Low-Dimensional Linear Programming with Violations

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Two decades ago, Megiddo and Dyer showed that linear programming in 2 and 3 dimensions (and subsequently, any constant number of dimensions) can be solved in linear time. In this paper, we consider linear programming with at most k violations: finding a point inside all but at most k of n given halfspaces. We give a simple algorithm in 2-d that runs in $O((n+k^2)\log n)$ expected time; this is faster than earlier algorithms by Everett, Robert, and van Kreveld (1993) and Matousek (1994) and is probably near-optimal for all $k << n/2$. A (theoretical) extension of our algorithm in 3-d runs in near $O(n + k^{11/4}n^{1/4})$ expected time. Interestingly, the idea is based on concave-chain decompositions (or covers) of the $\leq k$-level, previously used in proving combinatorial k-level bounds.

Applications in the plane include improved algorithms for finding a line that misclassifies the fewest among a set of bichromatic points, and finding the smallest circle enclosing all but k points. We also discuss related problems of finding local minima in levels.