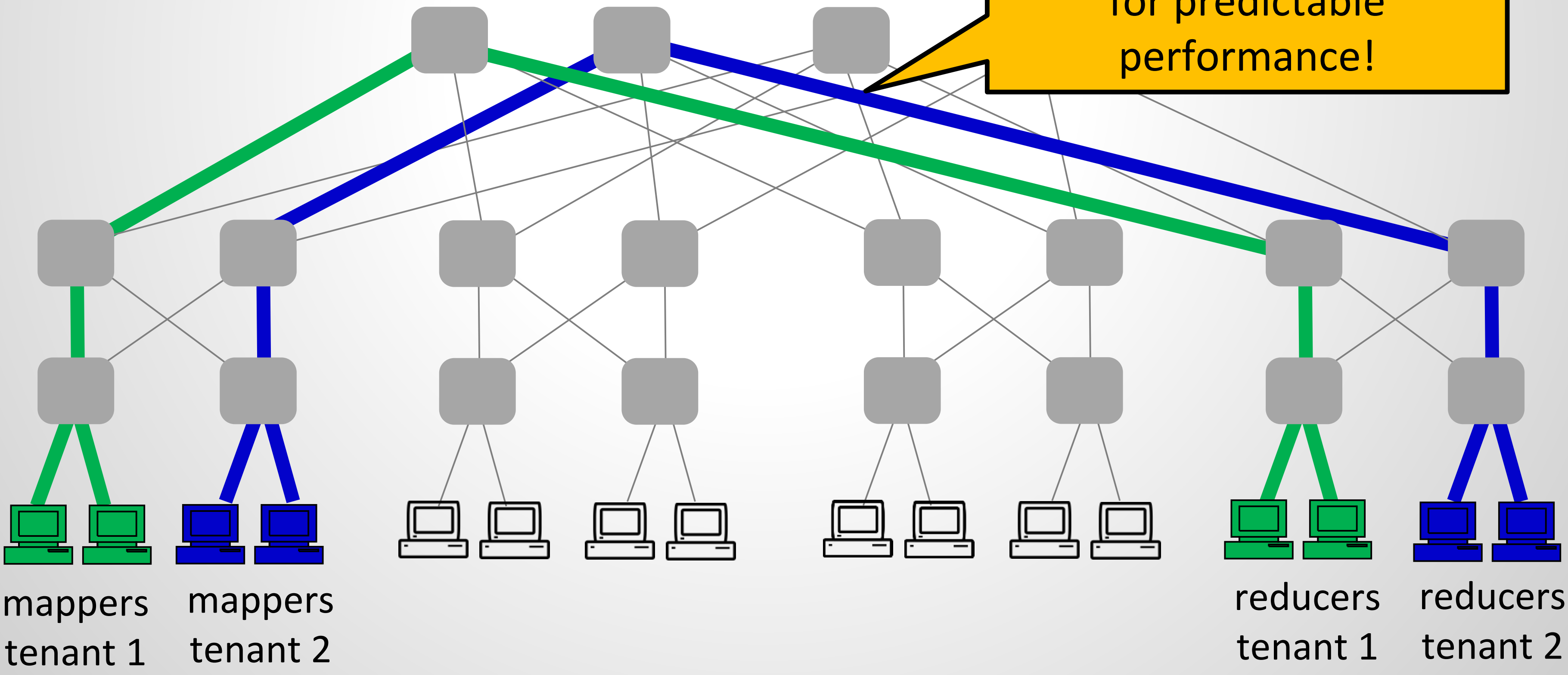
First Algorithmic Challenge:
Keep the traffic local!

Consider a simple data center hosting two tenants: green and blue


Bandwidth reservations for predictable performance!

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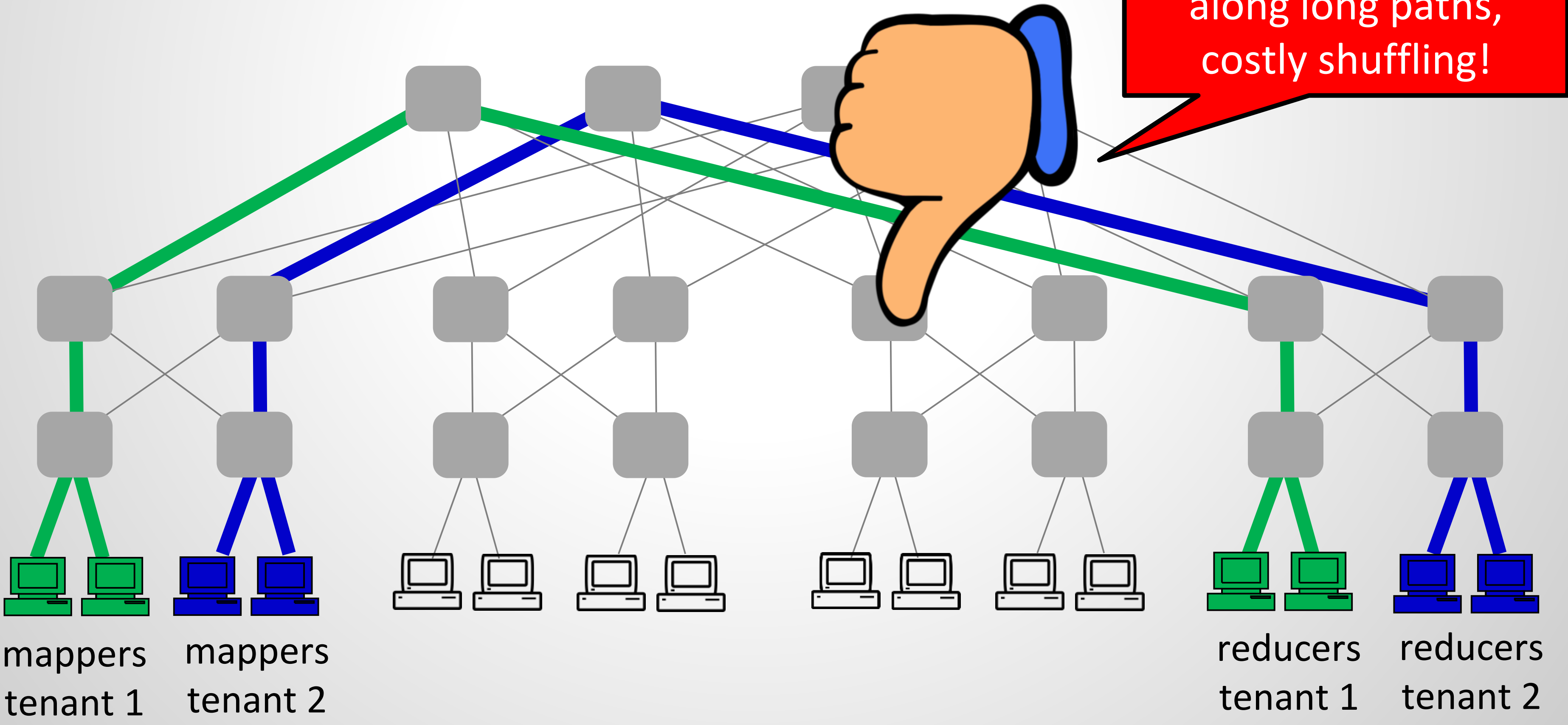

Bad embedding, distributed across pods: reservations along long paths, costly shuffling!

First Algorithmic Challenge:
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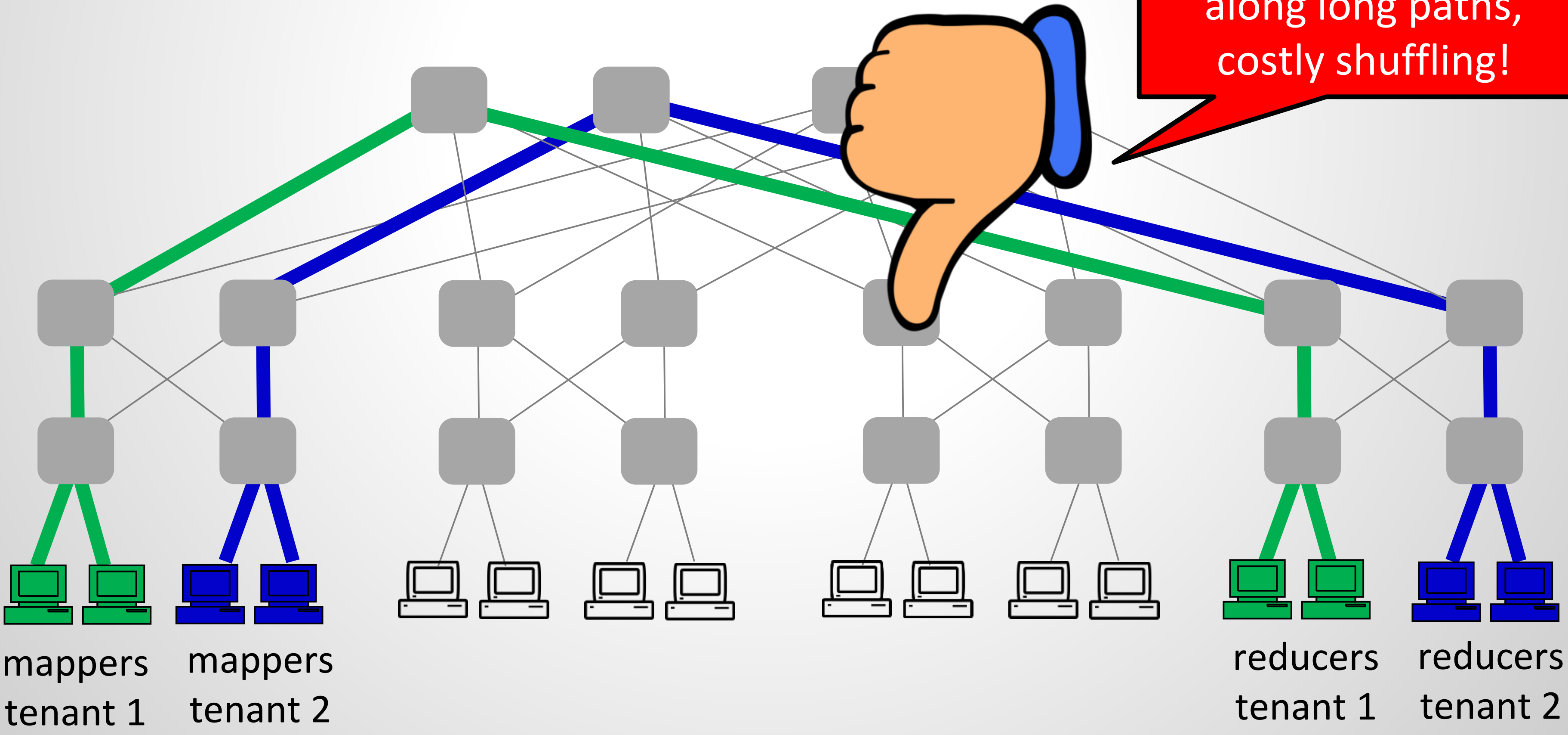
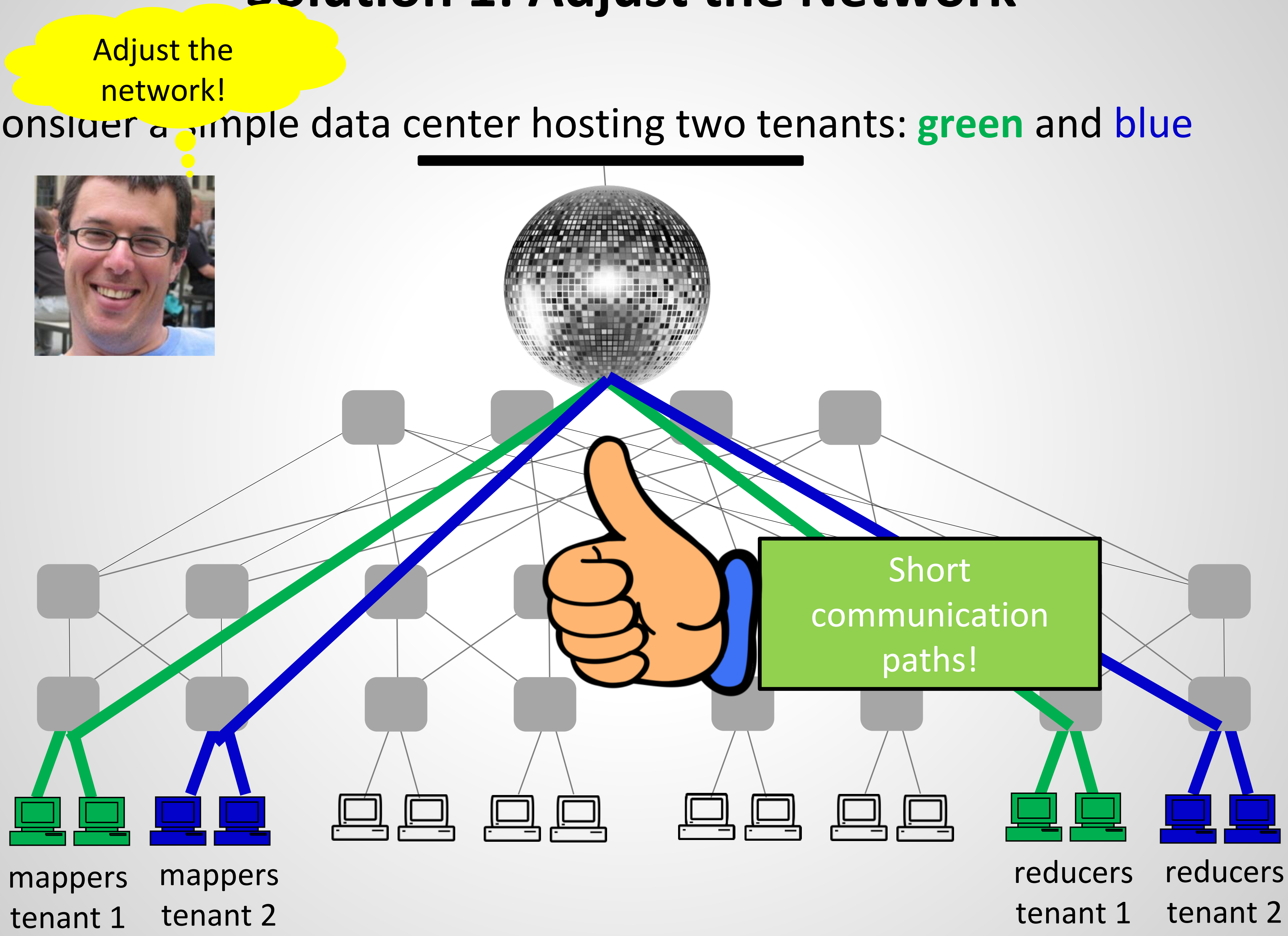
Solutions?!

Bad embedding, distributed across pods: reservations along long paths, costly shuffling!



Solution 1: Adjust the Network

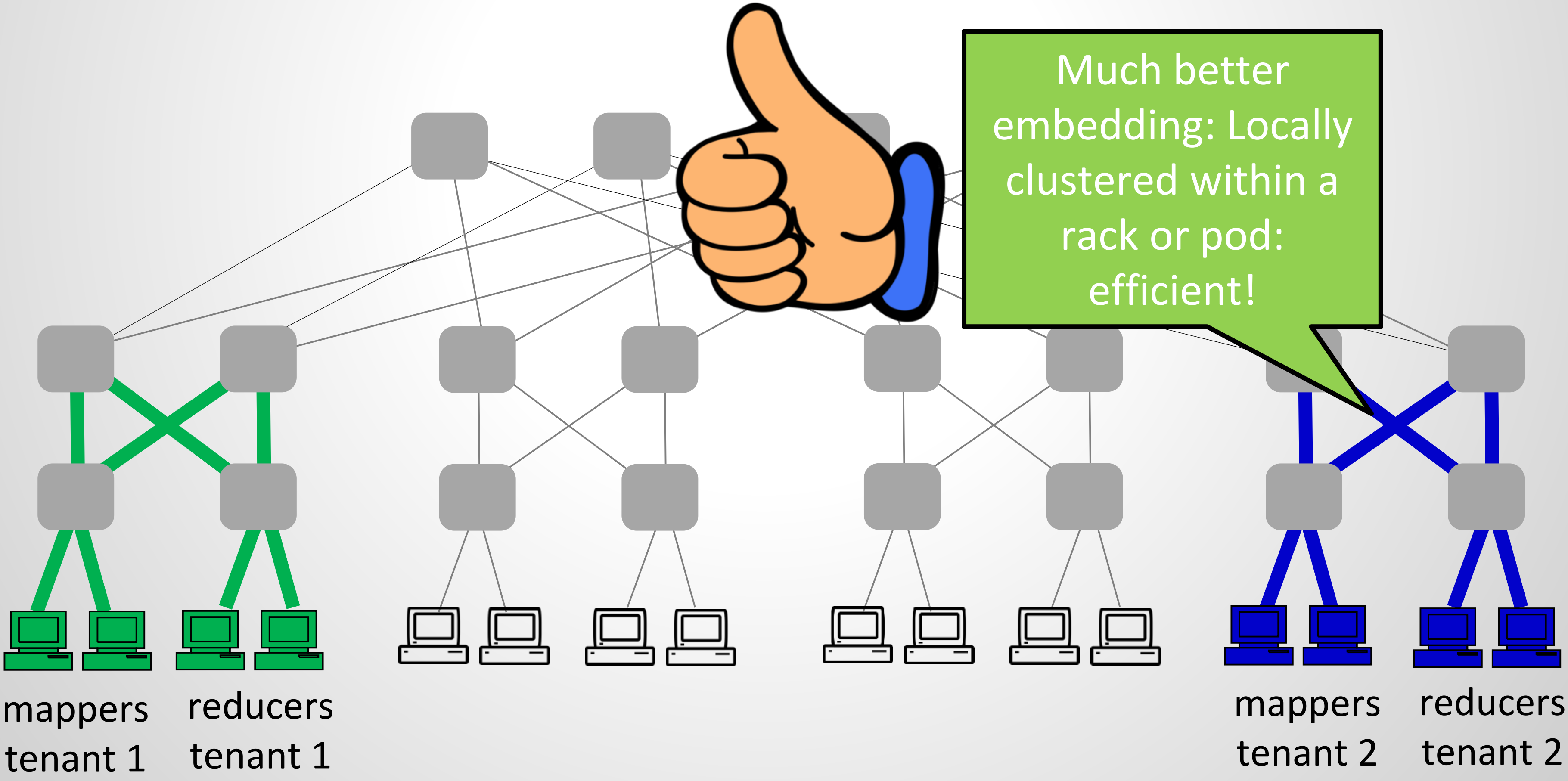
Consider a simple data center hosting two tenants: **green** and **blue**



Short communication paths!

Solution 2: Adjust Embedding

Consider a simple data center hosting two tenants: **green** and blue



Much better embedding: Locally clustered within a rack or pod: efficient!

Solution 2: Adjust Embedding

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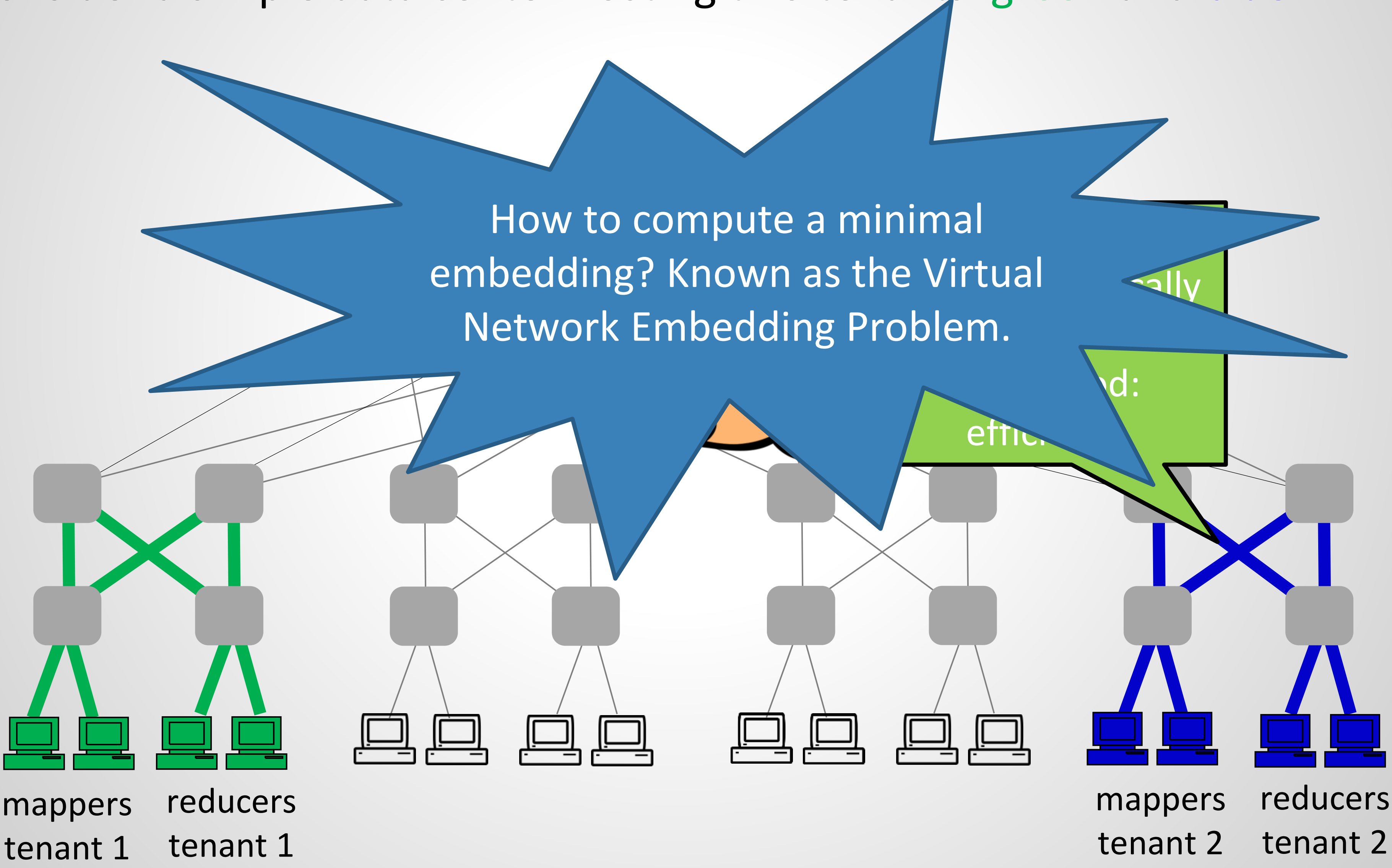


How to compute a minimal embedding? Known as the Virtual Network Embedding Problem.

Overview

PART I: Static Embeddings

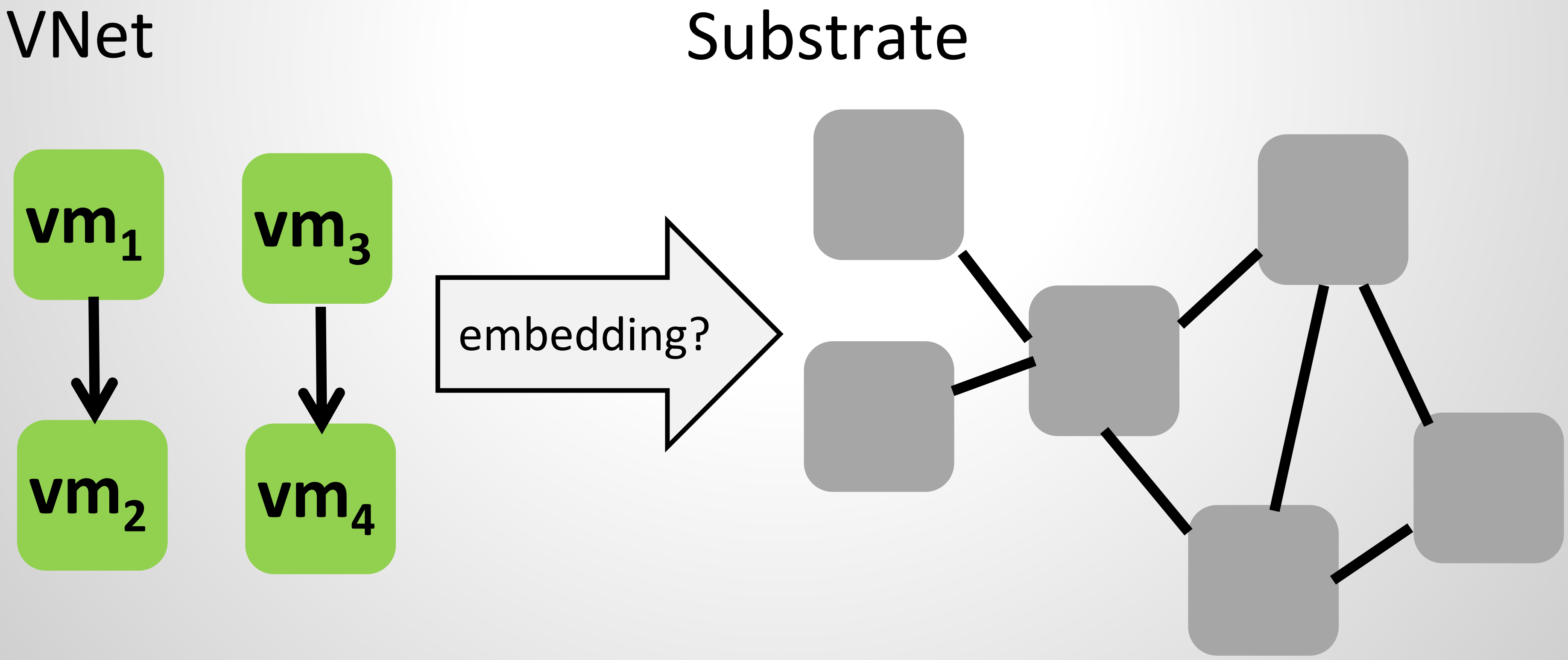
PART II: Reconfiguring Embeddings

PART III: A request comes seldom alone!

PART I:
Static Embeddings

The Virtual Network Embedding Problem

- 2 dimensions of flexibility:
  - Mapping of virtual nodes (to physical nodes)
  - Mapping of virtual links (to paths)



VNet

Substrate

The Virtual Network Embedding Problem

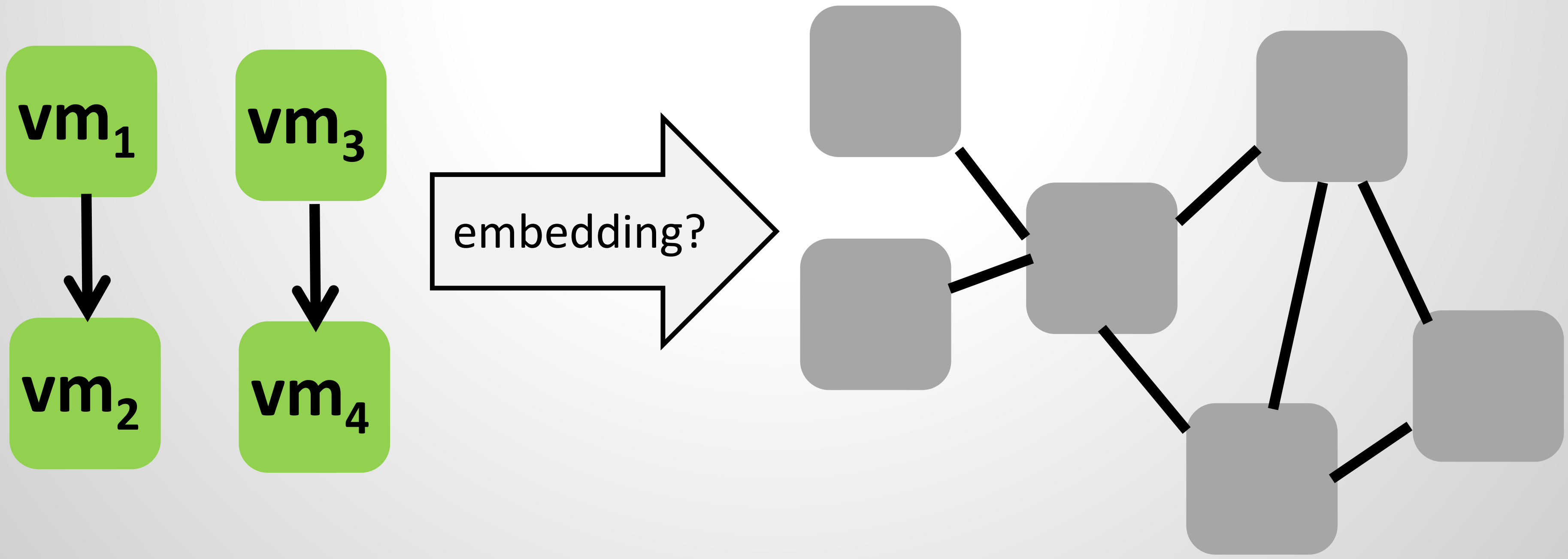
- 2 dimensions of flexibility:
  - Mapping of virtual nodes (to physical nodes)
  - [illegible] links (to paths)

aka “guest graph”

aka “host graph”

VNet

Substrate



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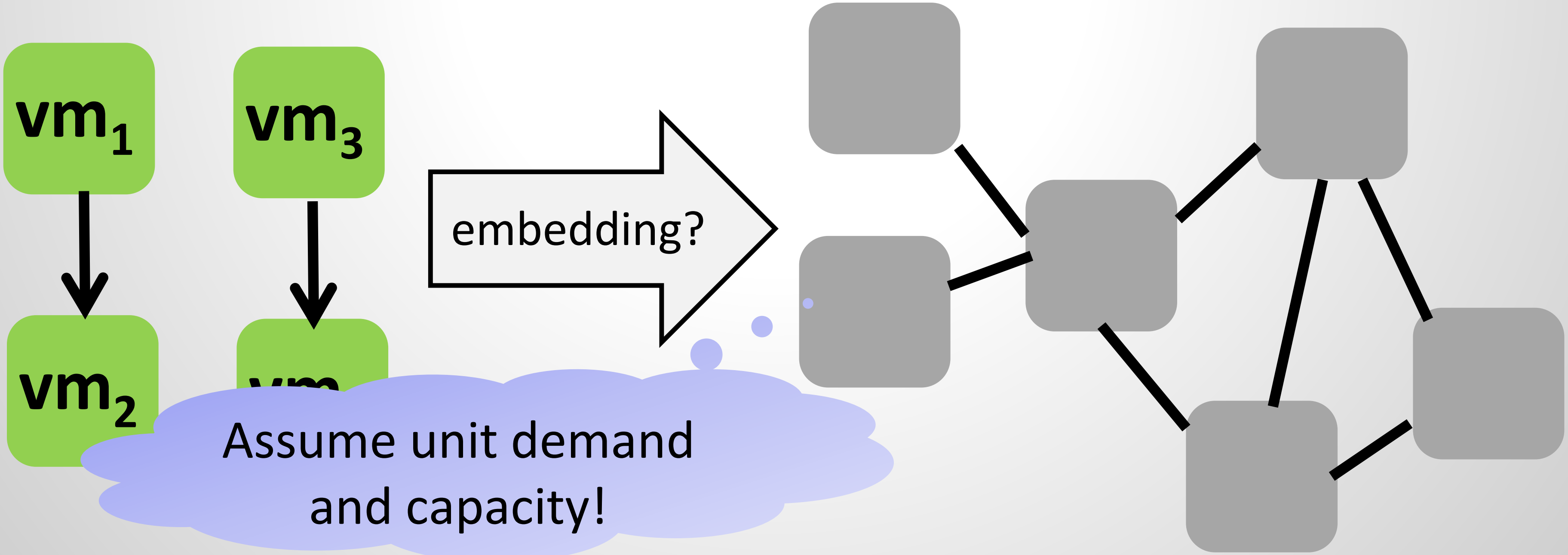

Assume unit demand and capacity!

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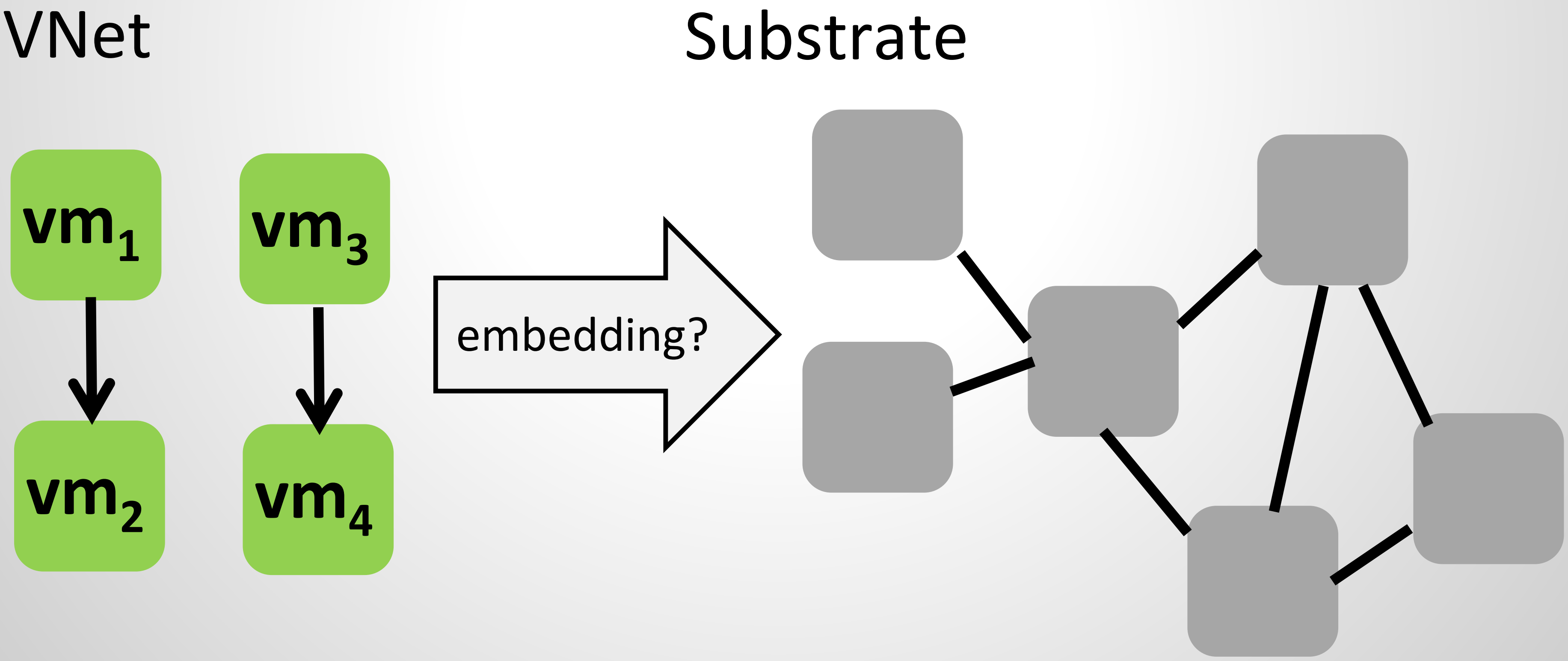

VNet

Substrate

vm_1

vm_2

vm_3

vm_4

embedding?

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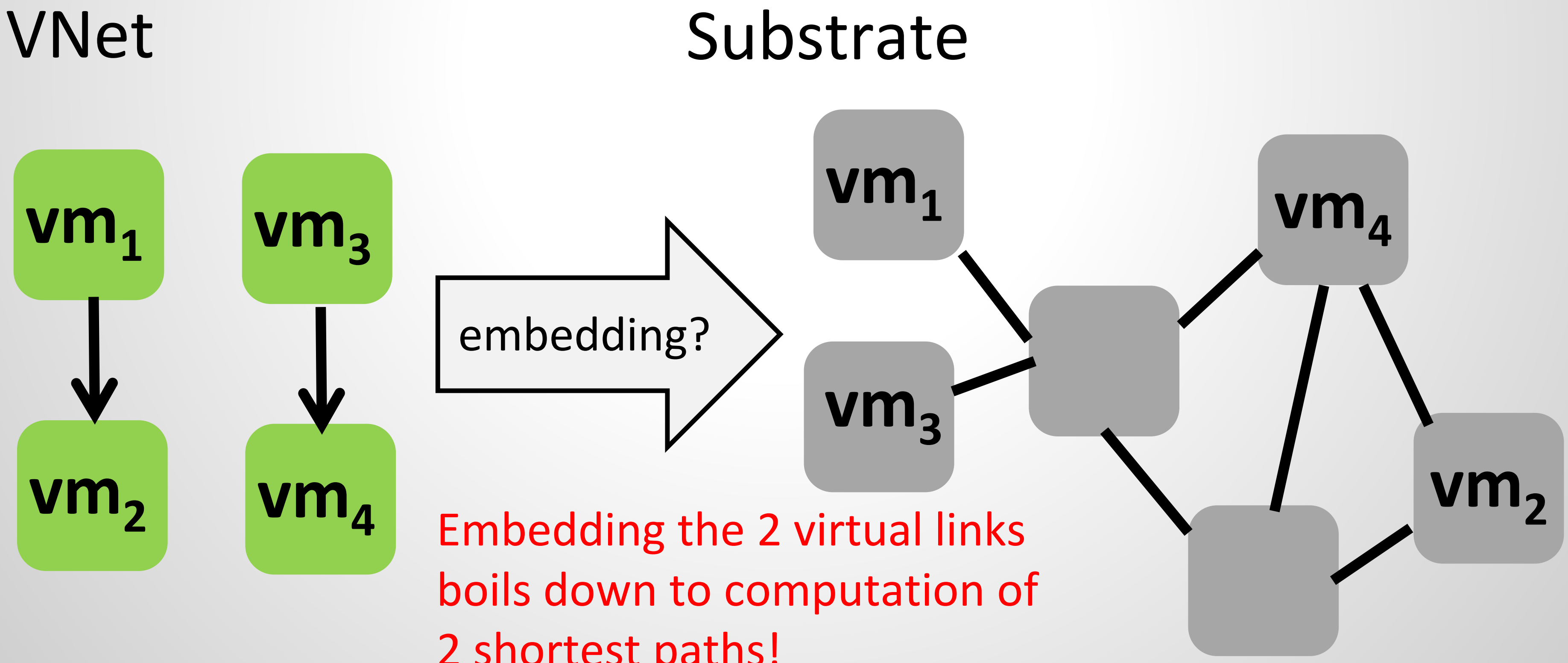

Embedding the 2 virtual links boils down to computation of 2 shortest paths!

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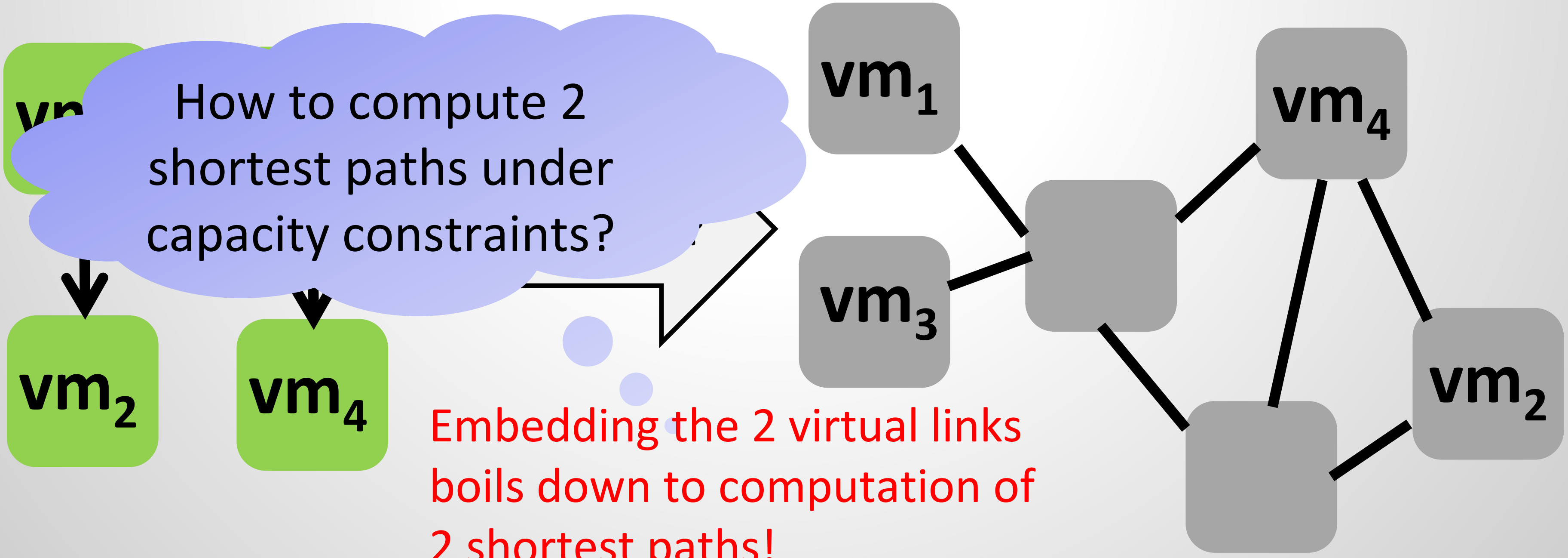

How to compute 2 shortest paths under capacity constraints?

Embedding the 2 virtual links boils down to computation of 2 shortest paths!

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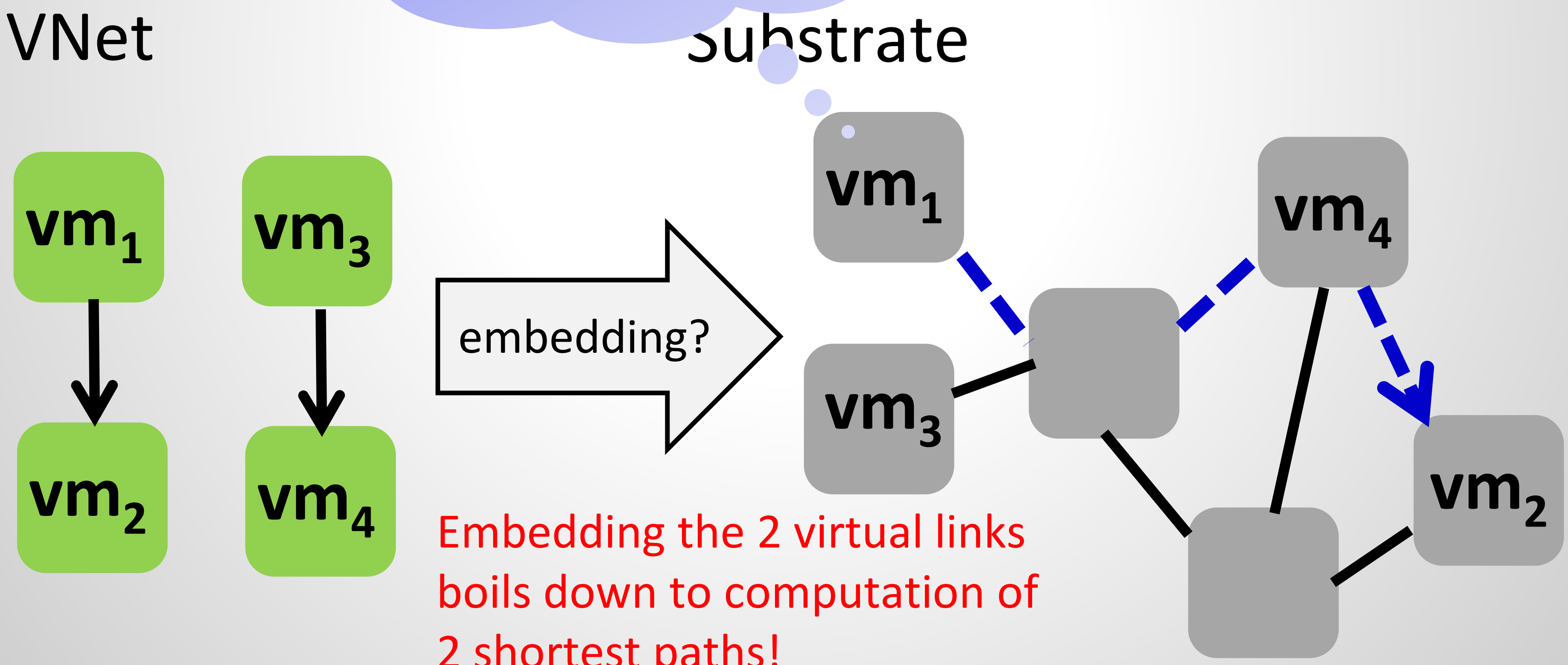
Let's try greedy!
First vm_1-vm_2.



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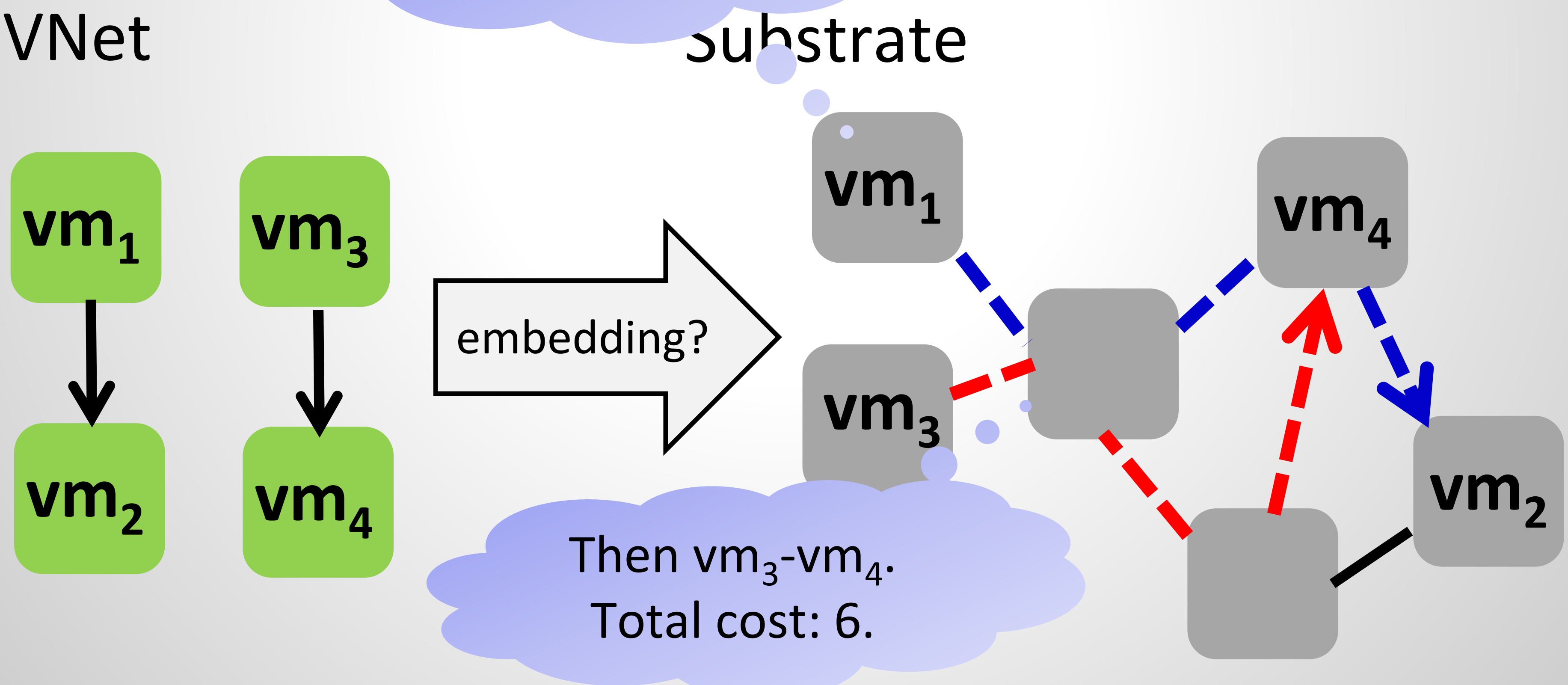

Then vm$_3$-vm$_4$.
Total cost: 6.

The Virtual Network Embedding Problem

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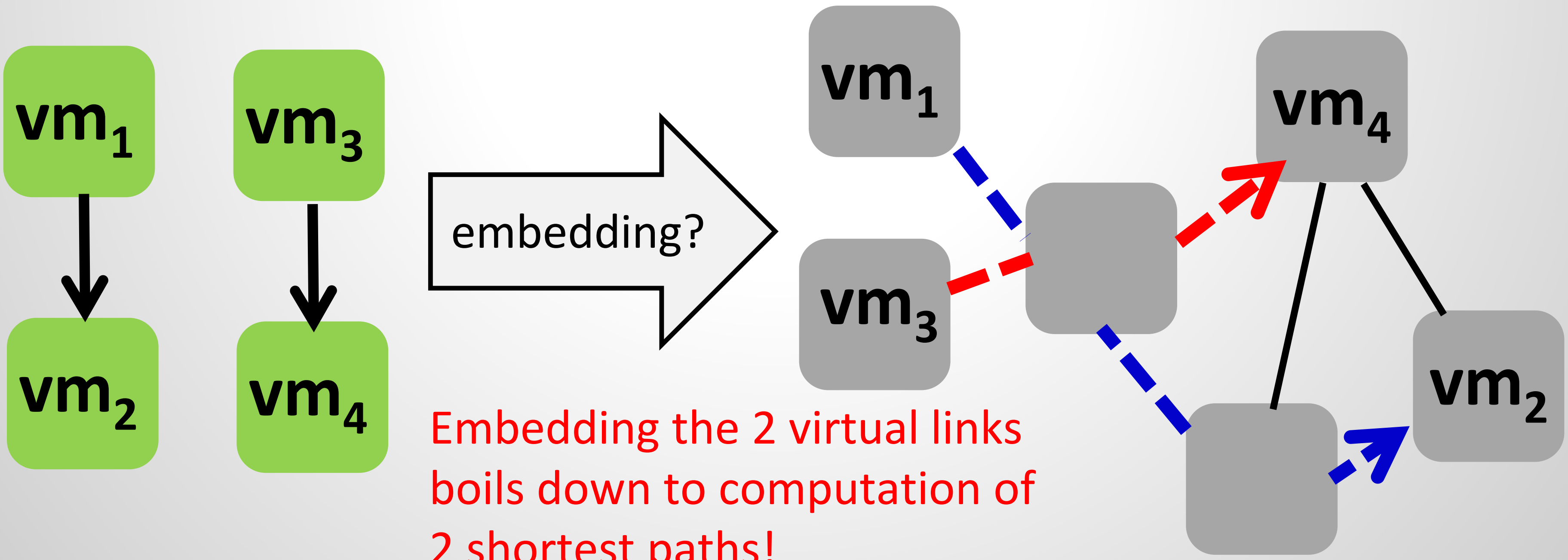
A better solution:
cost 5!

VNet

bst ate



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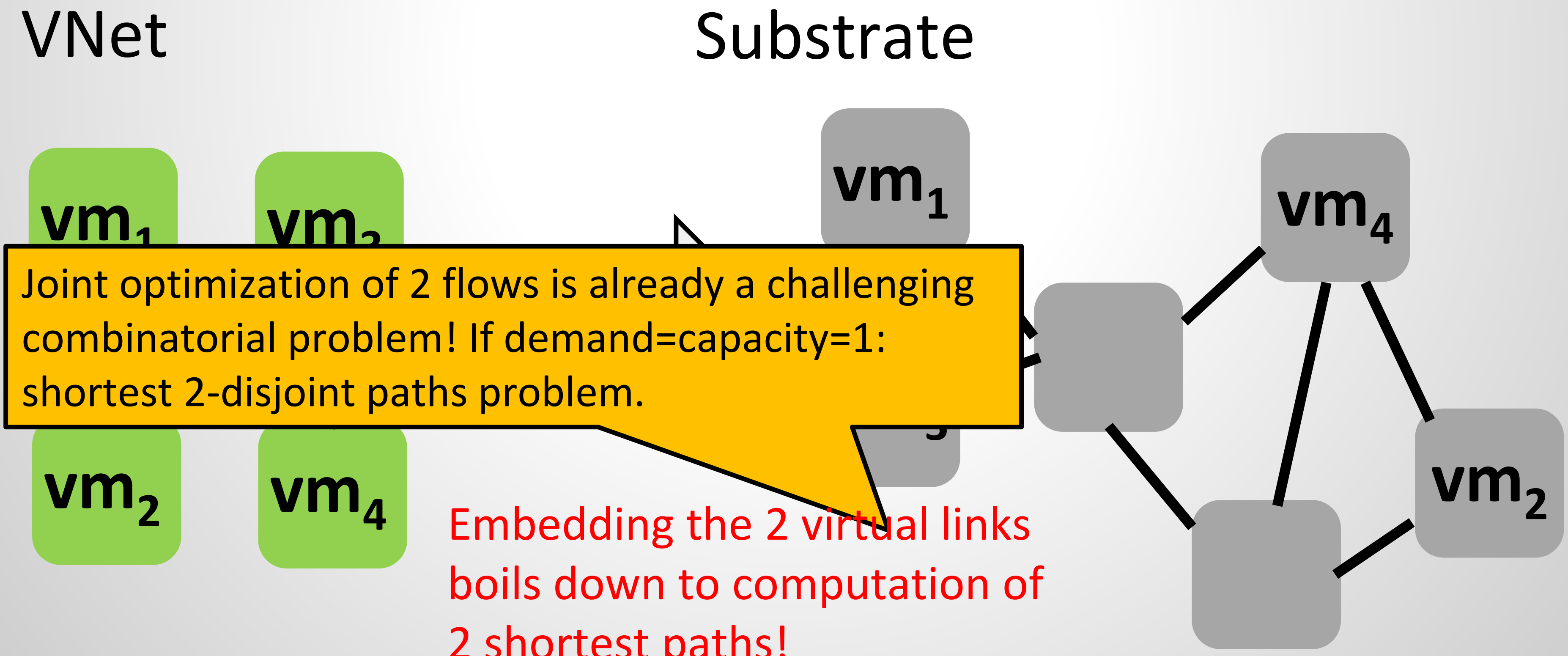

Joint optimization of 2 flows is already a challenging combinatorial problem! If demand=capacity=1: shortest 2-disjoint paths problem.

