Keynote Lecture on

Elements of Computational Metrology

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Abstract

Introduction

The past decade has seen the emergence of computational metrology as a separate discipline. It deals with fitting and filtering discrete geometric data that are obtained by measurements made on manufactured parts. It plays an important role in manufacturing industry. A manufactured part may be measured to characterize the manufacturing process that produced it or to assess the conformance of the part to designer-specified tolerances. In either case, measurements are made on the surface of the part and the measured data are then reduced to a few numbers or attributes by increasingly sophisticated computational techniques.

Before we proceed further, it is useful to establish some basic facts about manufacturing and measurement. These are best captured in the following two axioms.

- *Axiom of manufacturing imprecision*: All manufacturing processes are inherently imprecise and produce parts that vary.

In fact, this axiom of manufacturing imprecision contains a hidden primal fact that no man-made artifact has ideal geometric form. That is, no manufactured object can be perfectly cylindrical or perfectly spherical and so on. There is increasing experimental evidence that the geometry of manufactured surface behaves more like a *fractal*. This means that fractals offer a better approximation to manufactured surfaces than smooth surfaces. So, at least conceptually, a manufactured surface can be modeled as a fractal set and the only information we can obtain about it comes from a discrete set of points sampled on that surface.

- *Axiom of measurement uncertainty*: No measurement can be absolutely accurate and with every measurement there is some finite uncertainty about the measured attribute or measured value.

The measurement uncertainty axiom adds a sobering realization to our computational endeavor. If we take the discrete set of measurements as input to our computations, it is good to remember that these input values can never be taken as absolutely accurate. It is equally important to remember that results of our computations should be accompanied by statements about their uncertainty, which is partly inherited from the input uncertainty and partly attributable to the computational scheme itself.

It should be noted that these two axioms are independent. It is possible to build a modeling framework where manufacturing is imprecise but measurements have negligible uncertainty, and vice versa. But to model reality more closely, both axioms should be considered operative. With these preliminaries, we can now proceed to address fitting and filtering problems.

Fitting

We have seen already that no manufactured surface can have the platonic ideal form. Fitting is the task of associating ideal geometric forms to non-ideal forms (such as, for example, discrete set of points sampled on a manufactured surface). Engineers are interested in fitting for the following reasons:

- *Datum establishment*: Datum is a reference geometric object of ideal form established on one or more non-ideal geometric forms on a manufactured part. Datums are used for relative positioning of geometric objects in parts and assemblies of parts.
- *Deviation assessment*: It is often important to determine how far a manufactured surface has deviated from its intended ideal geometric form. This deviation can be quantified by fitting.

Historically such fitting was accomplished by the use of surface plates, collates, mandrels, and specialized measurement fixtures. More recently, manufacturing industry has started using modern measurement devices such as Coordinate Measuring Machines (CMM). This has accelerated the use of fitting by computation. Its initial success is placing increasingly complex demands on our ability to compute.

The computational scheme used for fitting is one of optimization. For example, it may be of interest to fit a plane to a set of points in space such that the sum of the squares of the distances of the points from the plane is minimized. This can be recognized as a least squares fitting problem. Such problems have been studied in science for over two centuries and we can draw from this wealth of knowledge to find satisfactory solutions. But there are also other seemingly simple fitting problems that can tax our computational skills. For example, an engineer may want to find the smallest cylinder (that is, a cylinder with the smallest diameter) that encloses a set of points in space because it gives him some quantitative information about how a part will fit in an assembly. This can be easily posed as a minimization problem, but computational methods to solve this problem are not simple.

It is customary to divide the fitting problems broadly into the following two categories on the basis of the objective function that is optimized.

- *Least squares fitting*: Here the objective is to find an ideal geometric object (a smooth curve or surface) that minimizes the sum of squared deviations of data points from this object. It includes linear least squares, total least squares, and non-linear least squares techniques.
- *Chebyshev fitting*: Here the objective is to minimize the maximum deviation. Some of these fitting problems have been studied by discrete and computational geometers in the last twenty years. This has added some valuable insight in designing algorithms to solve such problems.

National and international standards groups are actively working on standardized definitions for the objective functions and constraints for the fitting problems.

Finally, it may be added parenthetically that fitting is one of the most direct and practical means of educating engineering students in optimization and computational geometry.

Filtering

Filtering is the task of obtaining scale-dependent information from measured data. At a more mundane level, filtering can be used to remove noise and other unwanted information from the measured data. In the context of engineering metrology, engineers are interested in filtering mainly for the following two reasons.

- *Surface roughness*: Many engineering functions depend on how rough or smooth a piece of surface is. Designers define bounds on certain roughness parameters obtained by observation on rather small scale to ensure functionality of parts. These small-scale variations are subtracted from the surface measurement data before form and other deviations are assessed.
- *Manufacturing process diagnosis*: Manufacturing processes leave tool marks on surfaces. By measuring surfaces at fine scale, it is possible to track the detect tool erosion and its effect on the surface quality.

Historically, filtering techniques were pioneered by communication theorists. Developments in analog and digital signal processing strongly influenced how filtering was carried out in surface metrology. More recently, developments in digital image processing have been influencing computational surface metrology.

The computational scheme used for filtering is one of convolution. Engineers use the following two types of convolutions.

- *Convolution of functions*: Filtering is often implemented as discrete convolution of functions. In the most popular version, the measured data is convolved with the Gaussian function. It has a smoothing effect on the surface data.
- *Convolution of sets*: Morphological filters are implemented using Minkowski sums. These can be regarded as convolutions where the input set is convolved with a circular or flat structuring element.

Recent developments in wavelets seem to indicate that the scale-dependent information can be processed more effectively using wavelet filters.