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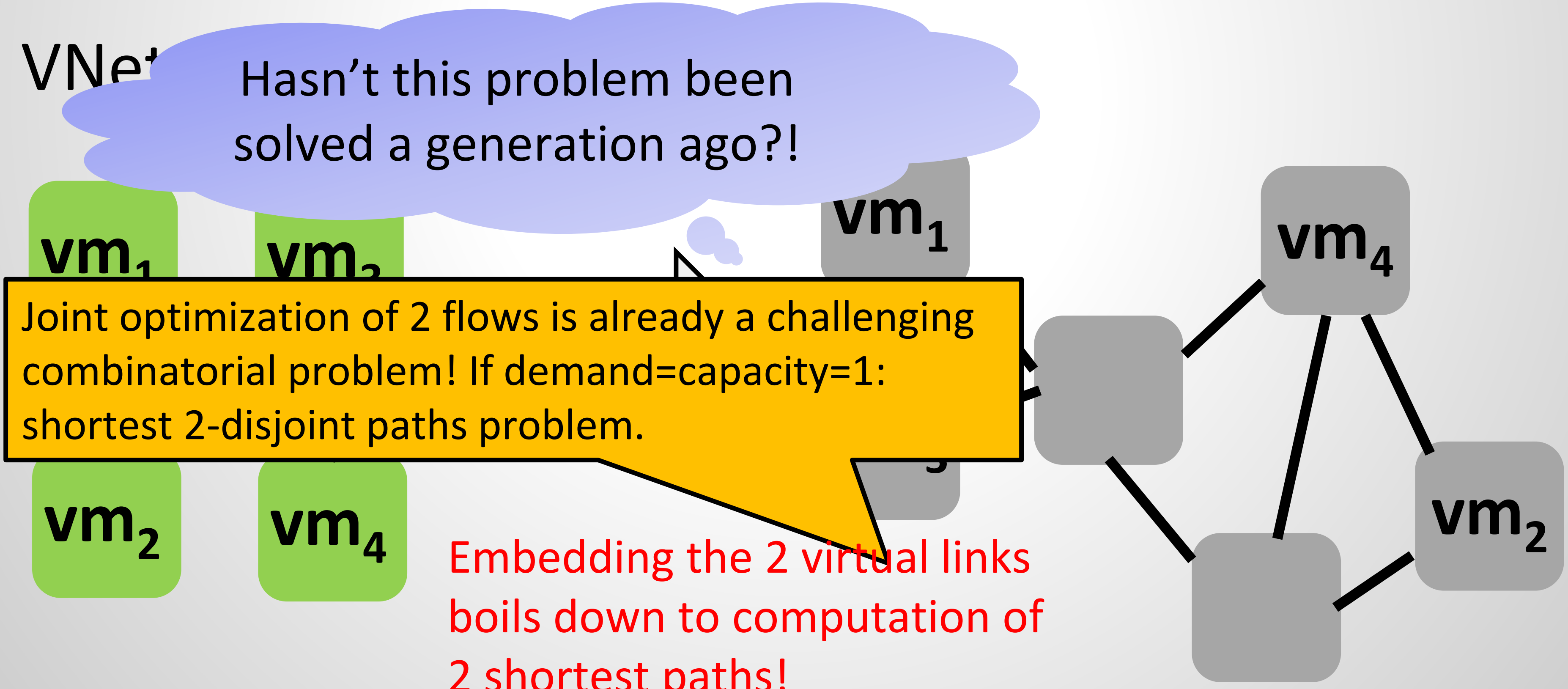

Hasn’t this problem been solved a generation ago?!

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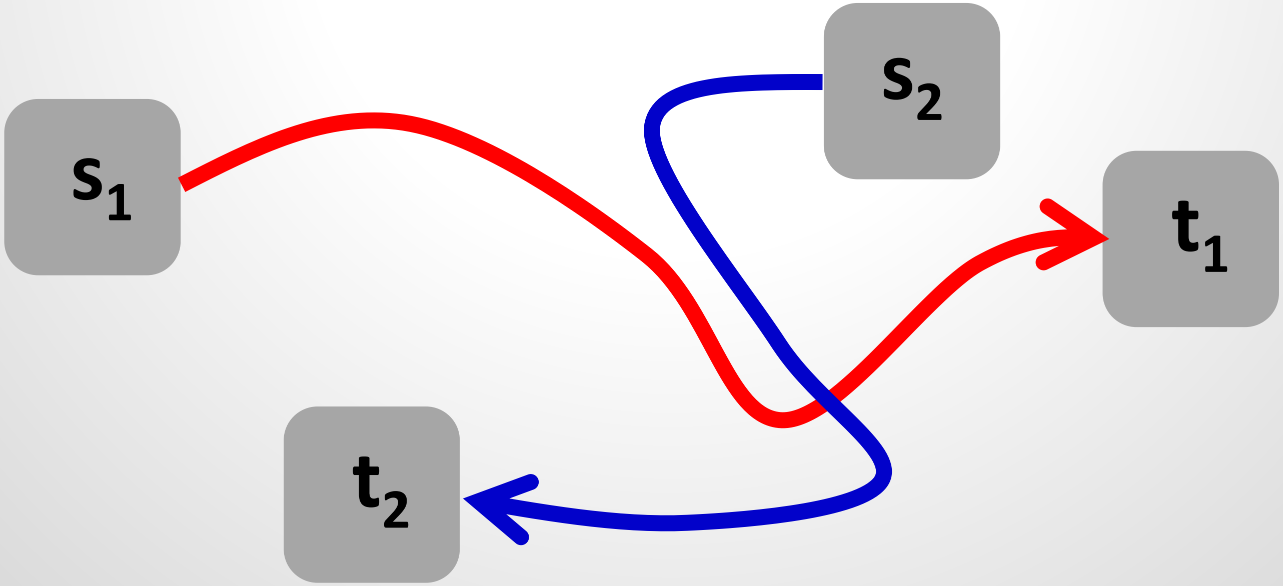
Embedding the 2 virtual links boils down to computation of 2 shortest paths!

Mapping virtual links: Already hard!

- ❑ Essentially a 2-disjoint shortest paths problem: a deep combinatorial problem
  - ❑ NP-hard on directed graphs
  - ❑ For undirected graphs:
    - ❑ Feasibililty more or less understood: Robertson&Seymour
    - ❑ Shortest paths: recent breakthrough, first polytime randomized algorithm (still slow: a theoretical result)
    - ❑ We are still looking for polytime deterministic algorithms!



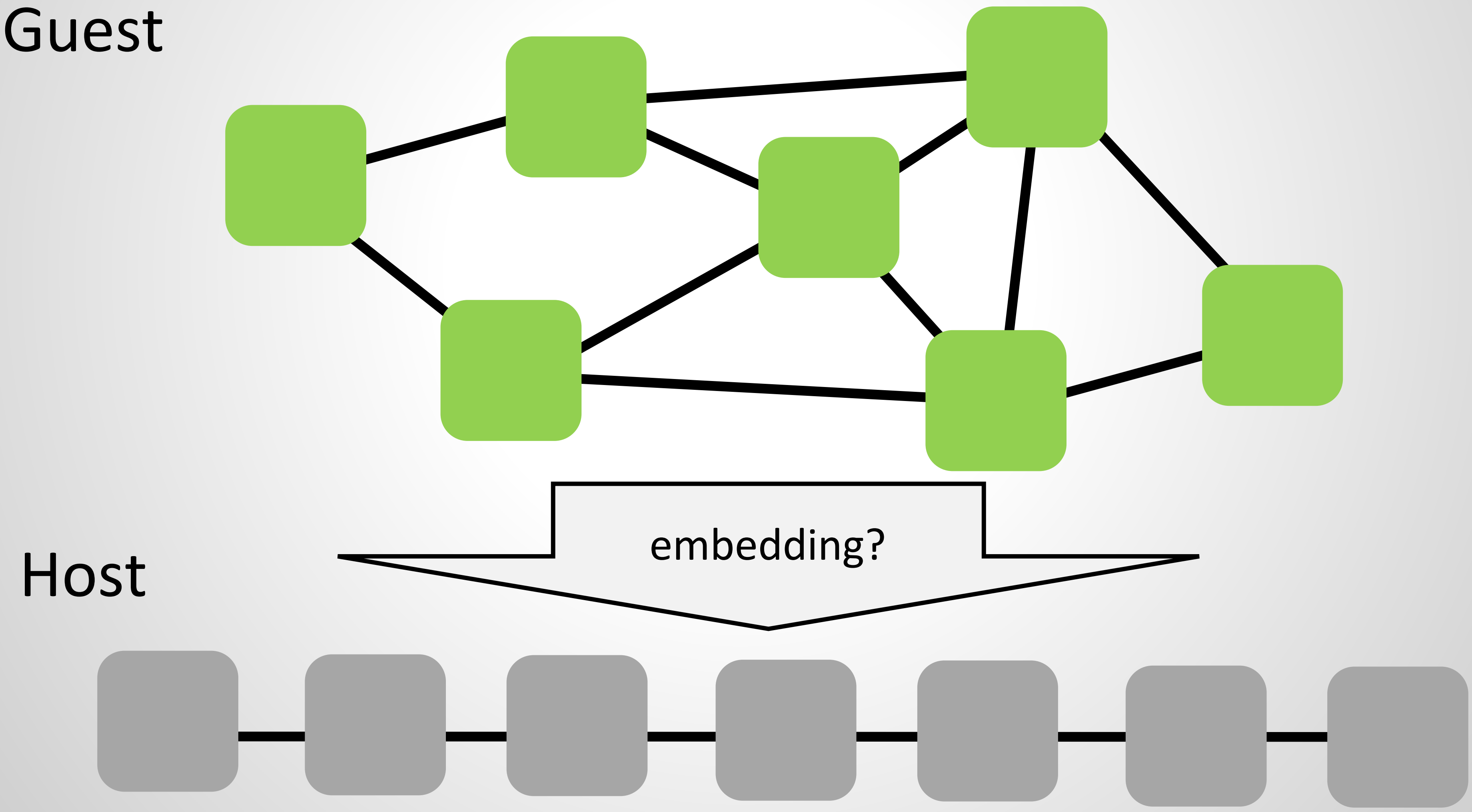
Therefore: Mapping Virtual Links is Challenging

Bad news: The Virtual Network Embedding Problem is hard even if endpoints are already mapped and given.

But maybe at least mapping nodes is simple?

Mapping Virtual Nodes

❑ Let’s start simple again: assume paths are trivial, e.g., the physical network (host graph) is a line



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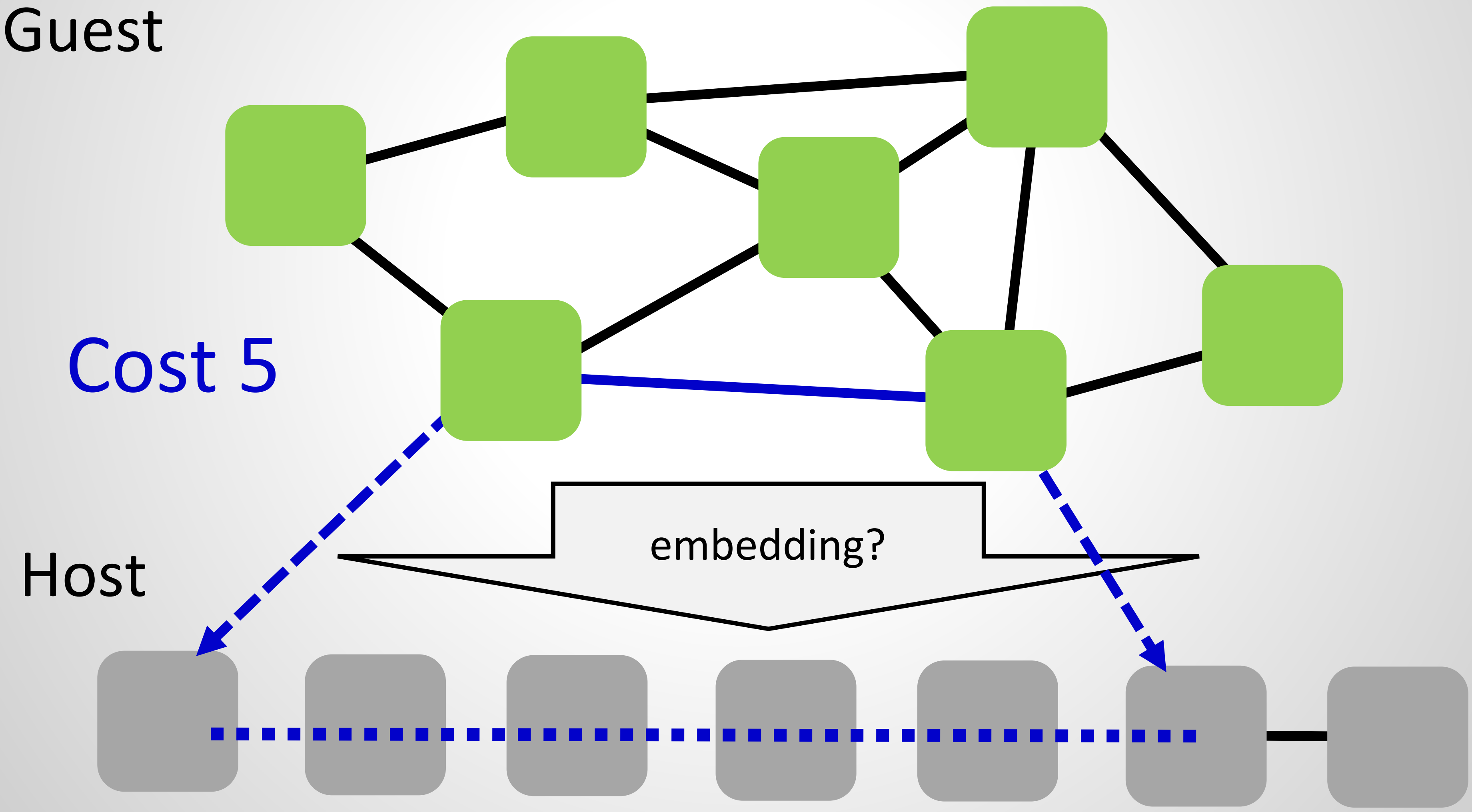

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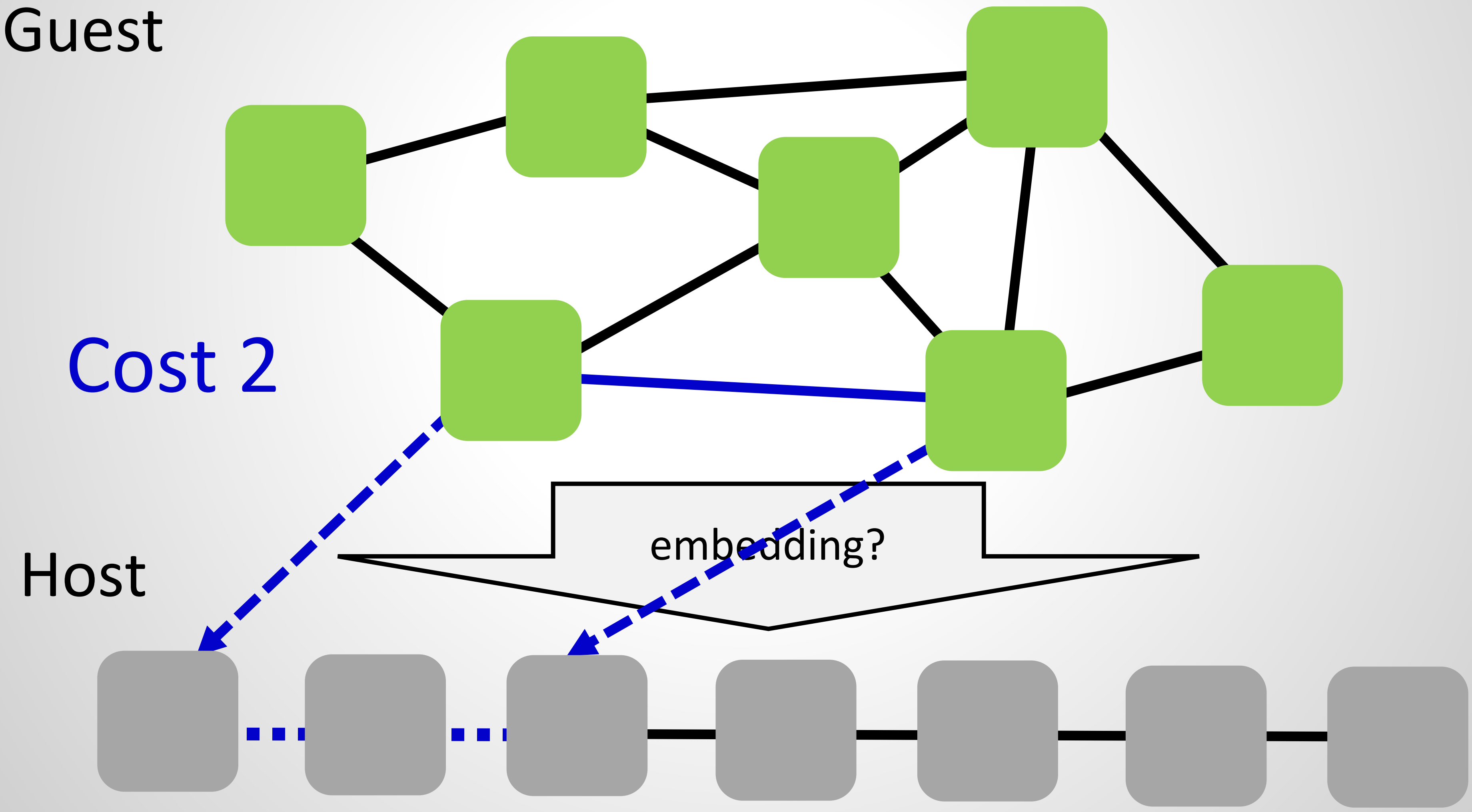

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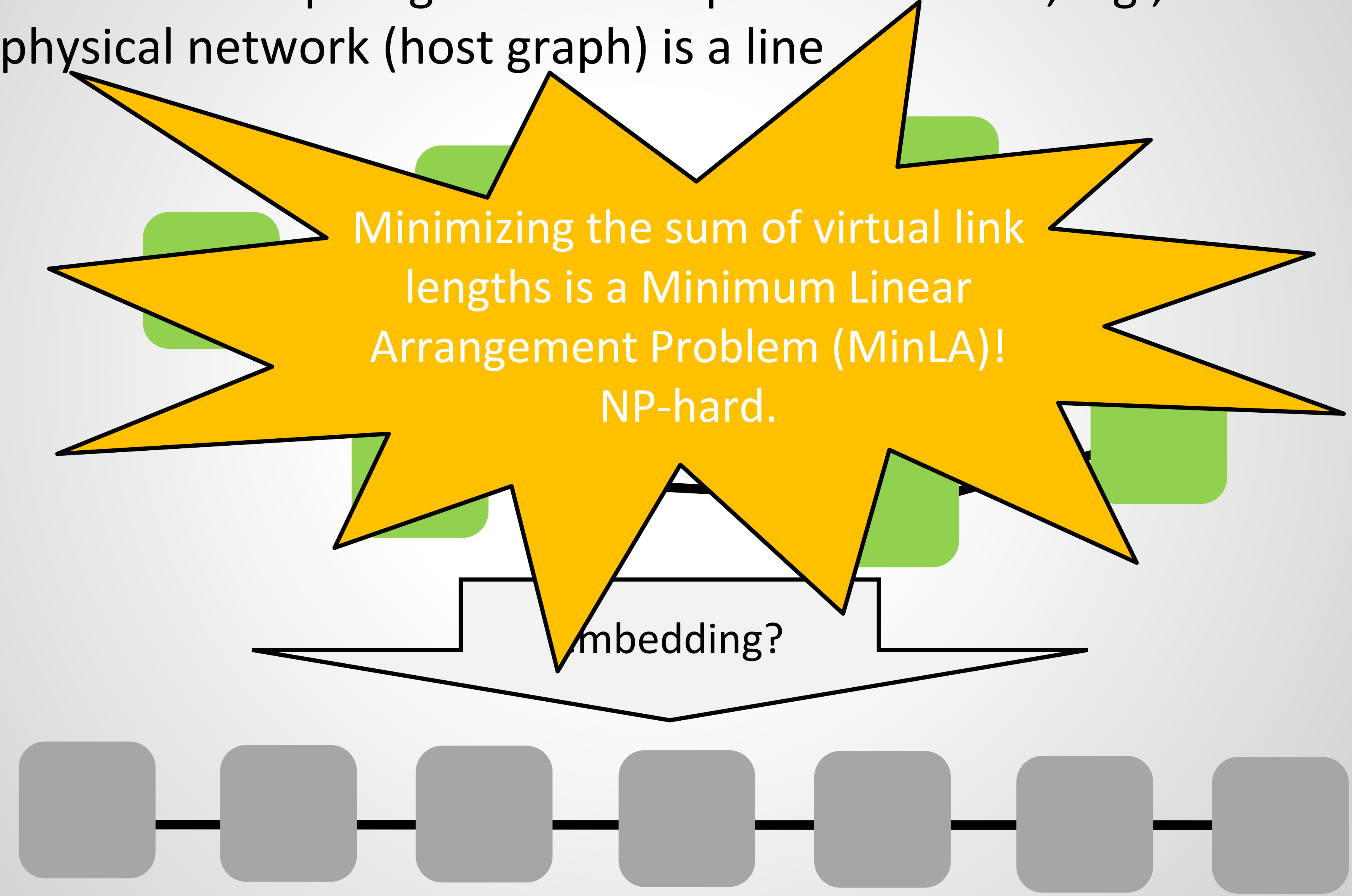


Minimizing the sum of virtual link lengths is a Minimum Linear Arrangement Problem (MinLA)! NP-hard.

Therefore: VNEP is Hard “in Both Dimensions”!

- We have seen examples that:
  - mapping virtual links is hard (even if nodes are given)
  - mapping virtual nodes is hard (even if links are trivial)

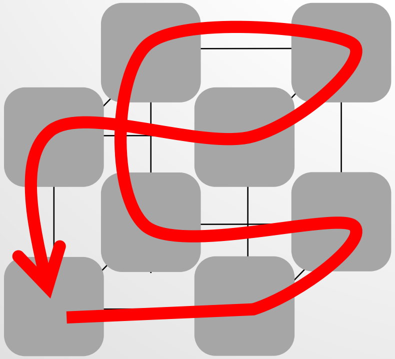
- Remark: the VNEP can also be seen as a generalization of the Subgraph Isomorphism Problem (SIP)

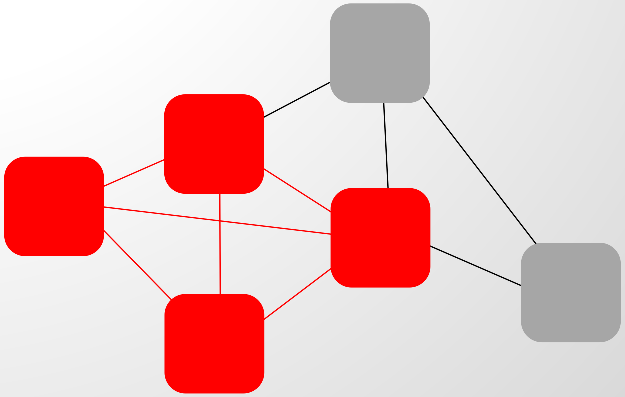
Known? Why is SIP NP-hard?

Therefore: VNEP is Hard “in Both Dimensions”!

- ❑ We have seen examples that:
  - ❑ mapping virtual links is hard (even if nodes are given)
  - ❑ mapping virtual nodes is hard (even if links are trivial)

- ❑ Remark: the VNEP can also be seen as a generalization of the Subgraph Isomorphism Problem (SIP)
  - ❑ The SIP problem: Given two graphs G,H, determine whether G contains a subgraph that is isomorphic to H?
  - ❑ NP-hard: “does G contain an n-node cycle?” is a Hamilton cycle problem (each node visited exactly once), a solution to “does G contain a k-clique?” solves maximum clique problem, etc.



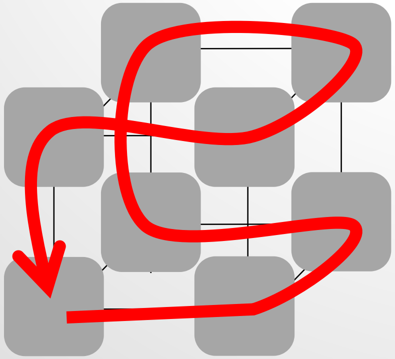


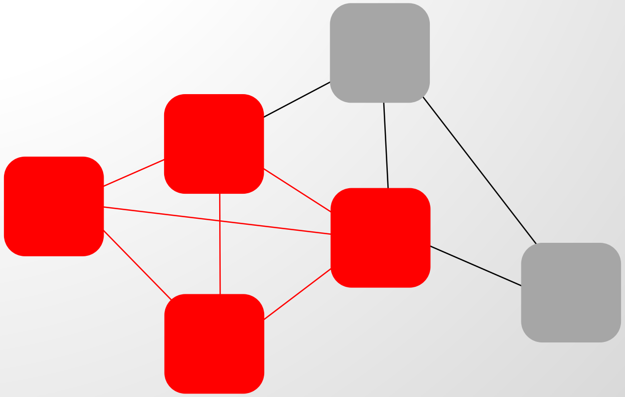
Therefore: VNEP is Hard “in Both Dimensions”!

- We have seen examples that:
  - mapping virtual links is hard (even if nodes are given)
  - mapping virtual nodes is hard (even if links are trivial)

- Remark: the VNEP can also be seen as a generalization of the Subgraph Isomorphism Problem (SIP)
  - The SIP problem: Given two graphs G,H, determine whether G contains a subgraph that is isomorphic to H?
  - N milton cycle problem (each node visited ?” solves maximum clique problem, etc.

So if SIP is hard, why is VNEP hard?





NP-Hardness: From SIP to VNEP

- Observe: VNEP is a generalization of SIP

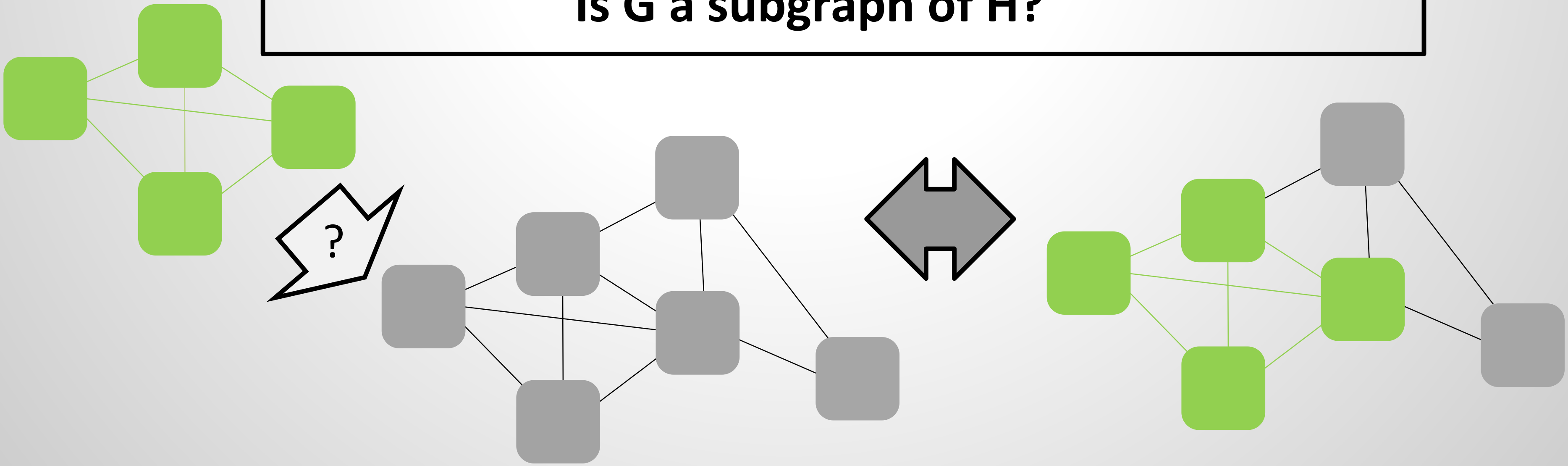
- For example:

Can VNet G=(V,E) be embedded in H at cost |E|?

(I.e., each virtual edge has length 1.)

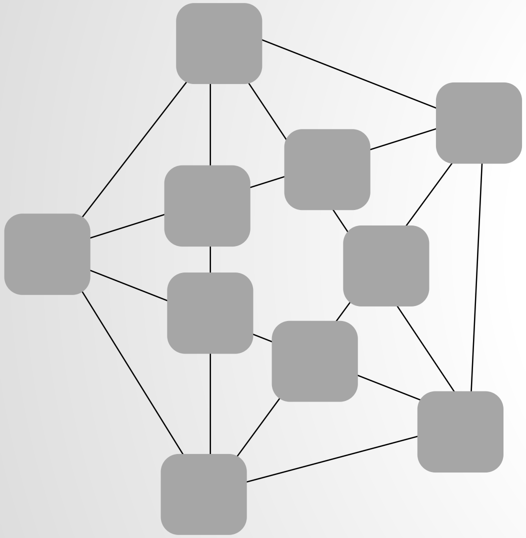
⇔

Is G a subgraph of H?

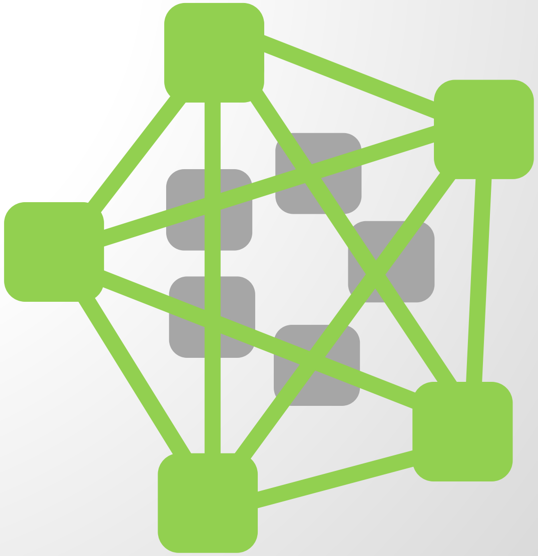


Remark: Graph Minors

Note: It is possible to embed a guest graph G on a host graph H, even though G is *not* a minor of H:



Assume planar host graph H: K_5 and $K_{3,3}$ minor-free…



… but it is possible to embed non-planar guest graph $G=K_5$!

Can we at least formulate a “fast” MIP?

?

Can we at least formulate a “fast” MIP?

- Recall: Mixed Integer Program (MIP)
  - Linear objective function (e.g., minimize embedding footprint)
  - Linear constraints (e.g., do not violate capacity constraints)

- Solved, e.g., with branch-and-bound search tree

Can we at least formulate a “fast” MIP?

- ❑ Recall: Mixed Integer Program (MIP)
  - ❑ Linear objective function (e.g., minimize
  - ❑ Linear constraints (e.g., do not violate capacity constraints)

One that provides good relaxations!

- ❑ Solved, e.g., with branch-and-bound search tree

Can we at least formulate a “fast” MIP?

- Recall: Mixed Integer Program (MIP)
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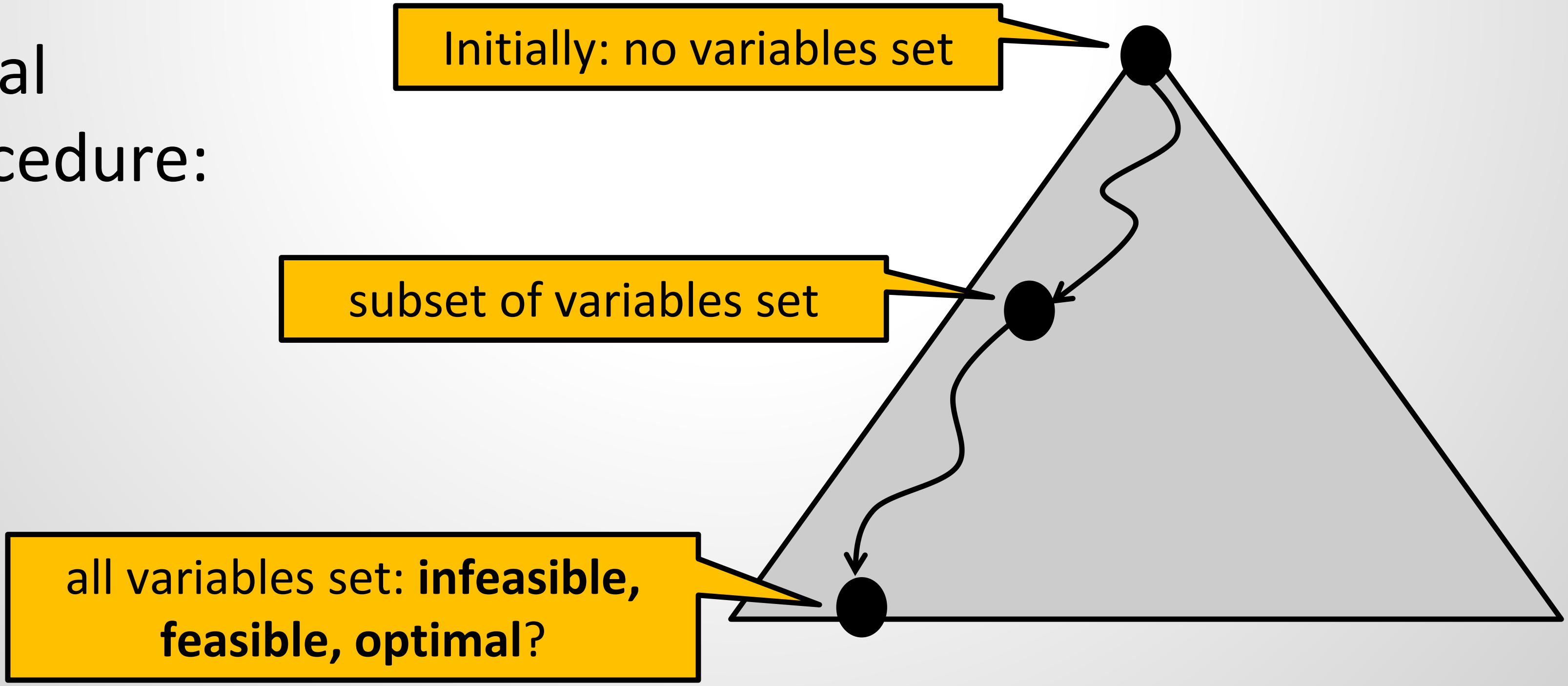
Usual procedure:



Initially: no variables set

subset of variables set

all variables set: **infeasible, feasible, optimal**?

Can we at least formulate a "fast" MIP?

- Recall: Mixed Integer Program (MIP)
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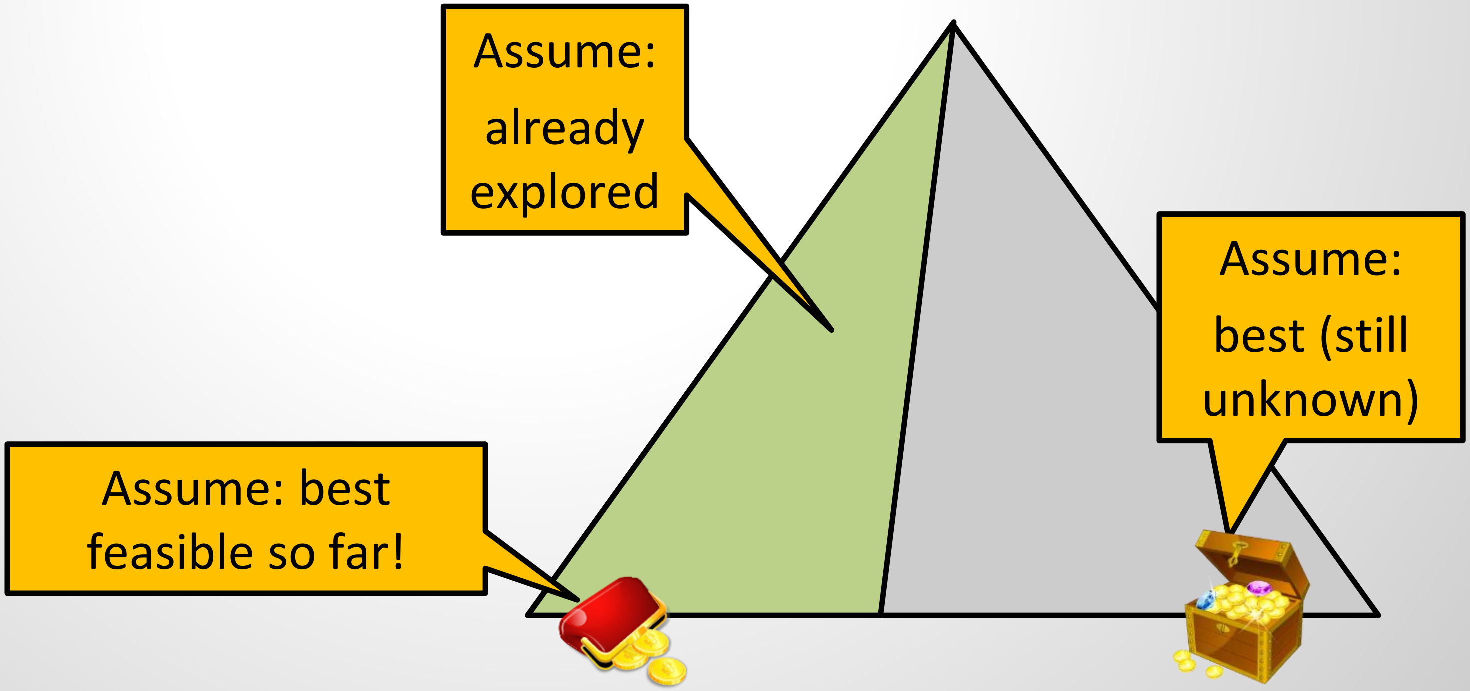


Assume: already explored

Assume: best (still unknown)

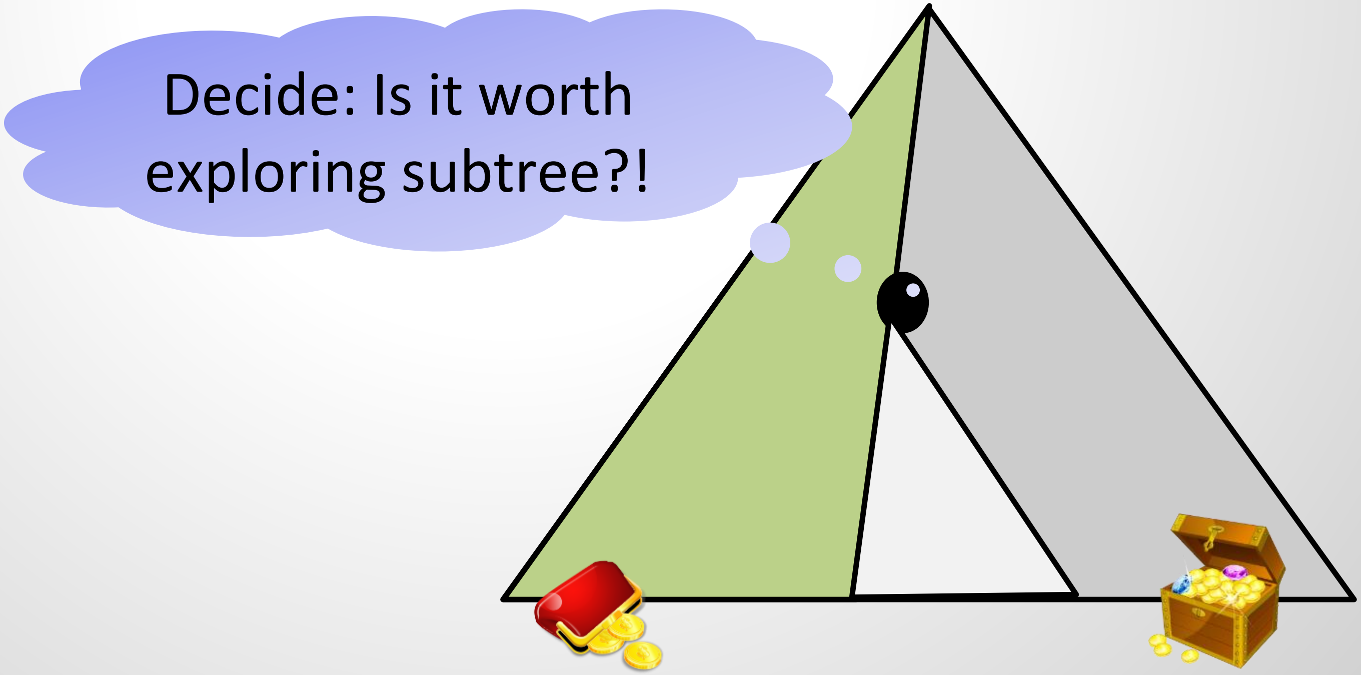
Assume: best feasible so far!

Can we at least formulate a "fast" MIP?

- Recall: Mixed Integer Program (MIP)
  - Linear objective function (e.g., minimize embedding footprint)
  - Linear constraints (e.g., do not violate capacity constraints)

- Solved, e.g., with branch-and-bound search tree

Usual procedure:

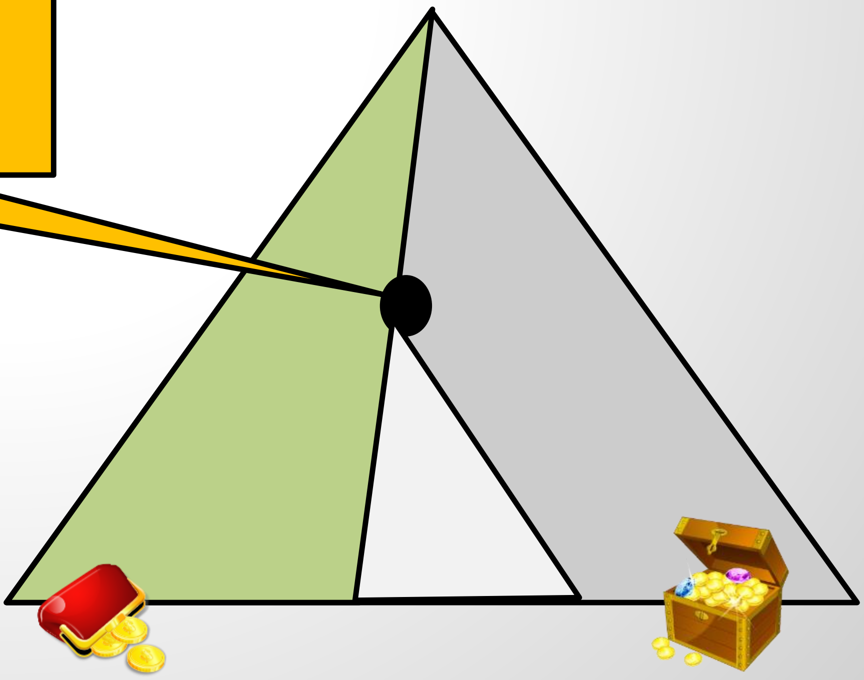


Decide: Is it worth exploring subtree?!

Can we at least formulate a “fast” MIP?

- Recall: Mixed Integer Program (MIP)
  - Linear objective function (e.g., minimize embedding footprint)
  - Linear constraints (e.g., do not violate capacity constraints)

- Solved, e.g., with branch-and-bound search tree



Usual trick: Relax! Solve LP (fast!), and if **relaxed solution** (more general!) **not better** then best solution so far: skip it!

Can we at least formulate a “fast” MIP?

- ❑ Recall: Mixed Integer Program (MIP)
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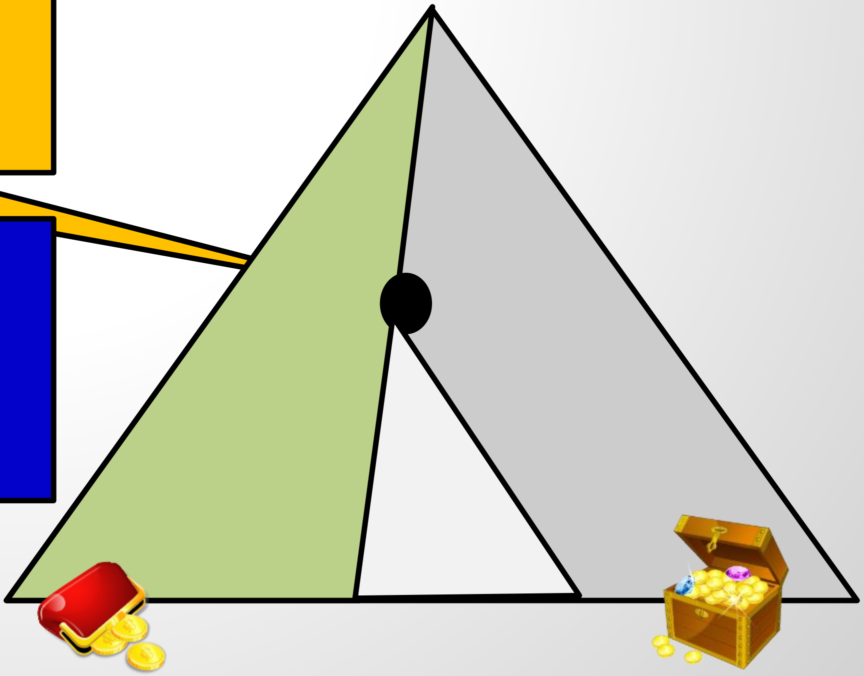
- ❑ Solved, e.g., with branch-and-bound search tree

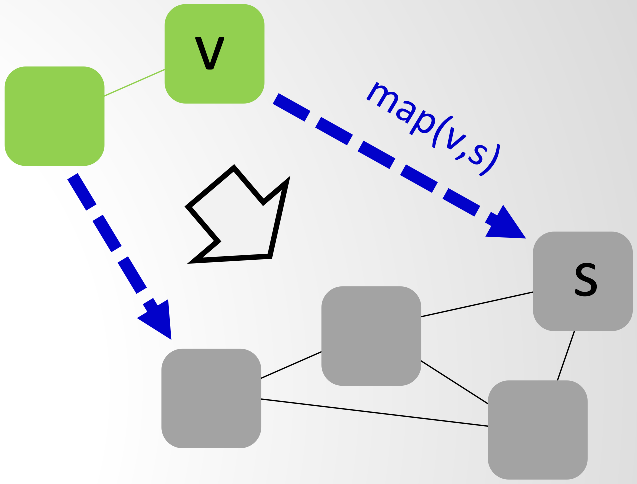
Usual trick: Relax! Solve LP (fast!), and if **relaxed solution** (more general!) **not better** then best solution so far: skip it!

Bottomline: If MIP provides «good relaxations», large parts of the search space can be pruned.



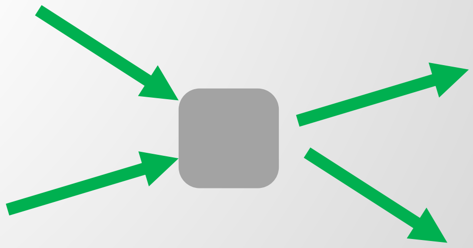
Can we at least formulate a “fast” MIP?

A typical MIP formulation:



❑ Introduce binary variables ***map(v,s)*** to map virtual nodes *v* to substrate node *s*

❑ Introduce flow variables for paths (say splittable for now)



❑ Ensure flow conservation: all flow entering a node must leave the node, unless it is the source or the destination

$$\Sigma_{u->v} f_{uv} = \Sigma_{v->w} f_{vw}$$

In Out

Can we at least formulate a “fast” MIP?

We get constraints like:

Assume bandwidth b requested from node s to node t.

In Out

$$\boxed{\forall v: \sum_{u} f_{uv} - f_{vu} \geq \mathrm{map}(s,v) * b - \mathrm{map}(t,v) * \infty}$$

What does this formula do and why is it correct?

Can we at least formulate a “fast” MIP?

We get constraints like:

Assume bandwidth b requested from node s to node t.

$$\forall v: \overset{\text{In}}{\sum_{u} f_{uv}} - \overset{\text{Out}}{f_{vu}} \geq map(s,v) * b - map(t,v) * \infty$$

If map(s,v)=1, i.e., s mapped to v: so **flow starts at v**, and hence outgoing flow must be larger than incoming flow (plus b).

Can we at least formulate a “fast” MIP?

We get constraints like:

Assume bandwidth b requested from node s to node t.

$$\forall v: \sum_{u} \overset{\text{In}}{f_{uv}} - \overset{\text{Out}}{f_{vu}} \geq \text{map}(s,v) *b - \text{map}(t,v) * \infty$$

If map(s,v)=0 and map(t,v)=0, i.e., **v is along the path** from s to t: then we have **flow conservation**: outgoing flow must equal incoming flow (here ≥, objective function will remove unnecessary flow).

Can we at least formulate a “fast” MIP?

We get constraints like:

Assume bandwidth b requested from node s to node t.

In Out

$$\forall v: \sum_u f_{uv} - f_{vu} \geq map(s,v) * b - map(t,v) * \infty$$

If map(t,v)=1, i.e., t mapped to v: so flow **terminates at node v**: so **no constraint**: minus infinity (but objective function will remove unnecessary flow).

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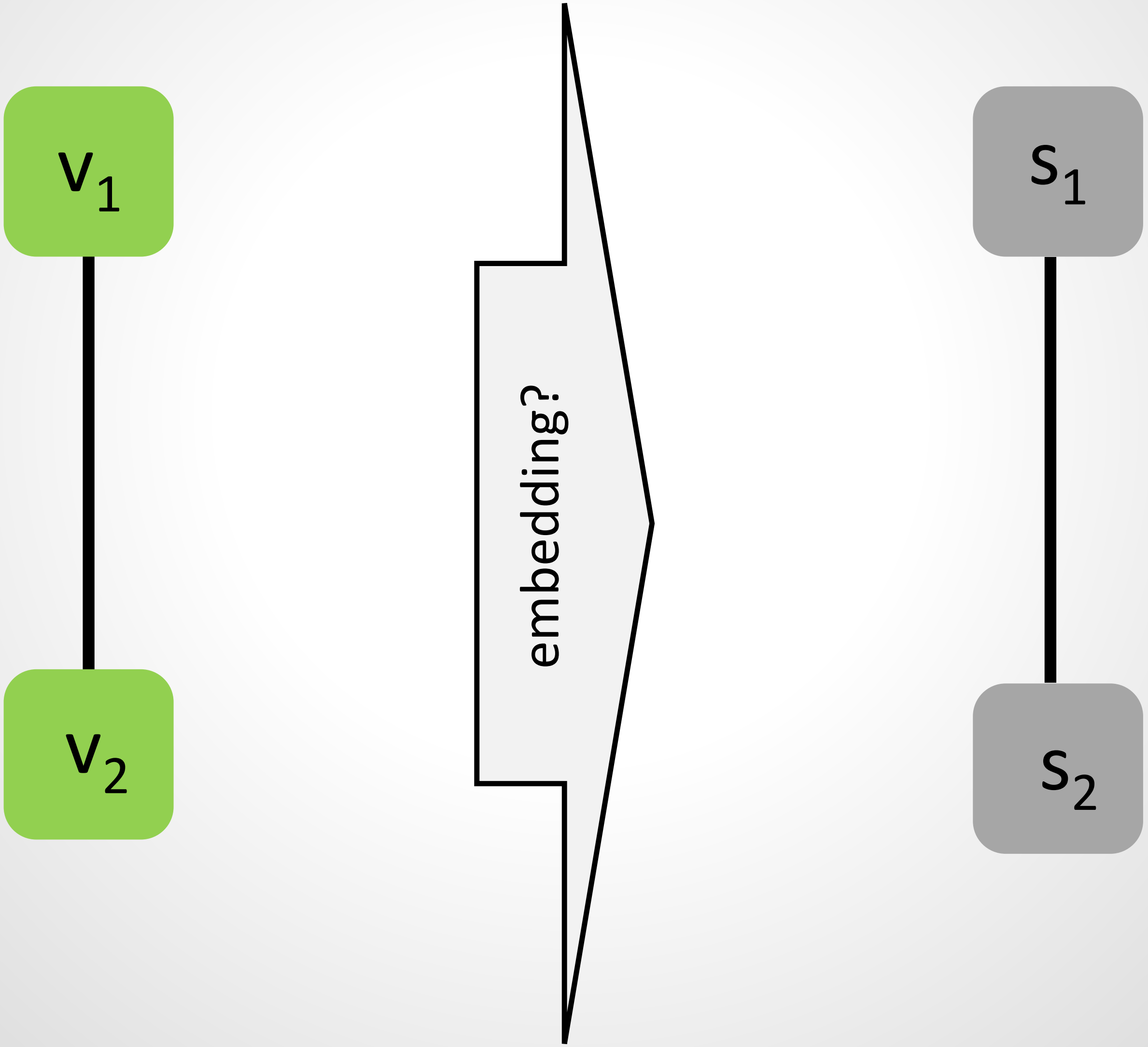
$$\forall v: \sum_{u} f_{uv} - f_{vu} \geq map(s,v) * b - map(t,v) * \infty$$

If map(t,v)=1, i.e., t mapped to v: so flow **terminates at node v**: so **no constraint**: minus infinity (but objective function will remove unnecessary flow).

Will such a MIP provide effective pruning?

What will happen in this example?



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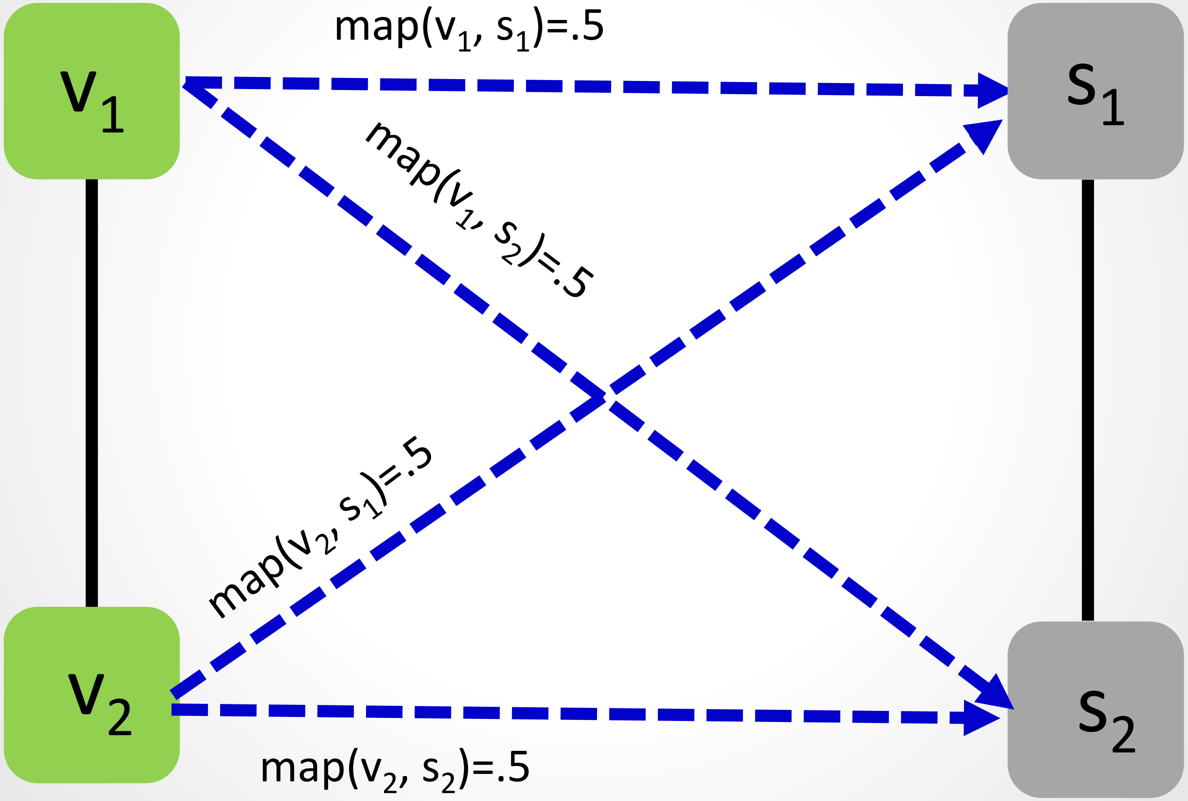

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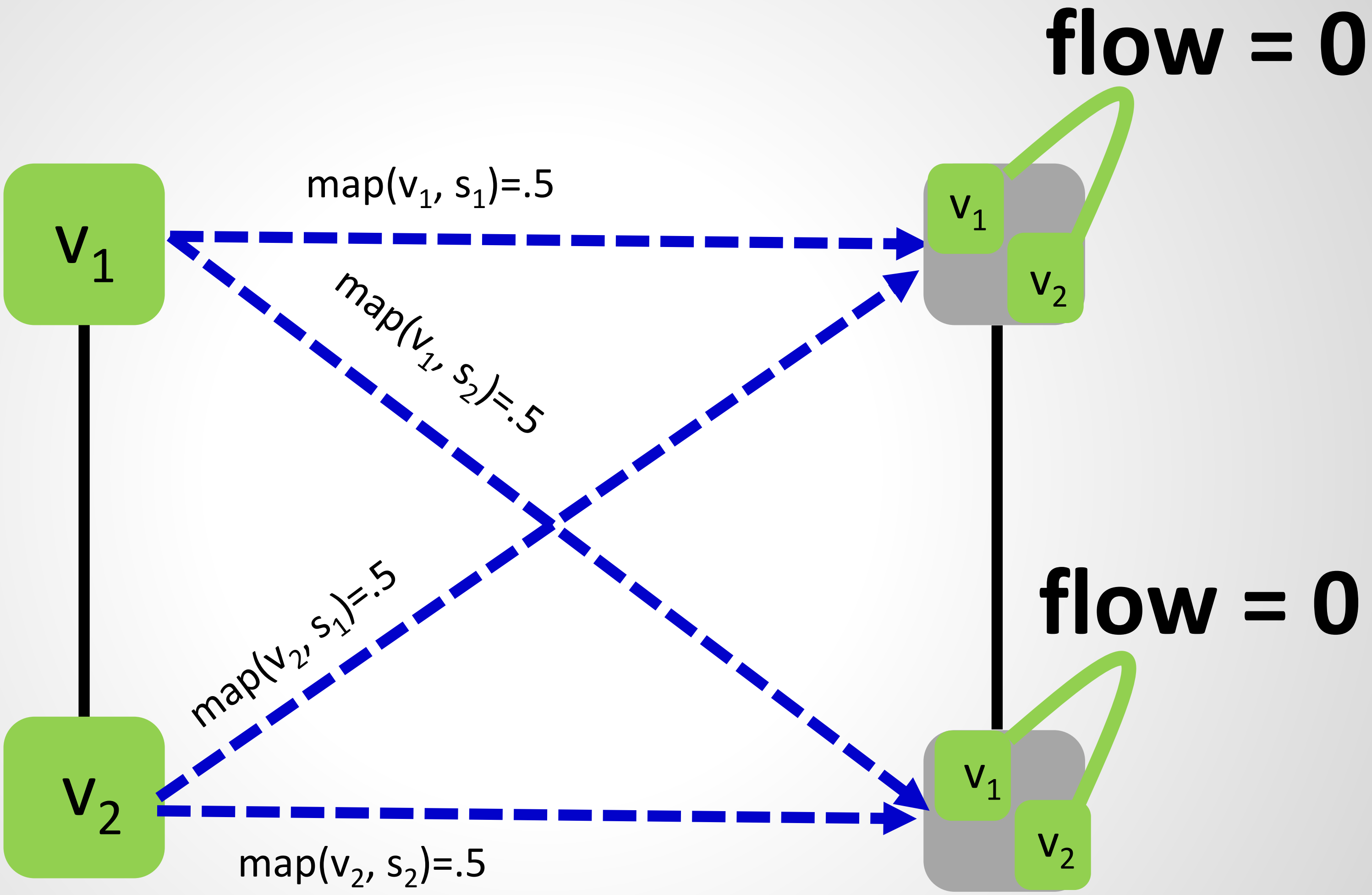


Minimal flow = 0: fulfills flow conservation! Relaxation useless: does not provide any lower bound or indication of good mapping!

What about using randomized rounding?

Recall: classic approxmation approach which: (i) computes a solution to the linear relaxation of the IP, (ii) decomposes this solution into convex combinations of elementary solutions, and (iii) probabilistically chooses any of the elementary solutions based on their weight.

What about using randomized rounding?

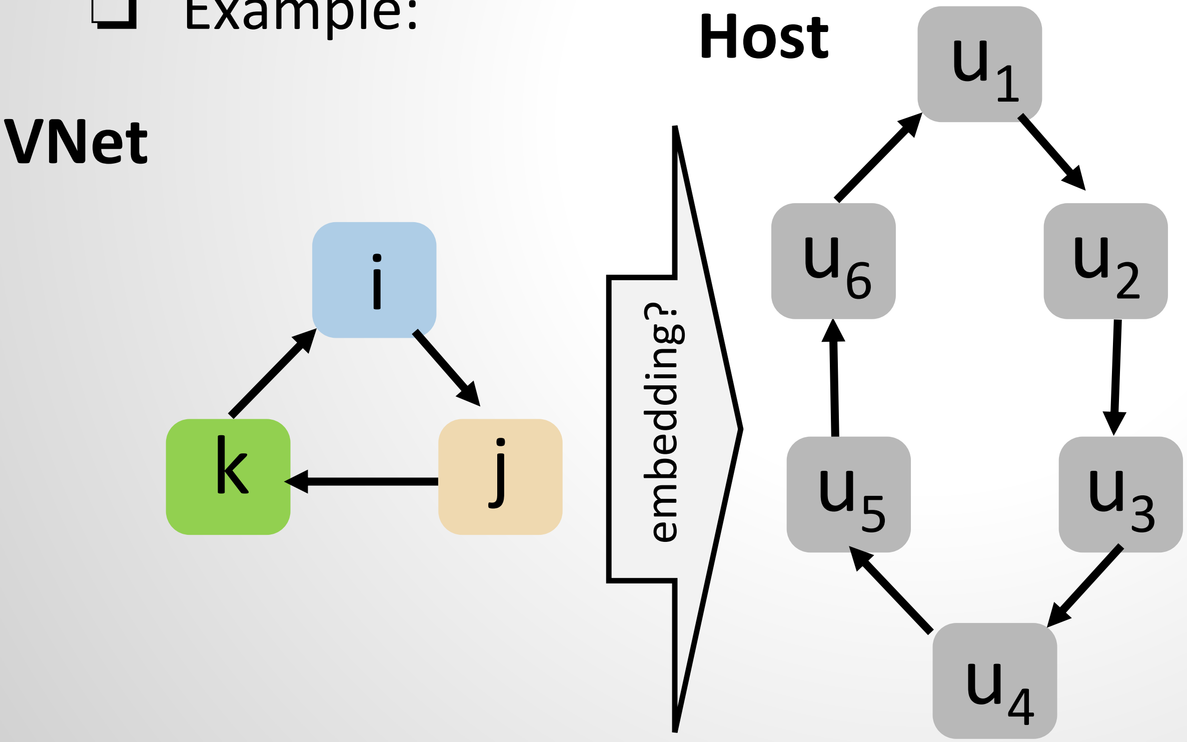
- Problem 1: relaxed solutions may not be very meaningful
  - see example for splittable flows before
- Problem 2: also for unsplittable flows, if using a standard Multi-Commodity Flow (MCF) formulation of VNEP, the integrality gap can be huge
  - Tree-like VNets are still ok
  - VNets with cycles: randomized rounding not applicable, since problem not decomposable

The linear solutions can be decomposed into convex combinations of valid mappings.

Non-Decomposability

- Relaxations of classic MCF formulation cannot be decomposed into convex combinations of valid mappings (so we need different formulations!)

- Example:



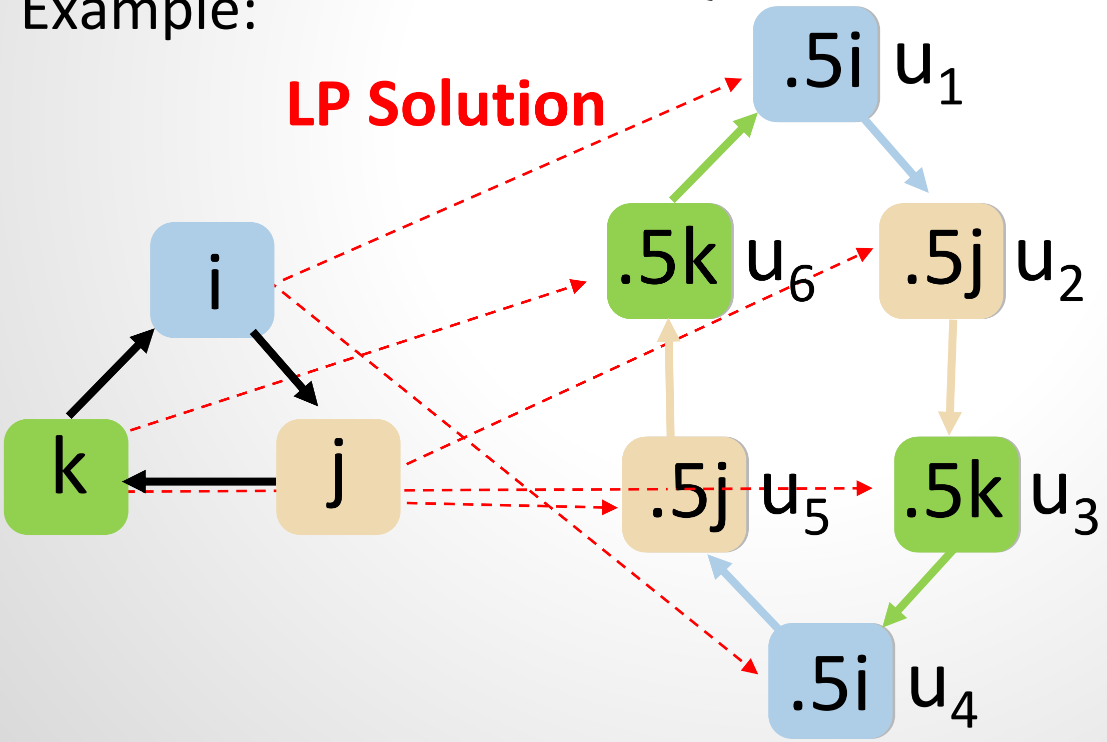
Non-Decomposability

❑ rmulation cannot be decomposed
valid mappings (so we need

Valid LP solution: virtual node mappings sum to 1 and each virtual node connects to its neighboring node **with half a unit of flow…**

❑ Example:

LP Solution



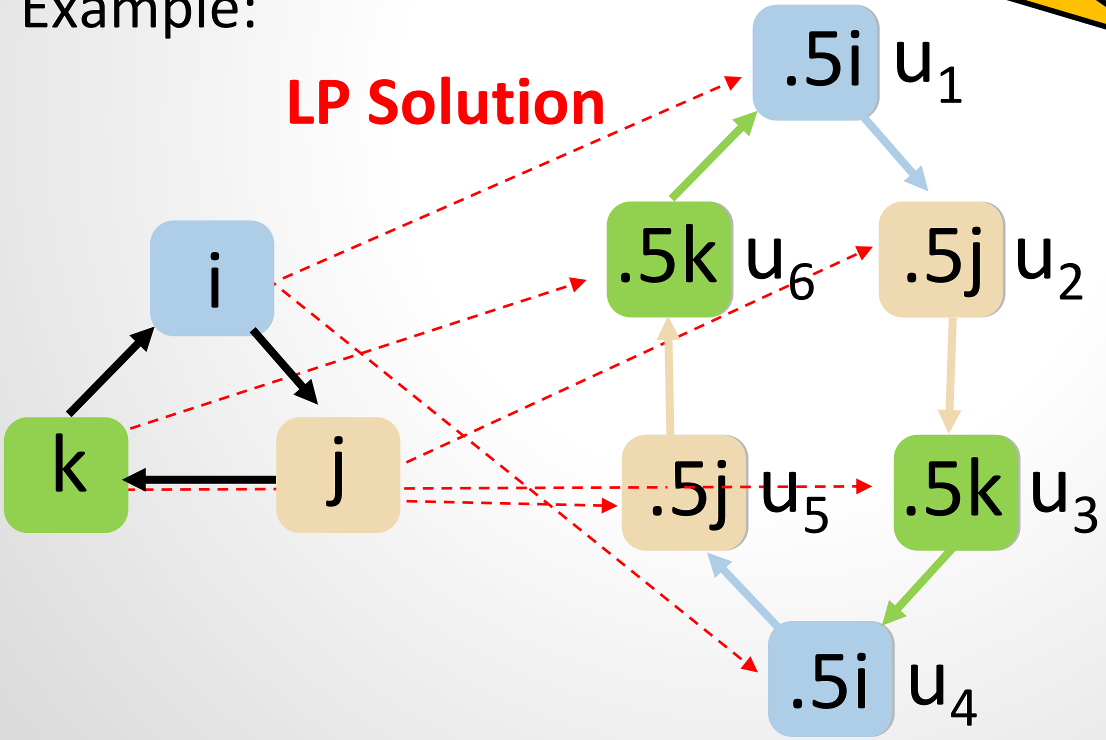
Impossible to decompose and extract **any single valid mapping**. **Intuition:** Node i is mapped to u_1 and the only neighboring node that hosts j is u_2, so i must be fully mapped on u_1 and j on u_2. Similarly, k must be mapped on u_3. But flow of virtual edge (k,i) leaving u_3 only leads to u_4, so **i must be mapped on both u_1 and u_4**. This is **impossible**, even if **capacities are infinite**.

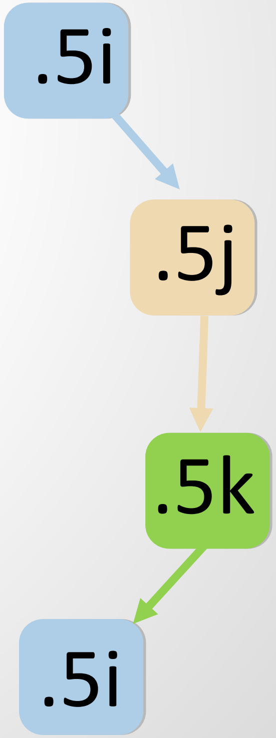
not be decomposed

s (so we need

Partial Decomposition

- Example:





Impossible to decompose and extract **any single valid mapping**. **Intuition:** Node i is mapped to u_1 and the only neighboring node that hosts j is u_2, so i must be fully mapped on u_1 and j on u_2. Similarly, k must be mapped on u_3. But flow of virtual edge (k,i) leaving u_3 only leads to u_4, so **i must be mapped on both u_1 and u_4**. This is **impossible**, even if **capacities are infinite**.

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Partial Decomposition

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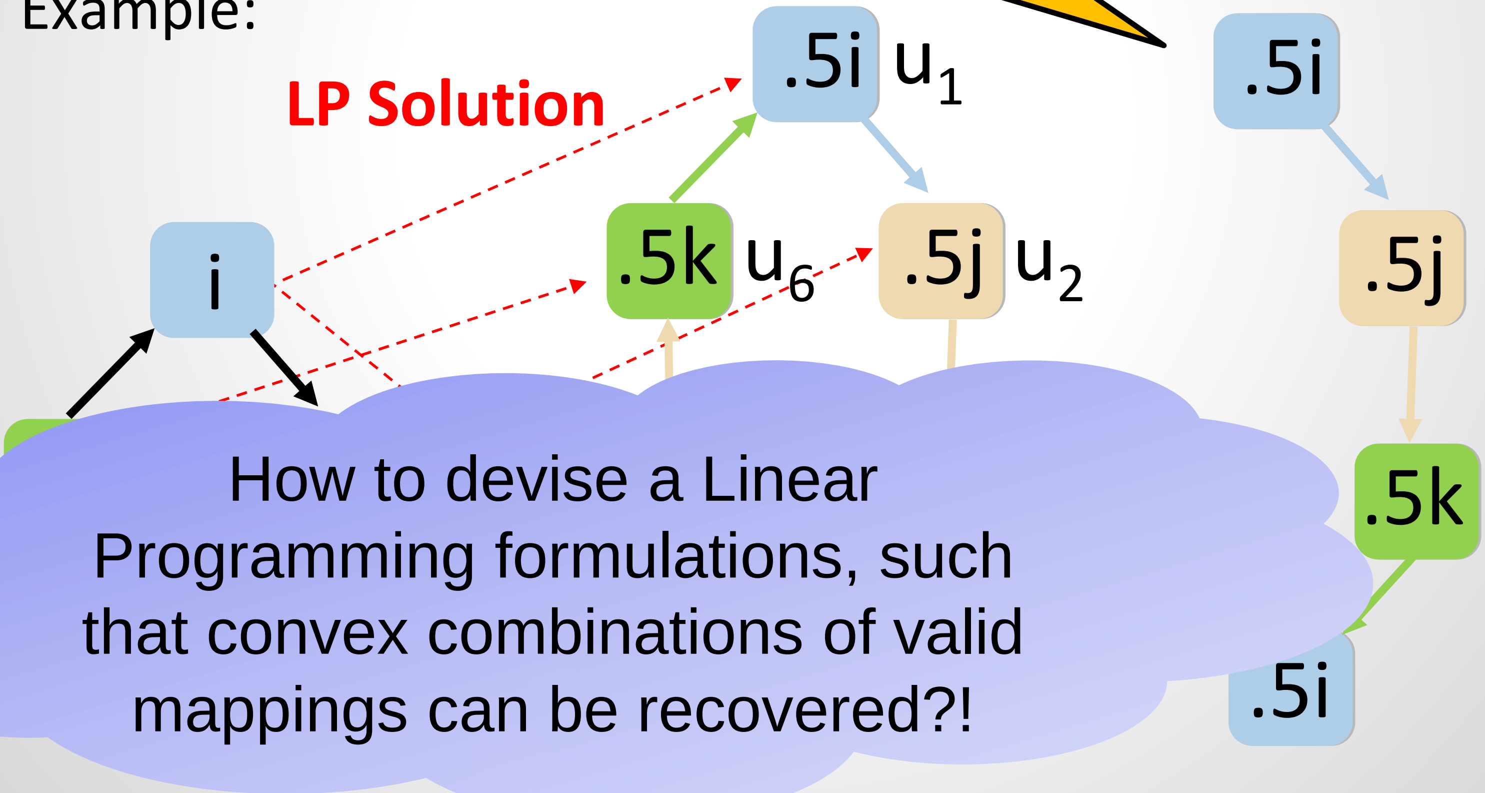


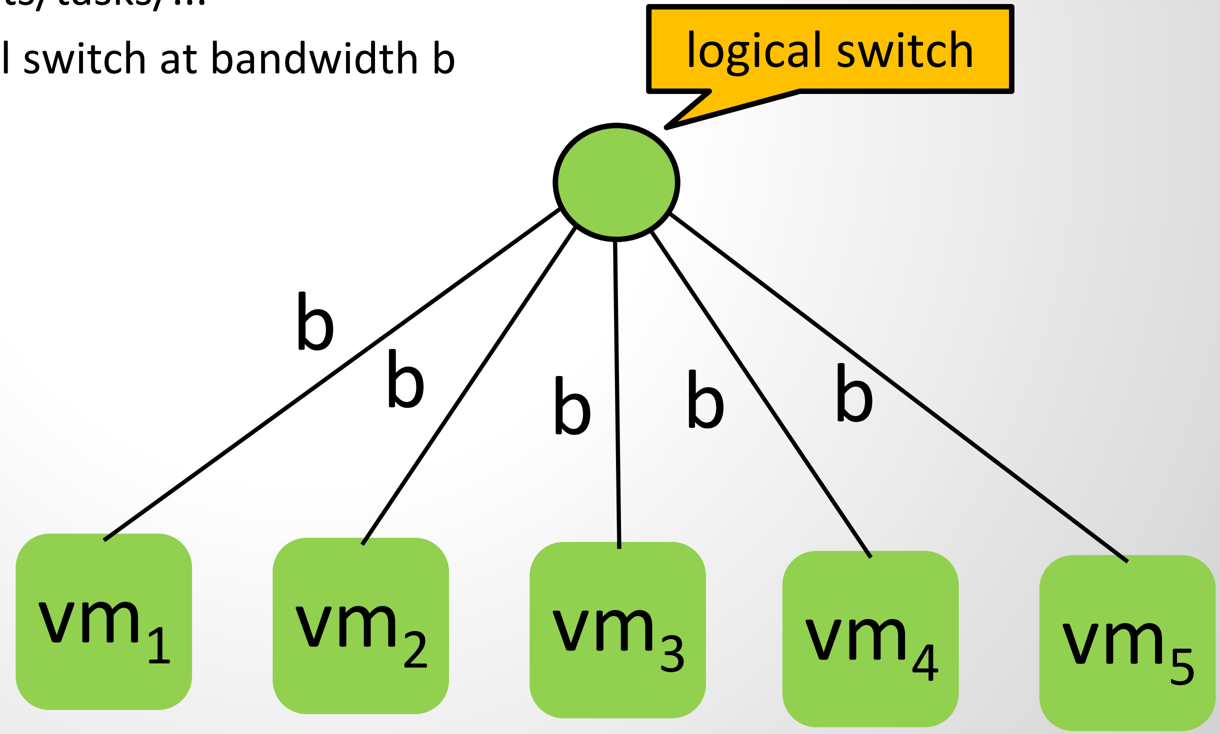
How to devise a Linear Programming formulations, such that convex combinations of valid mappings can be recovered?!

That's all Folks!

Wait a minute!
These problems need to be solved!
And they often can, even with guarantees.

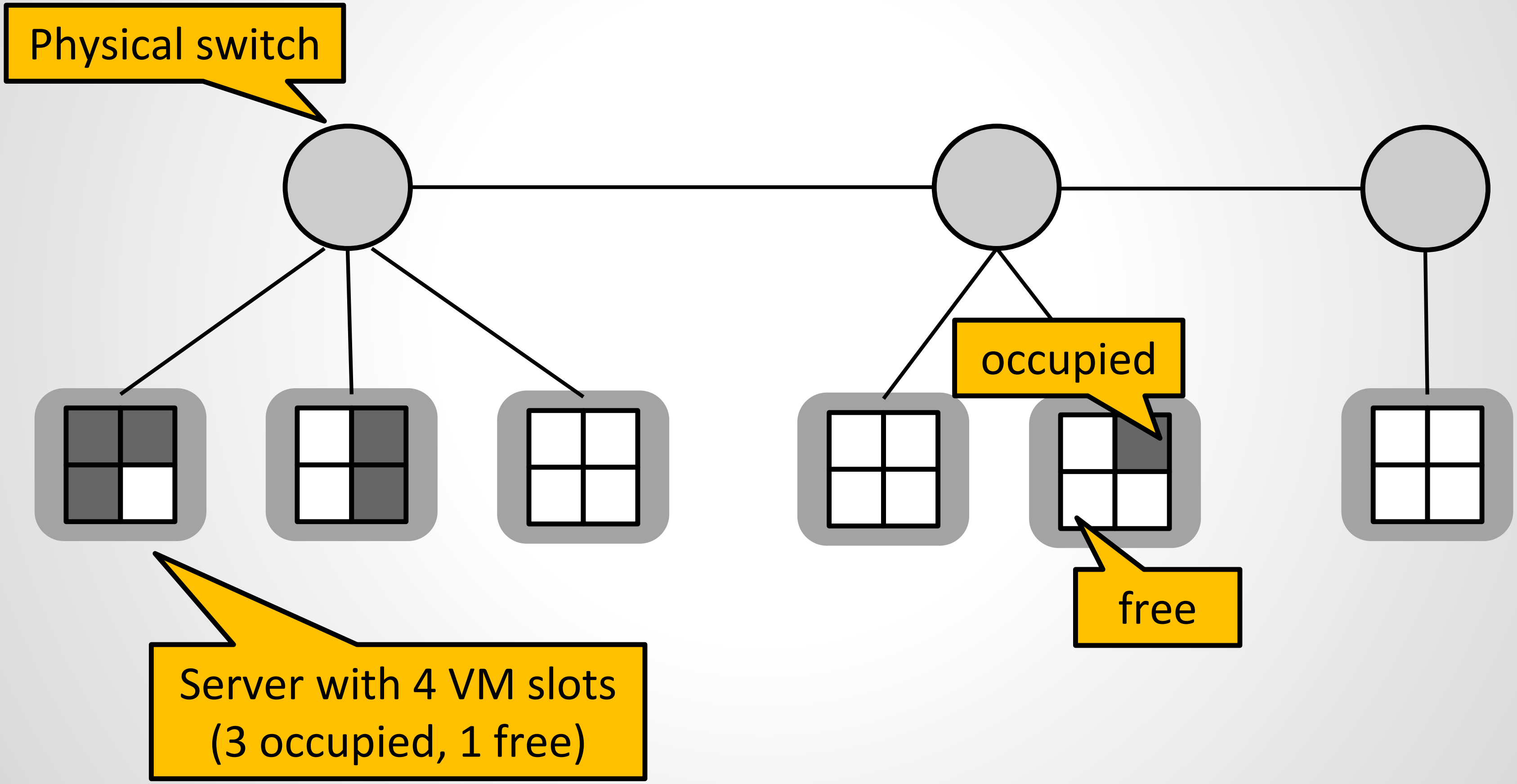
Theory vs Practice:
In Practice There is Hope!

- Guest graphs may not be general graphs, but e.g., virtual clusters: very simple and symmetric, used in context of batch processing
  - k VMs/compute-units/tasks/...
  - Connected to virtual switch at bandwidth b



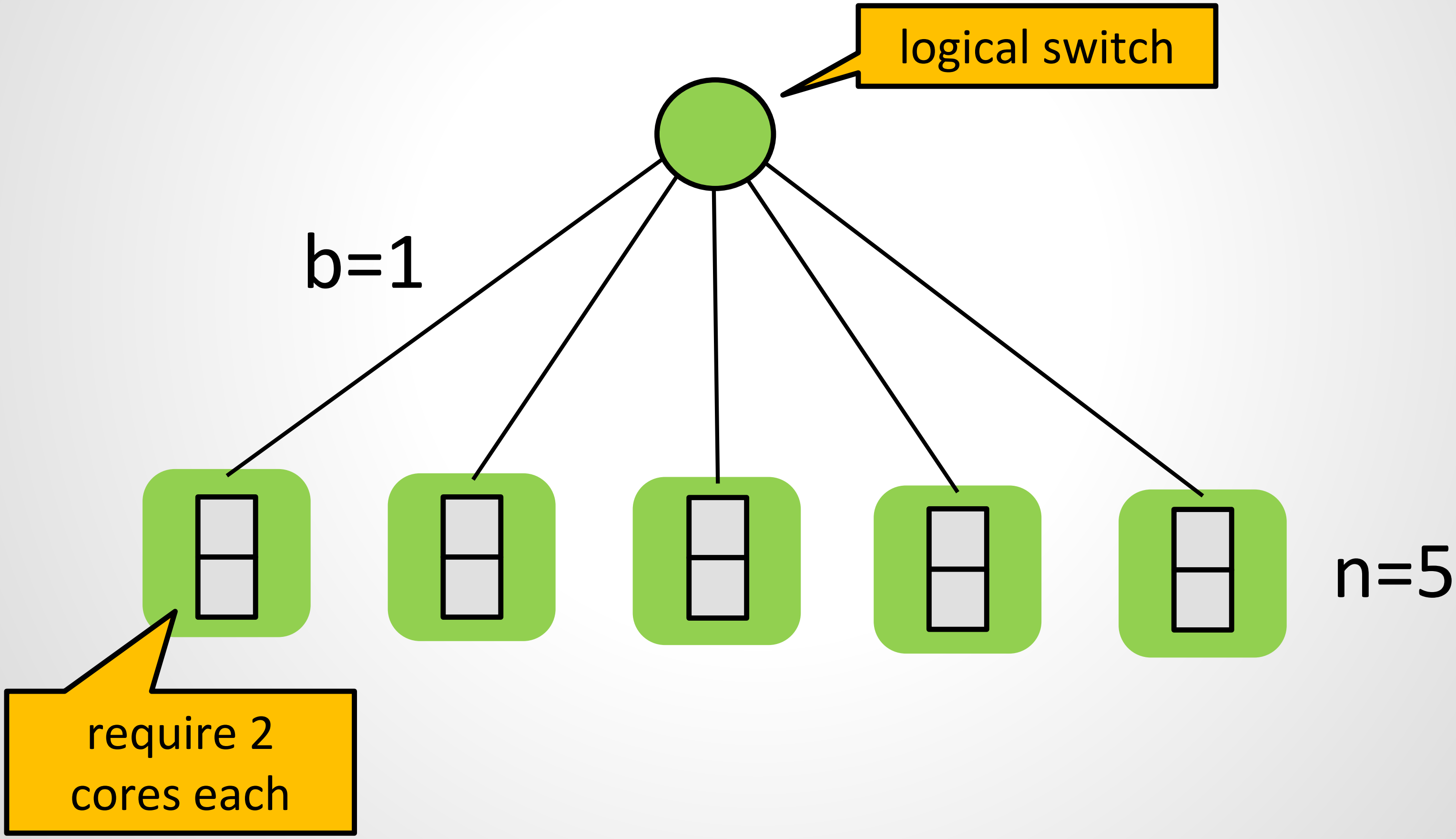
How to Embed a Virtual Cluster?

Consider host graph:

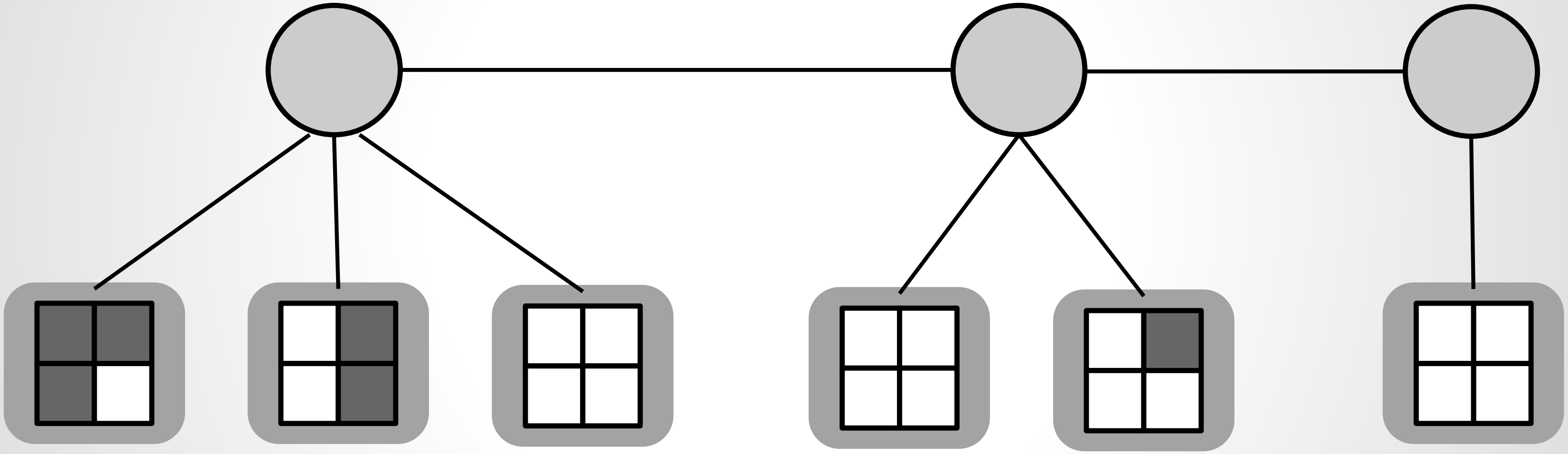


How to Embed a Virtual Cluster?

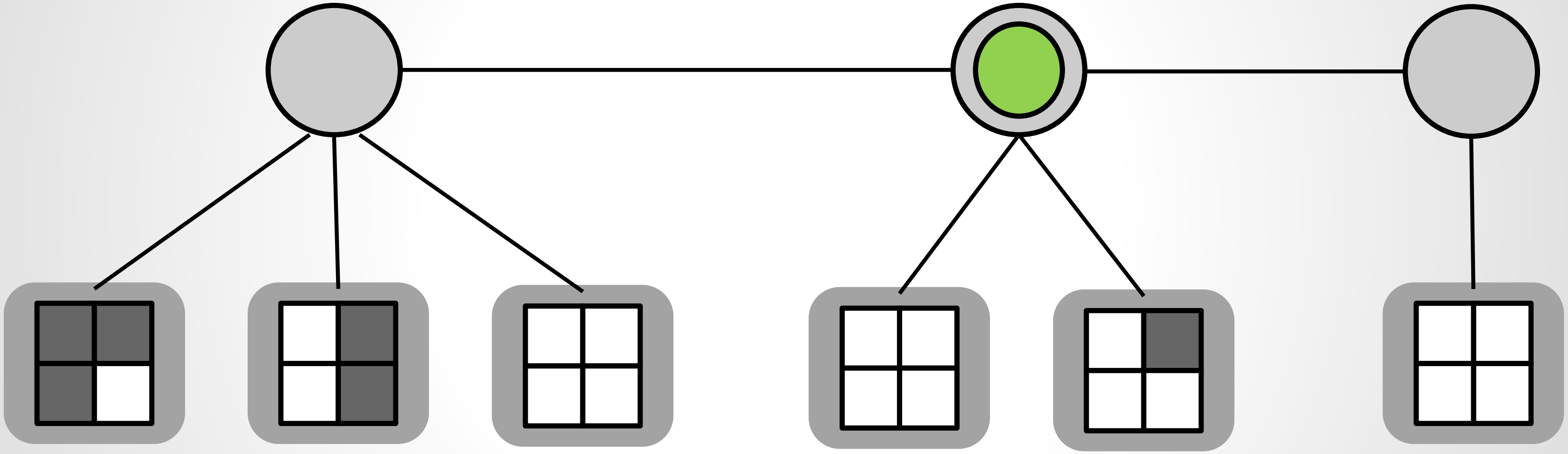
Consider guest graph:



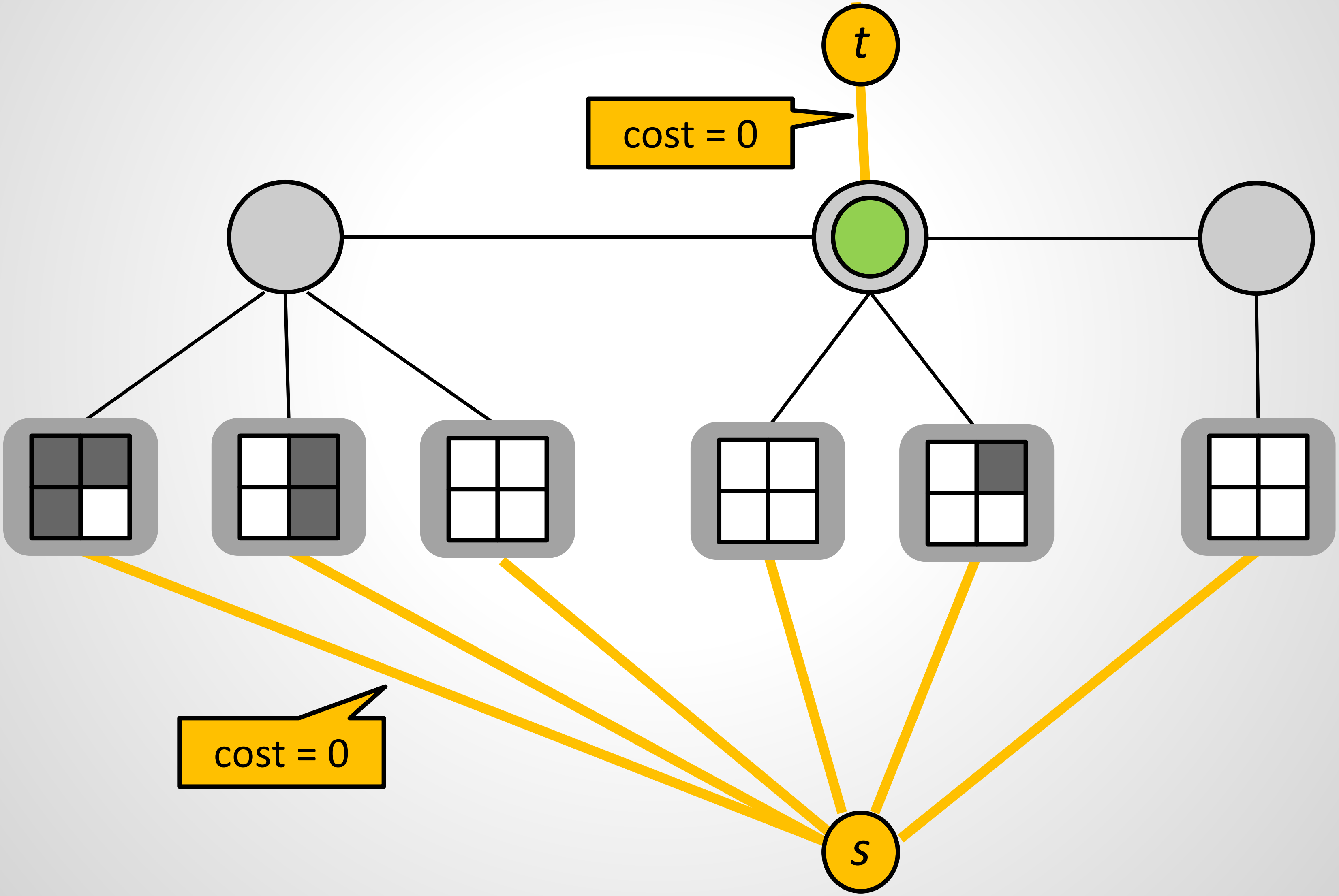
How to Embed a Virtual Cluster?



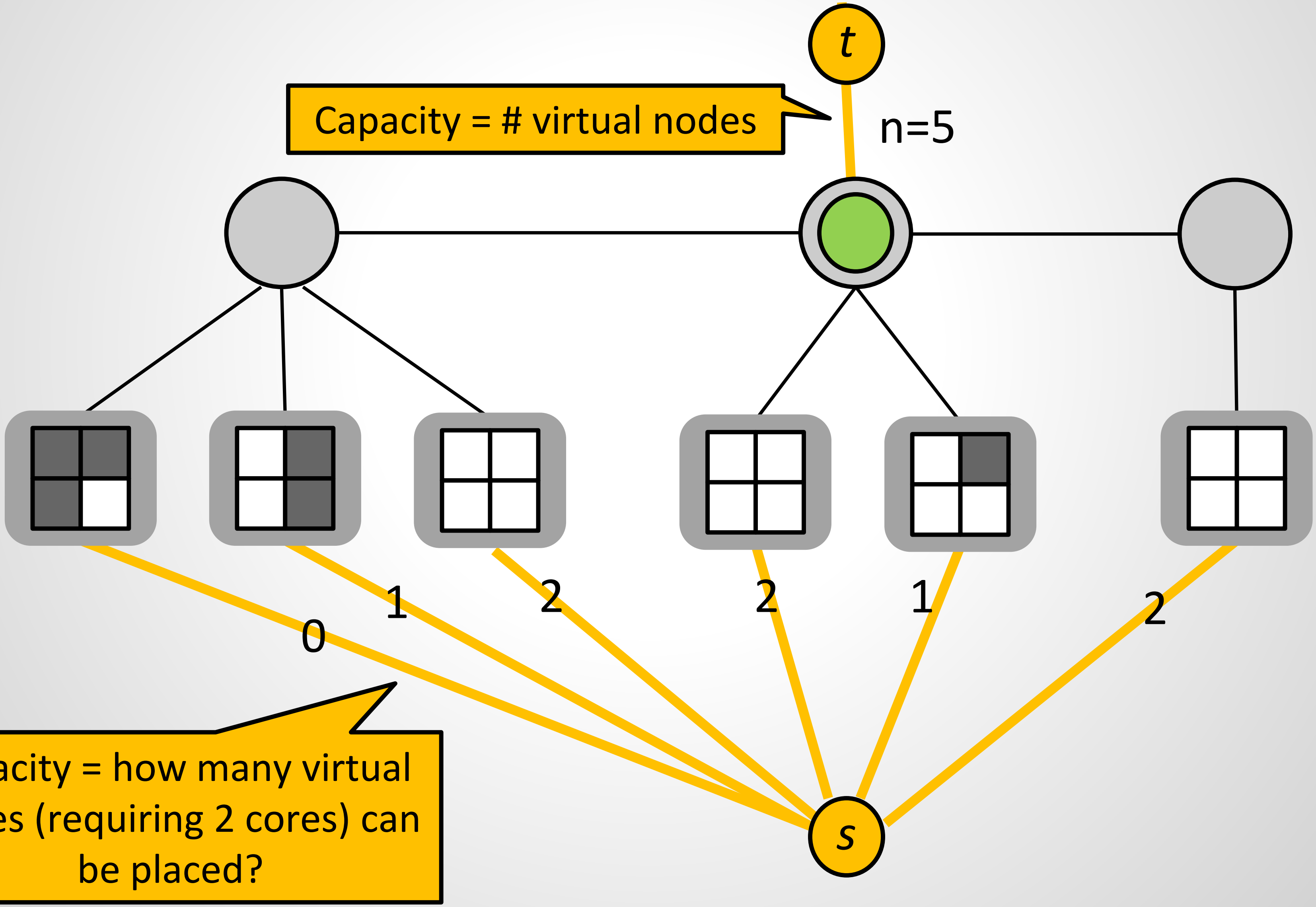
1. Place logical switch (try all options)



1. Place logical switch (try all options)
2. Extend network with artificial source s and sink t



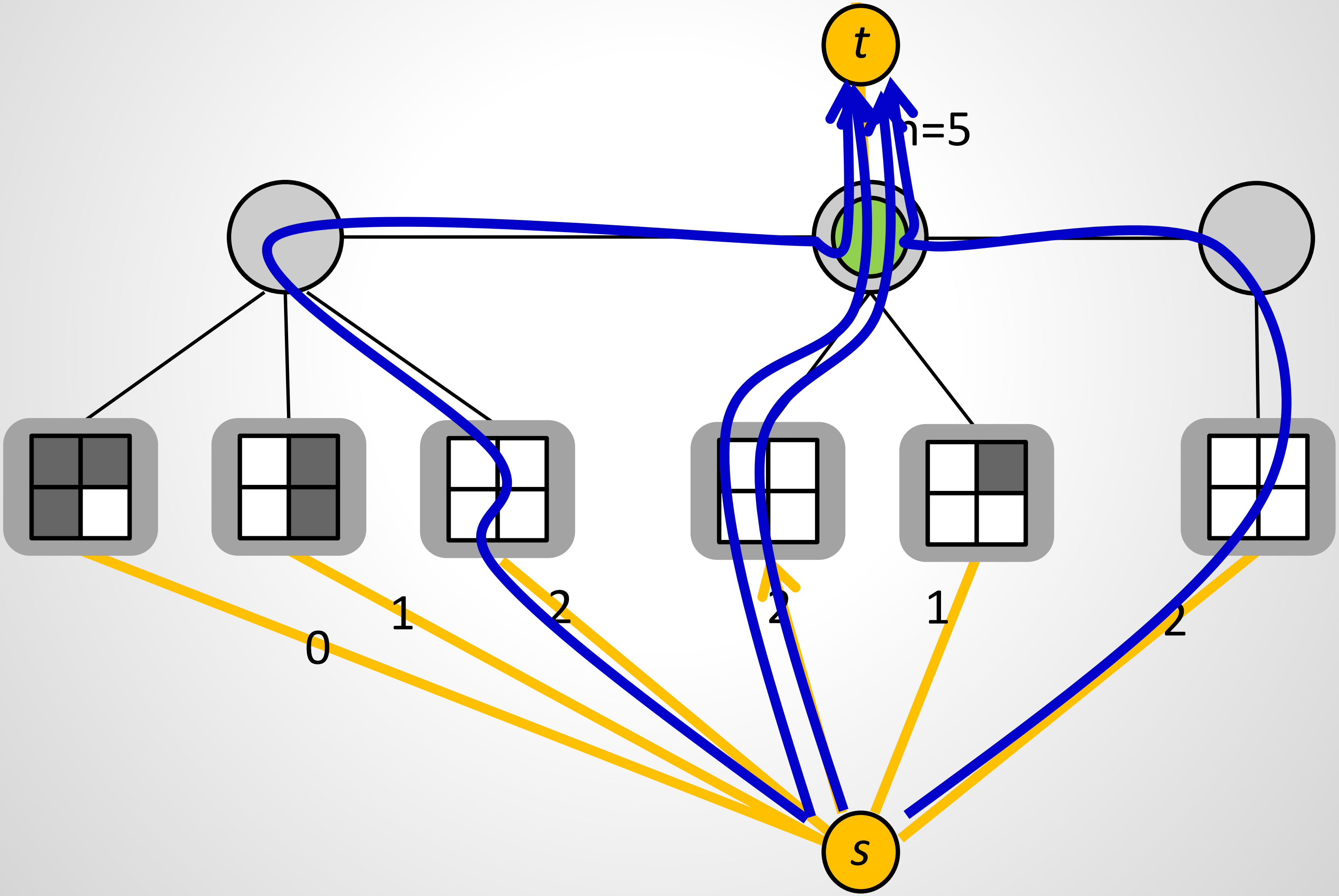
1. Place logical switch (try all options)
2. Extend network with artificial source s and sink t
3. Add capacities (recall that b=1, so each virtual node needs one unit of capacity)



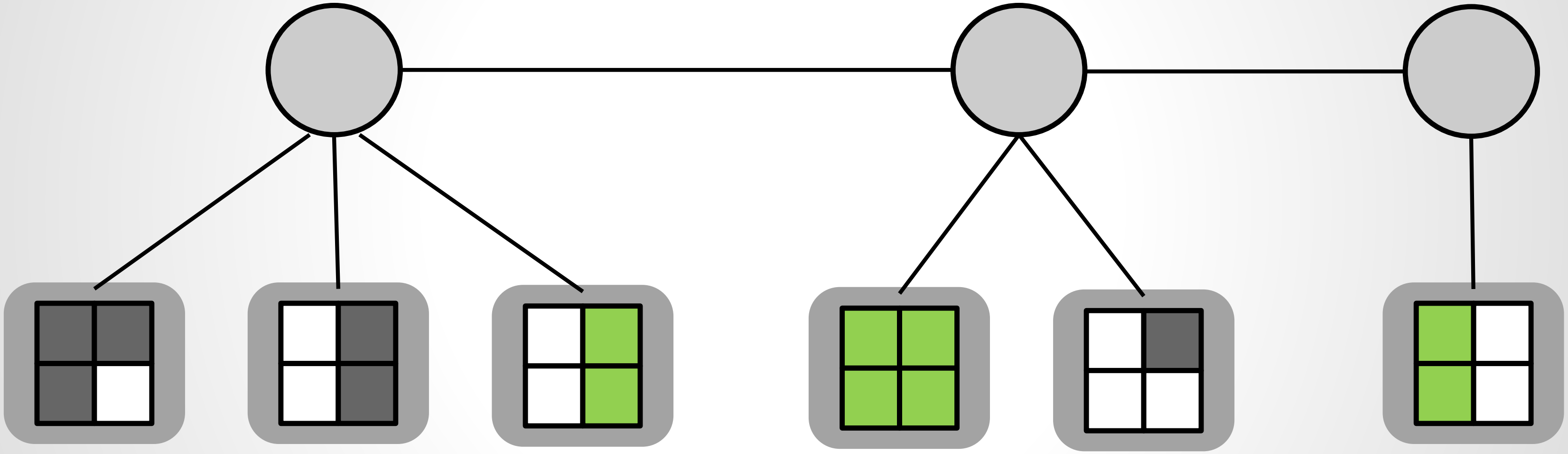
Capacity = # virtual nodes

capacity = how many virtual nodes (requiring 2 cores) can be placed?

Then: Compute min-cost max flow of size n from s to t (e.g., successive shortest paths): due to capacity constraints at most size n.



Then: Compute min-cost max flow of size n from s to t (e.g., successive shortest paths): due to capacity constraints at most size n.
… and assign virtual nodes (and edges) accordingly!



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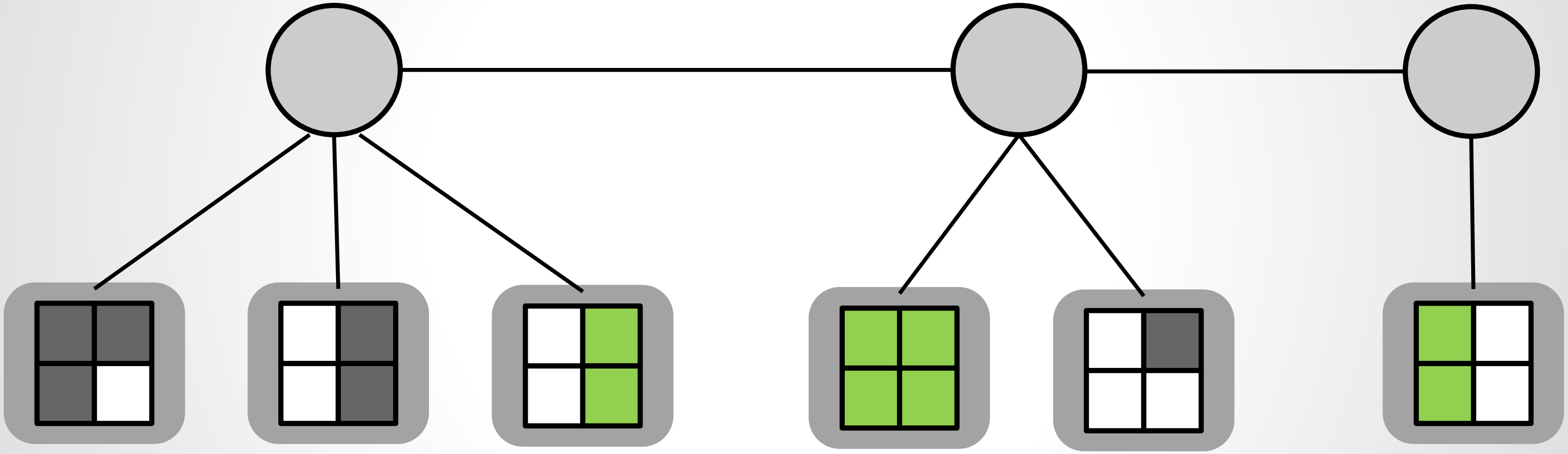
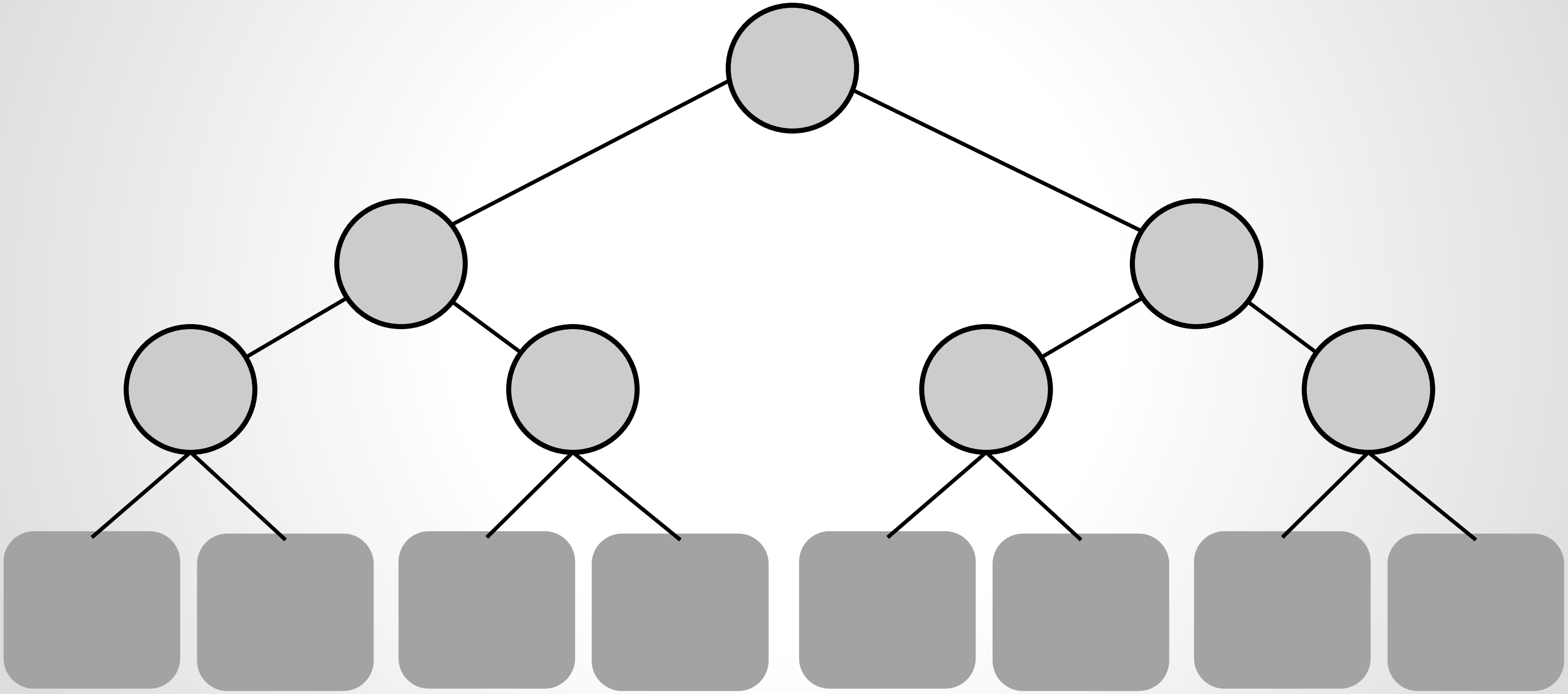


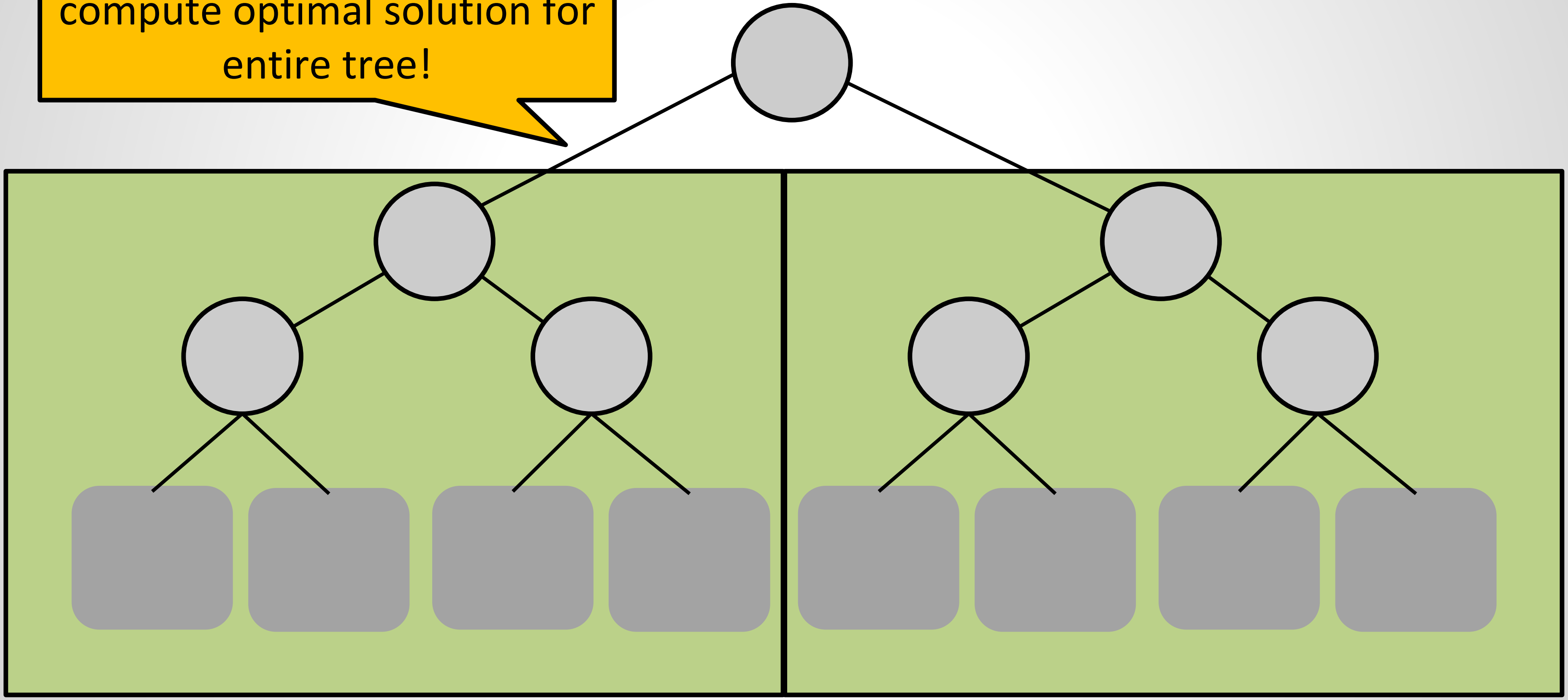
In fact: this physical network is even ***a tree***!
For trees with servers at leaves, even
simpler algorithms are possible. Ideas?

Dynamic Programming

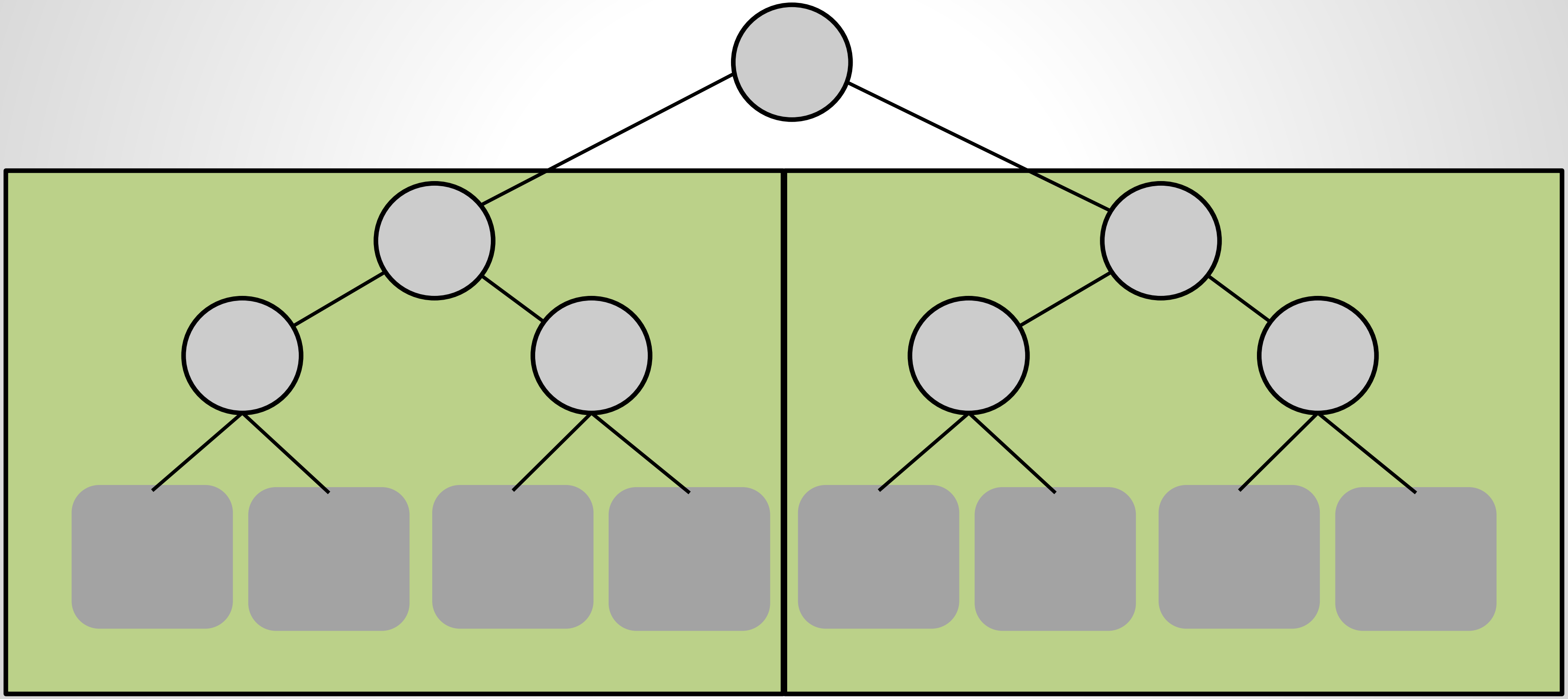


Dynamic Programming

Bottom-up programming: given optimal solution for subtrees, can quickly compute optimal solution for entire tree!



Dynamic Programming

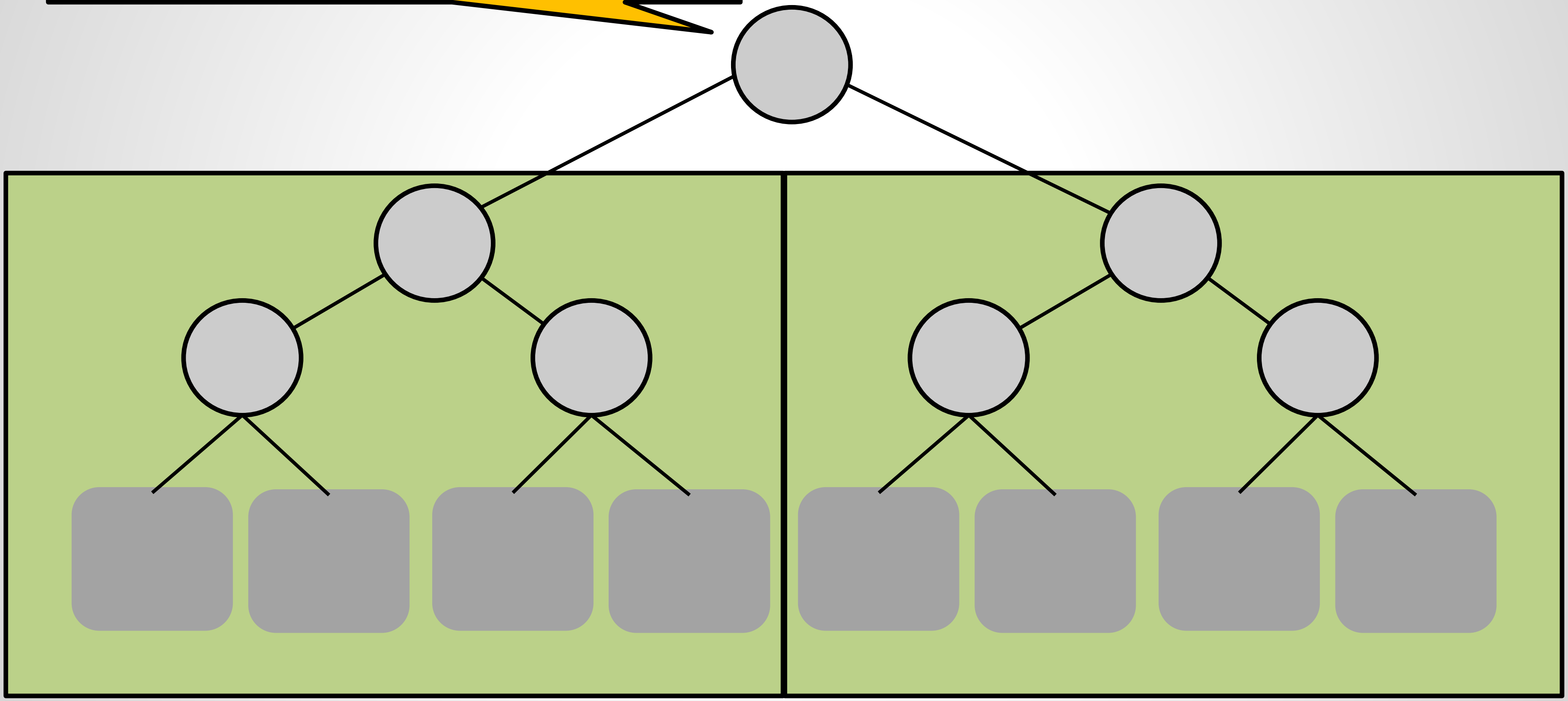


Given **optimal embedding** for x ∈ {0, ..., n} virtual nodes in left subtree...

Given **optimal embedding** for x ∈ {0, ..., n} virtual nodes in right subtree...

Dynamic Programming

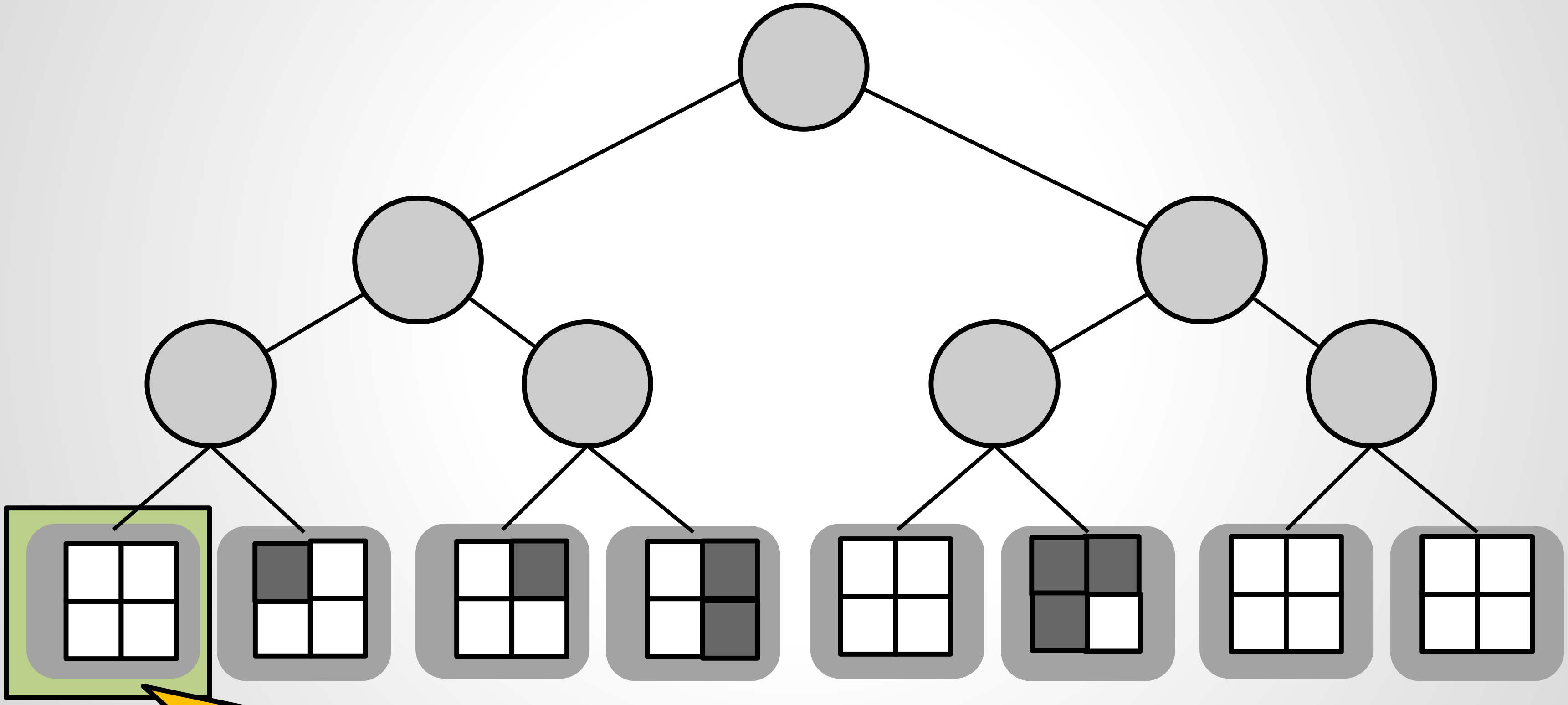
… can compute optimal embedding of x ∈ {0, …, n} virtual nodes in **entire subtree**!



Given **optimal embedding** for x ∈ {0, …, n} virtual nodes in left subtree…

Given **optimal embedding** for x ∈ {0, …, n} virtual nodes in right subtree…

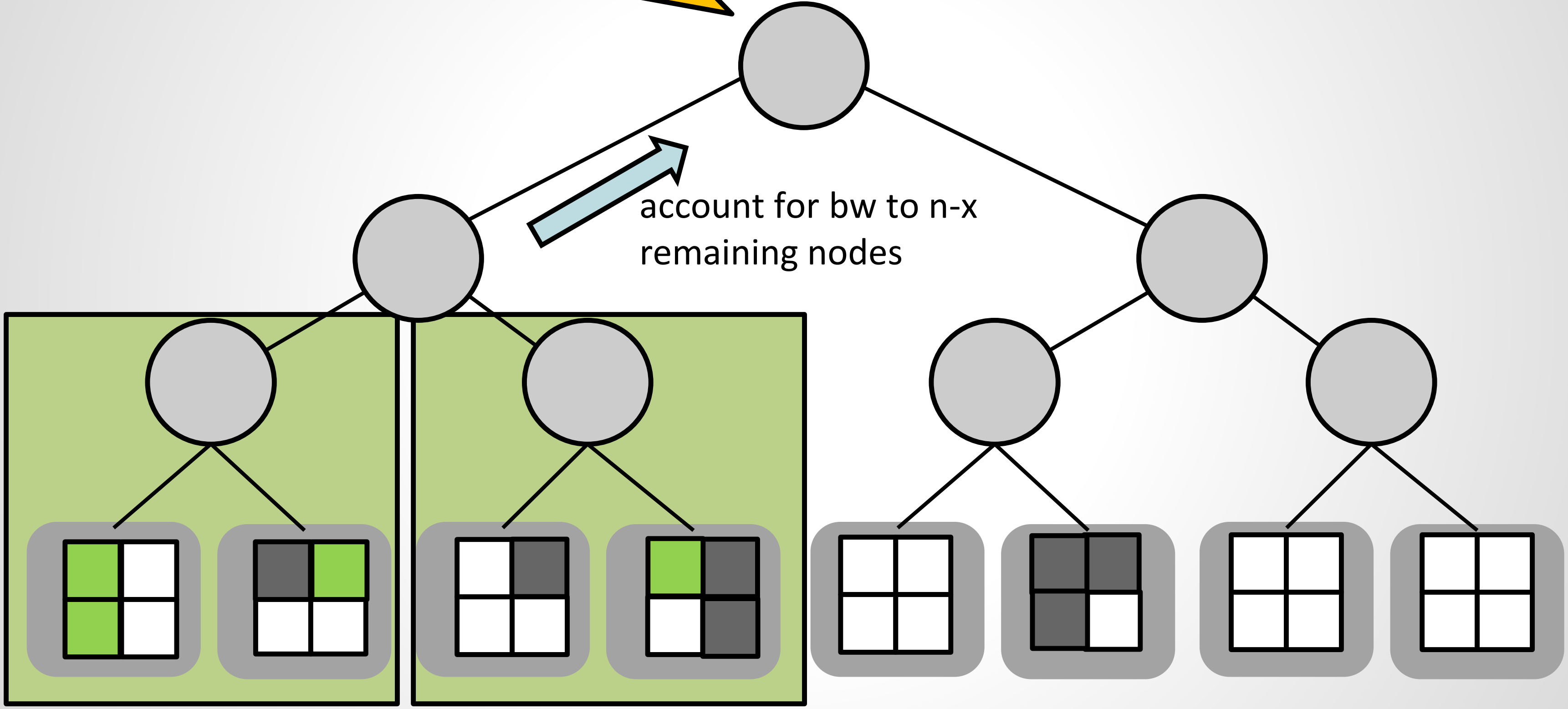
Dynamic Programming



Bottom-up «induction». Leaves easy: either x nodes fit server (cost 0) or not (cost ∞): opt[≤4] = 0 , opt[>4] = ∞

Dynamic Programming

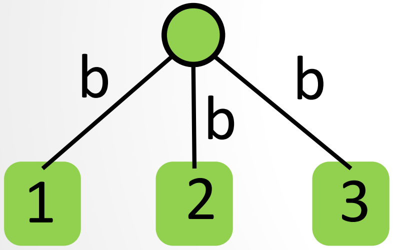
$\text{opt}(T,x)=\min_{0\leq y\leq x} \{\text{opt}(\text{left},y)+\text{opt}(\text{right},x-y\}+\text{bw}(T,x)$


account for bw to n-x remaining nodes

To compute cost of embedding x nodes in T, place y nodes on the left, x-y on the right subtree, and compute cost due to links across root.

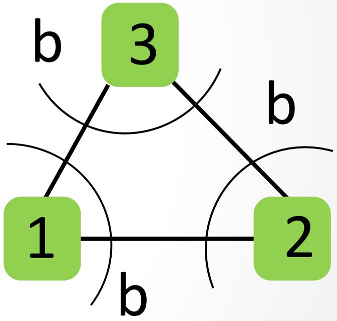
Remark on Virtual Cluster Abstraction

- Two interpretations:
  - Logical switch at unique location
  - Logical switch can be distributed («Hose model»)



Logical Switch Variant

vs



Hose Variant

Aggregated bw in/out node at most b.

Remark on Virtual Cluster Abstraction

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  - Logical switch at unique location
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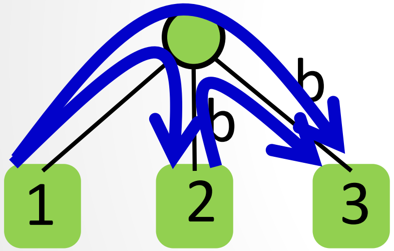

Logical Switch Variant

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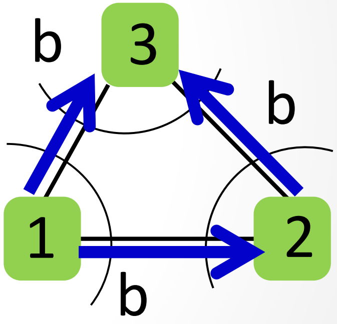

Hose Variant

Can serve the same communication patterns! (A **polytope of possible traffic matrices.**)

Example:

(1,2): b/2

(1,3): b/2

(2,3): b/2

Remark on Virtual Cluster Abstraction

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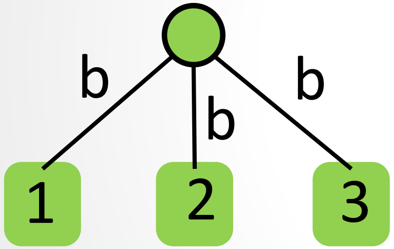
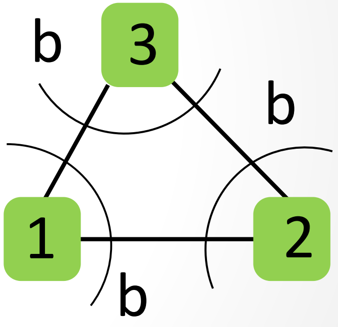


VS



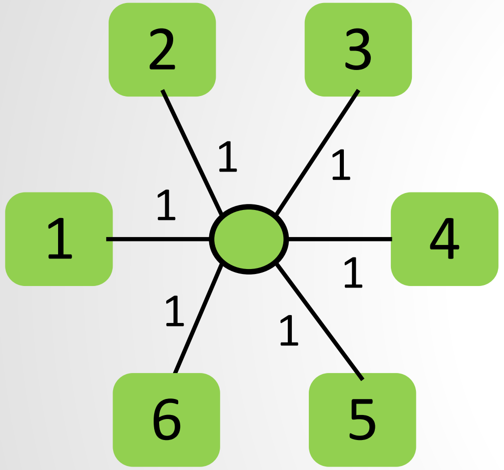
Hose Variant

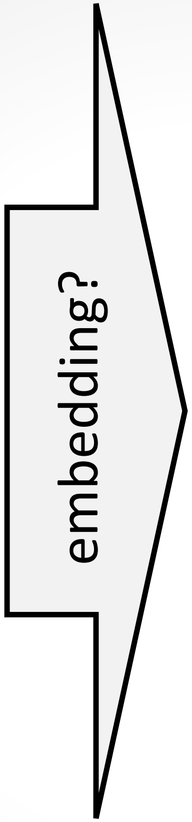
But **embedding costs** can be different if we do not insist on placing the logical switch explicitly! **Not on trees** though, and **not in uncapacitated networks**: without routing restrictions, optimal routing paths form a tree (**SymG=SymT** a.k.a. **VPN Conjecture**).

The Benefit of Hose Interpretation

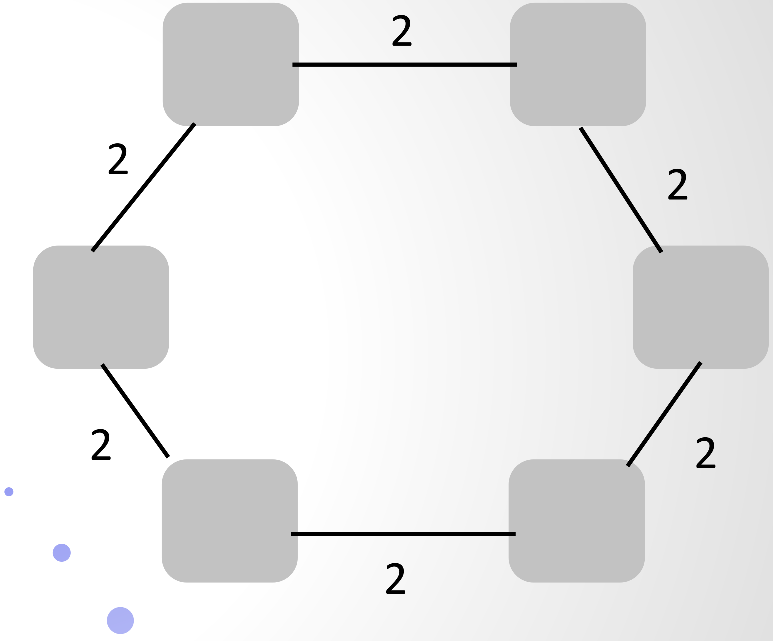
The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)





Host Graph: A Ring



How to embed as a star?

The Benefit of Hose Interpretation

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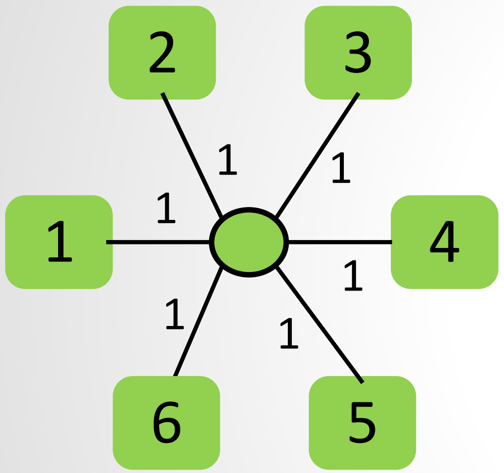
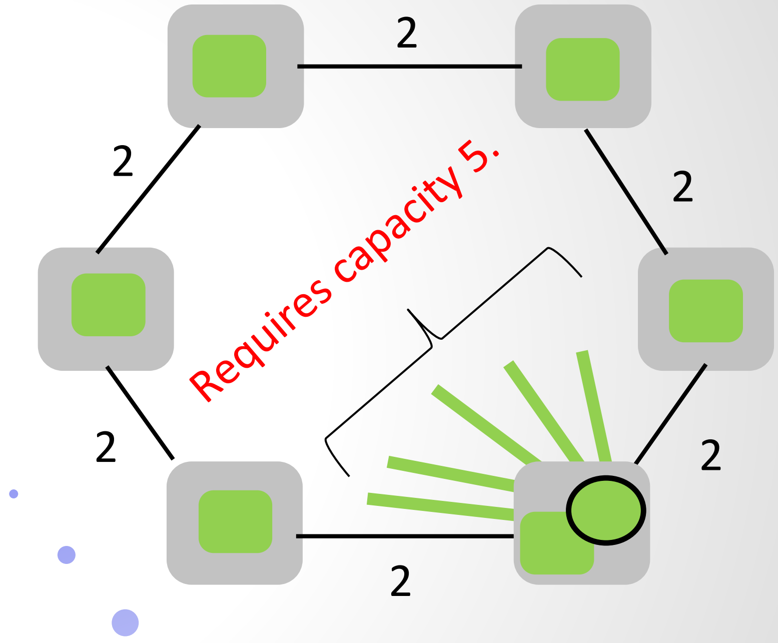


embedding?

Host Graph: A Ring



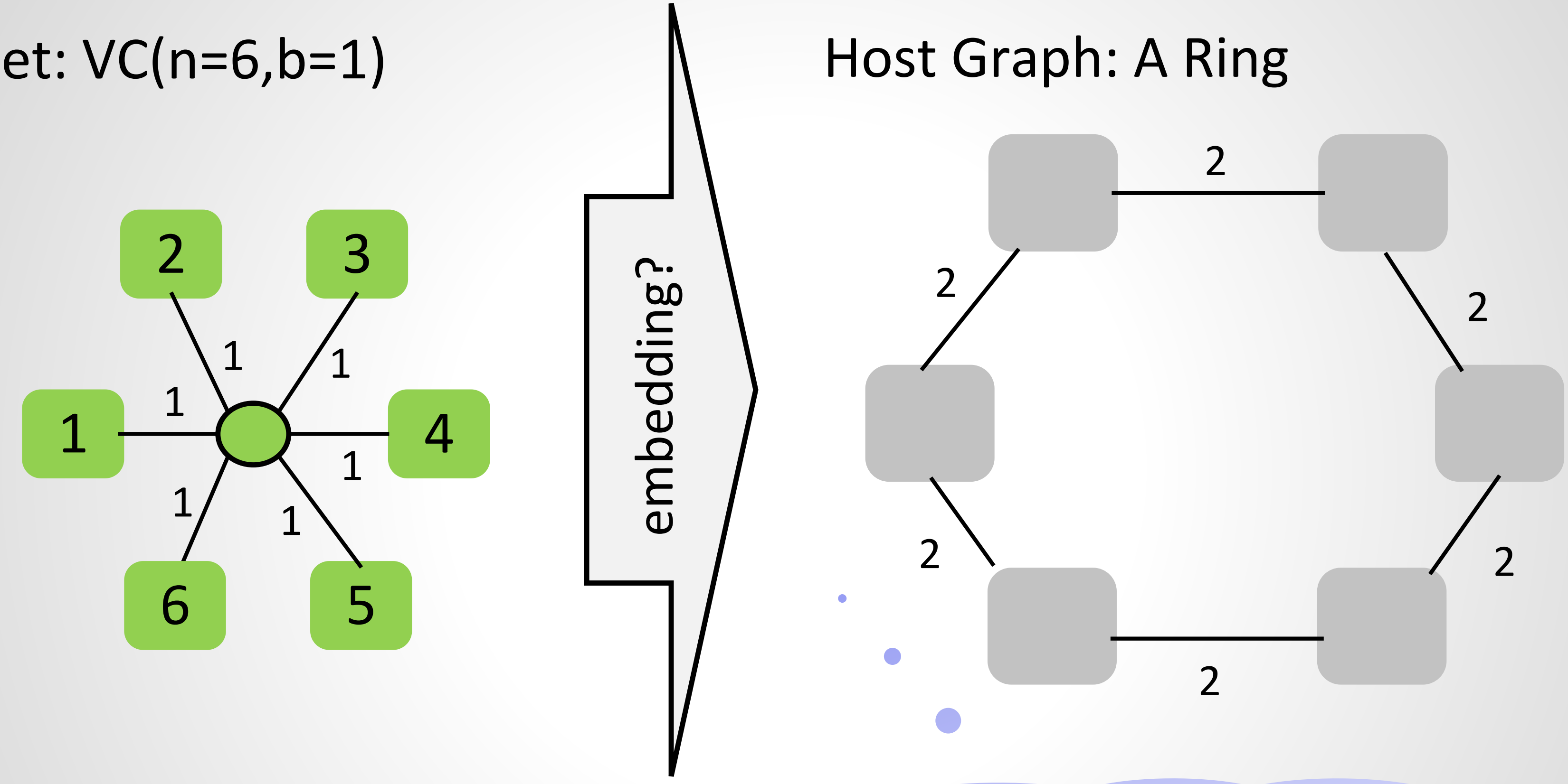
How to embed as a star?

Impossible: need at least 5 units of flow from/to node where ***star center is mapped***. However, capacity of incident links is only 4.

The Benefit of Hose Interpretation

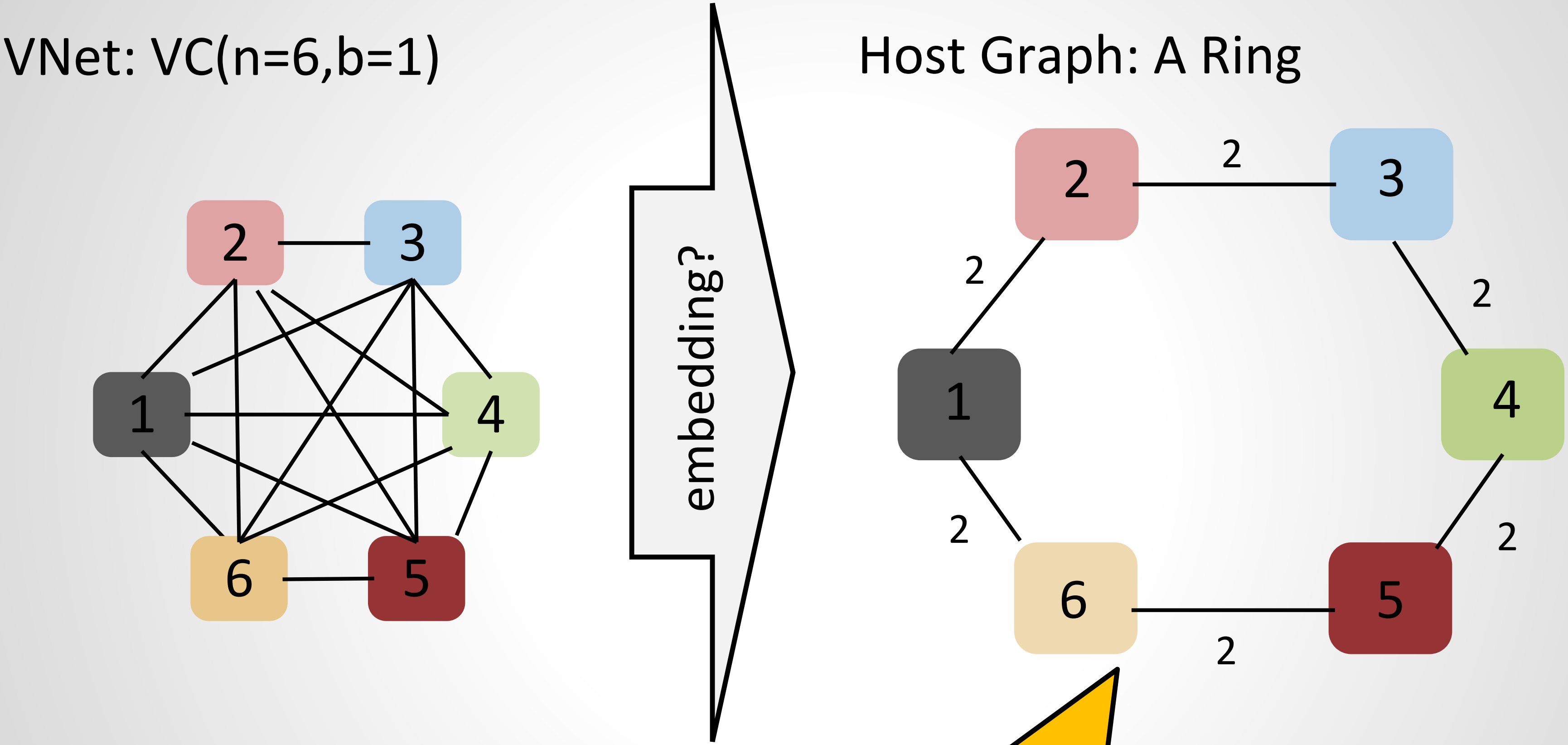
VNet: VC(n=6,b=1)

Host Graph: A Ring



How about hose?

The Benefit of Hose Interpretation



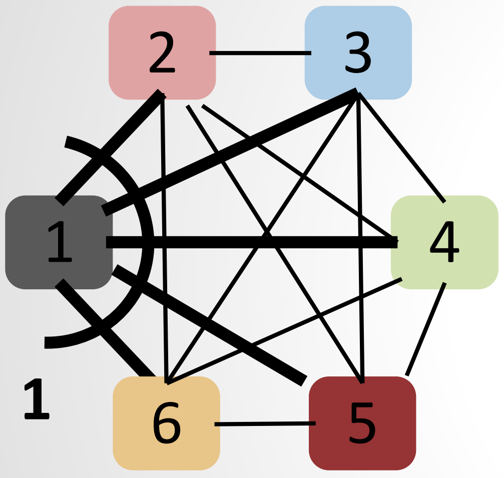
VNet: VC(n=6,b=1)

Host Graph: A Ring

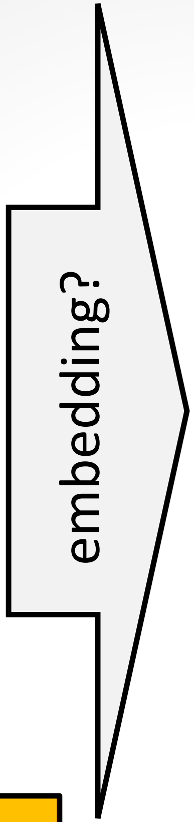
Node mapping! Now: How to embed these virtual links?

The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)



Recall: hose has total demand at most 1.



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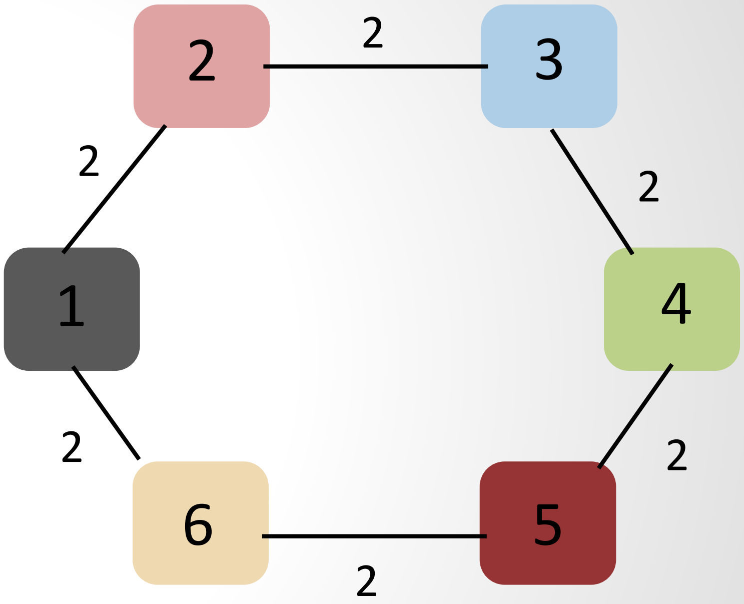

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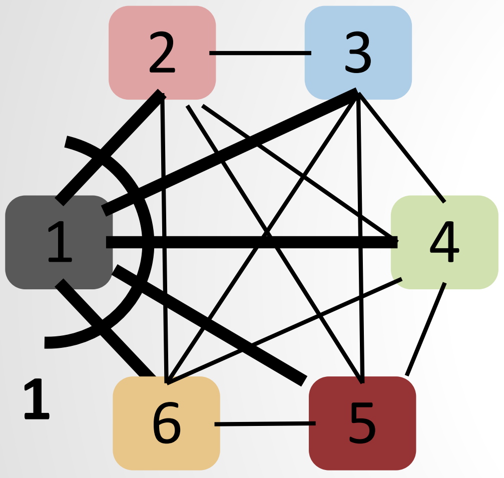

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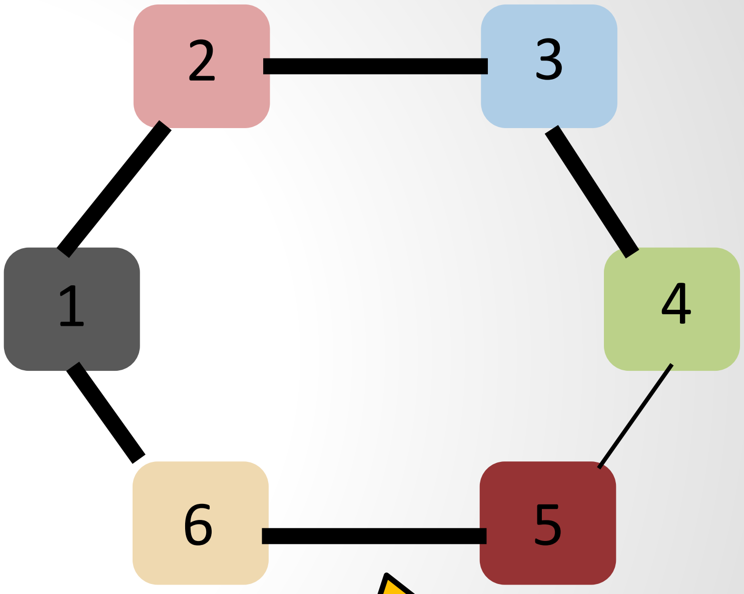
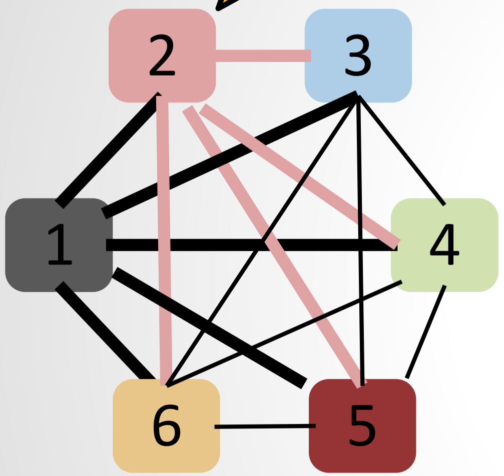
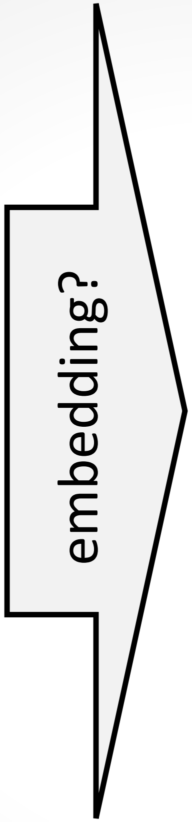


Virtual links from node 1 to {2,3,4,5,6} can be implemented along this route: fulfills capacity constraints under any traffic matrix fulfilling hose specification!

The Benefit of Hose Interpretation

Remaining virtual links to embed for virtual node 2.





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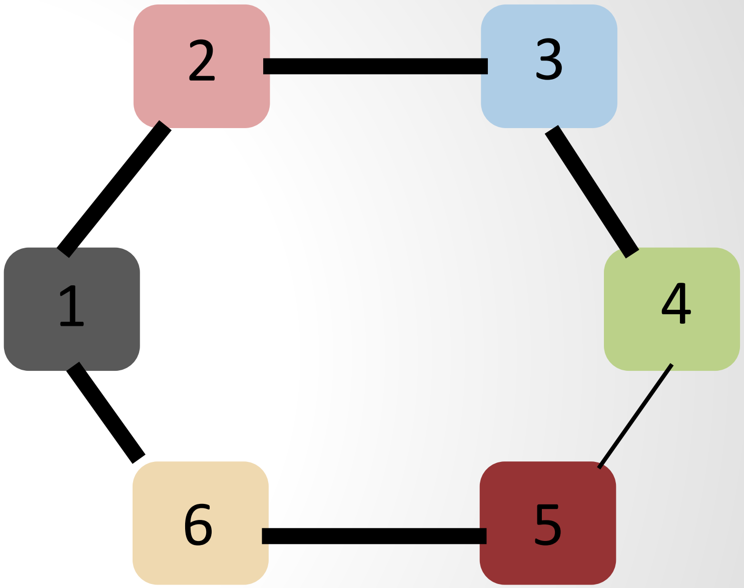

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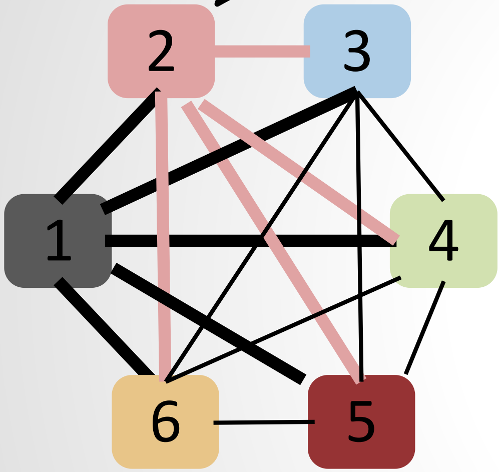
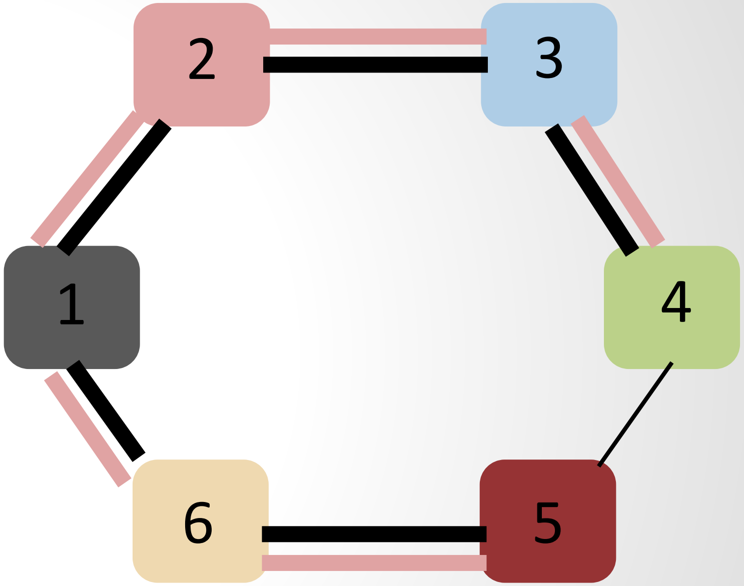


embedding?

Host Graph: A Ring



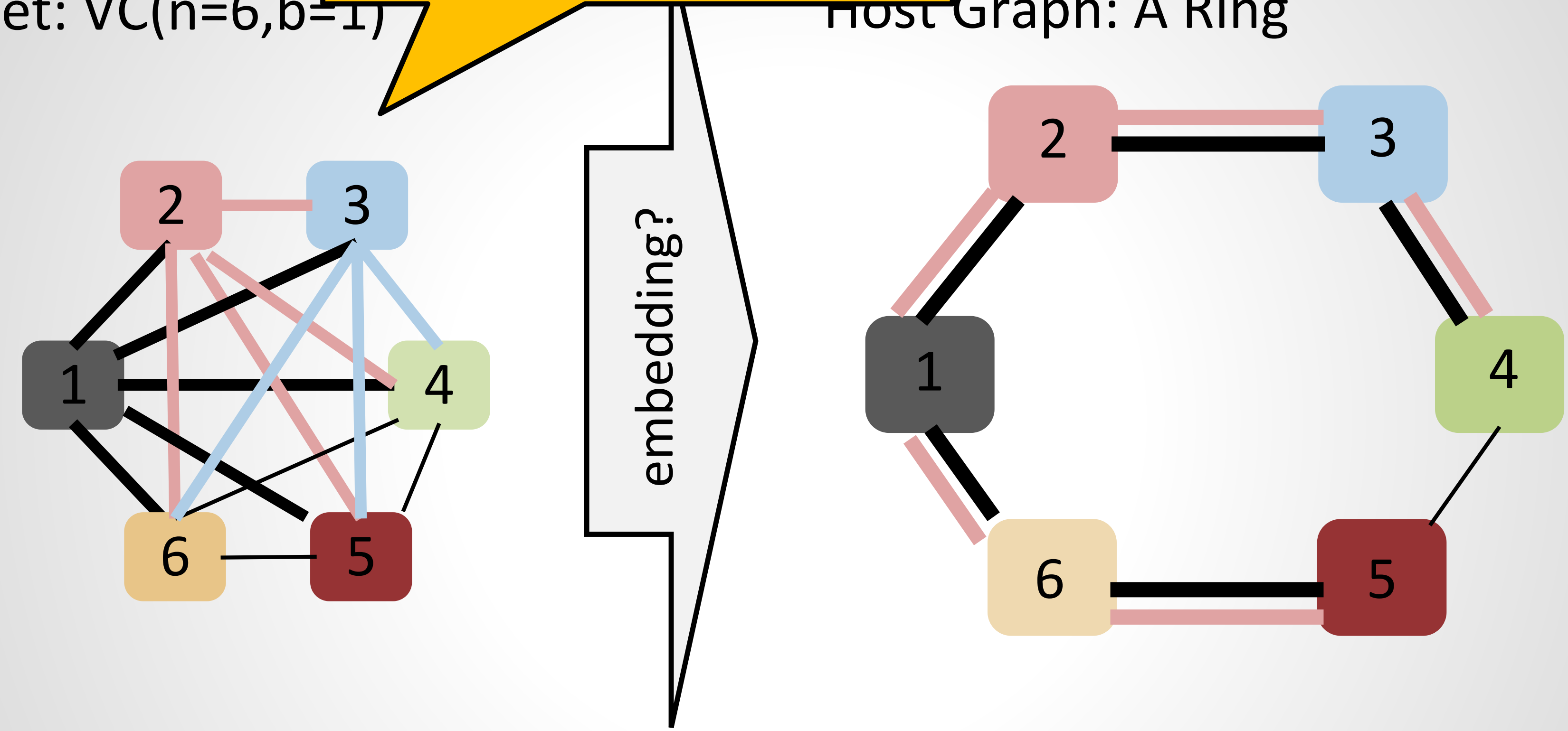
Can be implemented along this route: from node 2, reach nodes {3,4,5,6}.

The Benefit of Hose Interpretation

Remaining virtual links to embed for virtual node 3.

VNet: VC(n=6,b=1)

Host Graph: A Ring

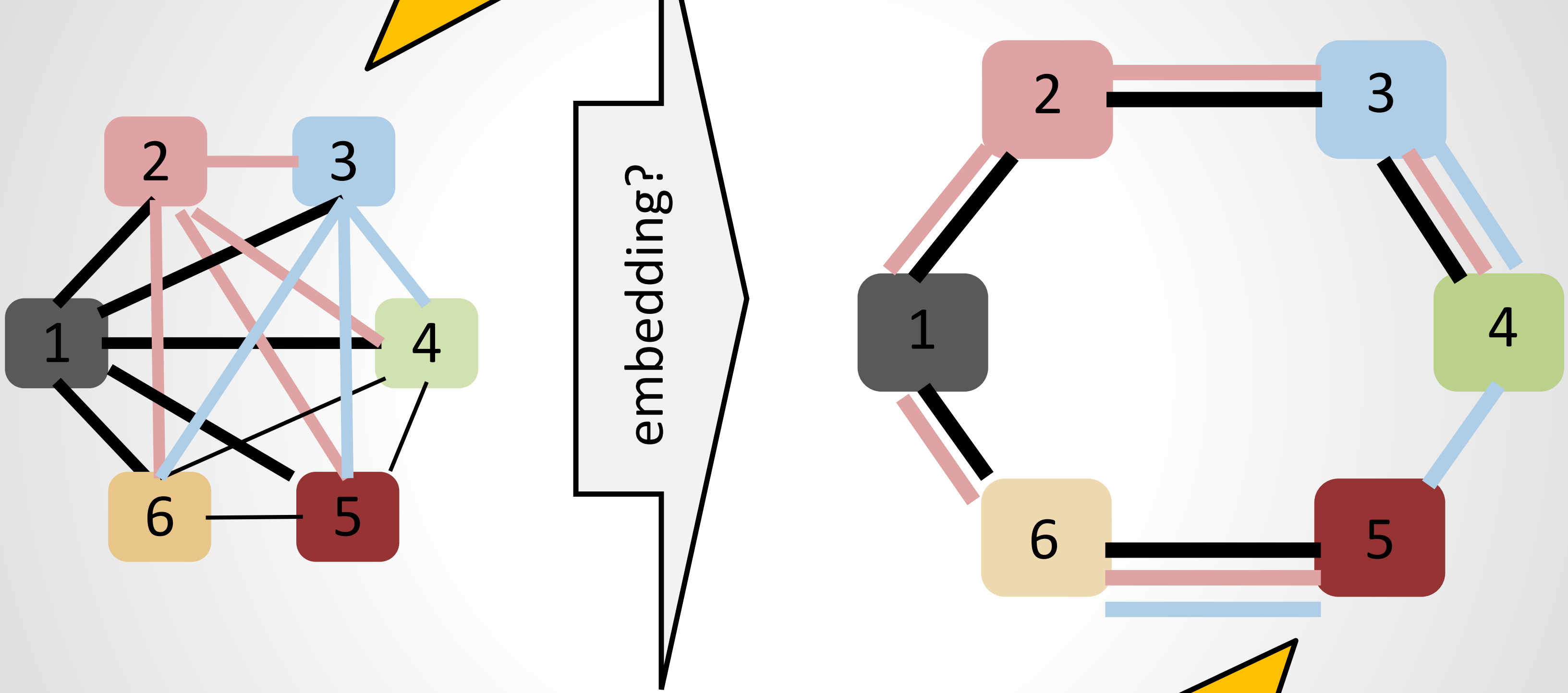


The Benefit of Hose Interpretation

Remaining virtual links to embed for virtual node 3.

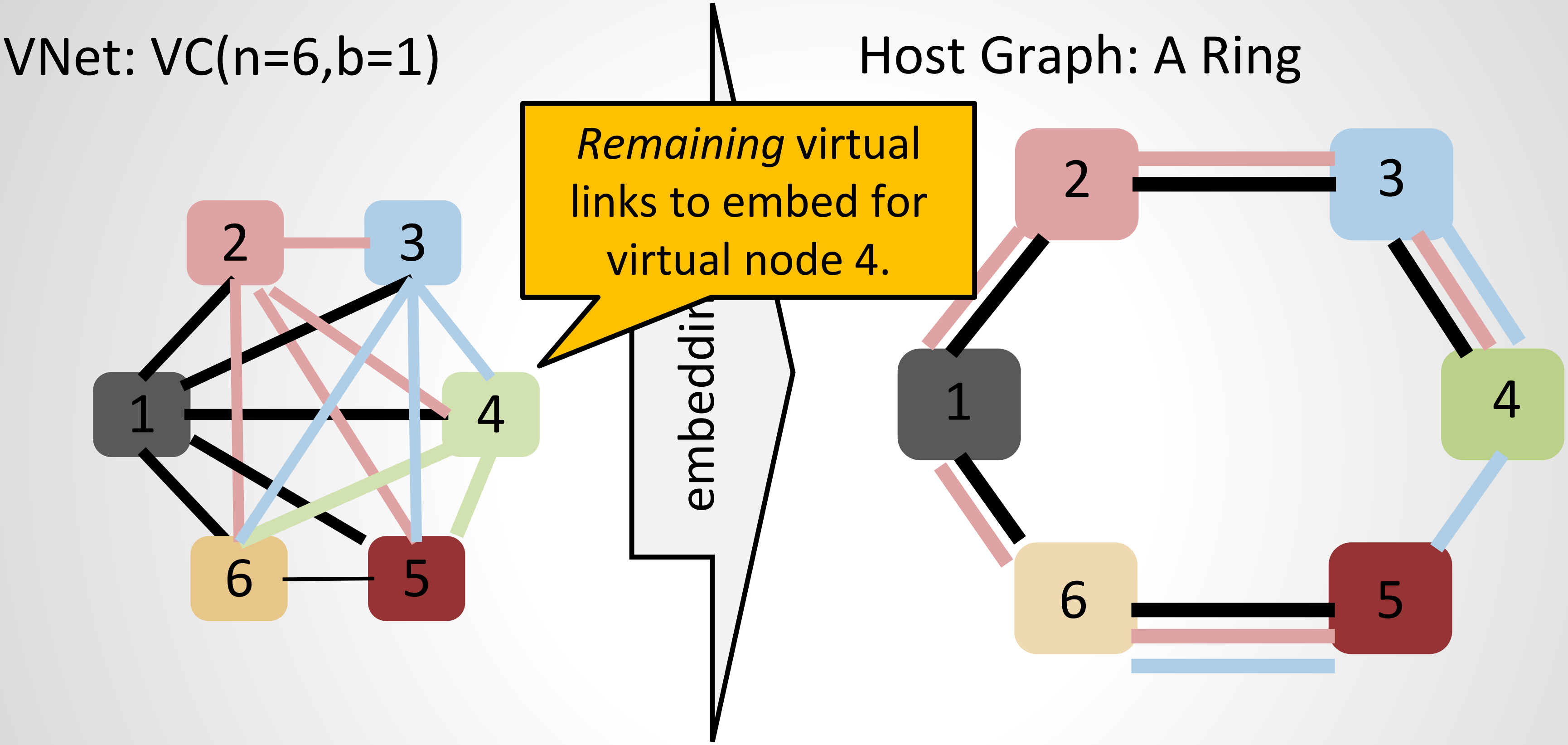
VNet: VC(n=6,b=1)

Host Graph: A Ring



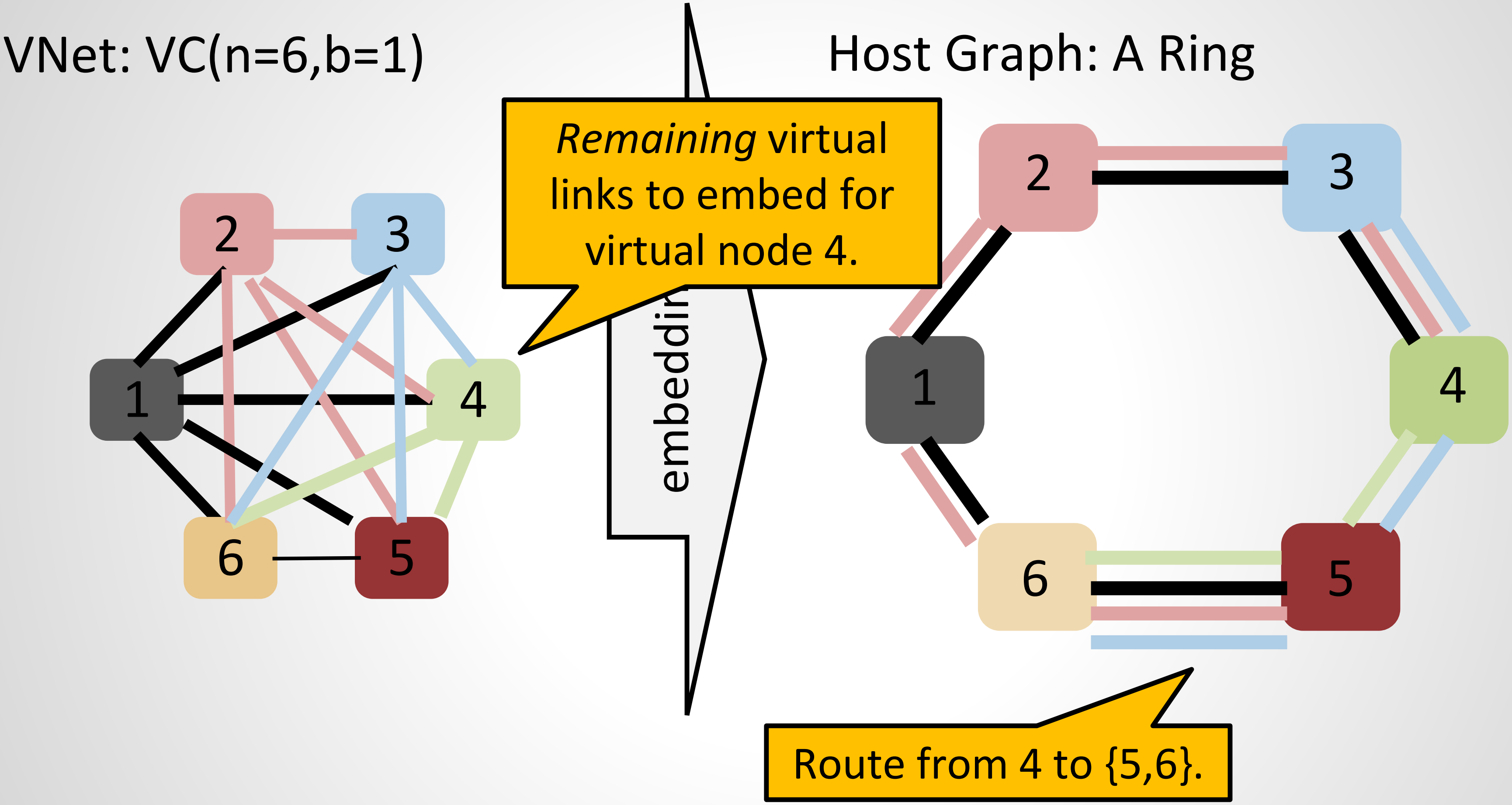
From 3, path reaches {4,5,6}.

The Benefit of Hose Interpretation



Remaining virtual links to embed for virtual node 4.

The Benefit of Hose Interpretation

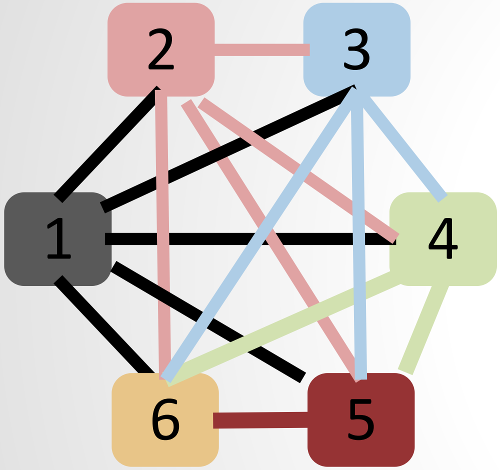


Remaining virtual links to embed for virtual node 4.

Route from 4 to {5,6}.

The Benefit of Hose Interpretation

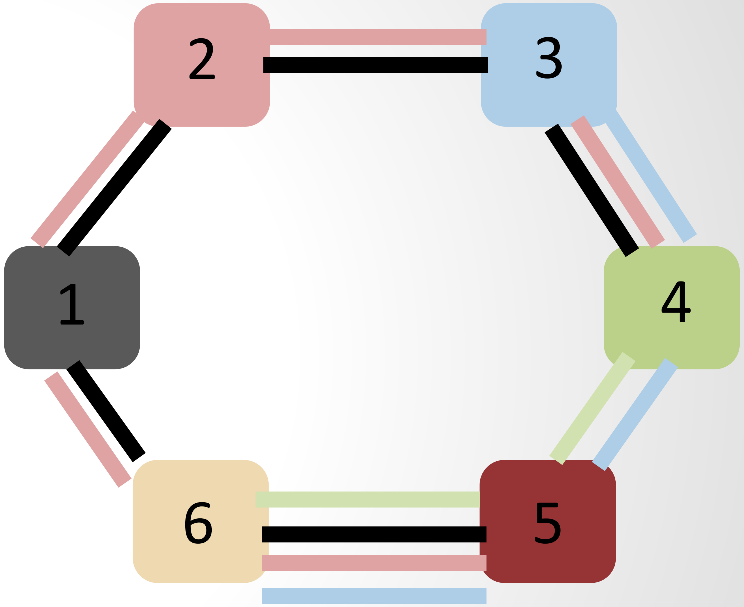
VNet: VC(n=6,b=1)



Remaining virtual link.

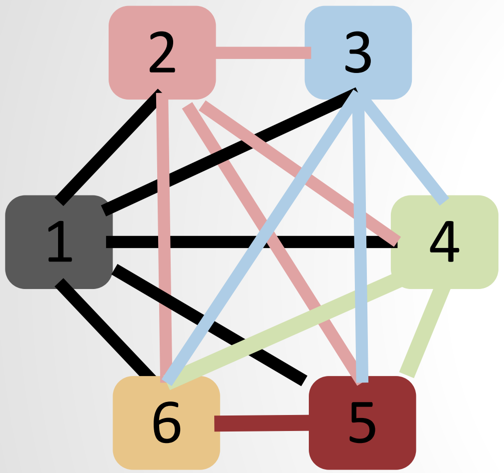
embedding?

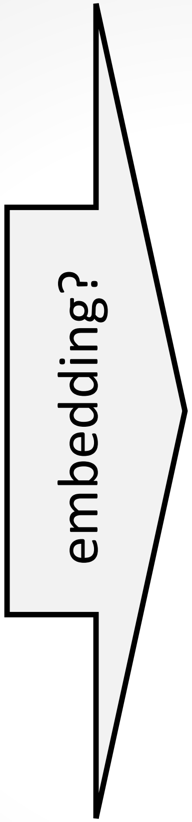
Host Graph: A Ring



The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)





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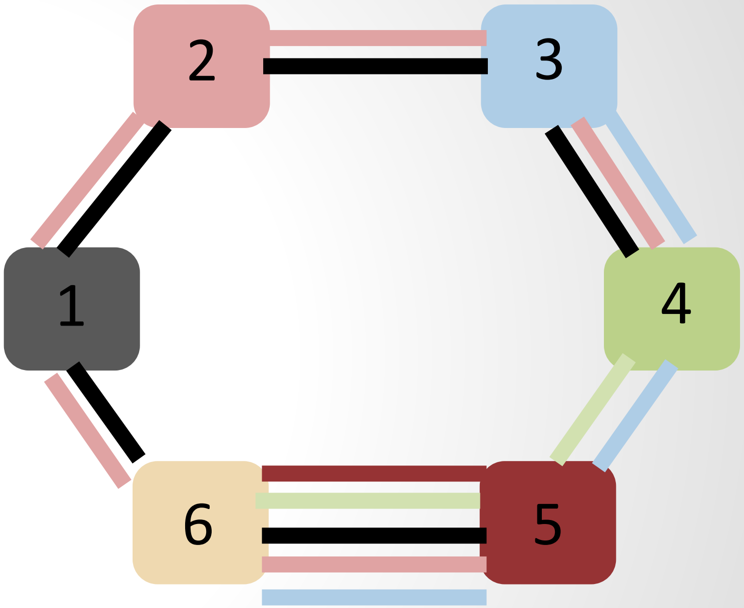
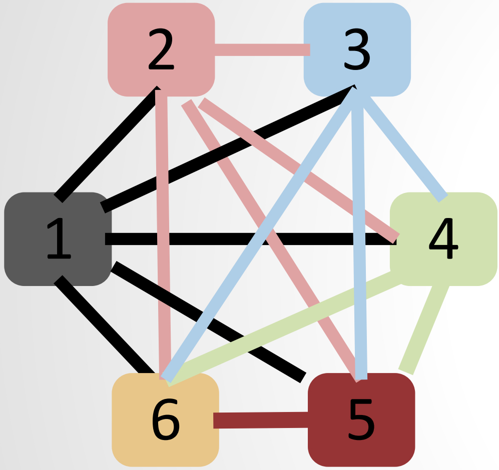
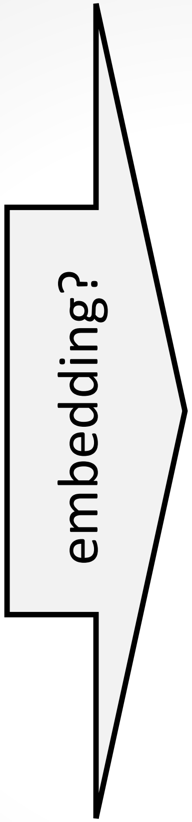


All vitual links mapped to routes!

The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)





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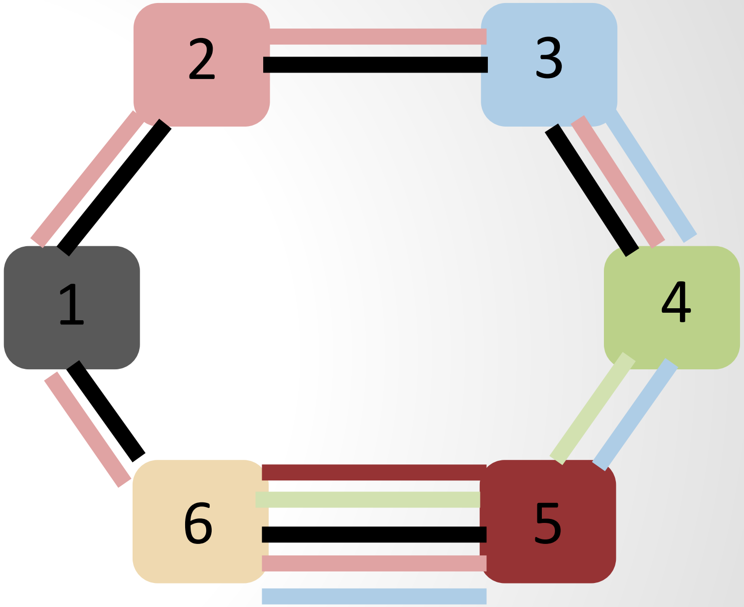
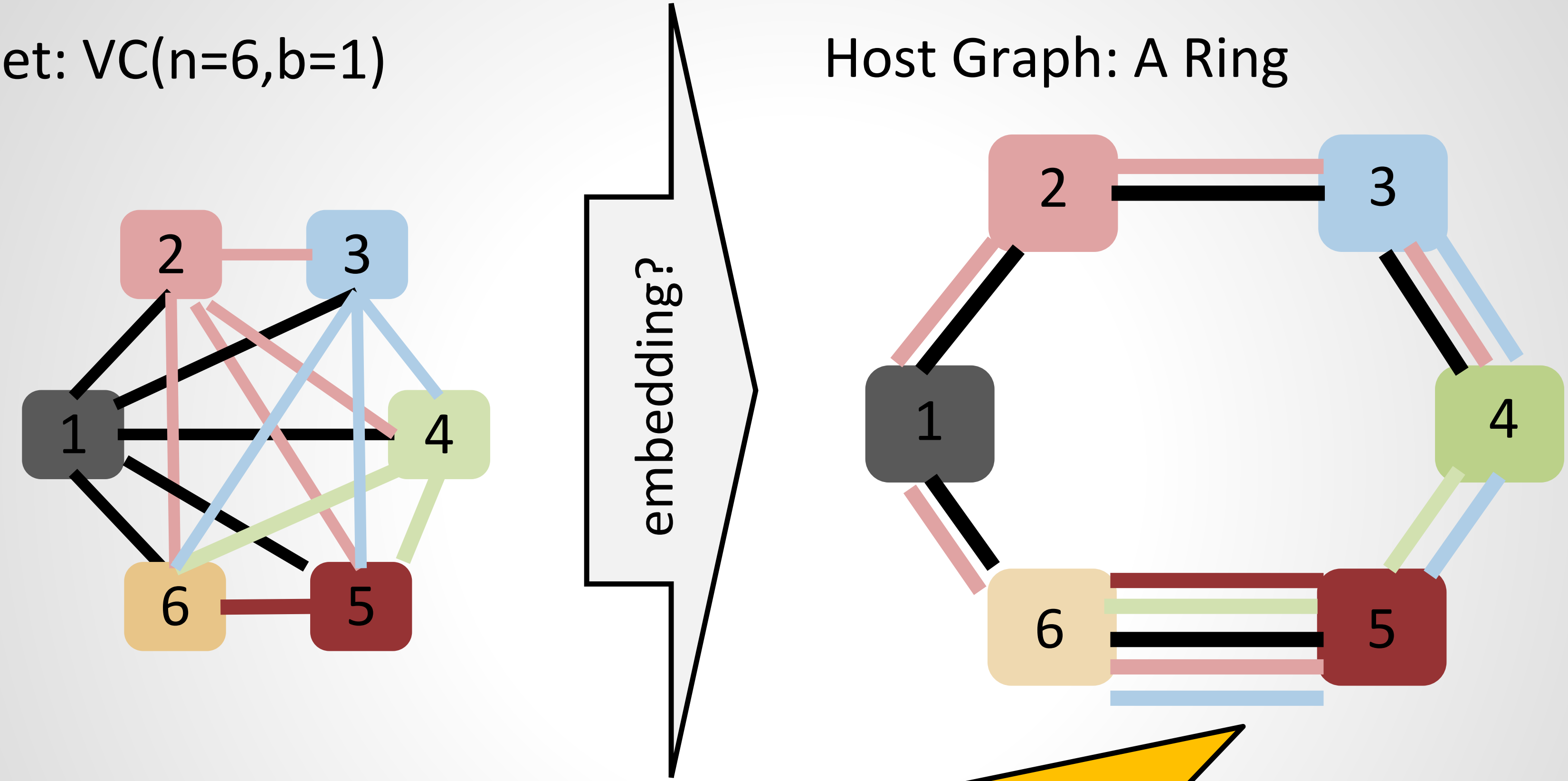


But wait: 5 paths on link {5,6}!
Can demand really be satisfied
given link capacity of 2?!

The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)

Host Graph: A Ring



Link {5,6} is used by routes: (1,5),(2,5),(3,6),(4,6),(5,6).

But by definition of the hose model, any traffic matrix M will respect:

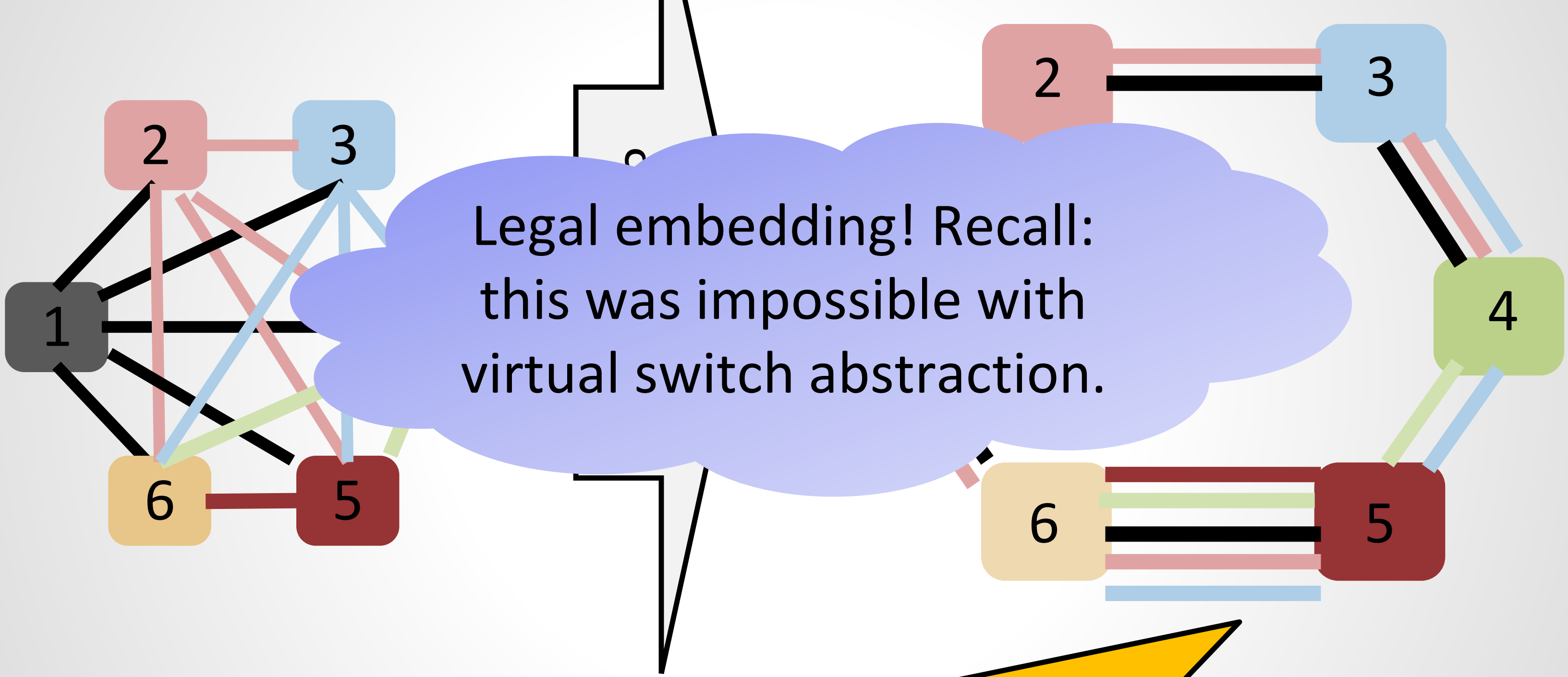
$M_{1,5}+M_{2,5} \leq 1$ *and* $M_{3,6}+M_{4,6}+M_{5,6} \leq 1$.

Hence $\Sigma\ M_{i,j} \leq 2$ *holds!*

The Benefit of Hose Interpretation

VNet: VC(n=6,b=1)

Host Graph: A Ring



Legal embedding! Recall: this was impossible with virtual switch abstraction.

Link {5,6} is used by routes: (1,5),(2,5),(3,6),(4,6),(5,6).

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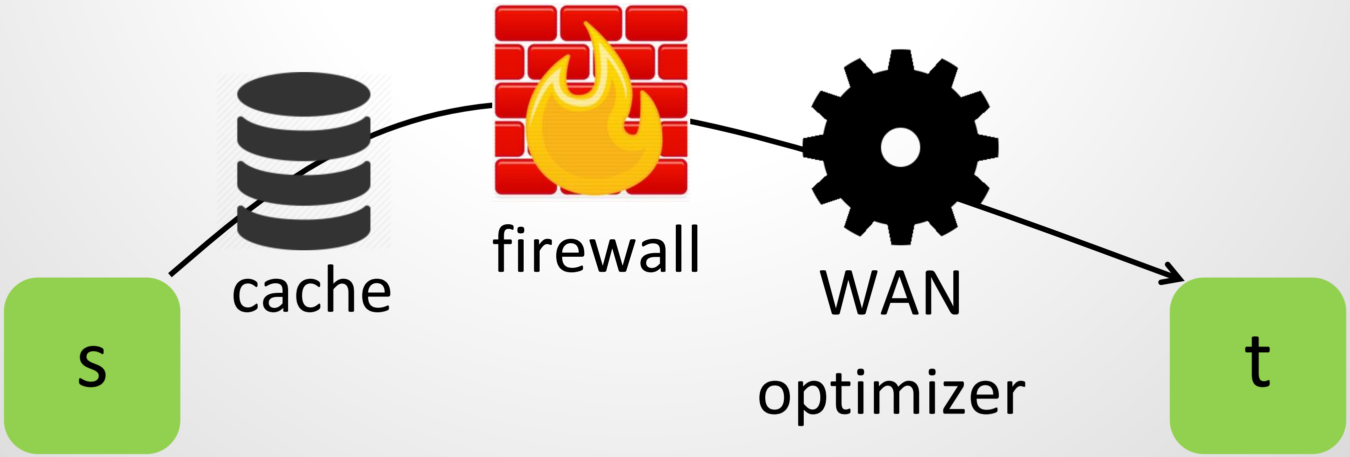
The Many Faces of the VNEP:
E.g., Service Chain Embedding

- ❑ Similar problems arise in many contexts

- ❑ For example, service chain embeddings: in a service chain, traffic is steered (e.g., using SDN) through a sequence of (virtualized) middleboxes to compose a more complex network service



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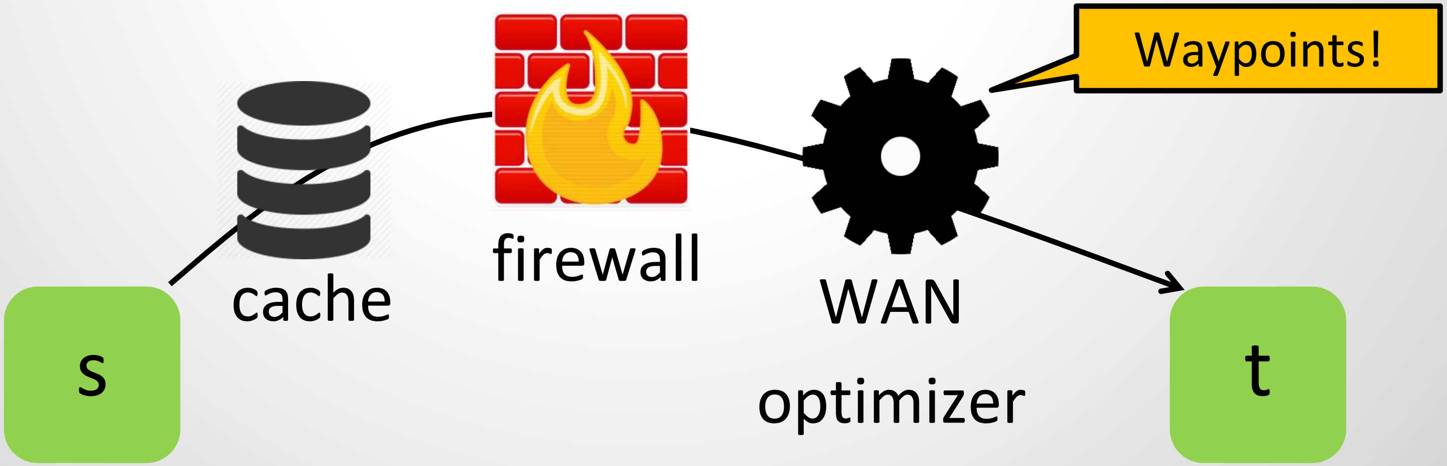

The Many Faces of the VNEP: E.g., Service Chain Embedding

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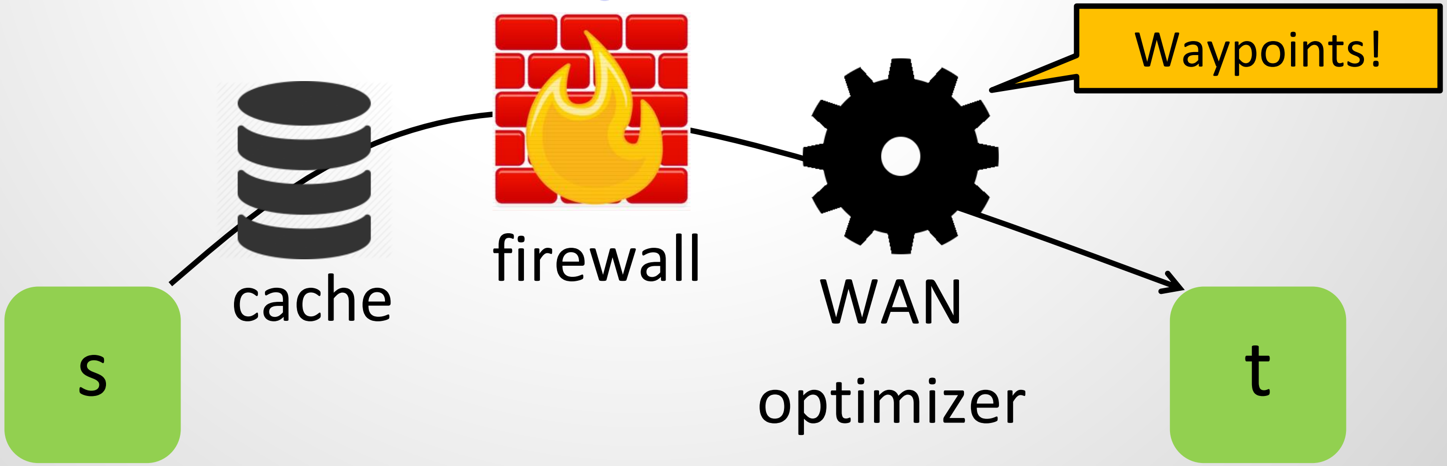
to compose a

more complex ervice

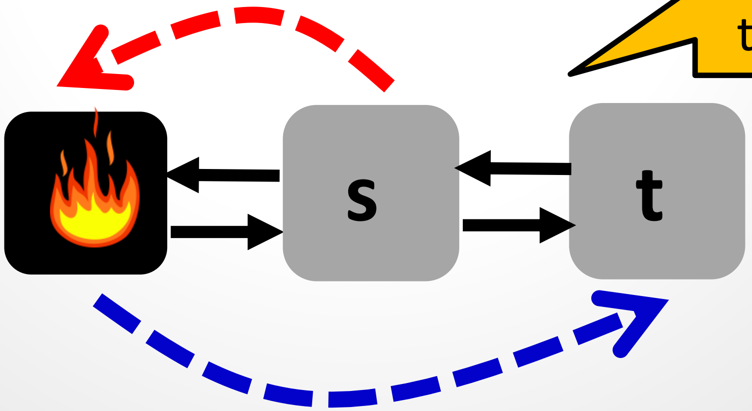
Interesting implication: routes from s to t become *walks* (rather than simple paths)! How to find shortest walks?



Routing Through Waypoints

❑ Traditionally: routes form simple paths (e.g., shortest paths)

❑ Novel aspect: routing through middleboxes may require more general paths, with loops: a walk

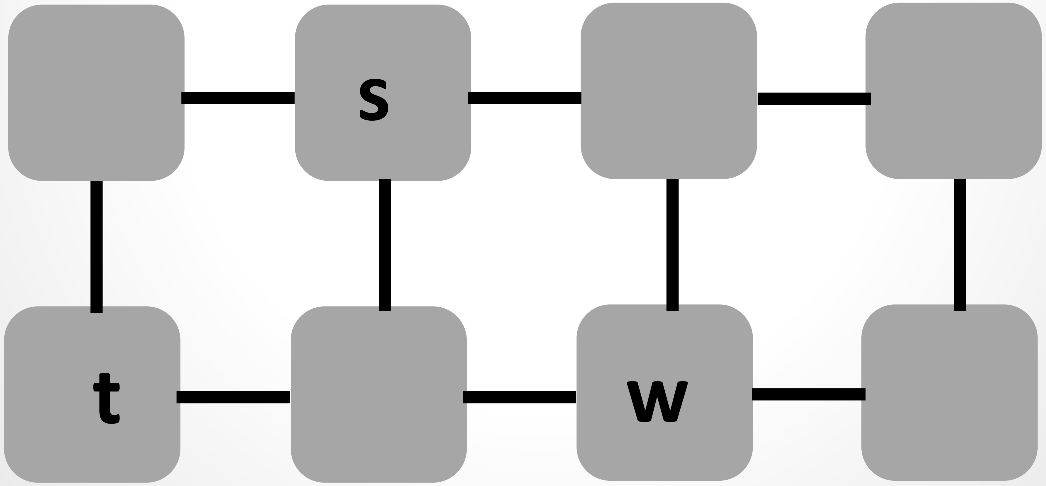


How to compute a shortest route through a waypoint?

Walking Through Waypoints

- Computing shortest routes through waypoints is non-trivial!

Assume unit capacity and demand for simplicity!



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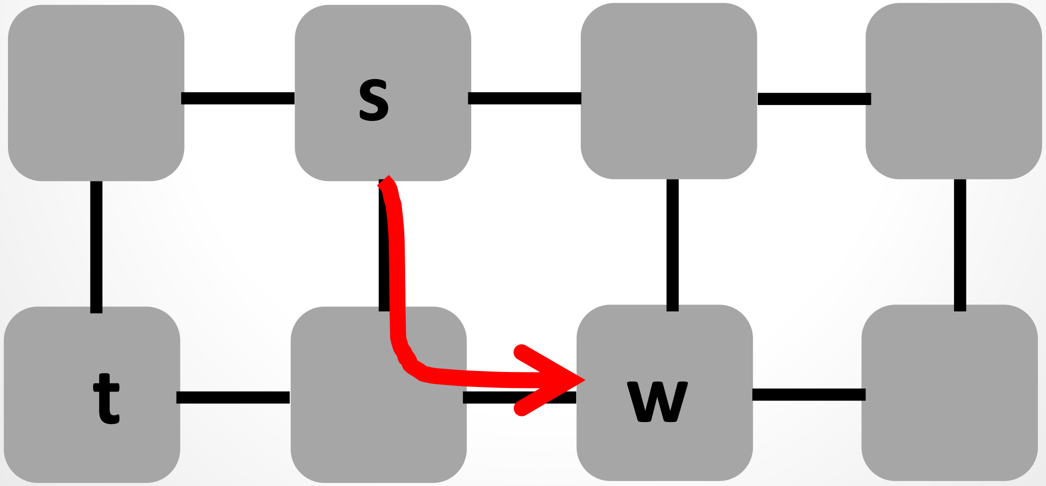

Greedy fails: **choose shortest path from s to w…**

Walking Through Waypoints

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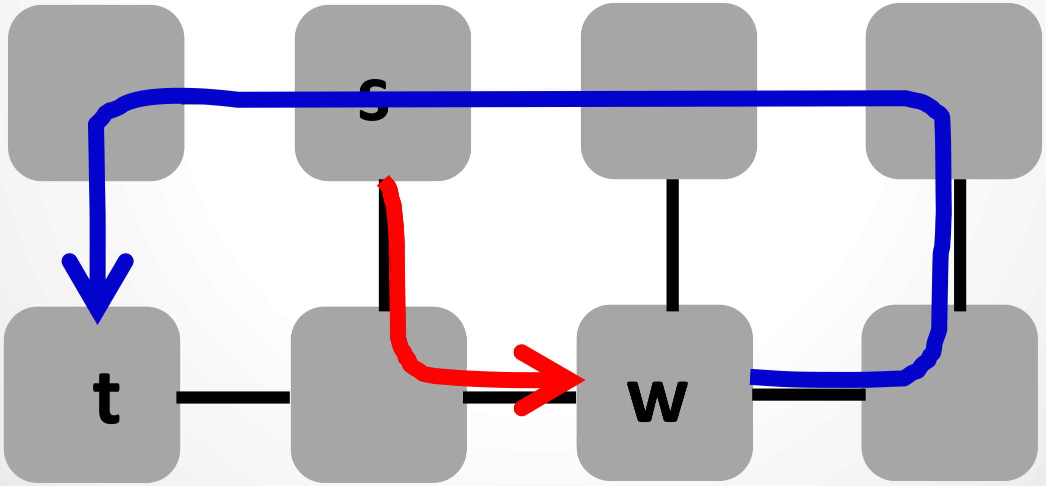

Greedy fails: **… now need long path from w to t**

Walking Through Waypoints

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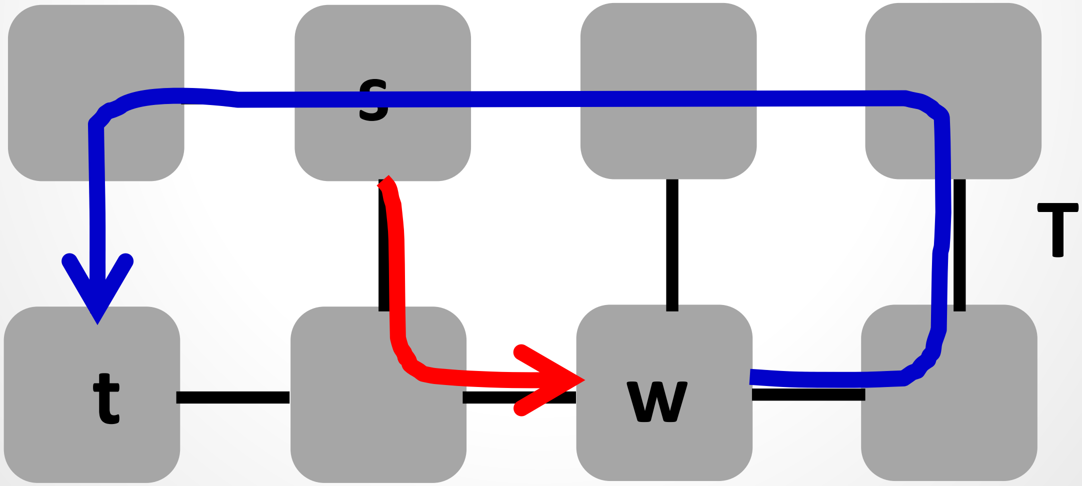

Total length: 2+6=8

Greedy fails: **… now need long path from w to t**

Walking Through Waypoints

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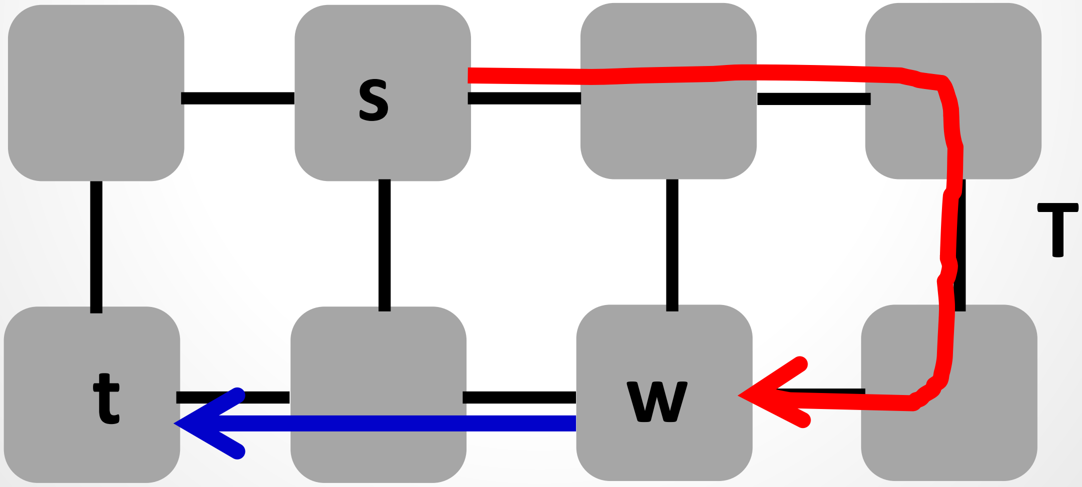

Total length: 4+2=6

A *better solution*: *jointly* optimize the two segments!

Walking Through Waypoints

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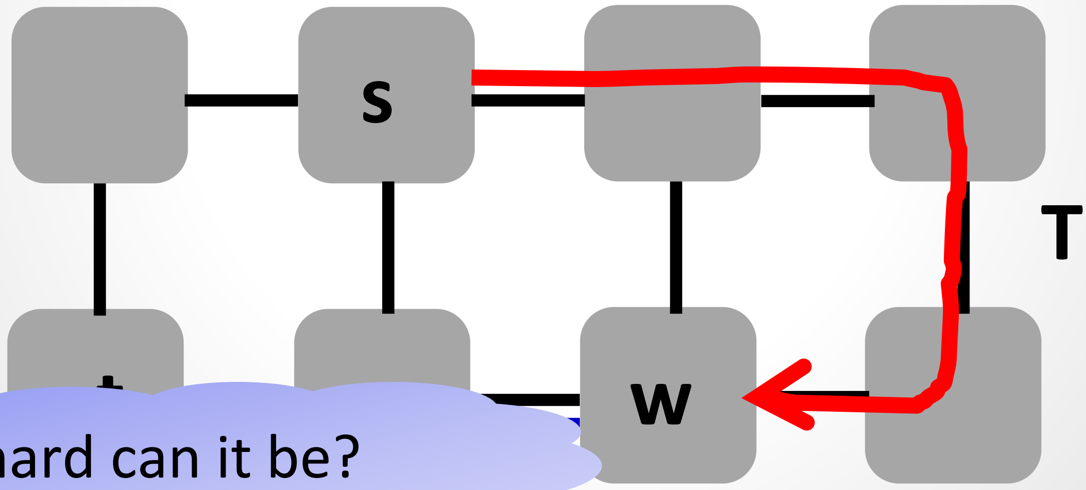


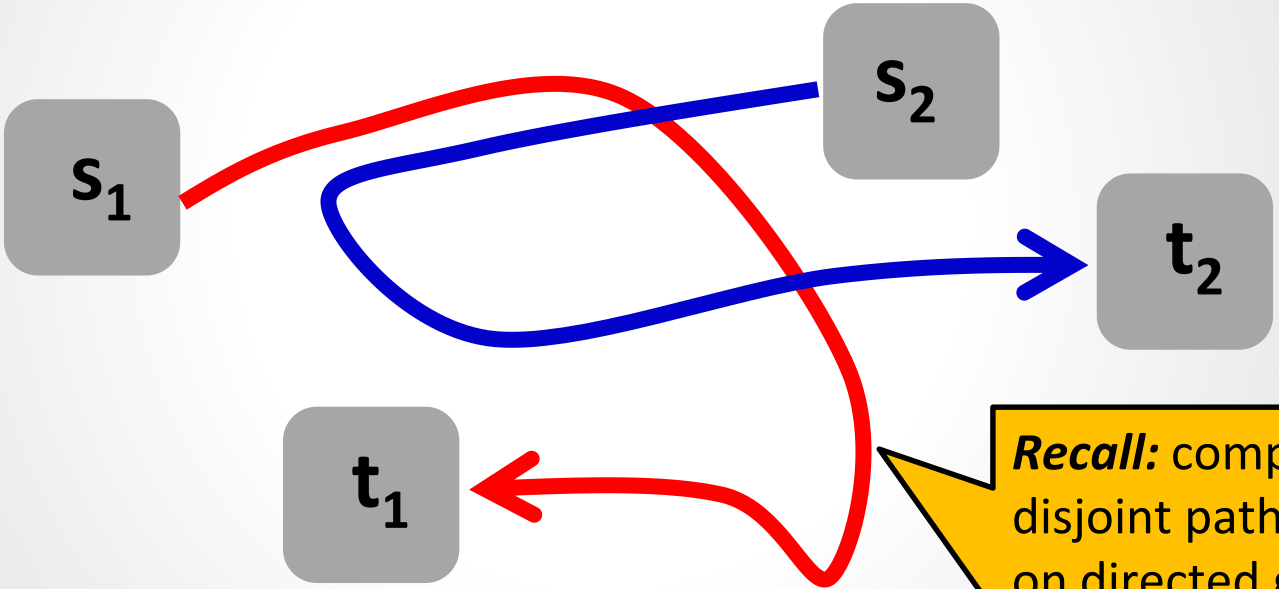
Total length: 4+2=6

How hard can it be?

A ***better solution***: *jointly* optimize the two segments!

NP-hard on Directed Networks: Reduction from Disjoint Paths Problem

Reduction: *From* joint shortest paths $(s_1,t_1),(s_2,t_2)$ *to* shortest walk (s,w,t) problem

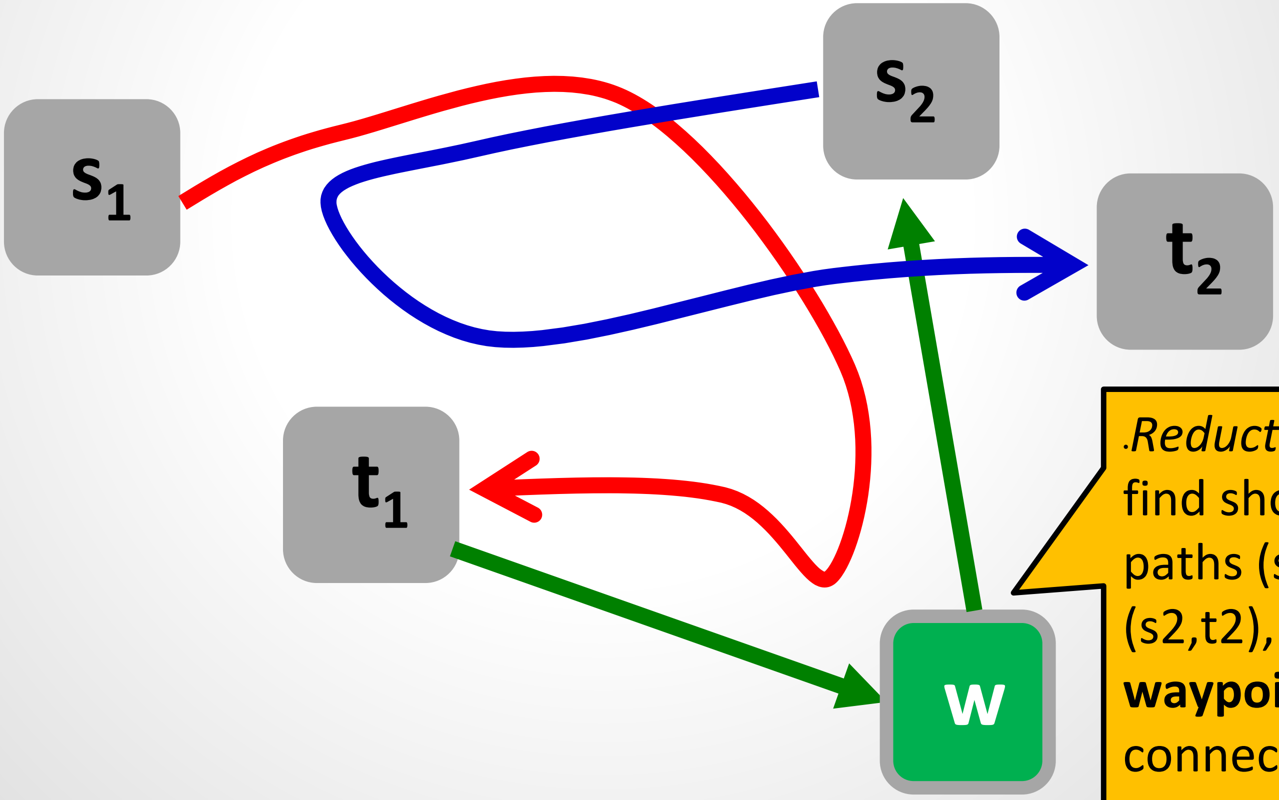


Recall: computing 2-disjoint paths NP-hard on directed graphs.

We show: If waypoint routing was be in P, we could solve it fast. ***Contradiction!***

NP-hard on Directed Networks: Reduction from Disjoint Paths Problem

Reduction: *From* joint shortest paths $(s_1,t_1),(s_2,t_2)$ *to* shortest walk (s,w,t) problem



.*Reduction:* To find shortest paths (s1,t1), (s2,t2), **introduce waypoint w** and connect t_1 to s_2 via w….

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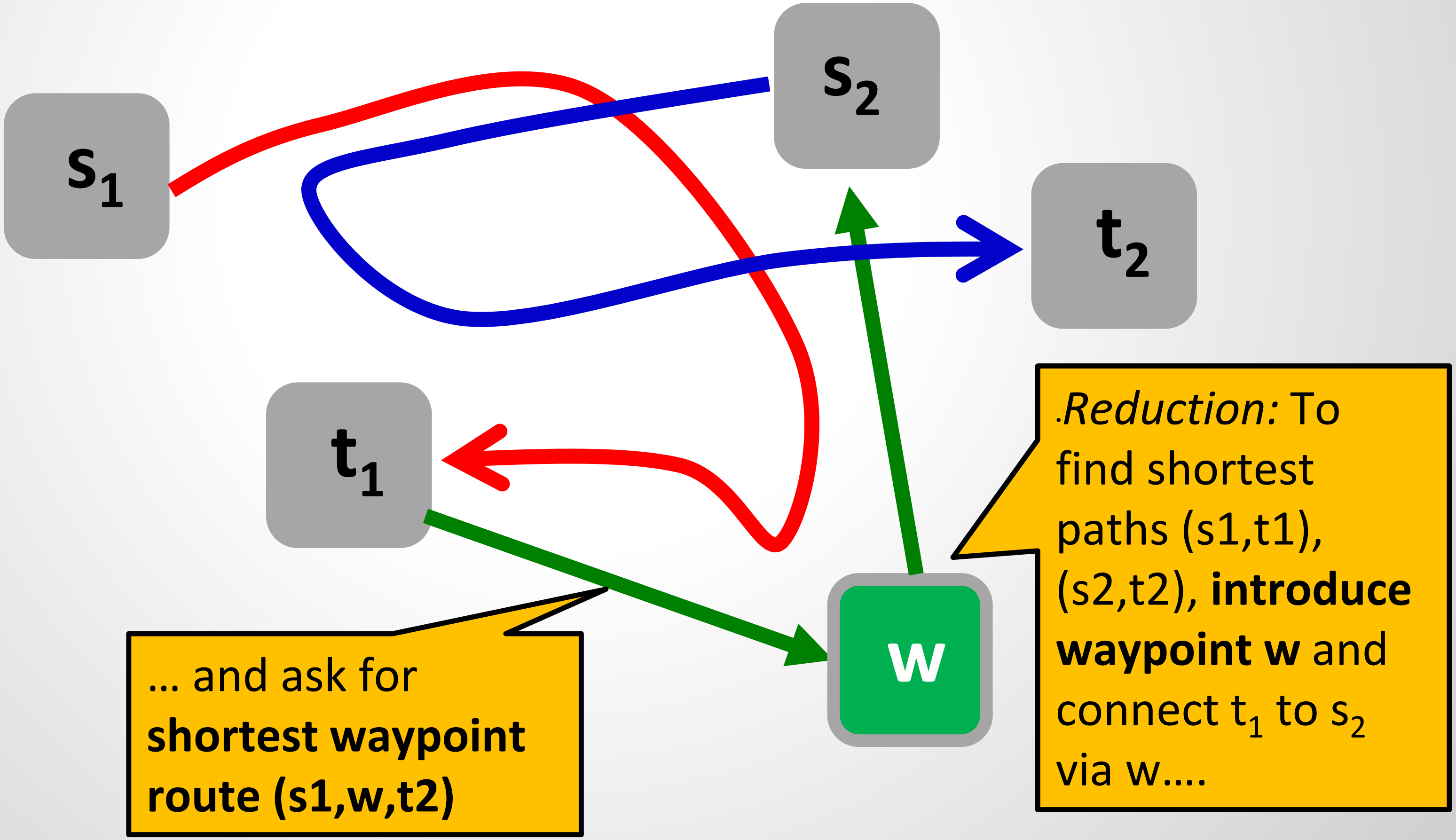

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… and ask for **shortest waypoint route (s1,w,t2)**

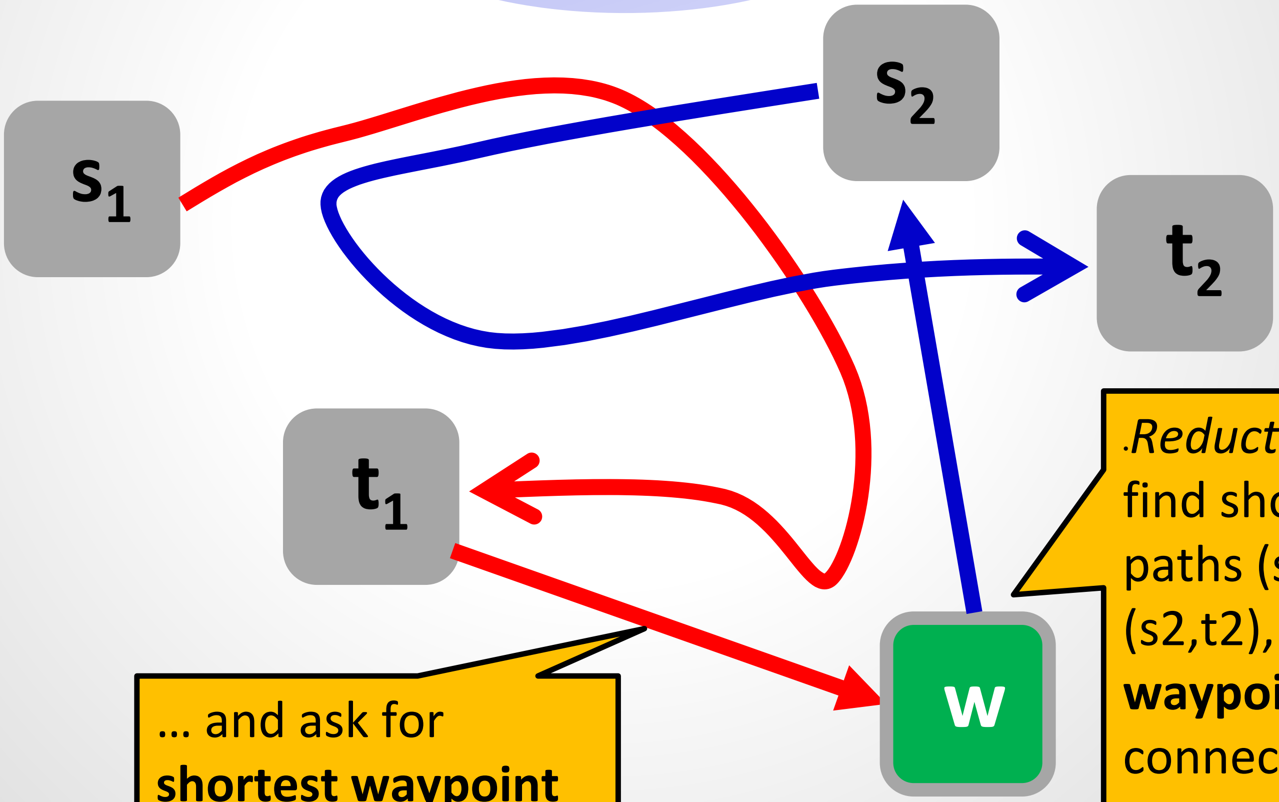
NP-hard on Direct Netw

F

Reduc

to shortest w

The walk (s_1,w,t_2) walk defines a (s_1,t_1) and a (s_2,t_2) path pair before/after the waypoint! Solves original problem: Contradiction!



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What about waypoint routes on *undirected* networks?

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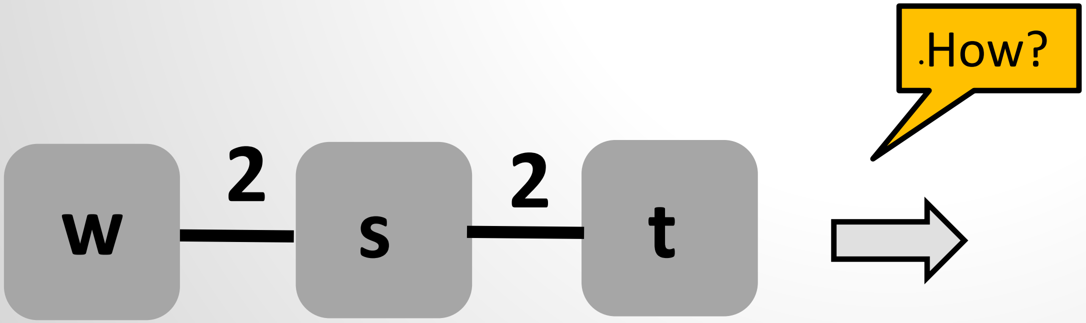
❑ Indeed, algorithm exists: We can reduce to edge-disjoint paths to compute a waypoint route!



Walks

Edge-Disjoint Paths

What about waypoint routes on *undirected* networks?

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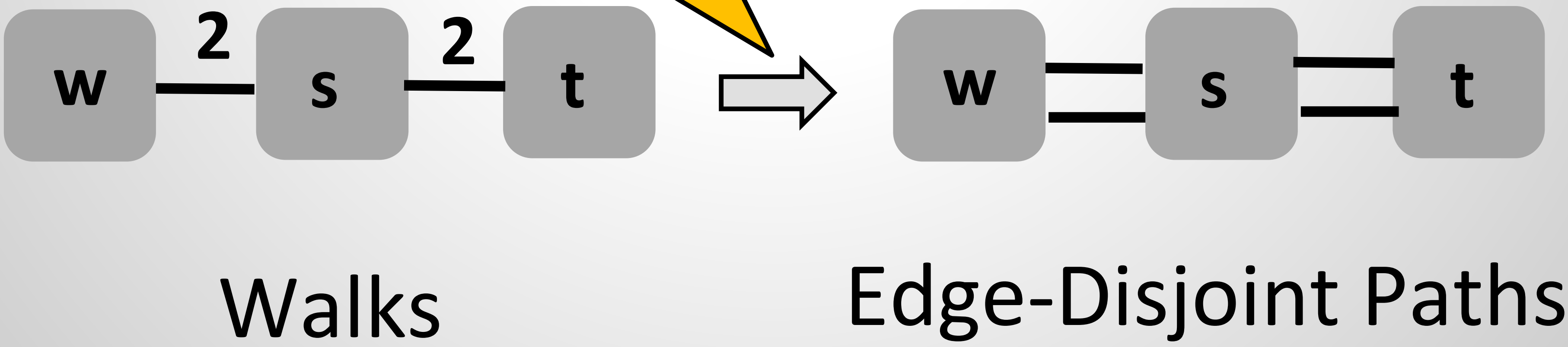
.Replace capacitated links with parallel links!



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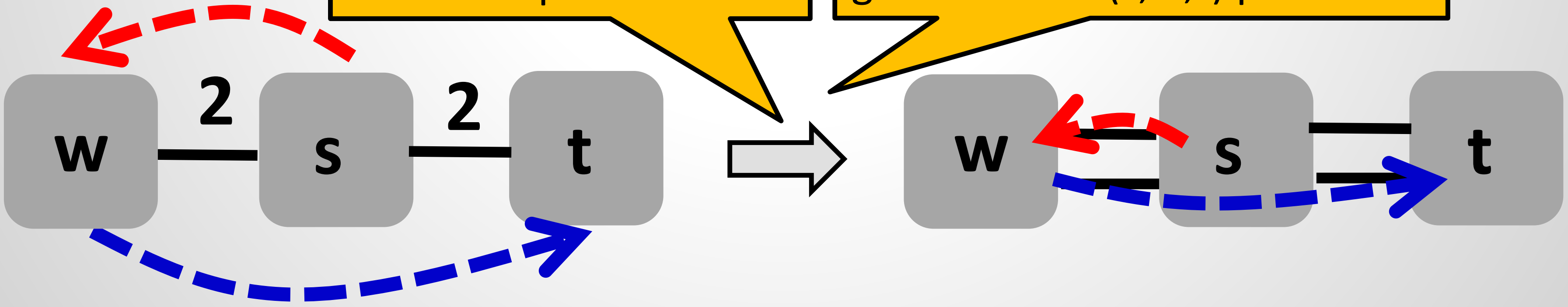


Replace capacitated links with parallel links!

Shortest paths (s,w),(w,t) will give shortest (s,w,t) path!

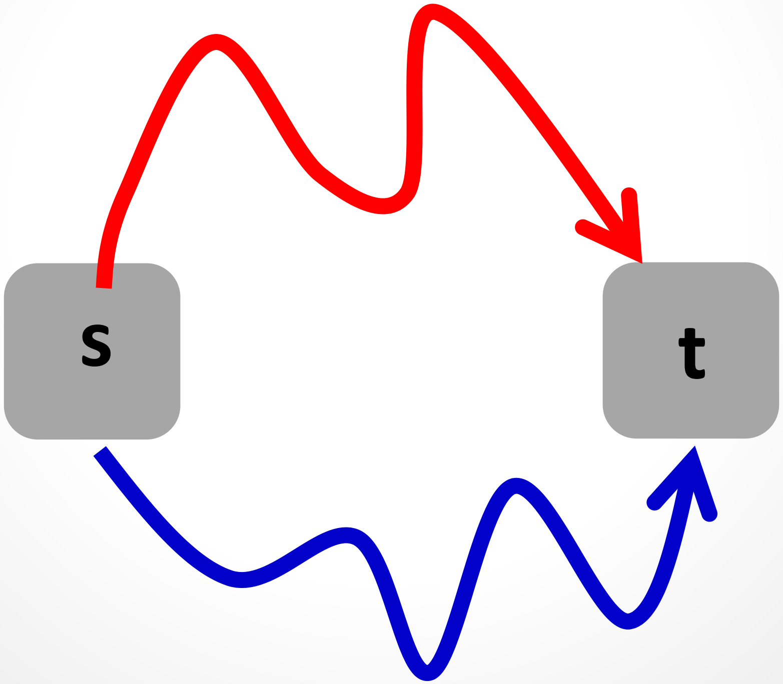
Walks

Edge-Disjoint Paths

Fast and Shortest Waypoint Routing on Undirected Networks: Suurballe’s Algorithm

Fast and Shortest Waypoint Routing on Undirected Networks: Suurballe's Algorithm

- ❑ Suurballe's algorithm: finds two (edge-)disjoint shortest paths between same endpoints:



Fast and Shortest Waypoint Routing on Undirected Networks: Suurballe’s Algorithm

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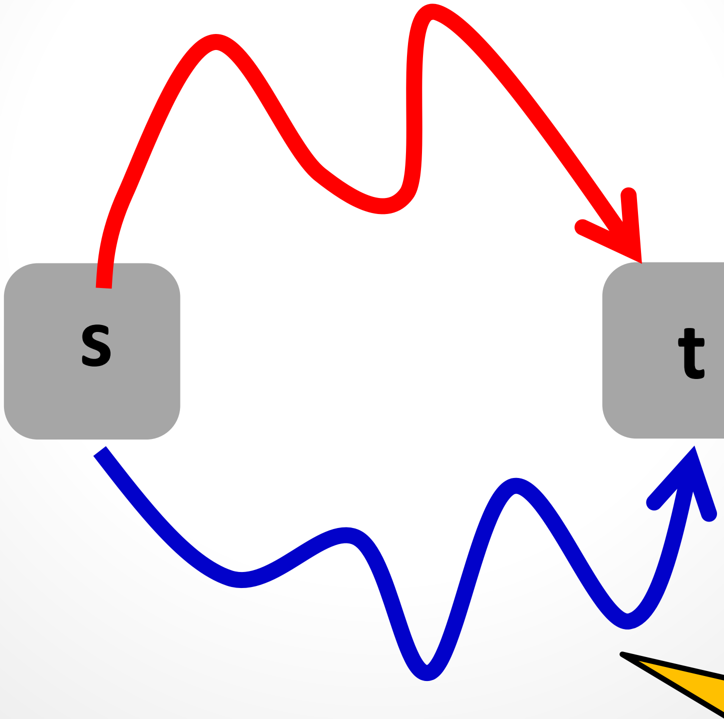
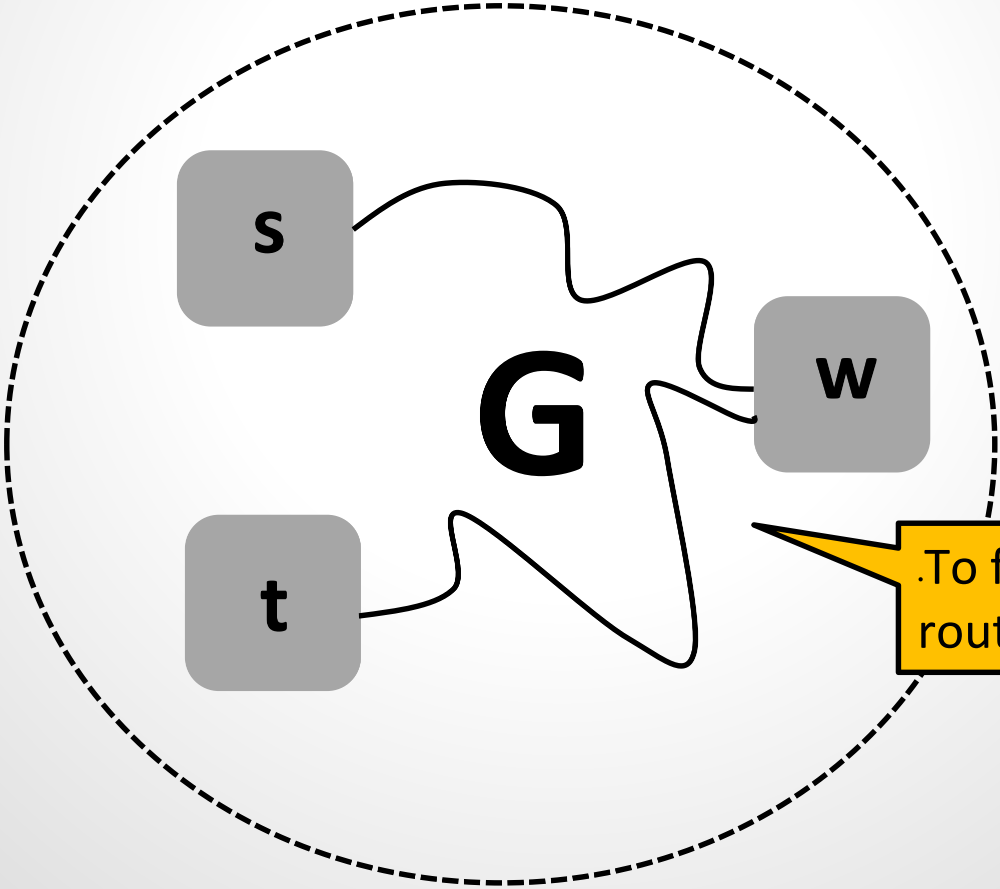


.How to compute a shortest (s,w,t) route with this algorithm??

Waypoint Routing on Steroids

❑ Reduction to Suurballe’s algorithm:



.To find shortest (s,w,t) route…

Waypoint Routing on Steroids

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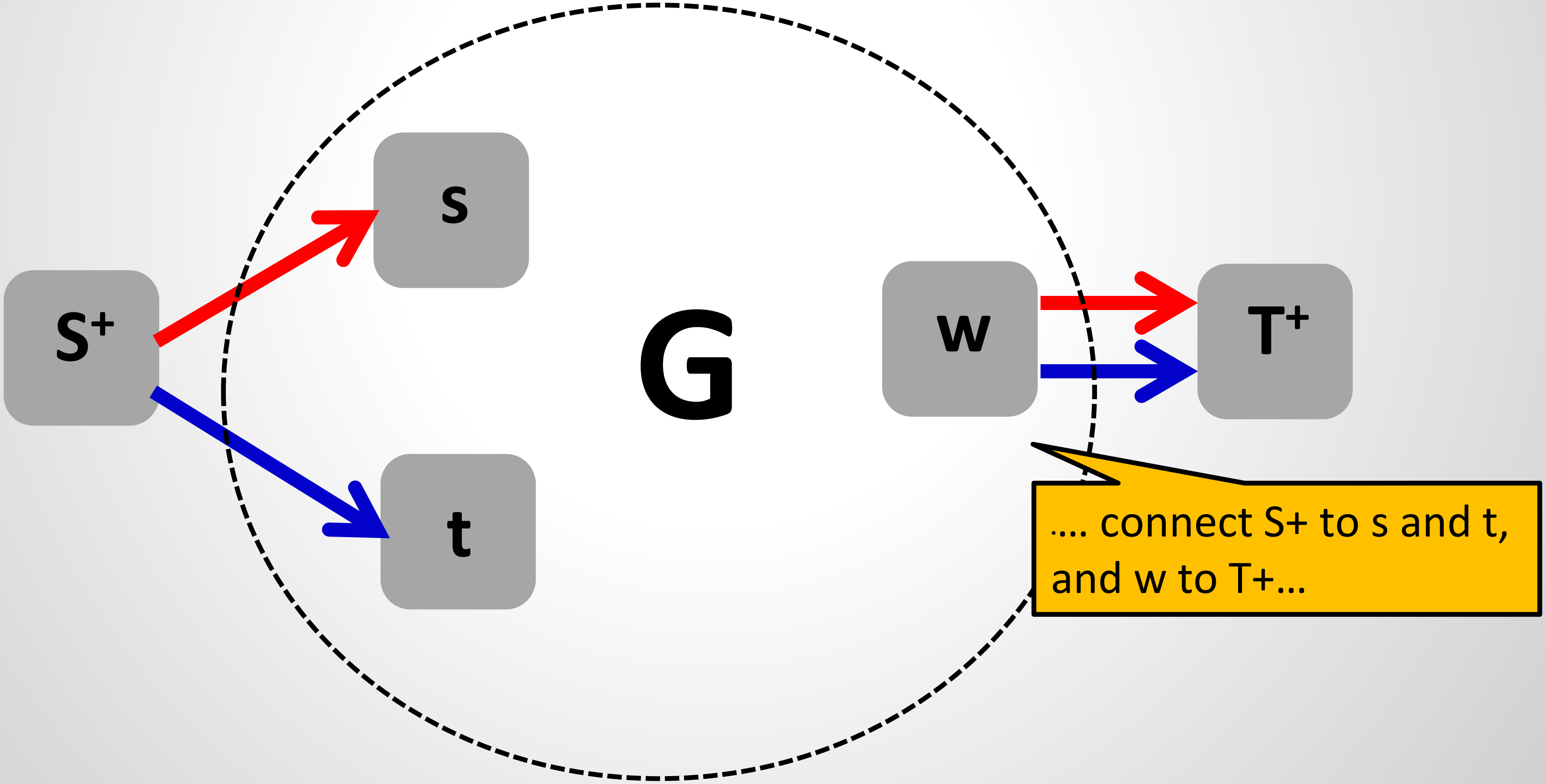
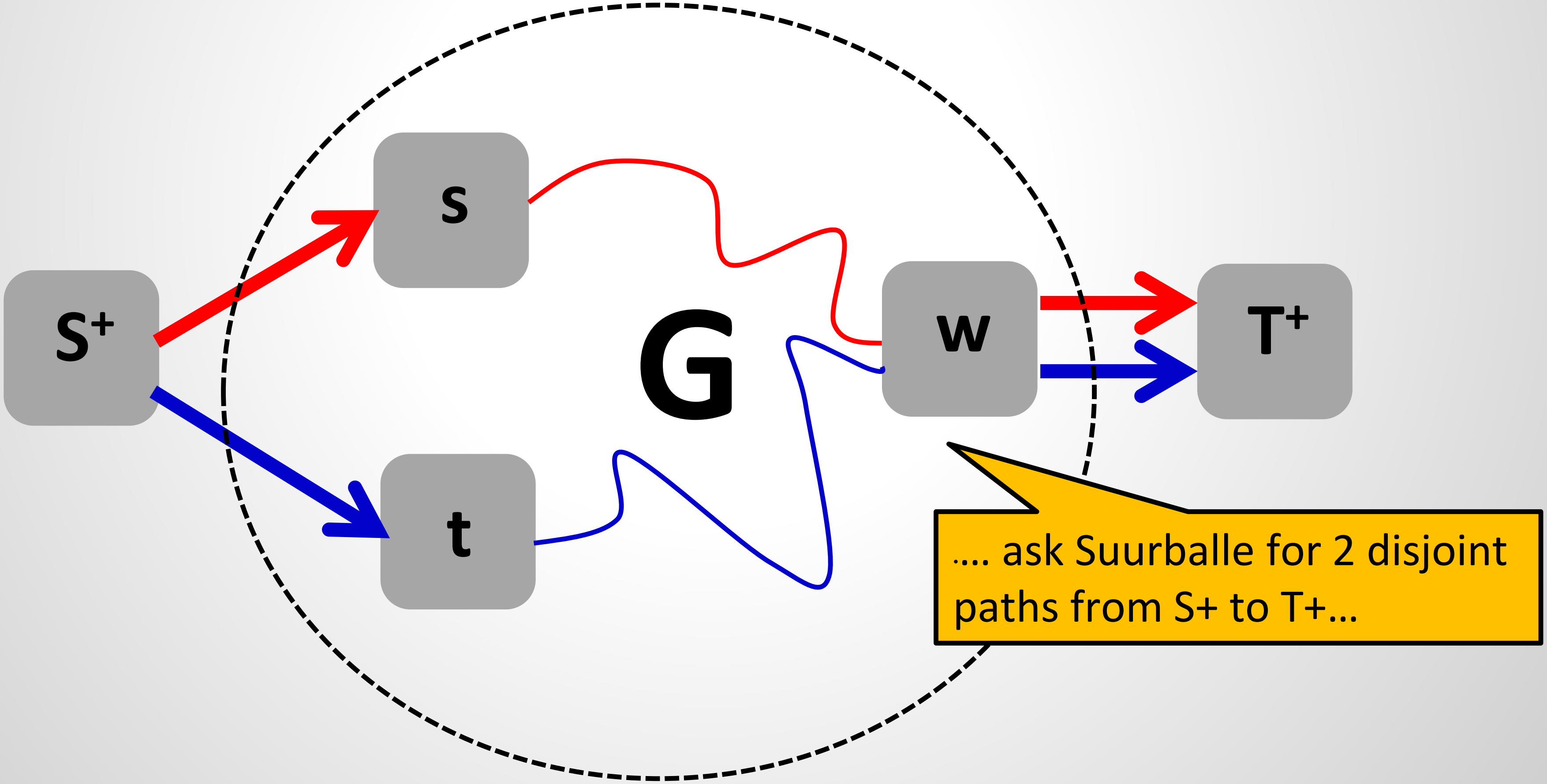


…. connect S+ to s and t, and w to T+…

Waypoint Routing on Steroids

❑ Reduction to Suurballe's algorithm:



…. ask Suurballe for 2 disjoint paths from S+ to T+…

Waypoint Routing on Steroids

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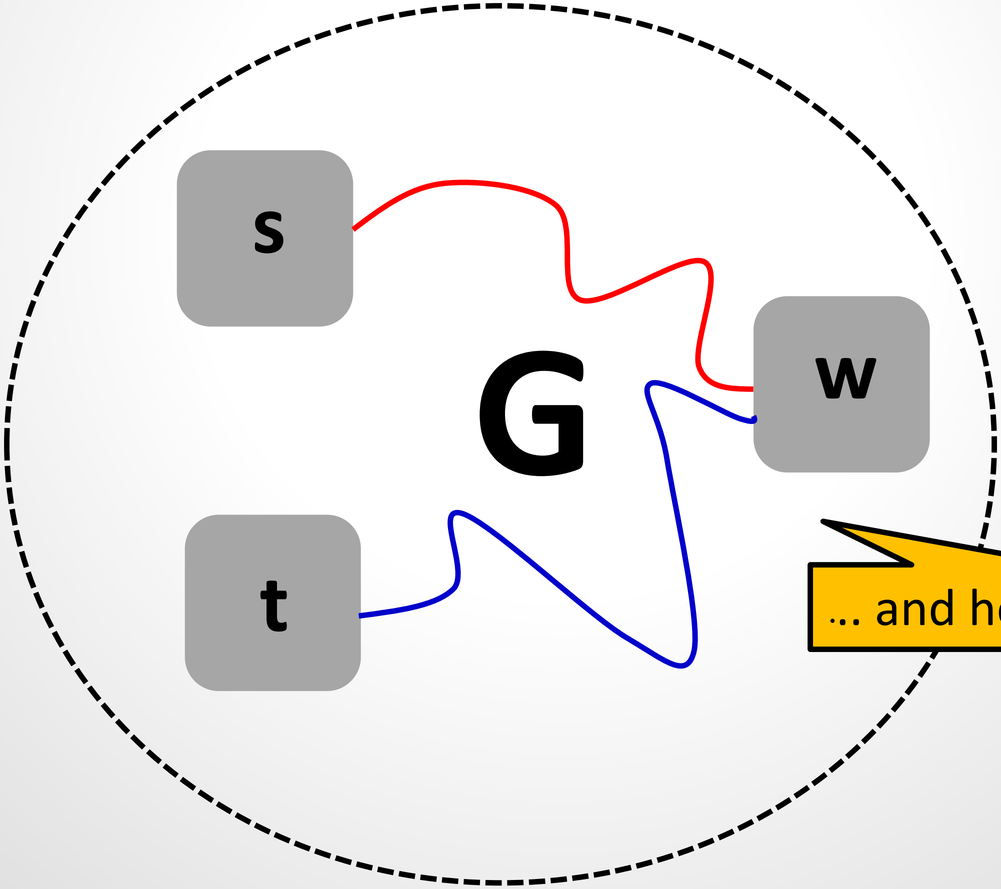
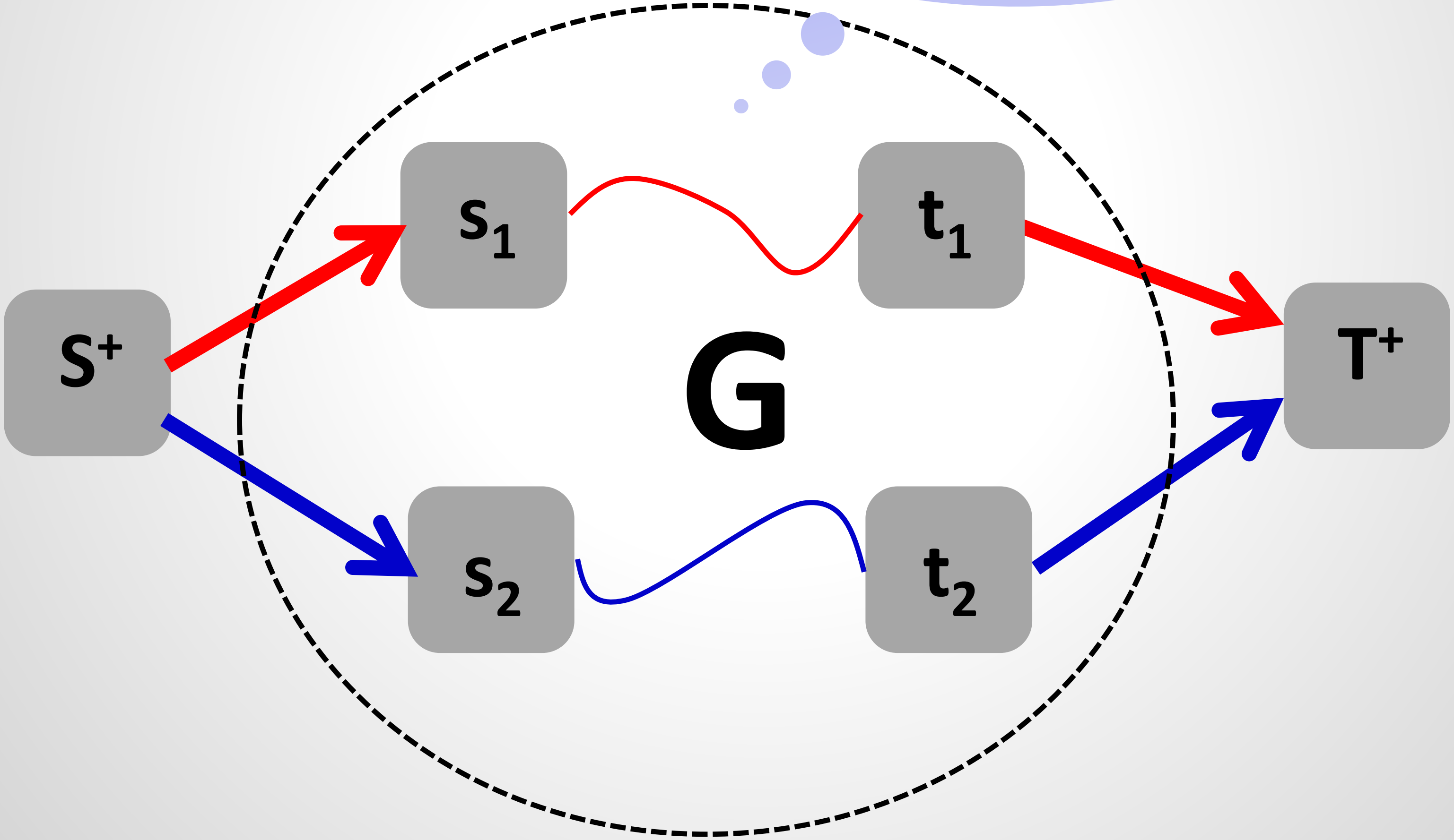


... and hence also (s,w,t).

Waypoint Routing on Steroids

- Reduction to Suur

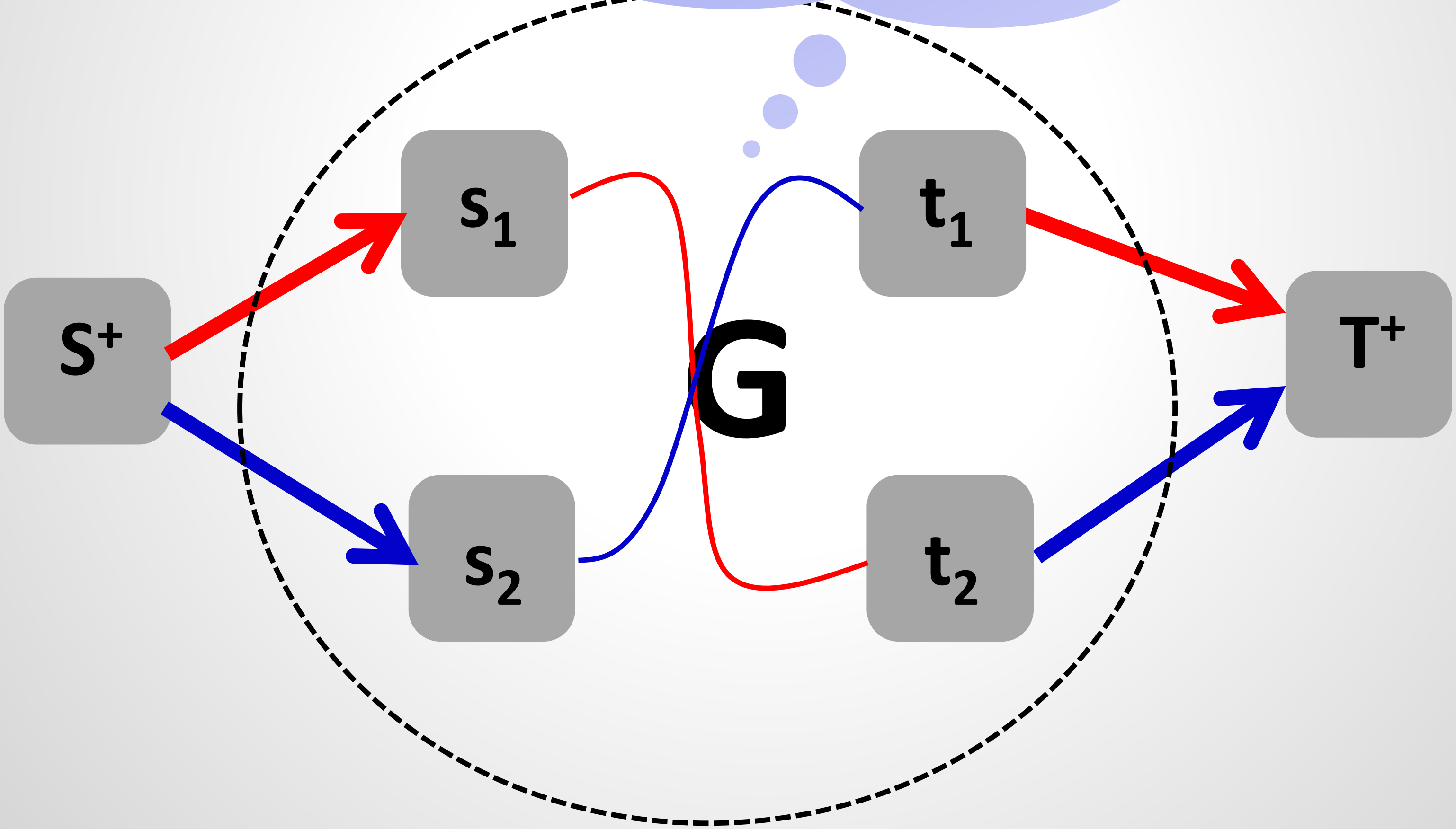
Can't I use Suurballe to efficiently compute disjoint paths as well?!



Waypoint Routing on Steroids

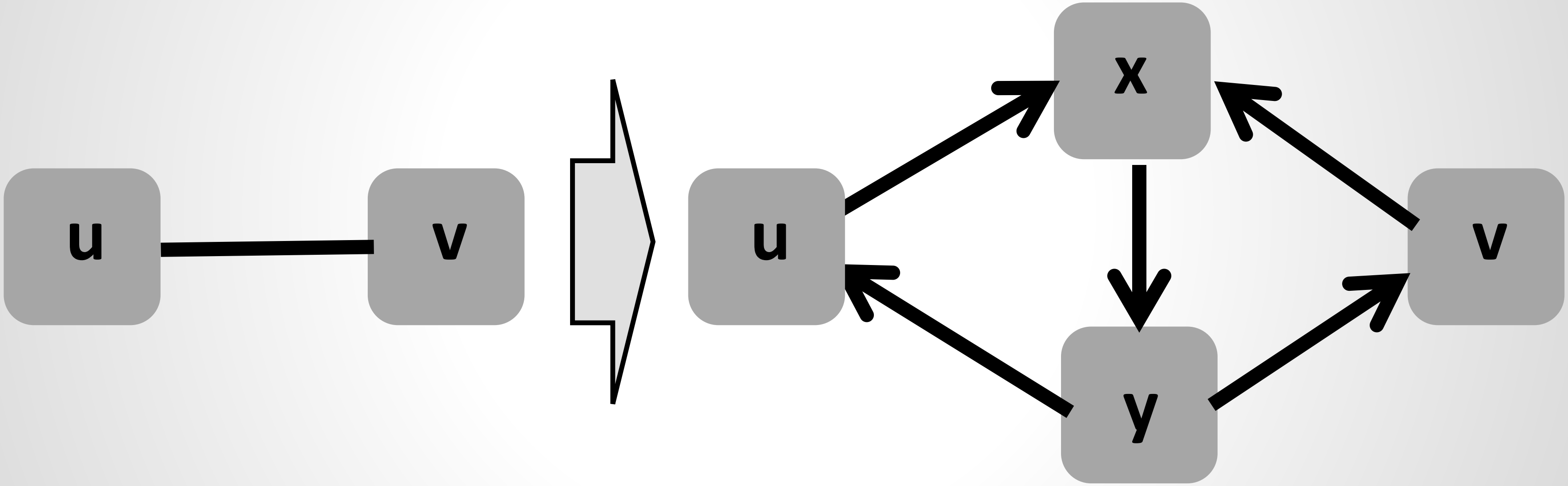
❑ Reduction to Suurb

No! Solves a much easier problem: 2 routes from $\{s_1,s_2\}$ to $\{t_1,t_2\}$.



Remarks: Under the rug…

- **Remark 1:** Suurballe is actually for directed substrate graphs, so need gadget to transform problem in right form:



- **Remark 2:** Suurballe: actually vertex disjoint
    - Suurballe & Tarjan: edge disjoint

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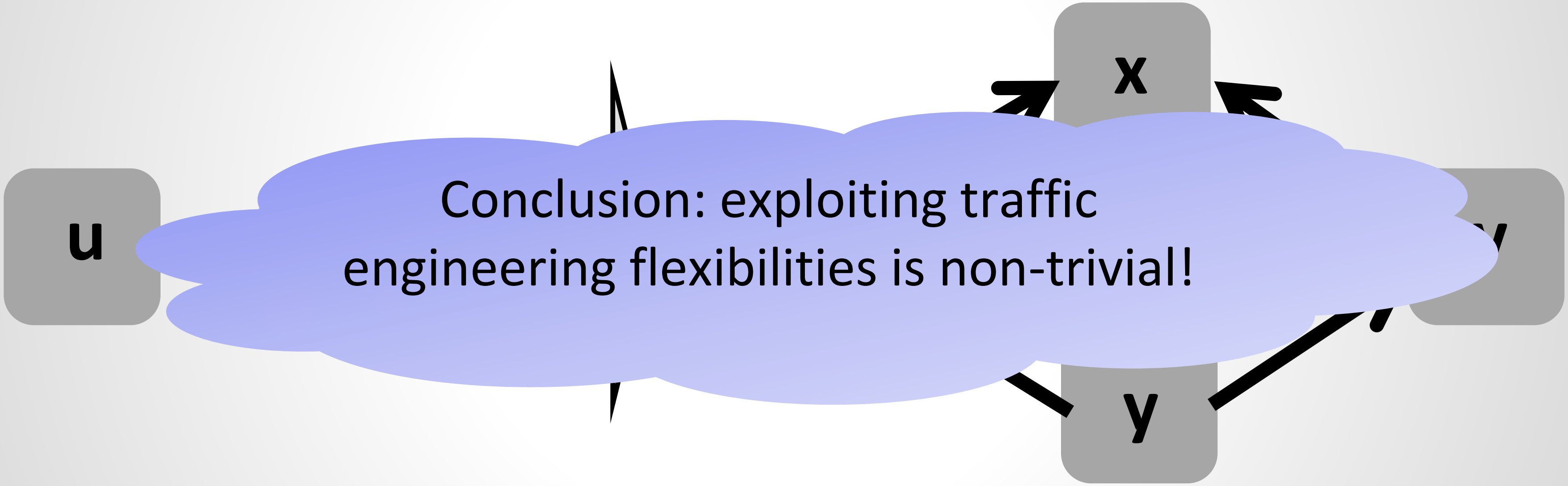


Conclusion: exploiting traffic engineering flexibilities is non-trivial!

- ❑ **Remark 2:** Suurballe: actually vertex disjoint
  - ❑ Suurballe & Tarjan: edge disjoint

PART II:
Dynamic Embeddings

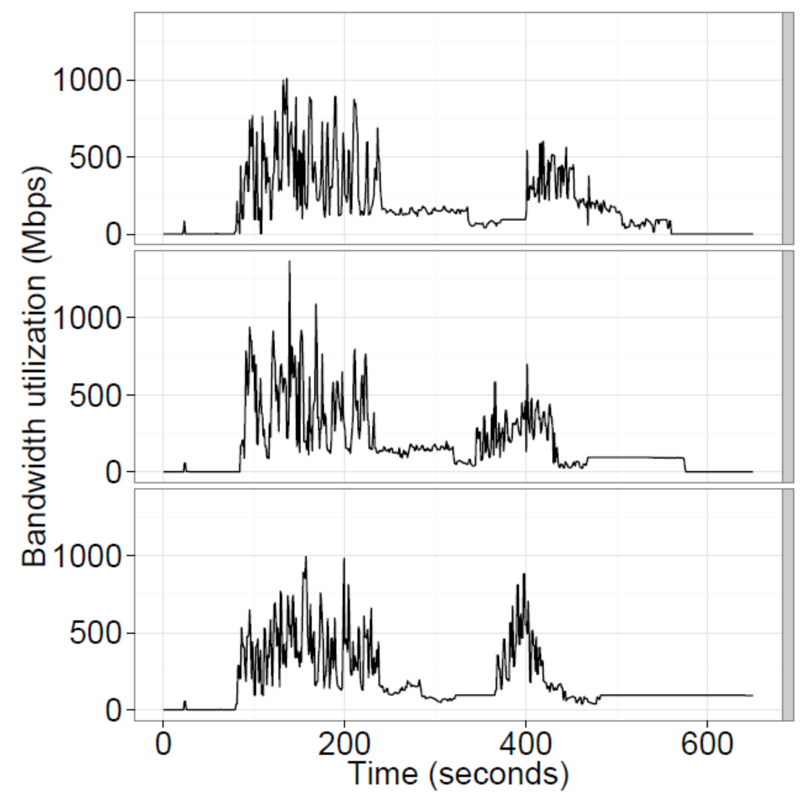
Real Communication Patterns Change over Time

- ❑ E.g., changing bandwidth demand: map reduce application cycles through phases of high and low bandwidth requirements

- ❑ E.g., long-running applications (e.g., streaming) change in popularity

- ❑ E.g., job churn: jobs terminate, new ones arrive

- ❑ E.g., large elephant flow degree, changing over time (cf ProjecToR at SIGCOMM 2016)



Bandwidth utilization of 3 different runs of the same **TeraSort workload** (**without interference**)

Real Communication Patterns Change over Time

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- E.g

Ideally, we want to place frequently communicating nodes close: may require adaptions over time!

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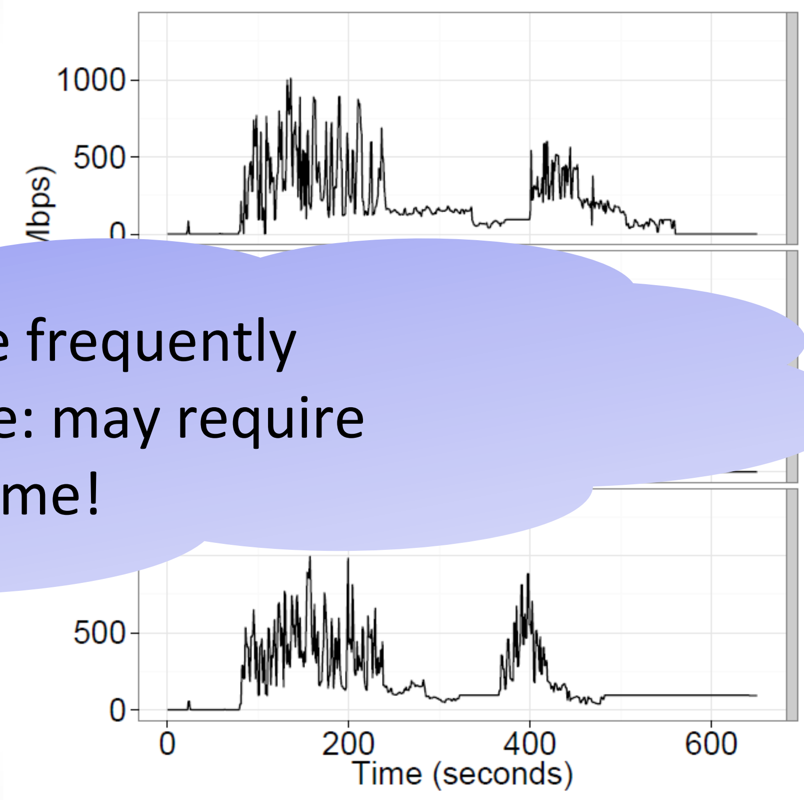

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  ones arr ve

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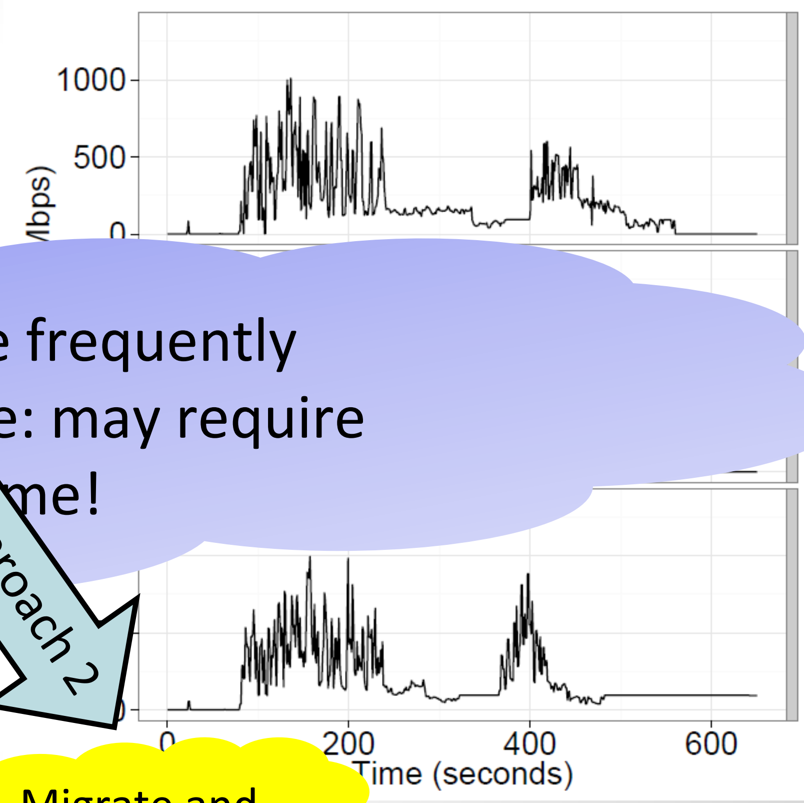

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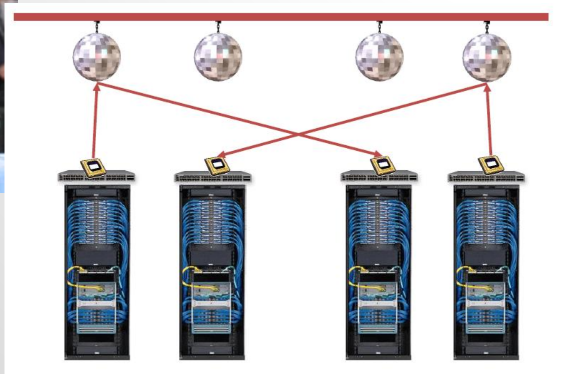
Approach 1

Approach 2

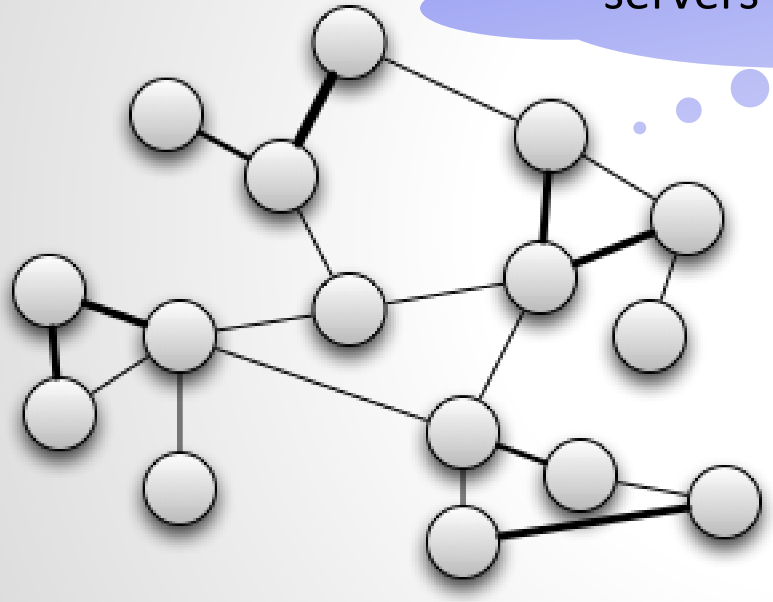
Adjust the network!



Migrate and collocate!

Essentially An Online *Re*Partitioning Problem

Communication @ time t:



How to embed pattern across ℓ=4 servers (or racks, pods, etc.) of size k=4?

Essentially An Online *Re*Partitioning Problem

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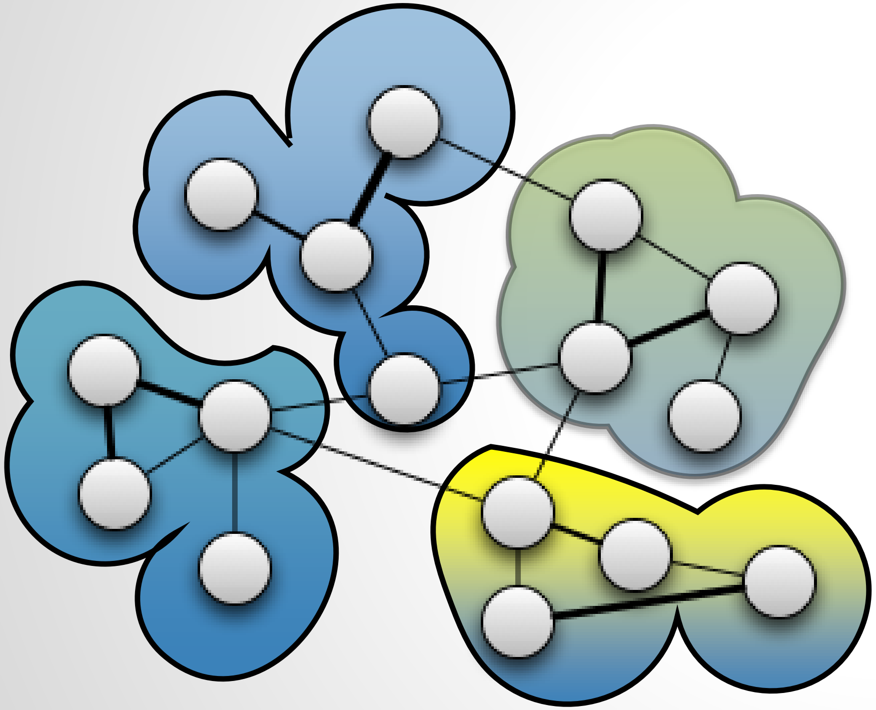
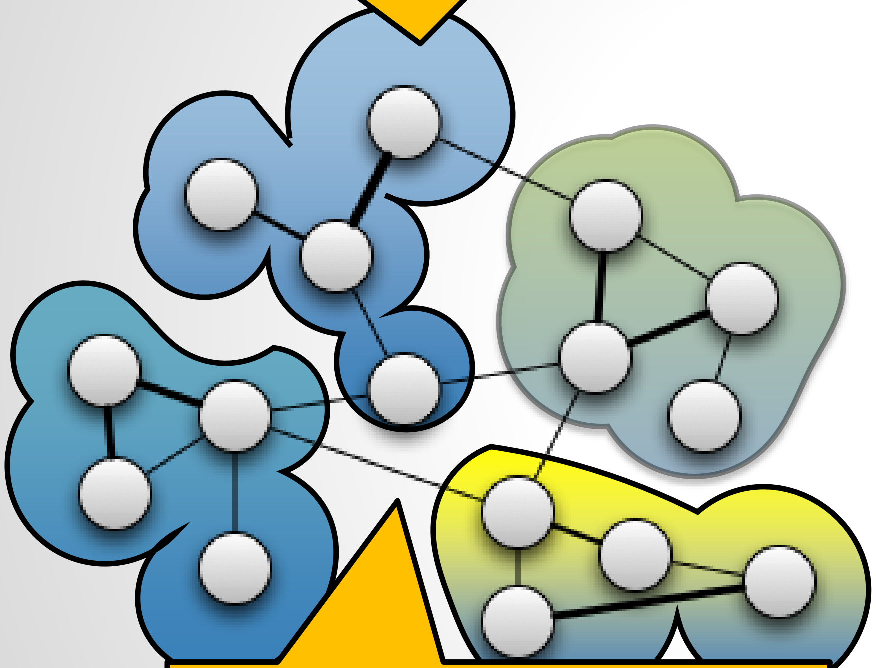


Essentially An Online *Re*Partitioning Problem

Most communication within cluster (intra-cluster)...

t:

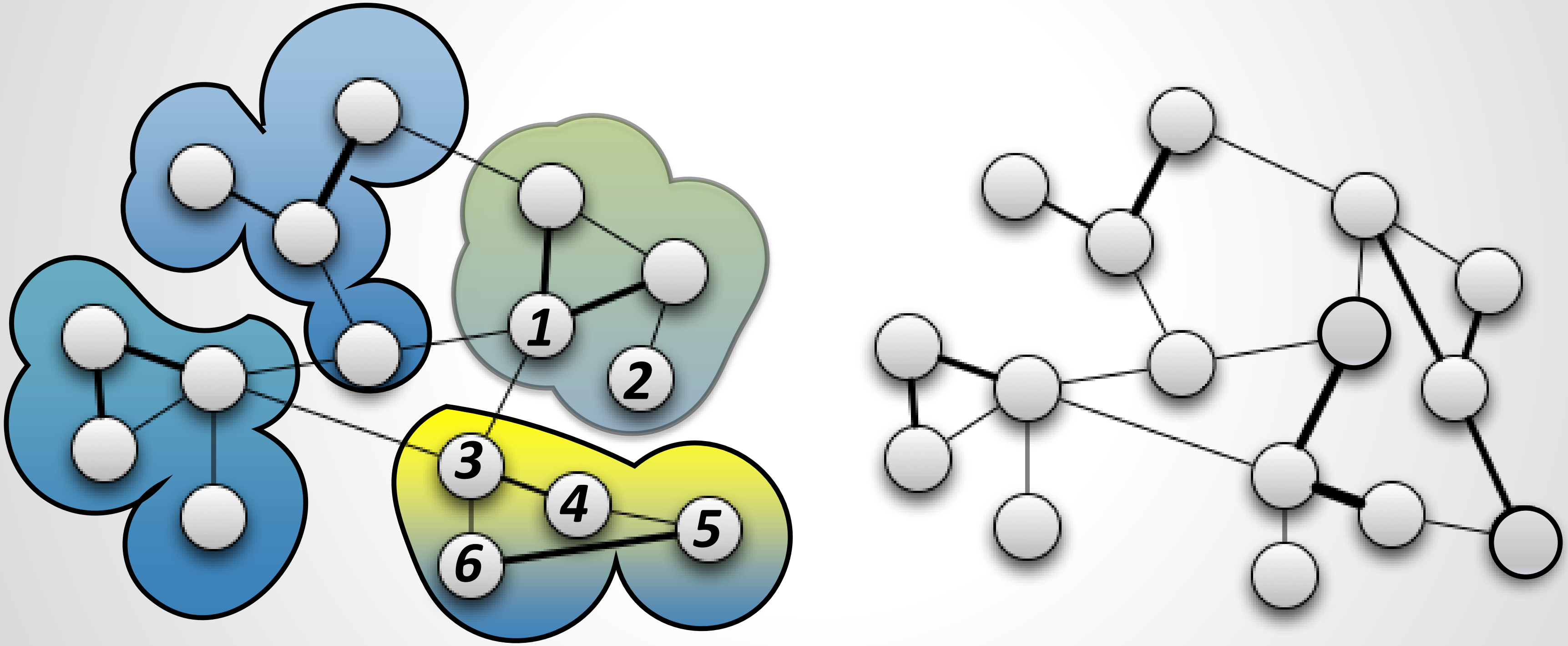


... little inter-cluster communication.

A classic (hard) combinatorial problem!

Essentially An Online *Re*Partitioning Problem

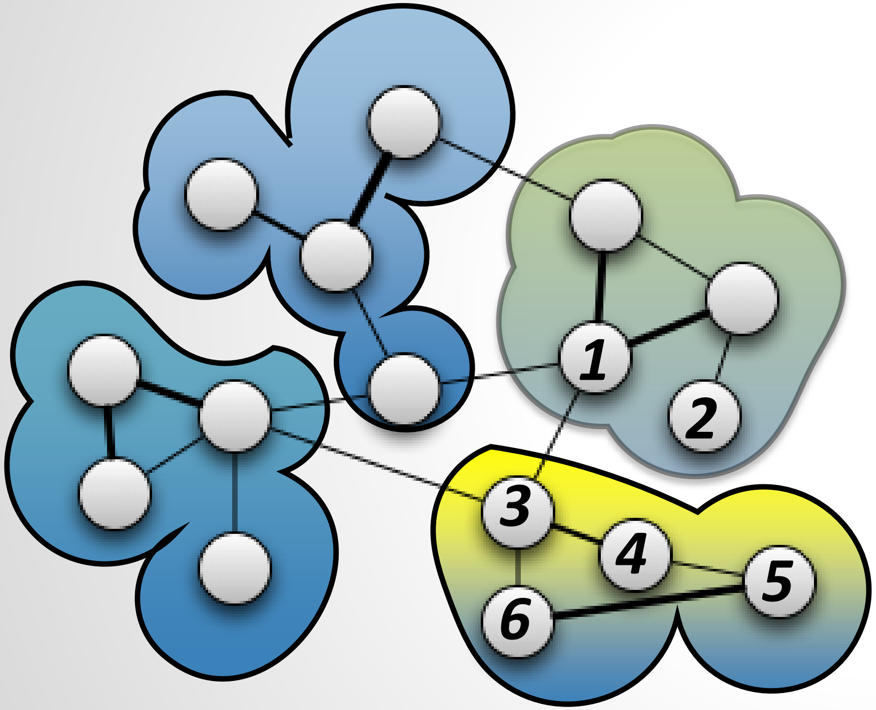
Communication @ time t: Communication @ time t+1:



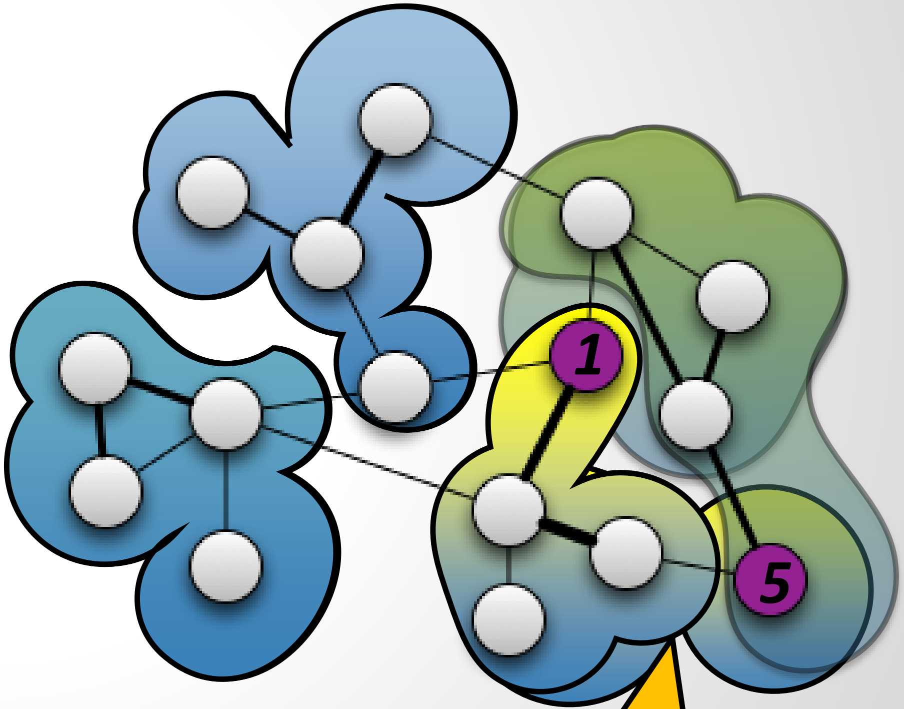
- ❑ Now assume: changes in communication pattern!
  - ❑ E.g., more communication (1,3),(3,4),(2,5) but less (5,6)

Essentially An Online *RePartitioning* Problem

Communication @ time t:



Communication @ time t+1:

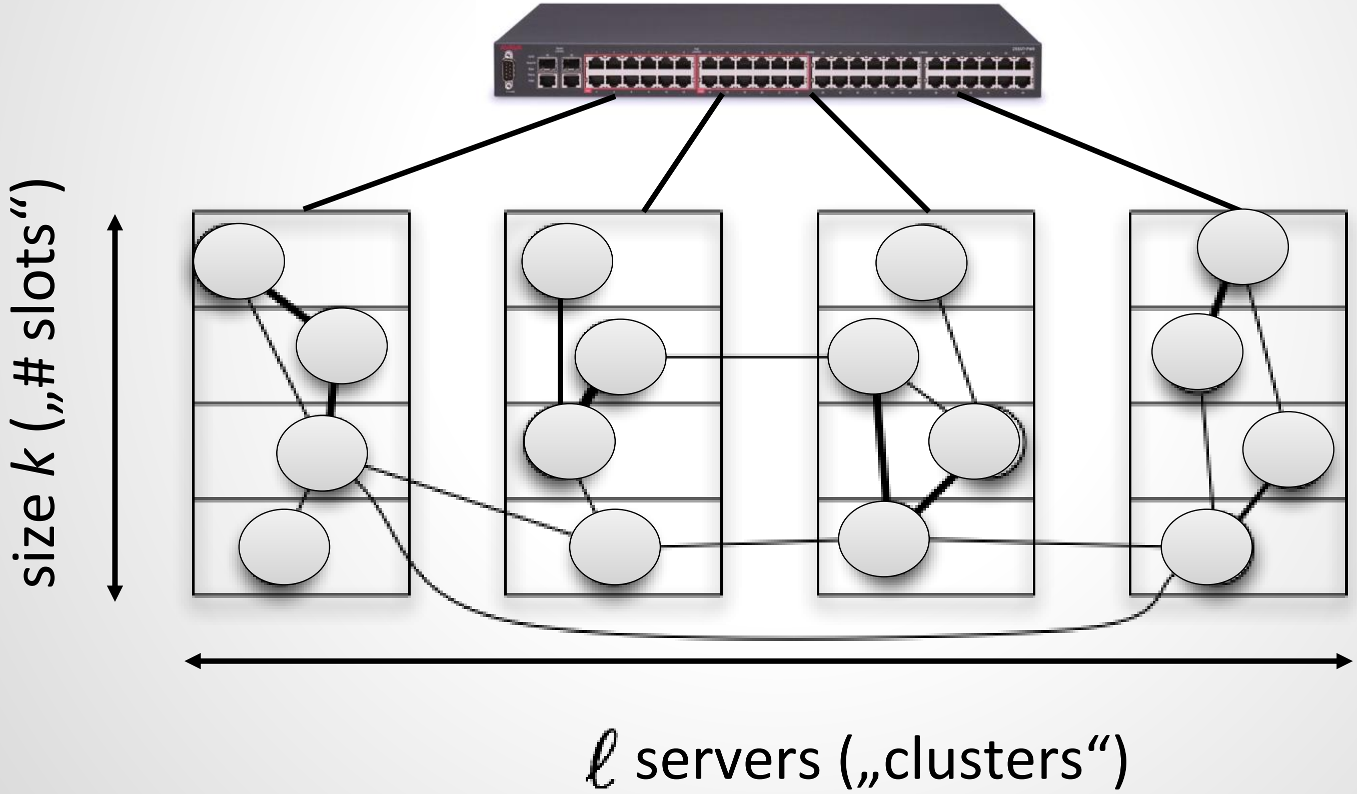


Makes sense that nodes 1 and 5 change clusters!

- ❑ Now assume: changes in communication pattern!
  - ❑ E.g., more communication (1,3),(3,4),(2,5) but less (5,6)

A Simple Model for the Tutorial

Consider a simple network, e.g., a single switch (e.g., a rack):



size k („# slots“)

ℓ servers („clusters“)

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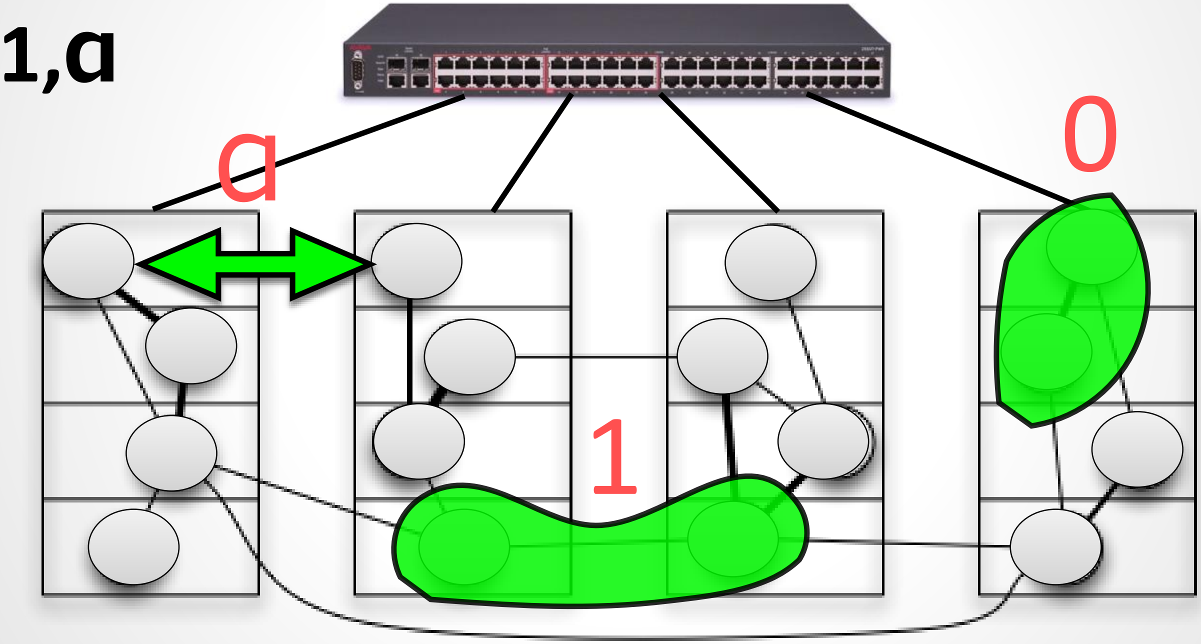
Costs: 0,1,α



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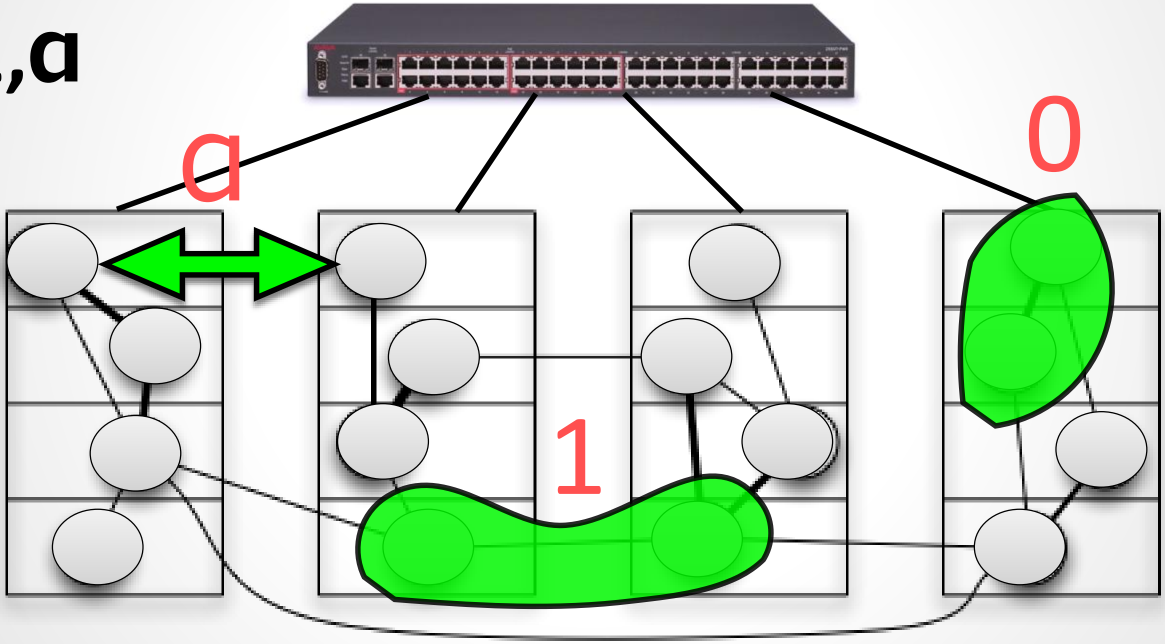

Objective: minimize total communication and migration cost!

More precisely: competitive ratio ρ = cost(ON)/cost(OPT)

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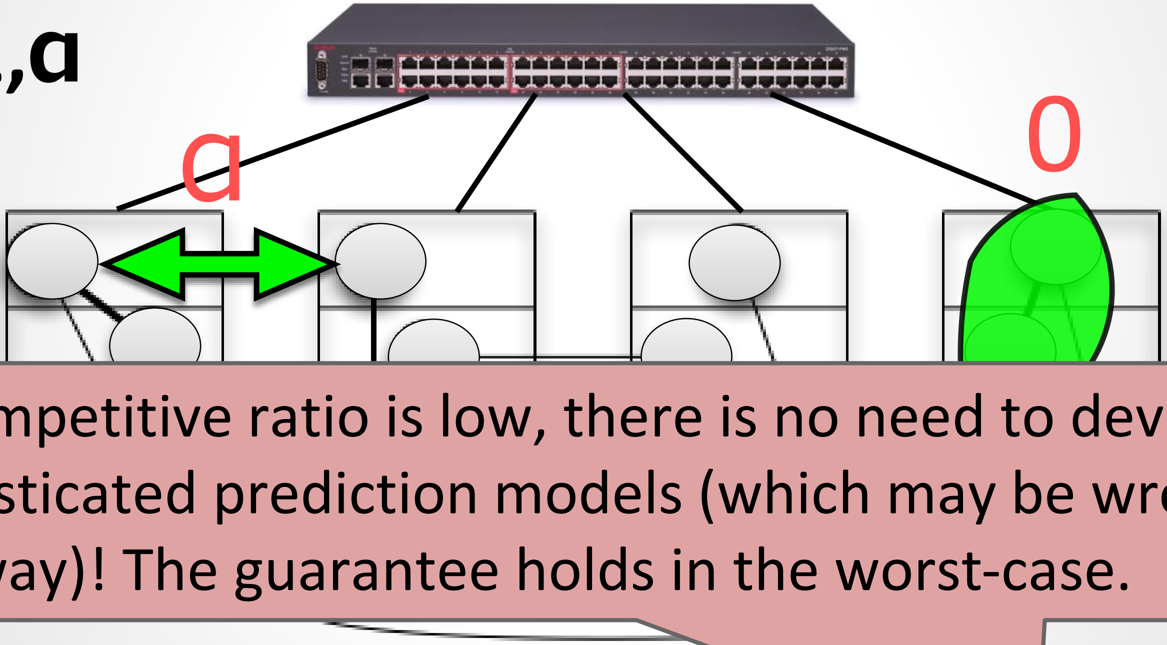

Nice: If competitive ratio is low, there is no need to develop any sophisticated prediction models (which may be wrong anyway)! The guarantee holds in the worst-case.

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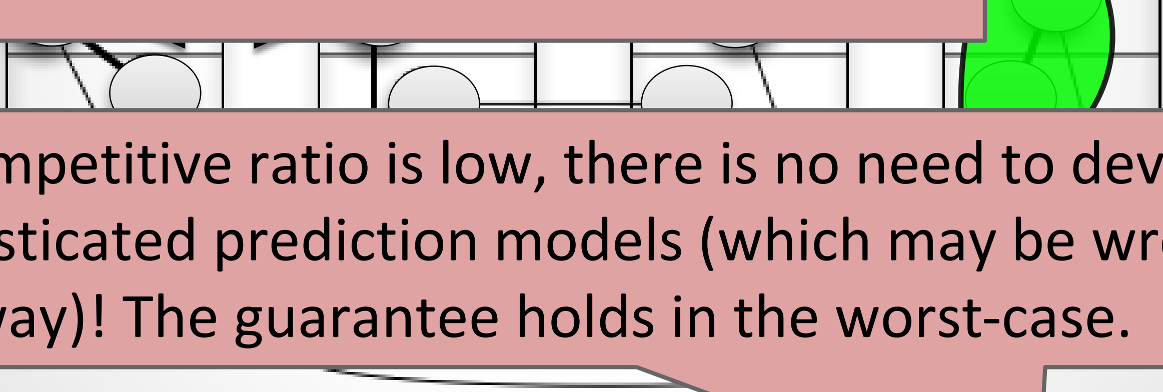
torial

C .g., a rack):

“Prediction is difficult,
especially about the future.”

Nils Bohr



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Adversarial Models

Weak adversary

- ❑ Chooses **request distribution D**
- ❑ Requests **sampled i.i.d.** from D
- ❑ Cannot react to online algo

Strong adversary

- ❑ Can generate **arbitrary request sequence σ**
- ❑ Knows and can react to online algo

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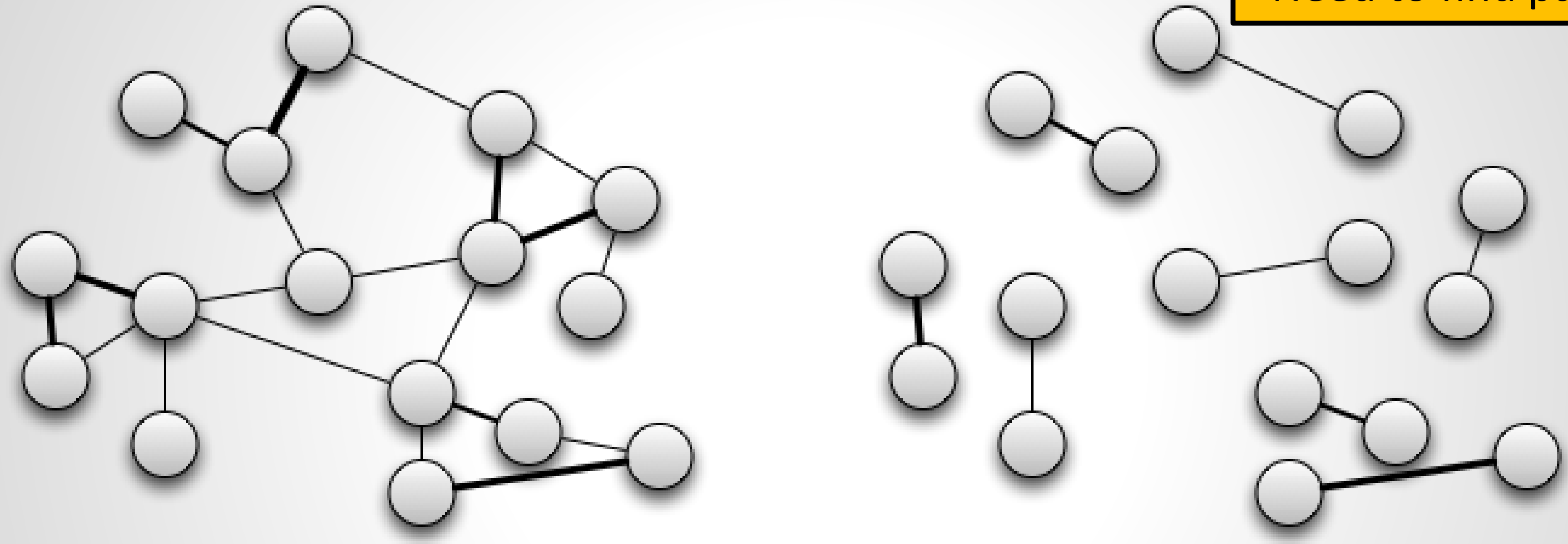
The Crux: Algorithmic Challenges

- ❑ Do not know D resp. σ ahead of time
- ❑ Upon each communication request (*u*,*v*):
  - ❑ Migrate *u* and *v* together? «Rent-or-buy»: migration cost should be amortized
  - ❑ Migrate where? *u* to *v*, *v* to *u*, both to a third cluster?
  - ❑ If cluster is full already: what to evict?

Example: Special Case *k*=2



Clusters of size 2:
Need to find pairs!

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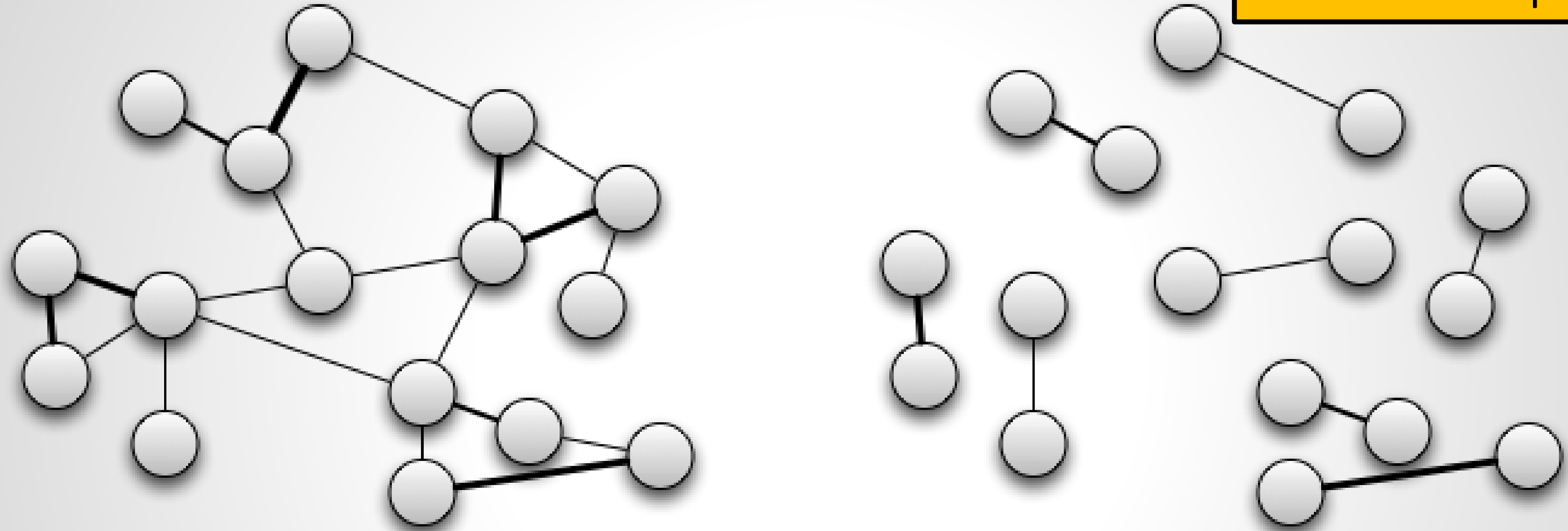


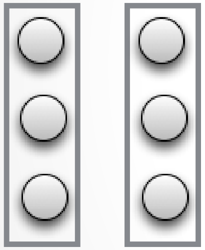
Clusters of size 2: Need to find pairs!

Clusters of size 2: A new type of online *re*matching problem!

It is hard to compete under !

- Assume two clusters: for offline algorithm they are of size k…

OFF:



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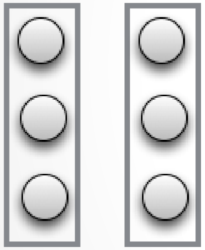
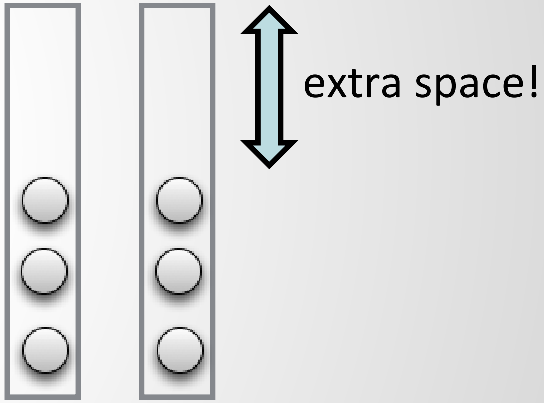


ON:



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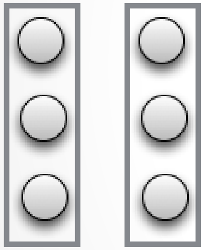

E.g., ON can even collocate all except for one!

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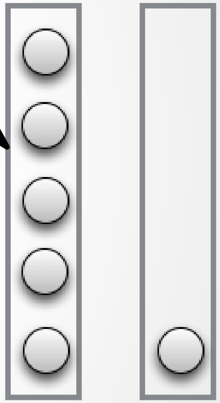

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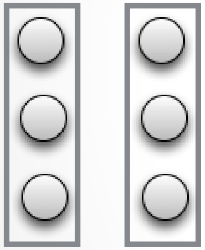

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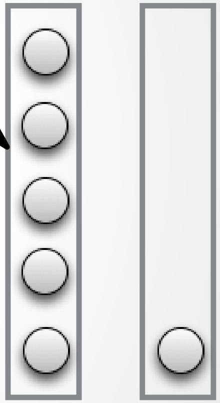

What is the achievable competitive ratio?

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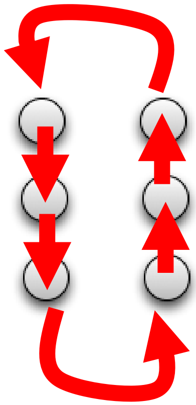

For the sake of lower bound, let us restrict the adversary more: can only ask for node pairs taken from a cyclic order: *k* pairs (resp. links) in total!

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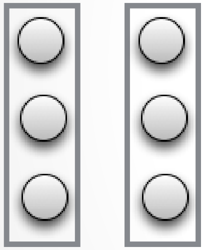
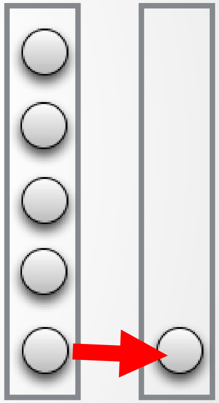


ON:



Adversary can always request an inter-cluster link: always exists!

Ouch! Cost 1 for each request.

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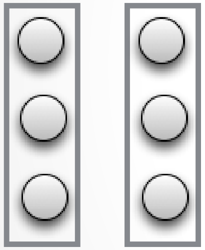

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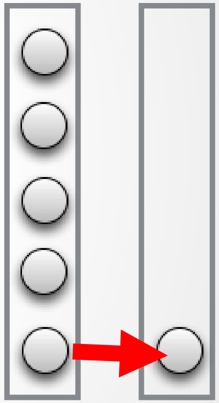

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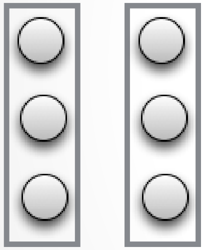
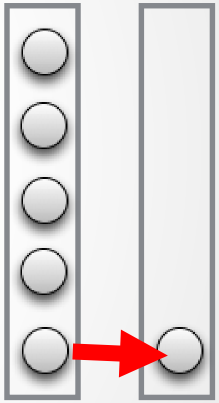


What is the cost of OFF?

ON:



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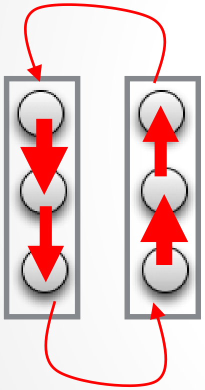

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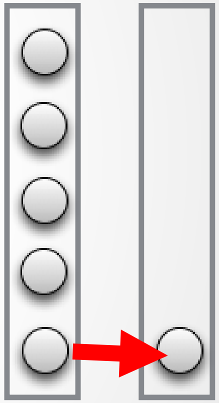

Ouch! Cost 1 for each request.

Move to configuration i ∈ {1,…,k} which is asked the least. Averaging argument: At least k times less communiation cost!

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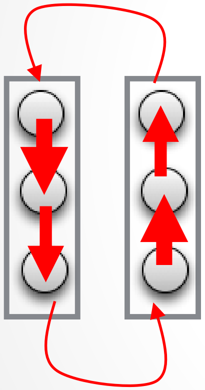

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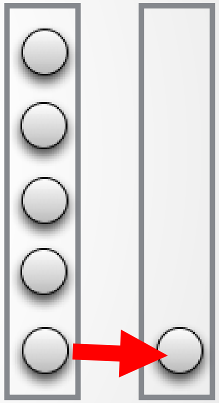


Ouch! Cost 1 for each request.

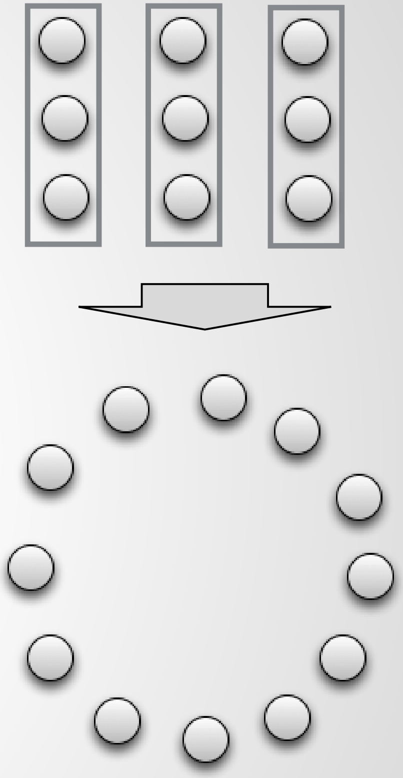
Lower bound of Ω(k) for competitive ratio, despite big augmentation!

A Simple $O(n^2)$ Upper Bound

At least it does not depend on time! ☺

A Simple $O(n^2)$ Upper Bound

- Algorithm DET:
  - Based on «growing communication components»
  - Cycles through phases
    - Initially in each phase: empty graph of n nodes



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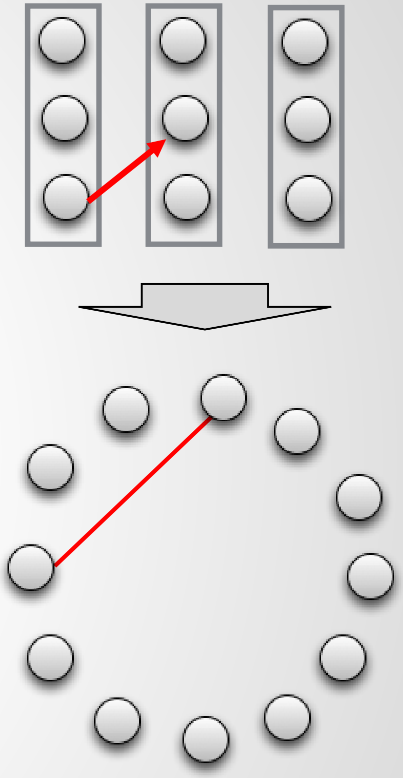

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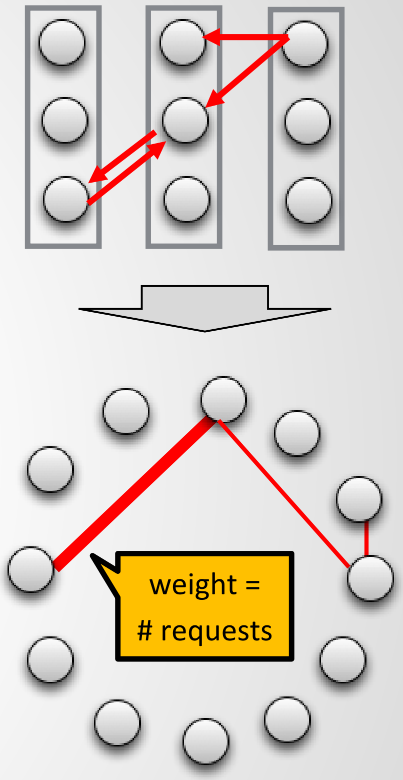

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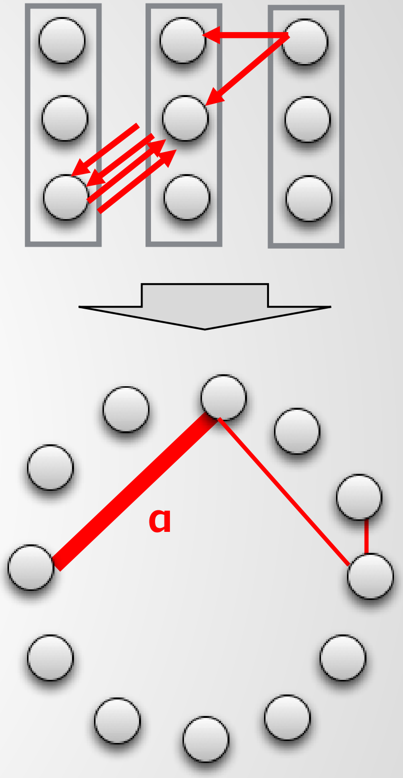

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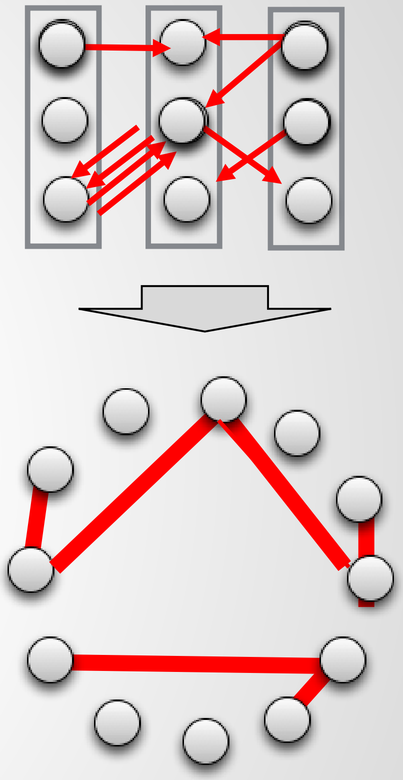

Components cannot be partitioned perfectly (first component alone too large)!

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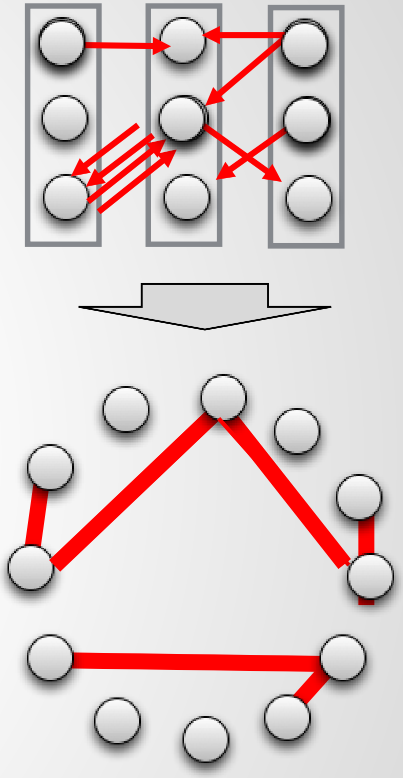
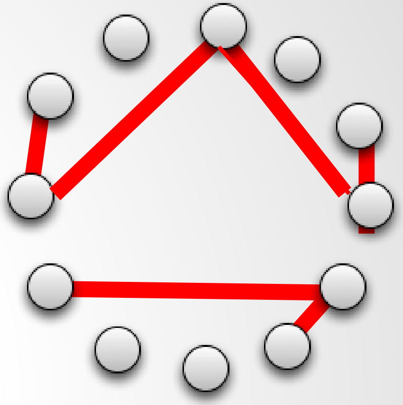
    - If an edge (u,v) weight reaches α, DET repartitions nodes, so that *all edges which have reached* α *so far are in same cluster*!
    - If this is not possible: phase ends



Competitive ratio?

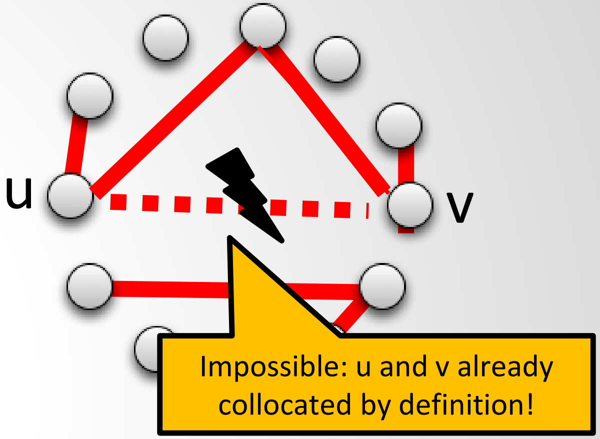
A Simple $O(n^2)$ Upper Bound

- ❑ Analysis (costs per phase):
  - ❑ Observe: edge weights always ≤ α: once reach α, their endpoints will always be collocated (by algorithm definition)



A Simple $O(n^2)$ Upper Bound

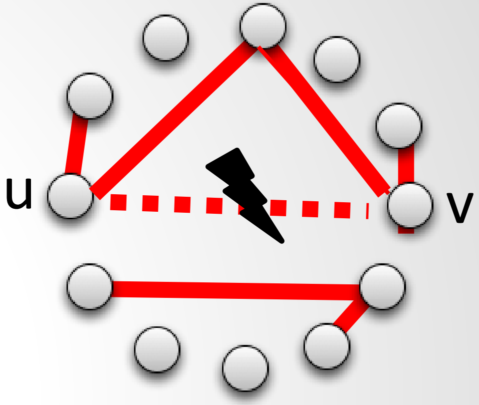
- ❑ Analysis (costs per phase):
  - ❑ Observe: edge weights always ≤ α: once reach α, their endpoints will always be collocated (by algorithm definition)
  - ❑ α-edges form a forest (so at most n many!): once two nodes (u,v) are connected by a path of α–edges, they are in a single cluster and will no longer communicate across clusters



Impossible: u and v already collocated by definition!

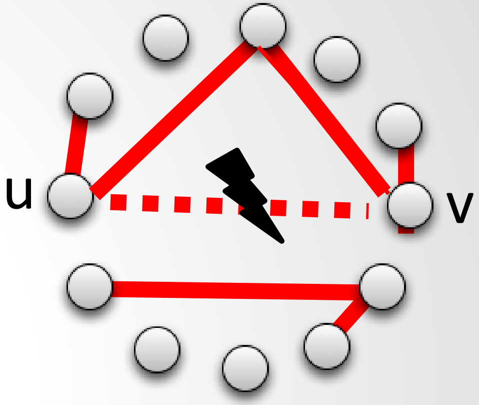
A Simple $O(n^2)$ Upper Bound

- Analysis (costs per phase):
  - Observe: edge weights always ≤ α: once reach α, their endpoints will always be collocated (by algorithm definition)
  - α-edges form a forest (so at most n many!): once two nodes (u,v) are connected by a path of α–edges, they are in a single cluster and will no longer communicate across clusters
  - Thus: ON cost per phase:
    - At most 1 reorganization per α-edge (at most n α-edges), so n times reconfig cost n·α, so $n^2α$
    - Communication cost: at most α per edge (at most n^2 many), so also at most $n^2α$



A Simple $O(n^2)$ Upper Bound

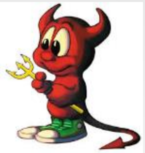
- ❑ Analysis (costs per phase):
  - ❑ Observe: edge weights always ≤ ɑ: once reach ɑ, their endpoints will always be collocated (by algorithm definition)
  - ❑ ɑ-edges form a forest (so at most n many!): once two nodes (u,v) are connected by a path of ɑ–edges, they are in a single cluster and will no longer communicate across clusters
  - ❑ Thus: ON cost per phase:
    - ❑ At most 1 reorganization per ɑ-edge (at most n ɑ-edges), so n times reconfig cost n·ɑ, so $n^2ɑ$
    - ❑ Communication cost: at most ɑ per edge (at most n^2 many), so also at most $n^2ɑ$
  - ❑ Costs of OFF per phase:
    - ❑ If OFF migrates any node, it pays at least ɑ
    - ❑ If not, it pays communication cost at least ɑ: the grown components do not fit clusters (intra-cluster edges only): definition of «end-of-phase»!



Upper bound of $O(n^2ɑ/ɑ)=O(n^2)$ for competitive ratio!

Known Results So Far

- ❑ Case k=2 („online rematching“): constant competitive ratio
- ❑ General case: with a little bit of augmentation: O(k log k) possible
  - ❑ Recall Ω(k) lower bound
  - ❑ Nice: independent of number of clusters!
  - ❑ Practically relevant: # VM slots per server usually small

What about  ?

What about ?

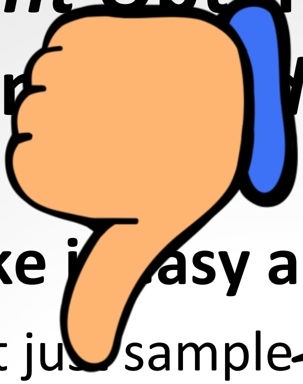
- Recall: weak adversary cannot choose request sequence but only the distribution
  - Adversary needs to sample i.i.d. from this distribution
  - Moreover: Adversary knows (deterministic or randomized) «learning» algorithm, i.e., chooses worst distribution

Any ideas?

The Crux: *Joint* Optimization of Efficient Learning *and* Searching

- **Naive idea 1: Take it easy and first learn distribution**
  - Do not move but just sample requests in the beginning: until exact distribution has been learned whp
  - Then move to the best location for good

The Crux: *Joint* Optimi Lear / S



Waiting can be very costly: maybe start configuration is very bad and others similarly good: takes long to learn, not competitive! Need to move early on, away from bad locations!

- ❑ **Naive idea 1: Take i asy an**
  - ❑ Do not move but ju sample requests in the beginning: until exact distribution has been learned whp
  - ❑ Then move to the best location for good

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- **Naive idea 2: Pro-actively always move to the lowest cost configuration seen so far**

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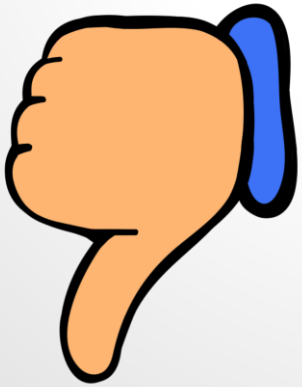


Bad: if requests are uniform at random, you should not move at all! Migration costs cannot be amortized. Crucial difference to classic distribution learning problems: guessing costs!

The Crux: *Joint* Optimization of Efficient Learning *and* Searching

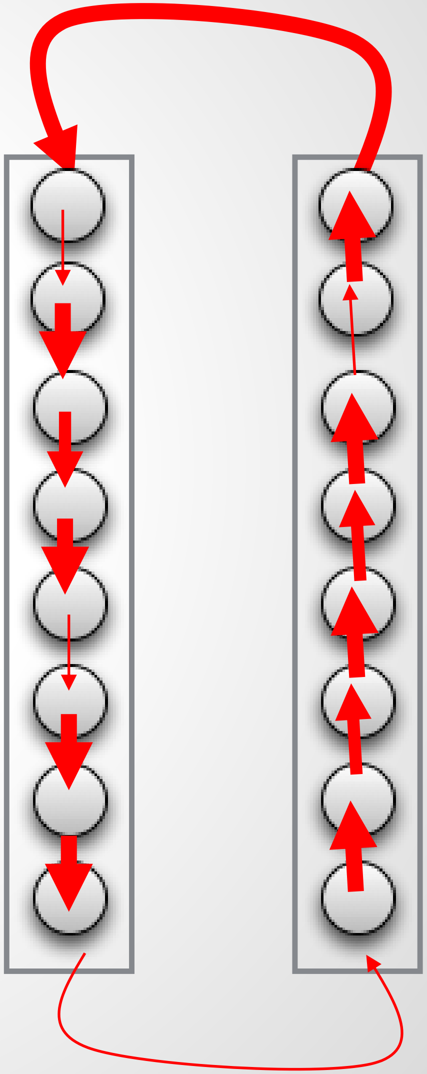
- ❑ **Naive idea 1: Take it easy and first learn distribution**
  - ❑ Do not move but just sample reques n the beginning: until exac tribution has learn o
  - ❑ Then move

- ❑ **Naive idea west cost configurat**
  - ❑ Bad, e.g., if requests a stribut niformly a random: better not to move at all (mo g costs can ot be amortized)

Only move when it pays off! But e.g., how to differentiate between uniform and „almost uniform“ distribution?

Example Learning Algorithm for Ring: Rotate Locally!

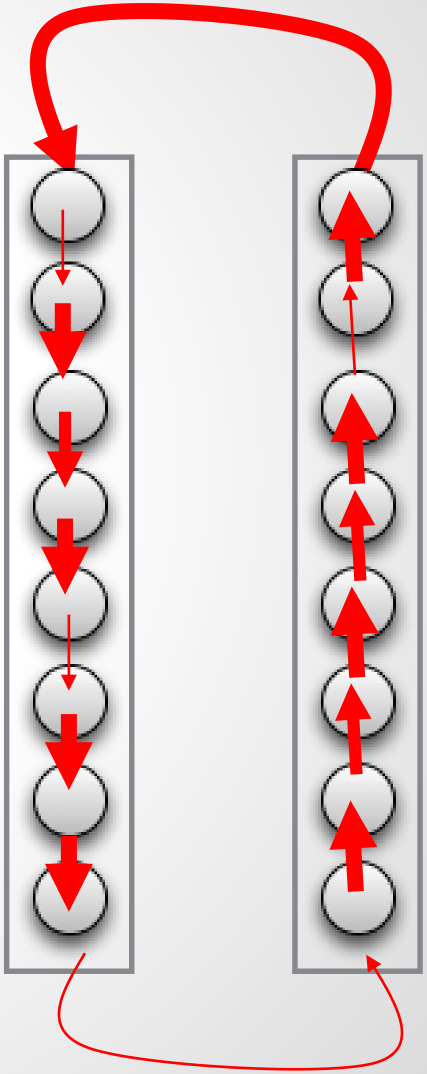
- ❑ Mantra of our algorithm: Rotate!
  - ❑ Rotate early, but not too early!
  - ❑ And: rotate locally



Example Learning Algorithm for Ring: Rotate Locally!

Define **conditions** for configurations: if met, **never go back** to it (we can afford it w.h.p.: seen enough samples)

- Mantra of gorithm: Rotate!
  - ❑ Rotate early, but not too early!
  - ❑ And: rotate locally



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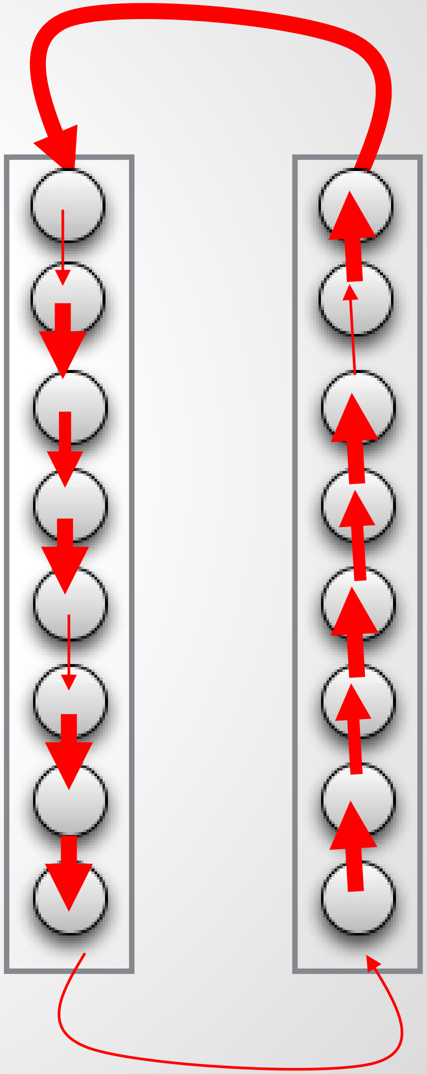

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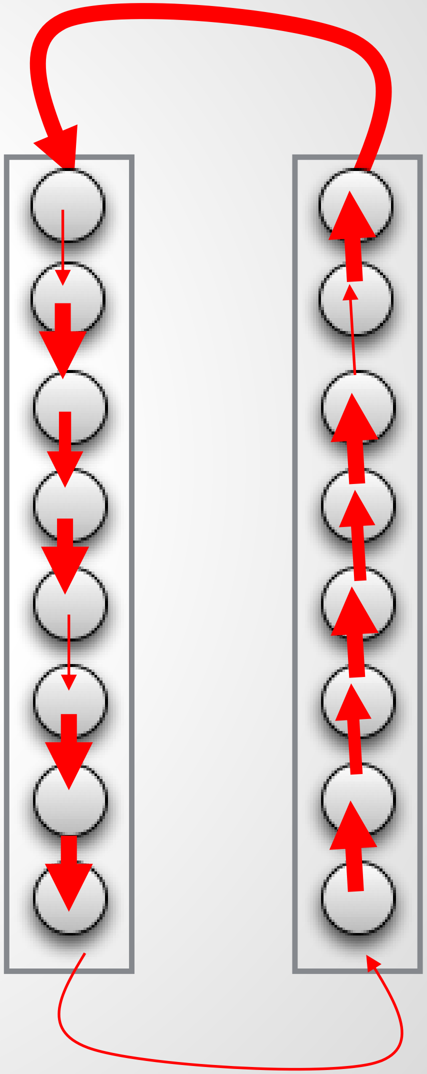
Growing radius strategy: allow to move further only once amortized!



Example Learning Algorithm for Ring: Rotate Locally!

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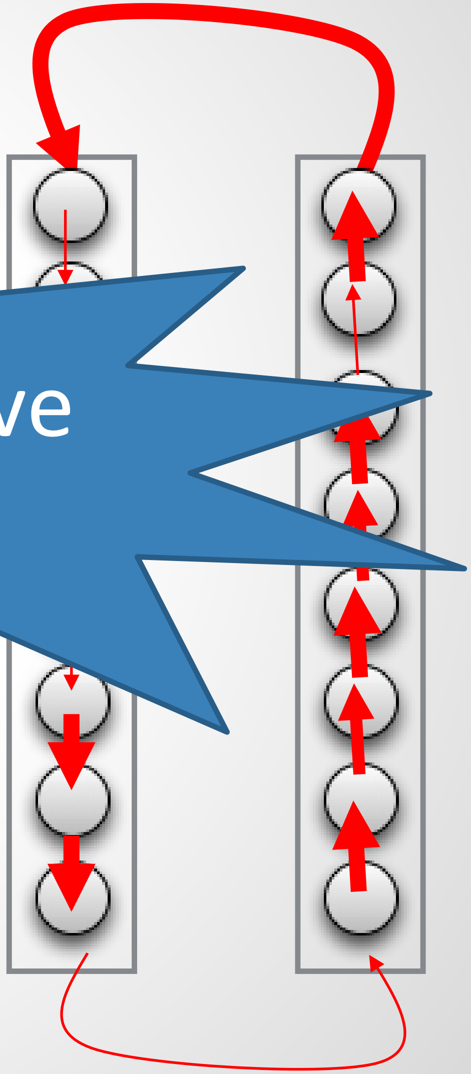
  - ❑ Ro te ly, but n ea
  - ❑ An ta

log(n)-competitive w.h.p.

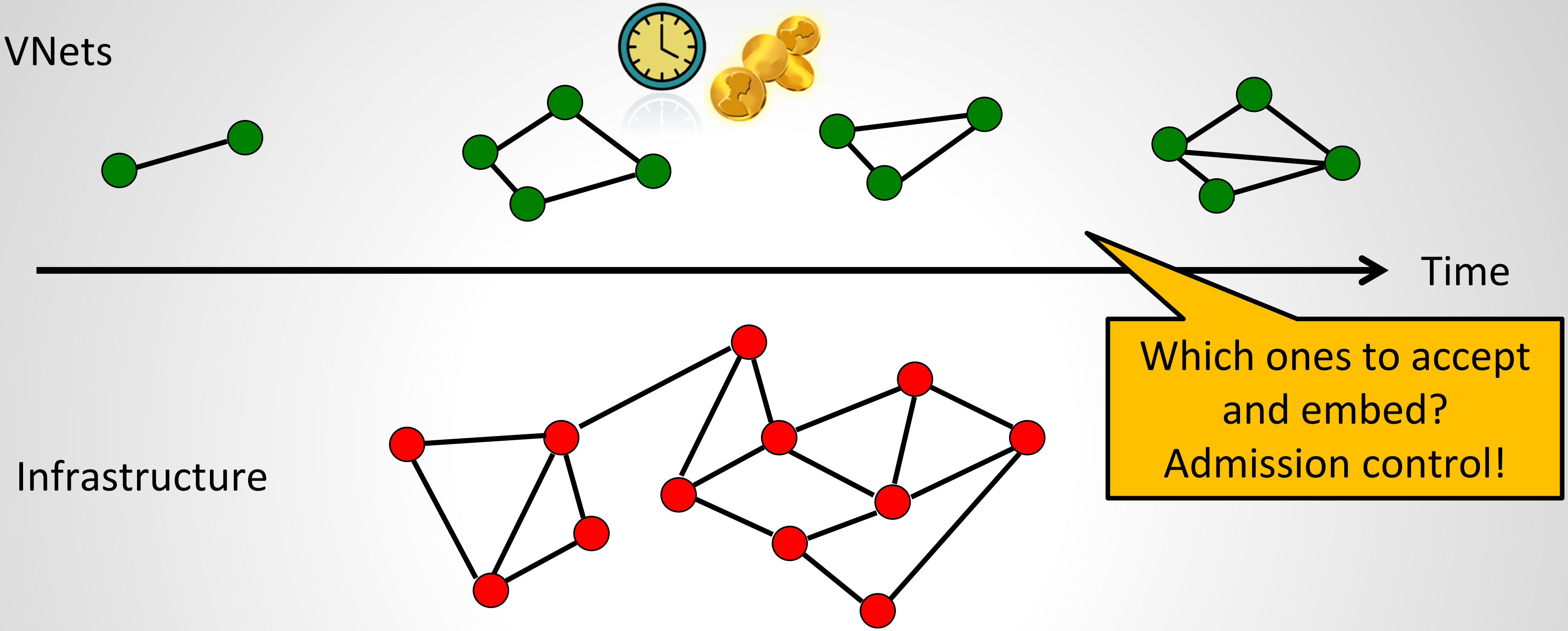
If cur configuration is eliminated, go to nearby configuration (in directed manner: no frequent back and forth)!

mo ler o onc ortized!



PART III:
Embeddings over Time

A VNet seldom comes alone!



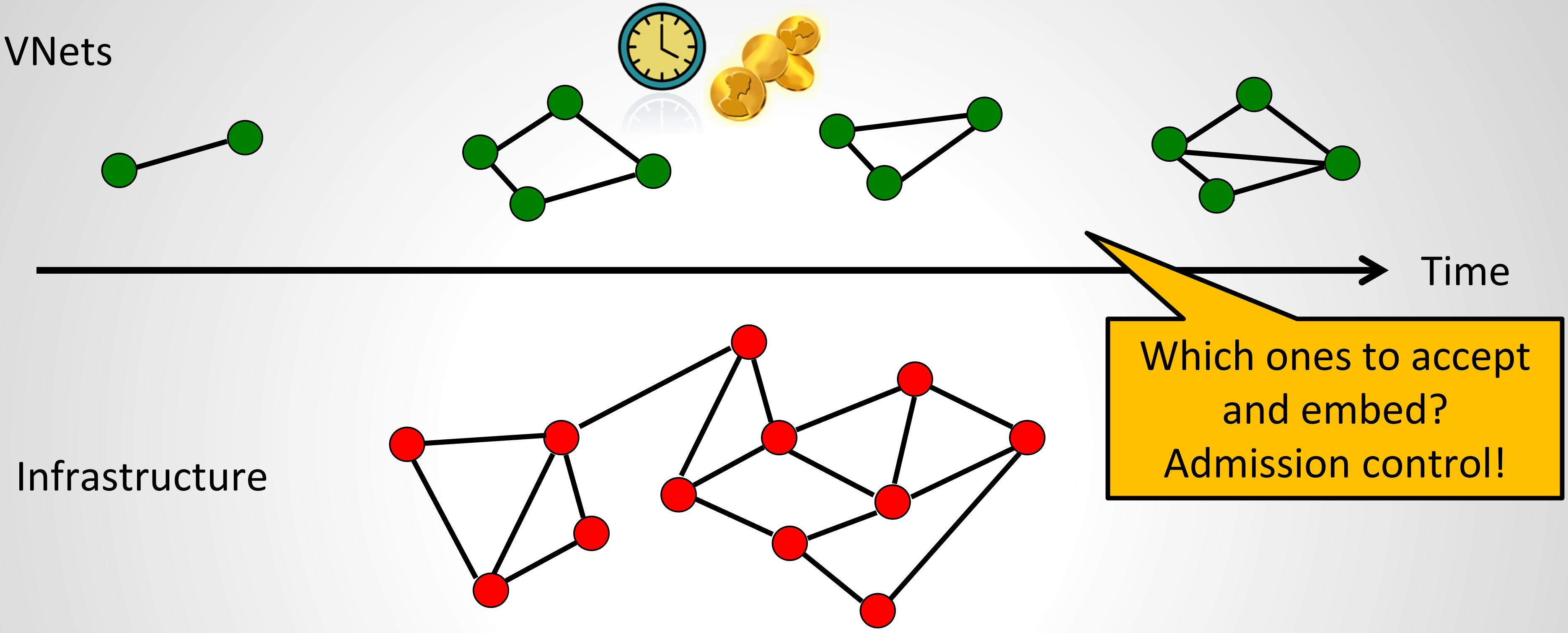
VNets

Time

Infrastructure

Which ones to accept and embed?
Admission control!

A VNet seldom comes alone!



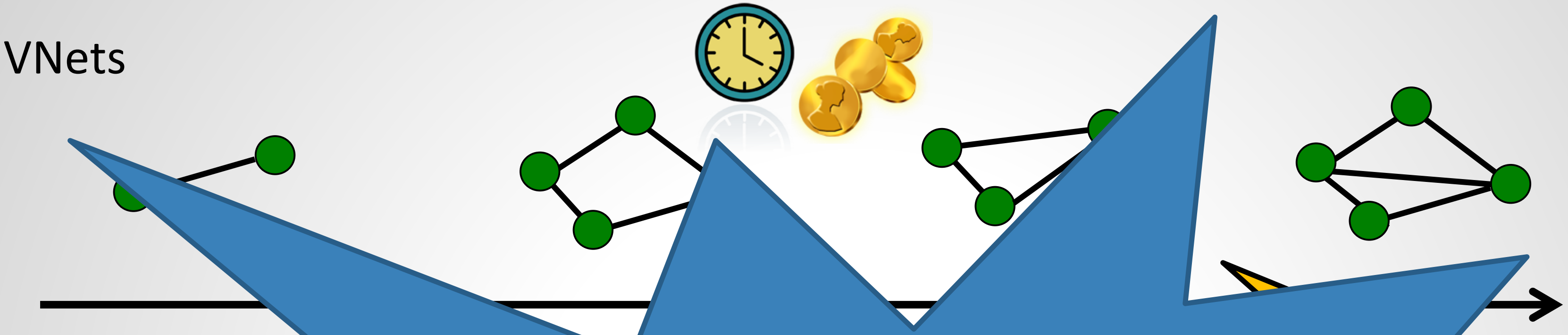
Which ones to accept and embed? Admission control!

Different flavors of VNets:

- ❑ Node mappings given or subject to optimization
- ❑ Different routing and traffic models
- ❑ Different prices and durations
- ❑ …

A VNet seldom comes alone!

VNets



ones to accept

ntrol!

Infrast

Can be solved by online approximation framework by Buchbinder and Naor.

Diff flavors of VNe

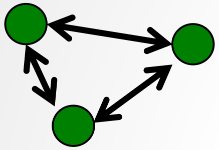
- ❑ Node mappings g u oti
- ❑ Different routin and traffic
- ❑ Different prices and duration
- ❑ …

Different VNet Flavors

❑ Traffic models

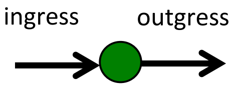
Customer Pipe

Traffic matrix: Bandwidth per VM pair (u,v)



Hose Model

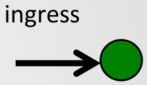
Per VM bandwidth: polytope of traffic matrices.



virtual switch

Aggregate Ingress

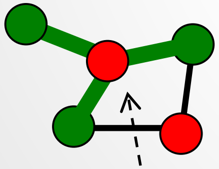
Only ingress specified: e.g., support multicast etc.



❑ Routing models

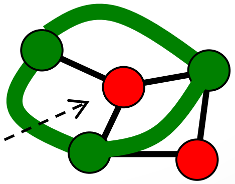
Tree

Steiner tree embedding



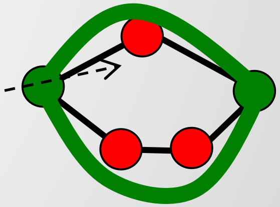
Single Path

Unsplittable paths



Multi-Path

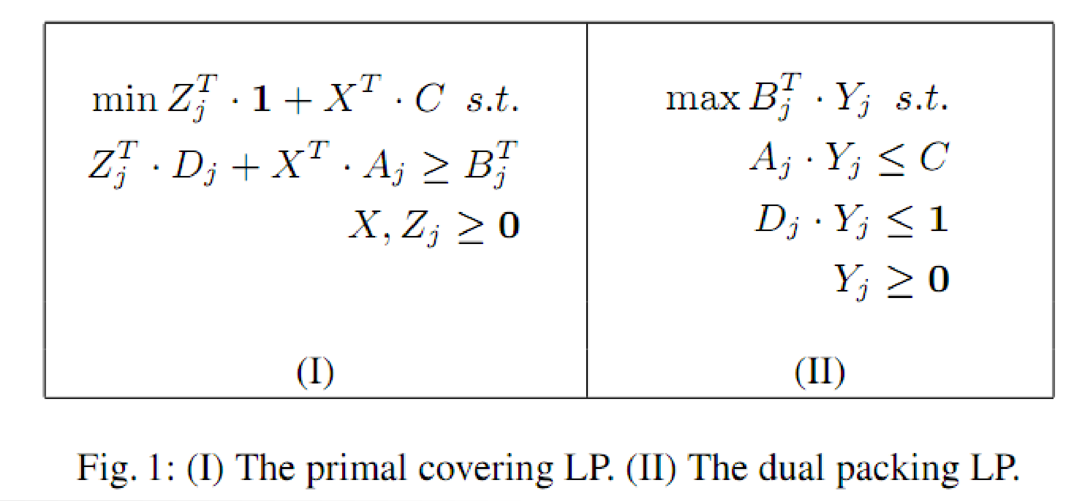
Splittable paths (more capacity)



Relay costs: e.g., depending on packet rate

Primal and Dual

Applying Buchbinder&Naor



$$\begin{aligned} \min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t. \\ Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T \\ X, Z_j \geq \mathbf{0} \end{aligned}$$

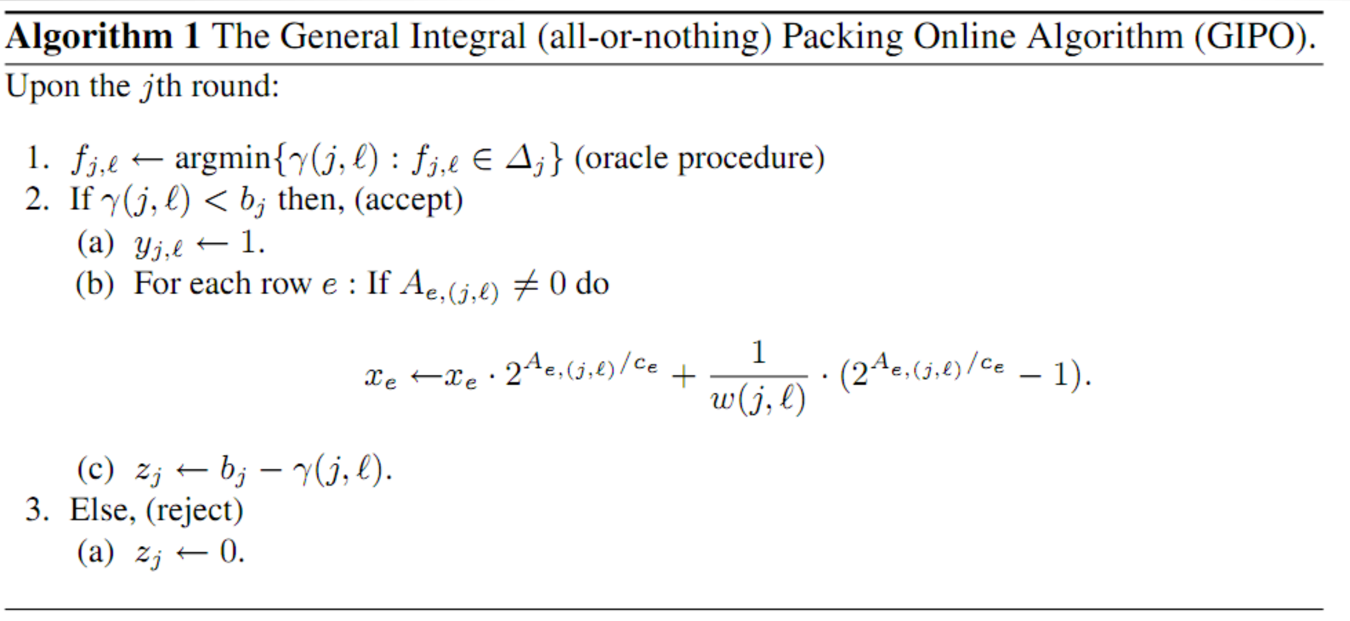
(I)

$$\begin{aligned} \max B_j^T \cdot Y_j \;\; s.t. \\ A_j \cdot Y_j \leq C \\ D_j \cdot Y_j \leq \mathbf{1} \\ Y_j \geq \mathbf{0} \end{aligned}$$

(II)

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

Algorithm



Algorithm 1 The General Integral (all-or-nothing) Packing Online Algorithm (GIPO).

```
Upon the $j$th round:

  1. $f_{j,\ell} \leftarrow \text{argmin}\{\gamma(j,\ell) : f_{j,\ell} \in \Delta_j\}$ (oracle procedure)
  2. If $\gamma(j,\ell) < b_j$ then, (accept)
     (a) $y_{j,\ell} \leftarrow 1$.
     (b) For each row $e$ : If $A_{e,(j,\ell)} \neq 0$ do
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     (c) $z_j \leftarrow b_j - \gamma(j,\ell)$.
  3. Else, (reject)
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```
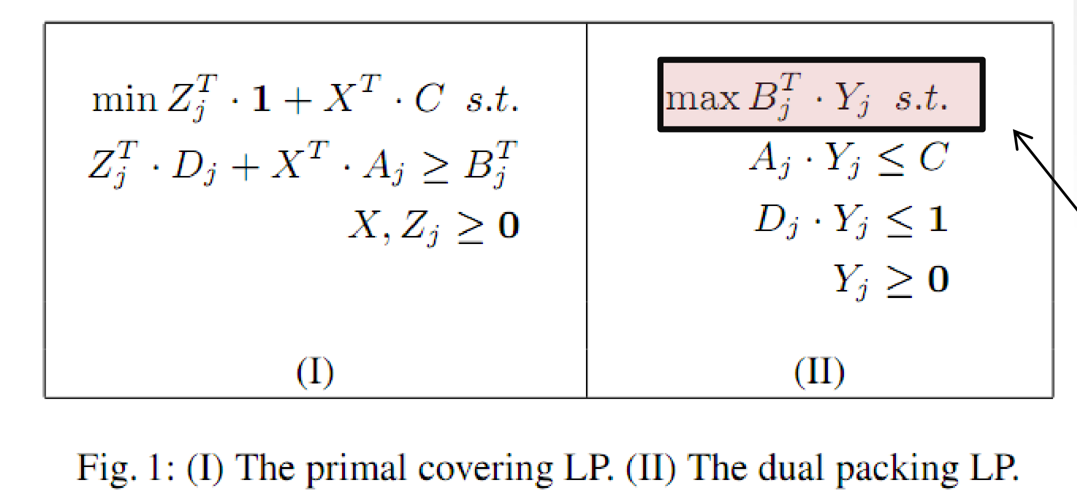

Primal and Dual

Applying Buchbinder&Naor



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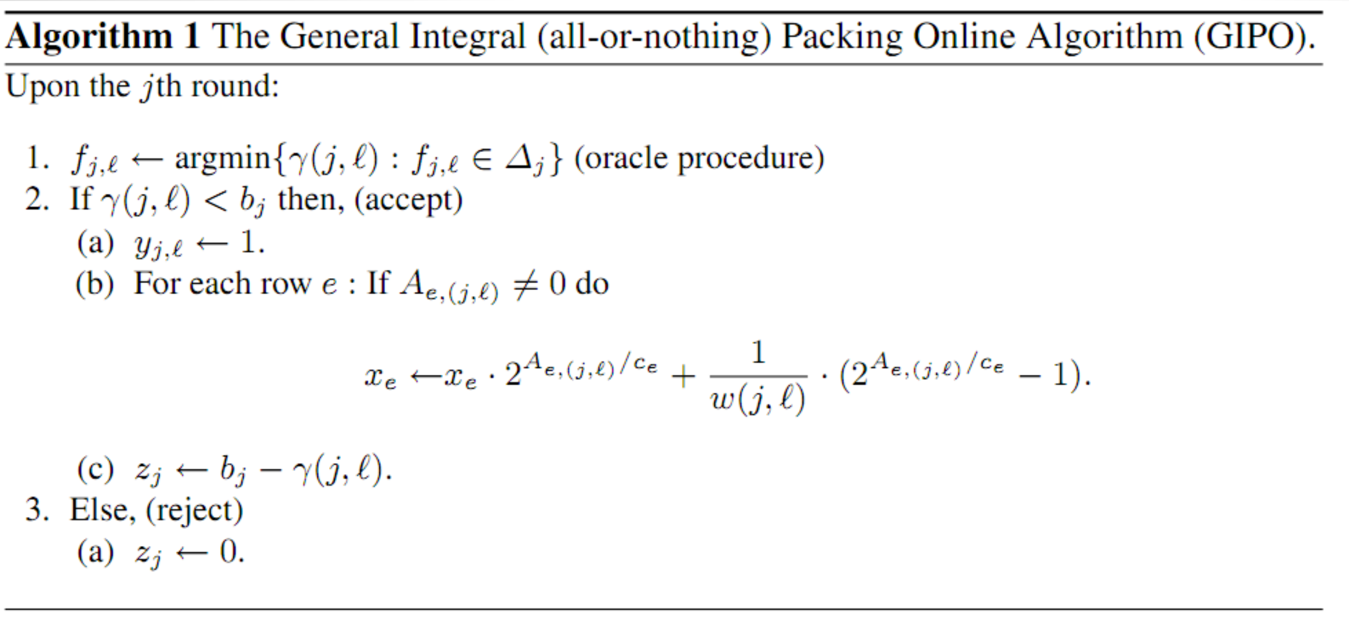
(II)

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

Formulate the packing (dual) LP: Maximize profit

(Note: dynamic LP!)

Algorithm



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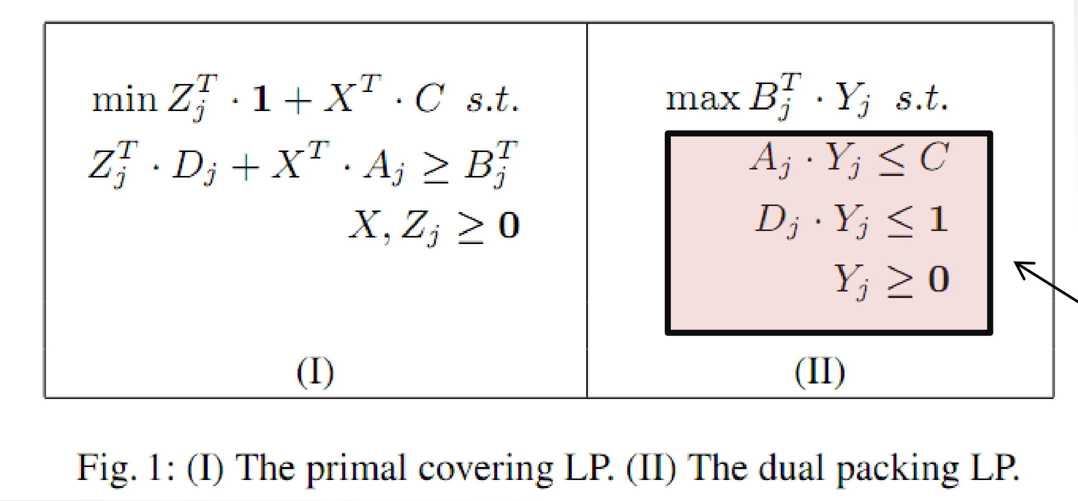
Primal and Dual

Applying Buchbinder&Naor



$$\min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t.$$
$$Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T$$
$$X, Z_j \geq \mathbf{0}$$

(I)

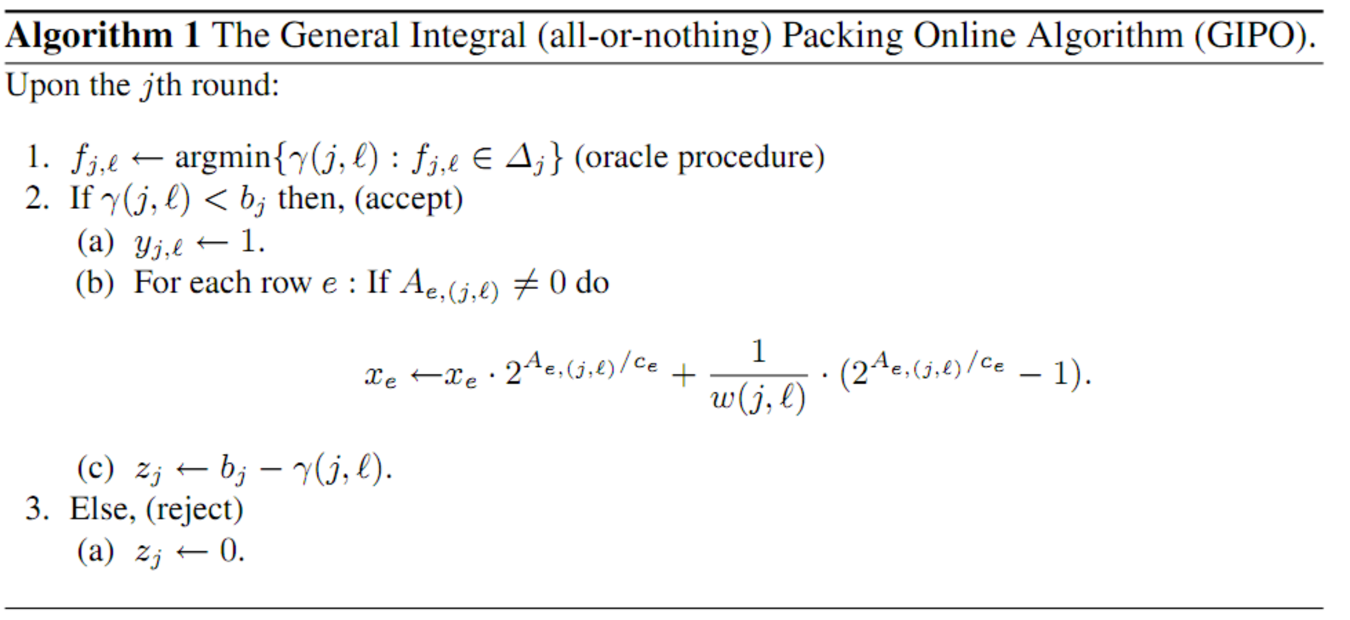
$$\max B_j^T \cdot Y_j \;\; s.t.$$
$$\boxed{\begin{aligned} A_j \cdot Y_j &\leq C \\ D_j \cdot Y_j &\leq \mathbf{1} \\ Y_j &\geq \mathbf{0} \end{aligned}}$$

(II)

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

s.t. constraints

Algorithm



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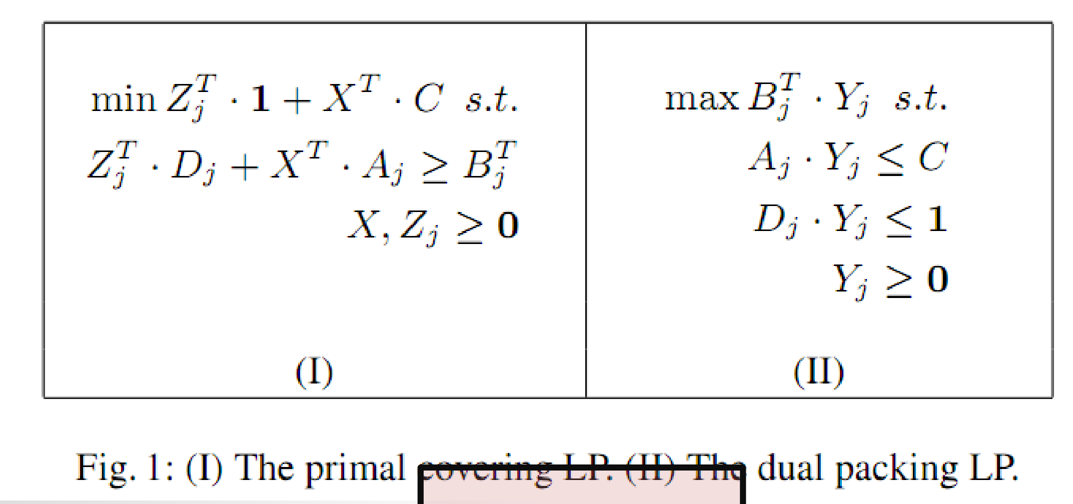
Primal and Dual

Applying Buchbinder&Naor



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(II)

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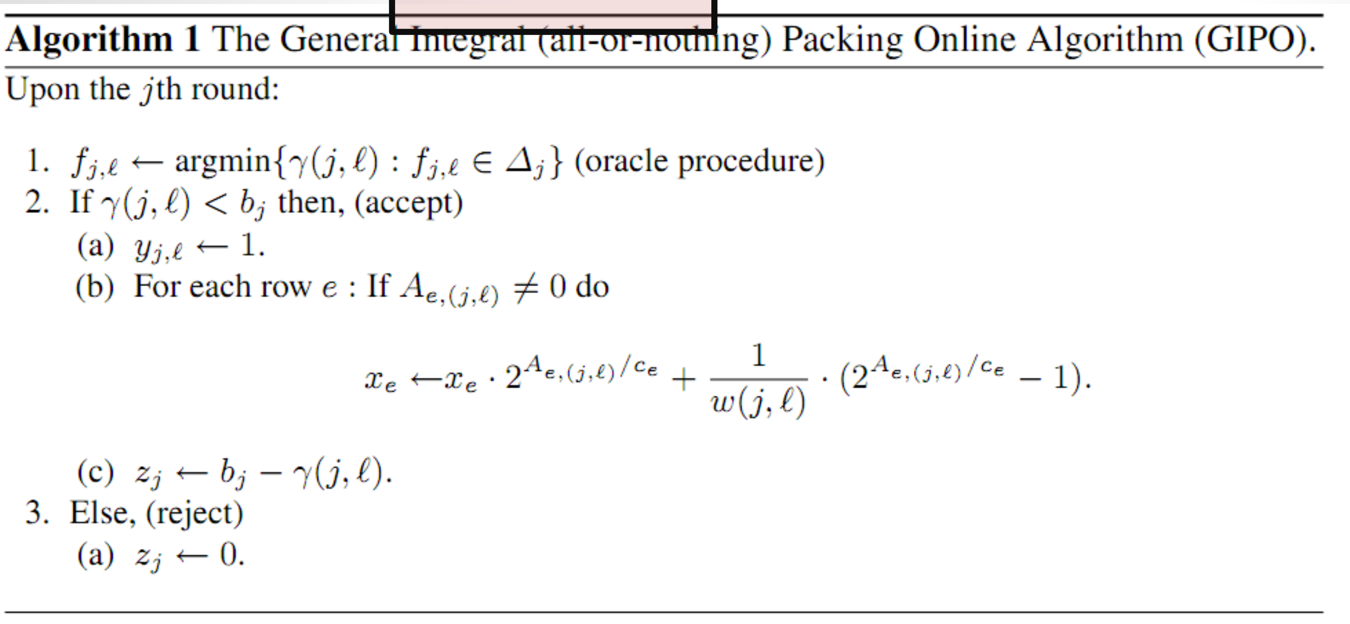
Buchbinder&Naor

Algorithm



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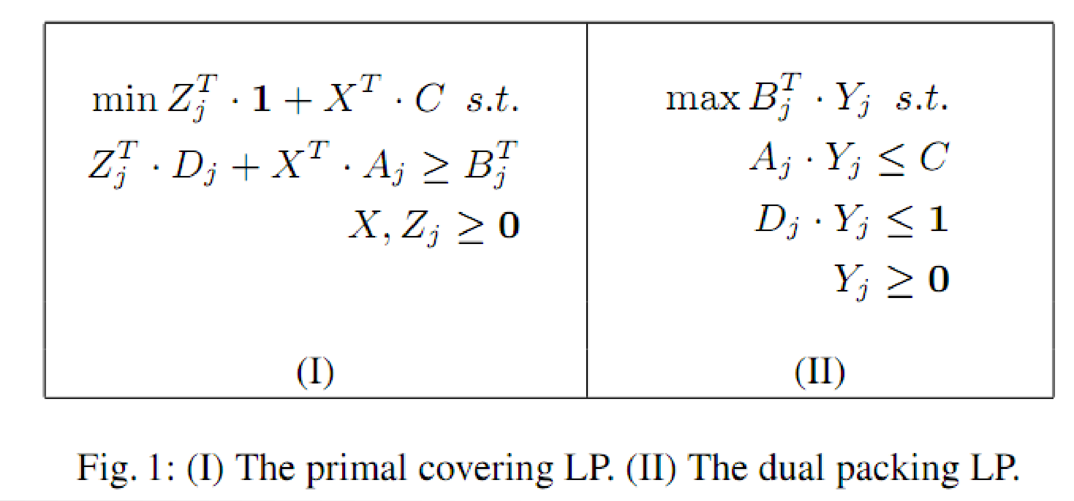
Primal and Dual

Applying Buchbinder&Naor



$\min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t.$ $Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T$ $X, Z_j \geq \mathbf{0}$	$\max B_j^T \cdot Y_j \;\; s.t.$ $A_j \cdot Y_j \leq C$ $D_j \cdot Y_j \leq \mathbf{1}$ $Y_j \geq \mathbf{0}$
(I)	(II)

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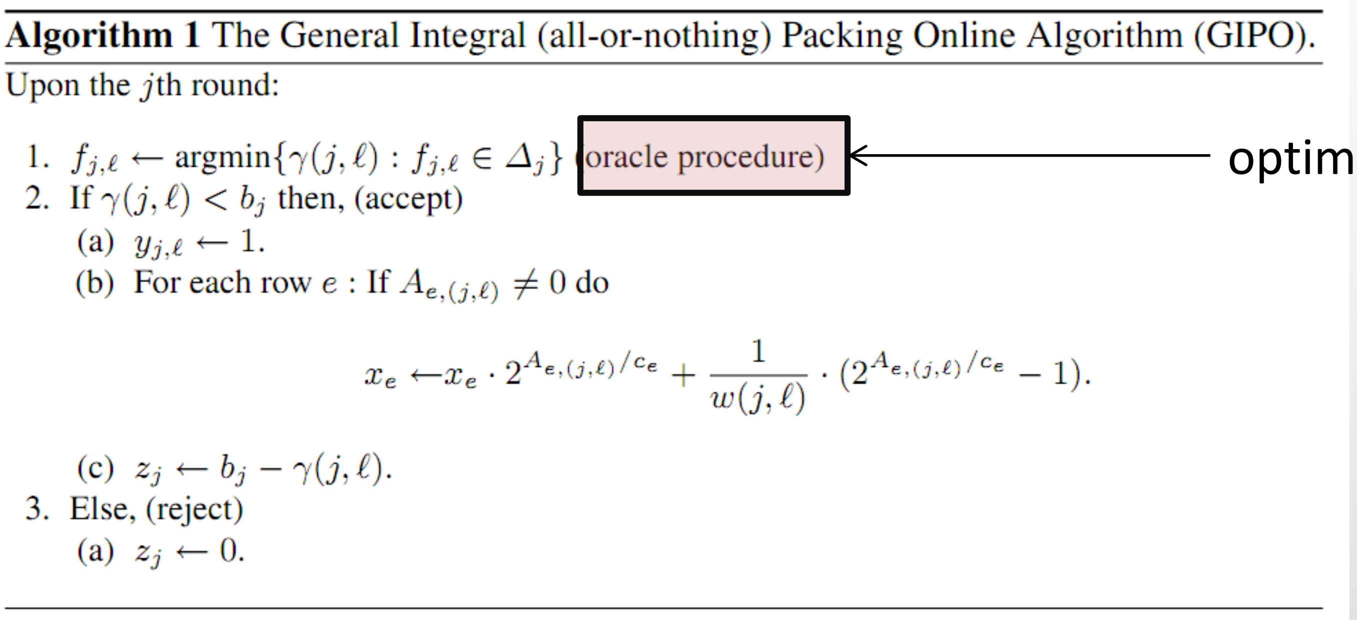
Algorithm



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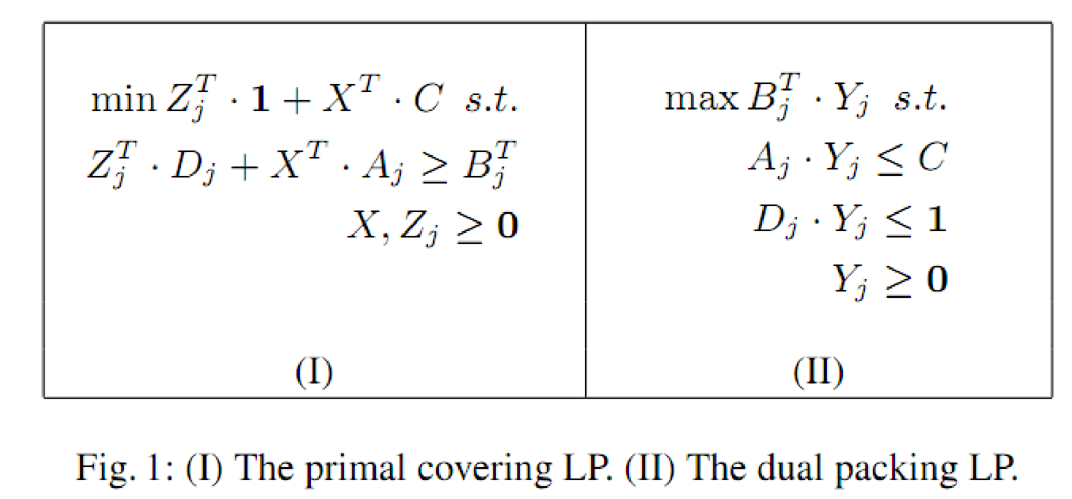
```

optimal embedding!

Primal and Dual

Applying Buchbinder&Naor



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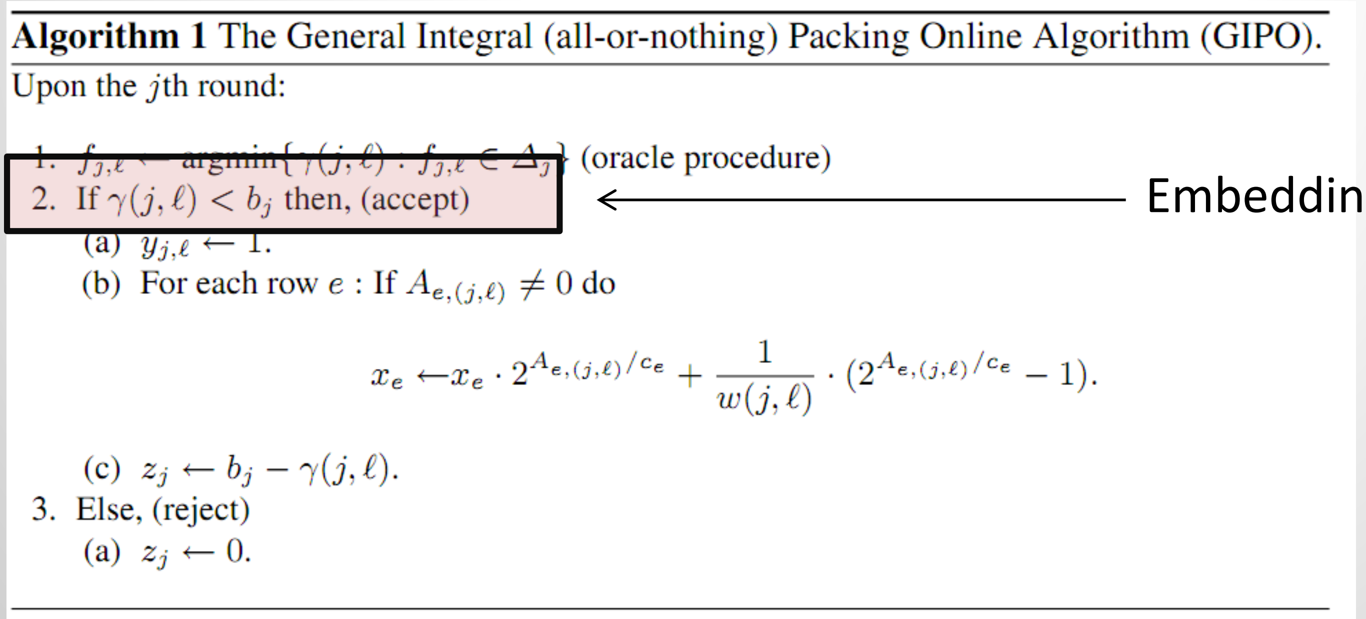
(I)

$$\begin{aligned} \max B_j^T \cdot Y_j \;\; s.t. \\ A_j \cdot Y_j \leq C \\ D_j \cdot Y_j \leq \mathbf{1} \\ Y_j \geq \mathbf{0} \end{aligned}$$

(II)

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

Algorithm



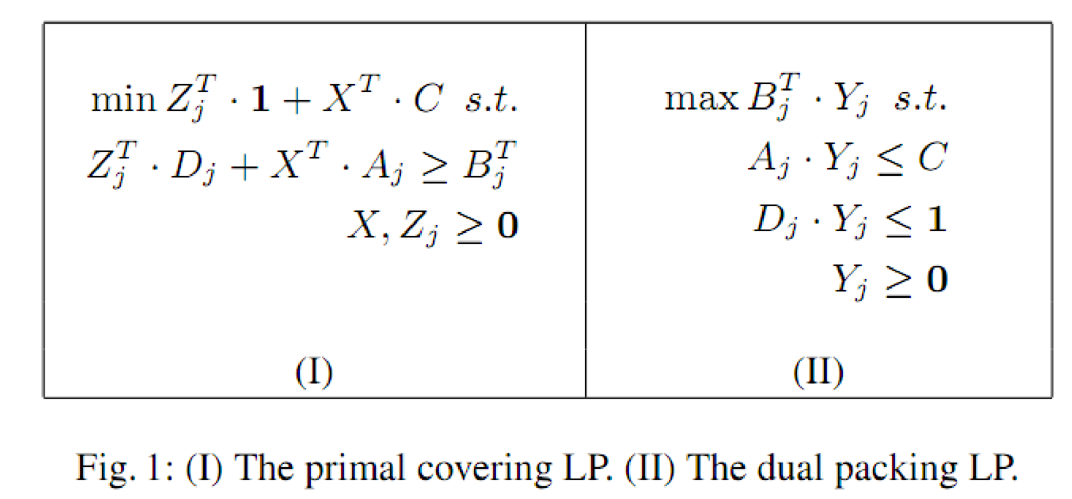
Algorithm 1 The General Integral (all-or-nothing) Packing Online Algorithm (GIPO).

```
Upon the $j$th round:
  1. $f_{j,\ell} \leftarrow \text{argmin}\{\gamma(j,\ell) : f_{j,\ell} \in \Delta_j\}$ (oracle procedure)
  2. If $\gamma(j,\ell) < b_j$ then, (accept)
     (a) $y_{j,\ell} \leftarrow 1$.
     (b) For each row $e$ : If $A_{e,(j,\ell)} \neq 0$ do
           $x_e \leftarrow x_e \cdot 2^{A_{e,(j,\ell)}/c_e} + \frac{1}{w(j,\ell)} \cdot (2^{A_{e,(j,\ell)}/c_e} - 1)$.
     (c) $z_j \leftarrow b_j - \gamma(j,\ell)$.
  3. Else, (reject)
     (a) $z_j \leftarrow 0$.
```

Embedding cost vs profit?

Primal and Dual

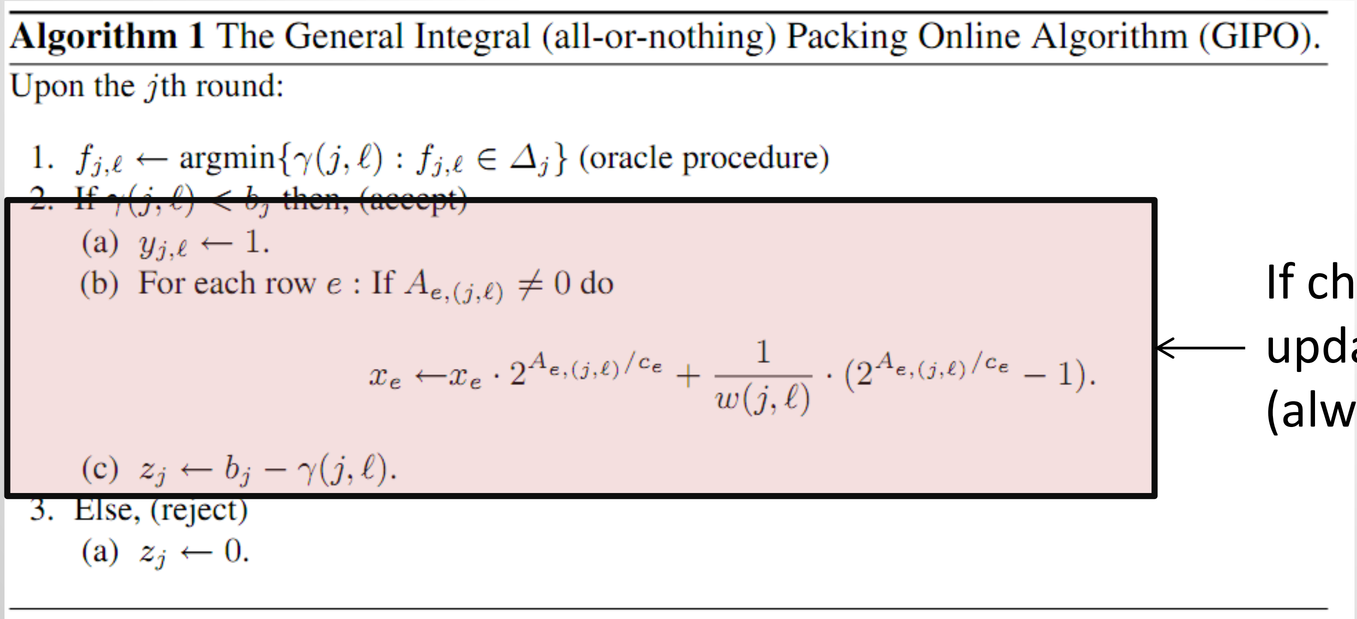
Applying Buchbinder&Naor



$$\begin{array}{r|r} \min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t. & \max B_j^T \cdot Y_j \;\; s.t. \\ Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T & A_j \cdot Y_j \leq C \\ X, Z_j \geq \mathbf{0} & D_j \cdot Y_j \leq \mathbf{1} \\ & Y_j \geq \mathbf{0} \\ \text{(I)} & \text{(II)} \end{array}$$

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

Algorithm



Algorithm 1 The General Integral (all-or-nothing) Packing Online Algorithm (GIPO).

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   (c) $z_j \leftarrow b_j - \gamma(j,\ell)$.
3. Else, (reject)
   (a) $z_j \leftarrow 0$.
```
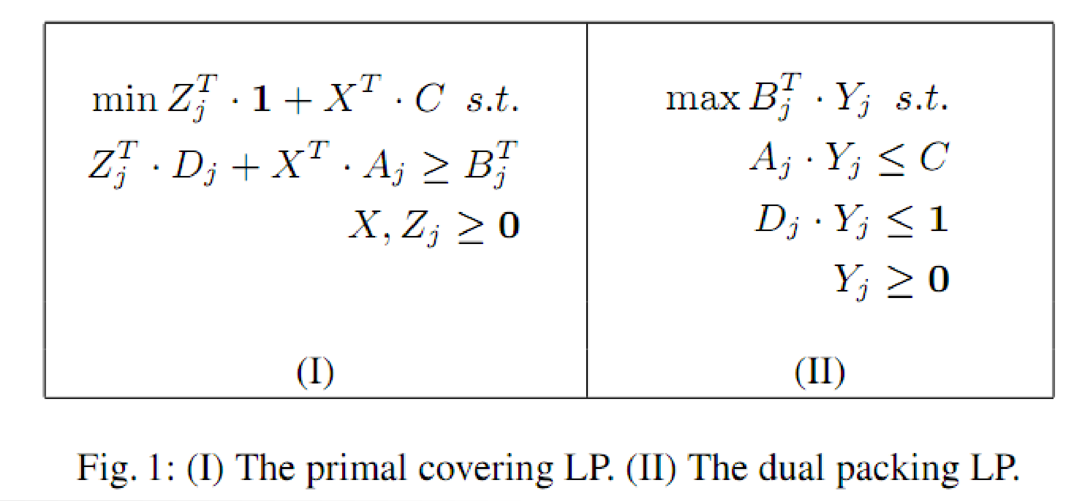

If cheap: accept and update primal variables (always feasible solution)

Primal and Dual

Applying Buchbinder&Naor



$$\min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t.$$
$$Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T$$
$$X, Z_j \geq \mathbf{0}$$

(I)

$$\max B_j^T \cdot Y_j \;\; s.t.$$
$$A_j \cdot Y_j \leq C$$
$$D_j \cdot Y_j \leq \mathbf{1}$$
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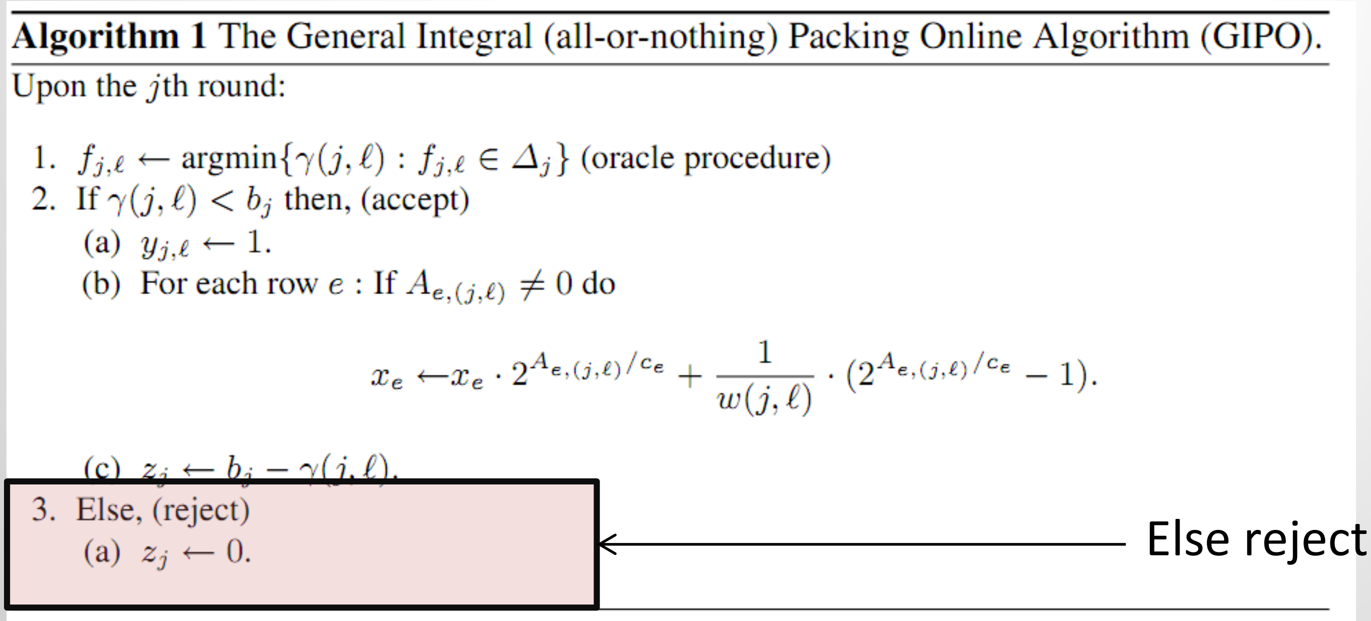
(II)

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

Algorithm



```
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  3. Else, (reject)
     (a) $z_j \leftarrow 0$.
```
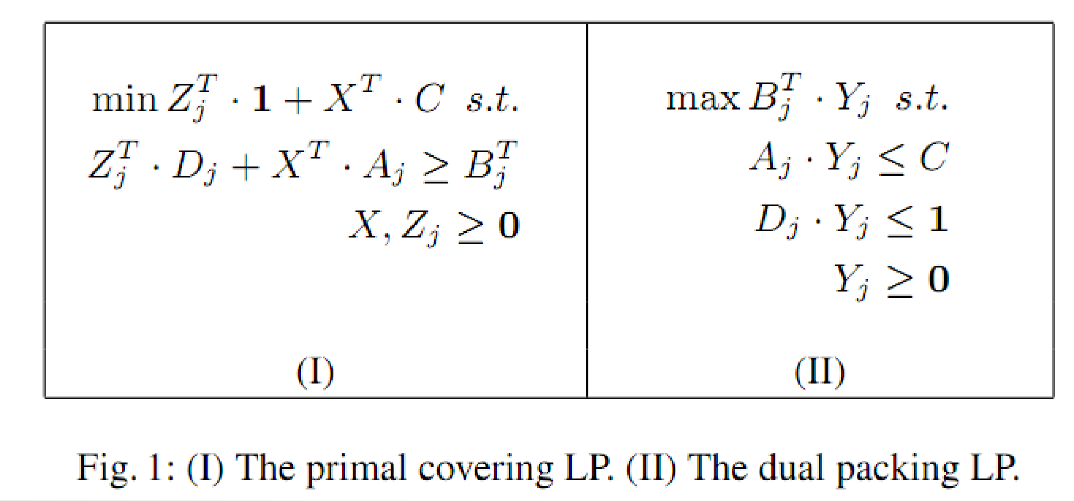

Else reject

Primal and Dual

Applying Buchbinder&Naor



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(I)

$$\begin{aligned} \max B_j^T \cdot Y_j \;\; s.t. \\ A_j \cdot Y_j \leq C \\ D_j \cdot Y_j \leq \mathbf{1} \\ Y_j \geq \mathbf{0} \end{aligned}$$

(II)

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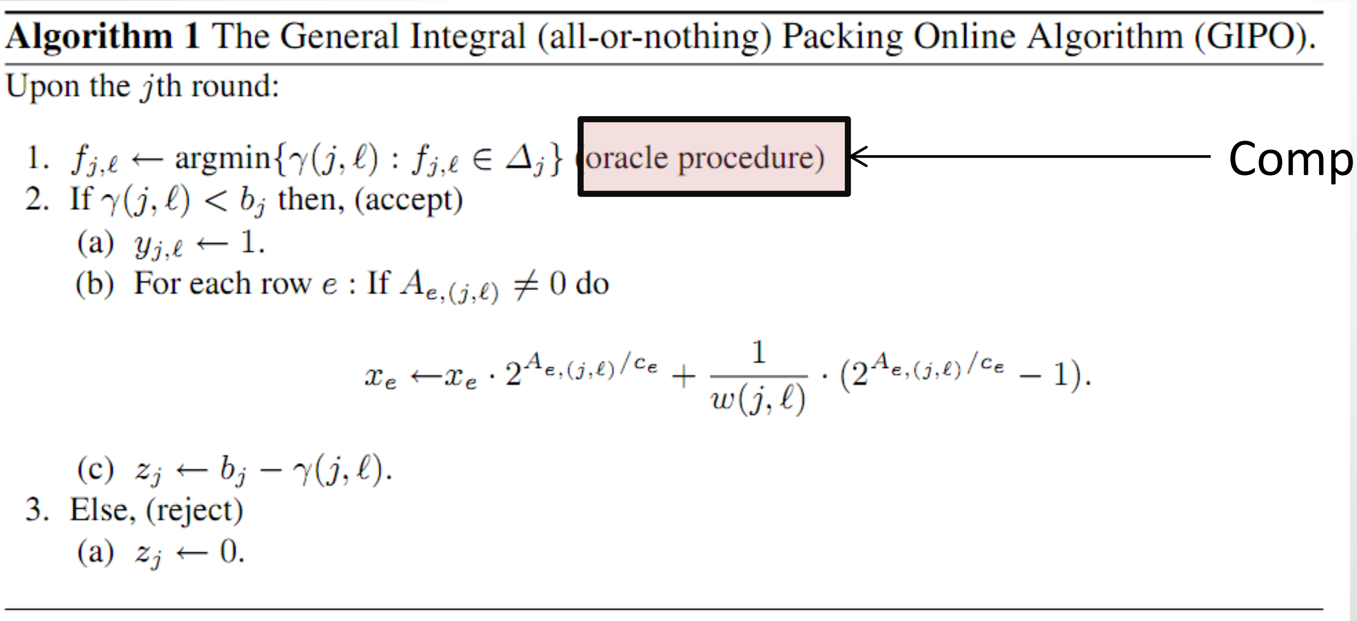
Algorithm



Algorithm 1 The General Integral (all-or-nothing) Packing Online Algorithm (GIPO).

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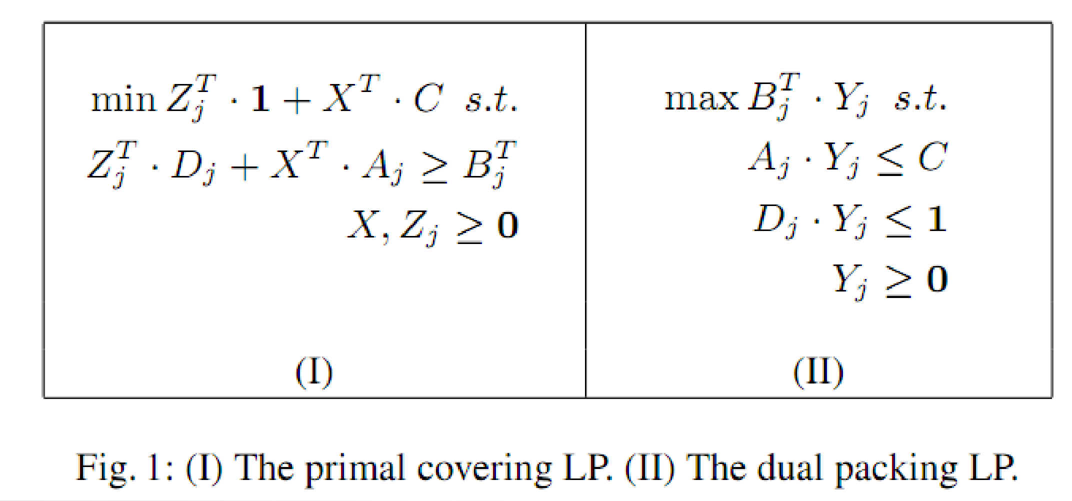
```

Computationally hard!

Primal and Dual

Applying Buchbinder&Naor



(I)	(II)
$\min Z_j^T \cdot \mathbf{1} + X^T \cdot C \;\; s.t.$ $Z_j^T \cdot D_j + X^T \cdot A_j \geq B_j^T$ $X, Z_j \geq \mathbf{0}$	$\max B_j^T \cdot Y_j \;\; s.t.$ $A_j \cdot Y_j \leq C$ $D_j \cdot Y_j \leq \mathbf{1}$ $Y_j \geq \mathbf{0}$

Fig. 1: (I) The primal covering LP. (II) The dual packing LP.

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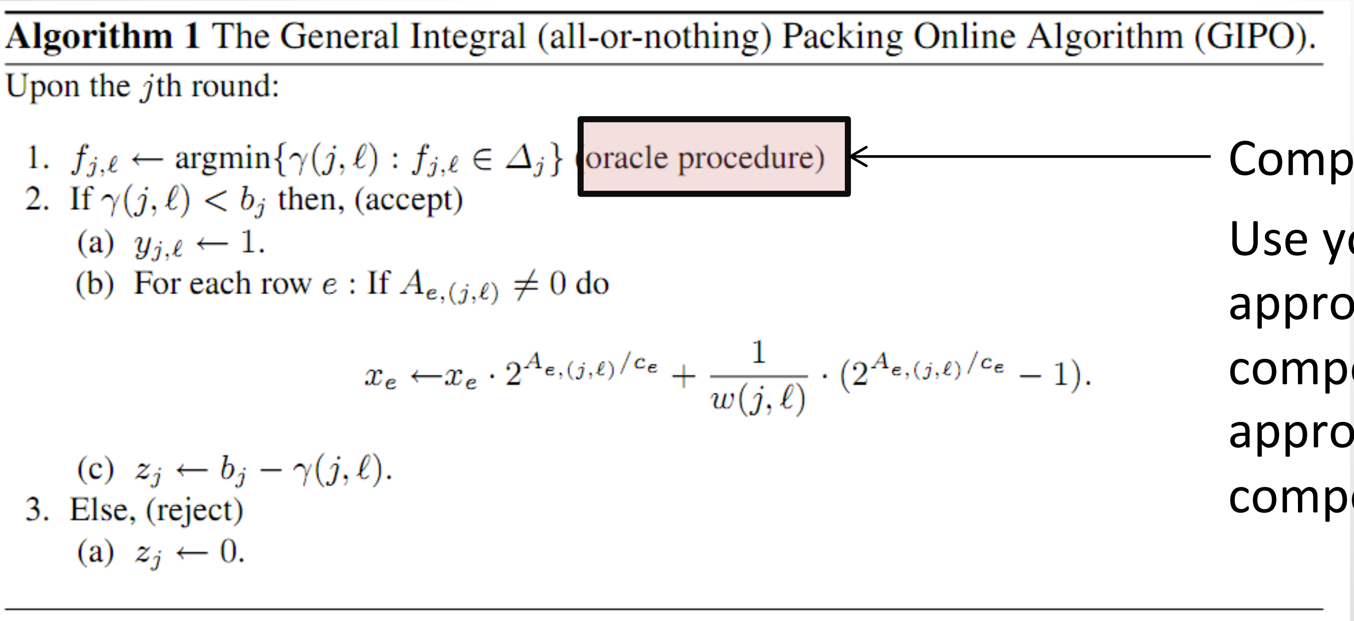

Algorithm 1 The General Integral (all-or-nothing) Packing Online Algorithm (GIPO).

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     (c) $z_j \leftarrow b_j - \gamma(j,\ell)$.
  3. Else, (reject)
     (a) $z_j \leftarrow 0$.
```

Computationally hard!

Use your favorite approximation algorithm! If competitive ratio ρ and approximation r, overall competitive ratio ρ*r.

Online VNet Admission Control

Algorithm comes in 2 flavors:

- Bicriteria guarantees: Obtain constant fraction of the optimal benefit while augmenting resources by a logarithmic factor.
- Fractional guarantees resp. limited resource consumption: The online algorithm achieves a logarithmic competitive ratio without resource augmentation, but either:
  - may serve a fraction of a request (associated benefit is assumed to be the same fraction of the benefit of the request)
  - the allowed traffic patterns of a request consumes at most a logarithmic fraction of every resource (in which case the algorithm rejects the request or fully embeds it)

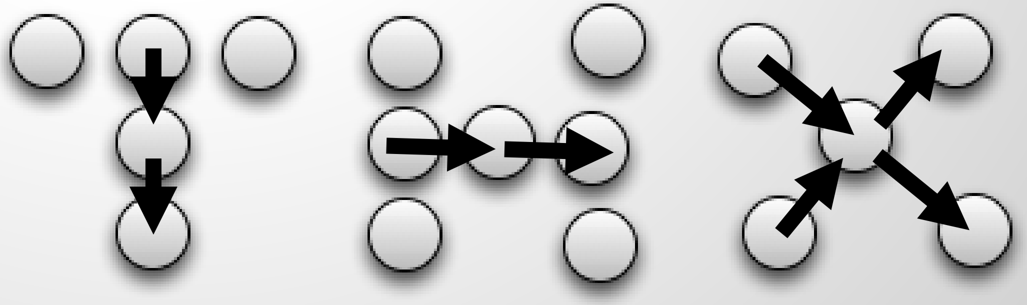
Summary

❑ Predictable performance requires isolation of all resources

❑ Static embeddings

❑ Reconfiguring embeddings

❑ Embeddings over time



Further Reading

Network Hypervisor Performance:

❑ Logically Isolated, Actually Unpredictable? Measuring Hypervisor Performance in Multi-Tenant SDNs Arsany Basta, Andreas Blenk, Wolfgang Kellerer, and Stefan Schmid. ArXiv Technical Report, May 2017.

Virtual Network Embedding:

❑ Beyond the Stars: Revisiting Virtual Cluster Embeddings Matthias Rost, Carlo Fuerst, and Stefan Schmid. ACM SIGCOMM Computer Communication Review (**CCR**), July 2015.

❑ Service Chain and Virtual Network Embeddings: Approximations using Randomized Rounding Matthias Rost and Stefan Schmid. ArXiv Technical Report, April 2016.

❑ Charting the Complexity Landscape of Waypoint Routing Saeed Akhoondian Amiri, Klaus-Tycho Foerster, Riko Jacob, and Stefan Schmid.
ArXiv Technical Report, May 2017

❑ Competitive and Deterministic Embeddings of Virtual Networks
Guy Even, Moti Medina, Gregor Schaffrath, and Stefan Schmid.
Journal Theoretical Computer Science (**TCS**), Elsevier, 2013.

Online Collocation:

❑ Online Balanced Repartitioning
Chen Avin, Andreas Loukas, Maciej Pacut, and Stefan Schmid.
30th International Symposium on Distributed Computing (**DISC**), Paris, France, September 2016.

❑ Competitive Clustering of Stochastic Communication Patterns on the Ring
Chen Avin, Louis Cohen, and Stefan Schmid.
5th International Conference on Networked Systems (**NETYS**), Marrakech, Morocco, May 2017

Network Design:

❑ Demand-Aware Network Designs of Bounded Degree
Chen Avin, Kaushik Mondal, and Stefan Schmid.
ArXiv Technical Report, May 2017.

Further Reading

Network Hypervis

Hypervisor performance and interference analysis

❑ Logically Isolated, Actually Unpredictable? Measuring Hypervisor Performance in Multi-Tenant SDNs Arsany Basta, Andreas Blenk, Wolfgang Kellerer, and Stefan Schmid. ArXiv Technical Report, May 2017.

Virtual Network Embedding:

❑ Beyond the S ngs Matthias R CM SIGCOMM Computer Communication Review (**CCR**), July 2015.

Star vs Hose embedding

Including decomposability (study of integrality gap)

❑ Service Chain and Virtual Network Embeddings: Approximations using Randomized Rounding Matthias Rost and Stefan Schmid. ArXiv Technical Report, April 2016.

❑ Charting the Complexity Landscape of Waypoint Routing Saeed Akhoondian Amiri, Klaus-Tycho Foerster, Riko Jacob, and Stefan Schmid.
ArXiv Technical Report, May 2017

Relationship to disjoint paths

❑ Competitive and Deterministic Embeddings of Virtual Networks
Guy Even, Moti Medina, Gregor Schaffrath, and Stefan Schmid.
Journal Theoretical Computer Science (**TCS**), Elsevier, 2013.

Online primal-dual

Online Collocation:

❑ Online Balanced Repartitioning
Chen Avin, Andreas Loukas, Maciej Pacut, and Stefan Schmid.
30th International Symposium on Distributed Computing (**DISC**), Paris, F

Adaptive VM migration to minimize communication costs

❑ Competitive Clustering of Stochastic Communication Patterns on the Ring
Chen Avin, Louis Cohen, and Stefan Schmid.
5th International Conference on Networked Systems (**NETYS**), Marrakech, Morocco, May 2017

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A model and analysis motivated by ProjectToR