

# Computational Geometric Techniques for Sculptured Surface Manufacturing and CAD/CAM

Yuan-Shin Lee, Ph.D., P.E.  
North Carolina State University  
Raleigh, NC 27695-7906  
U. S. A.

E-mail: [yslee@ncsu.edu](mailto:yslee@ncsu.edu)

<http://www.ie.ncsu.edu/yslee>

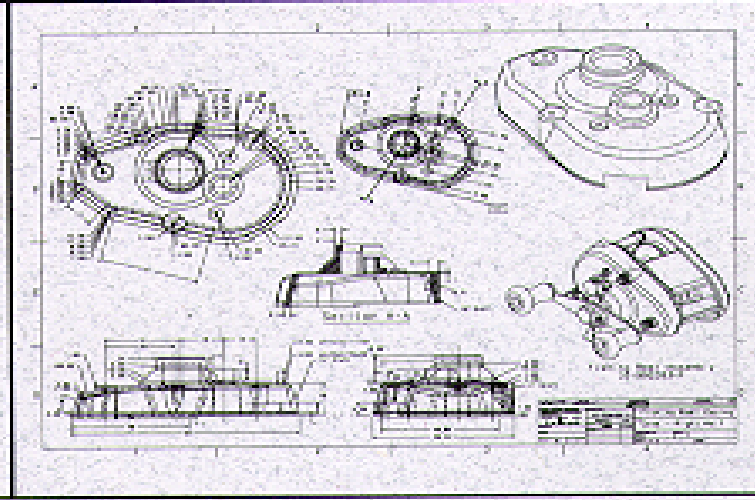
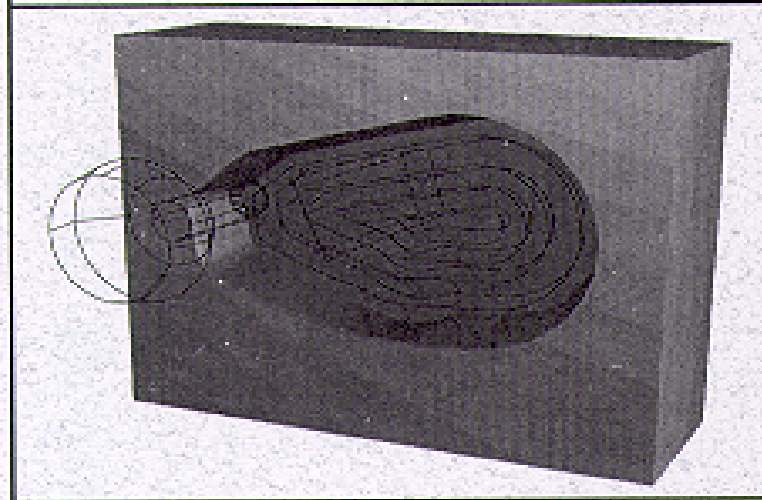
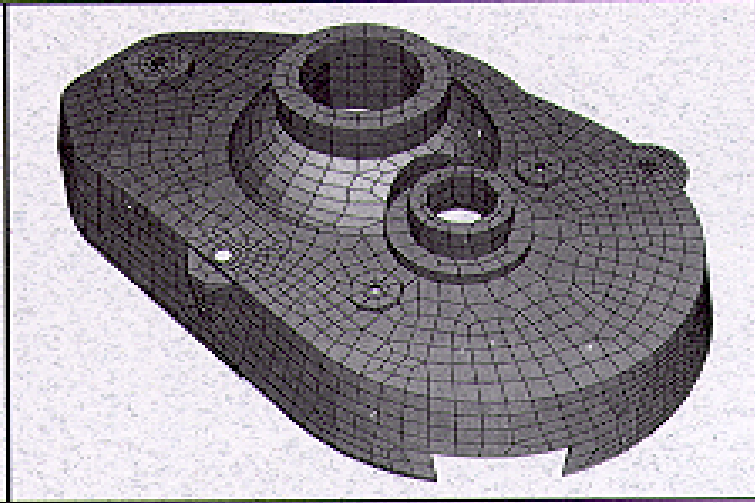
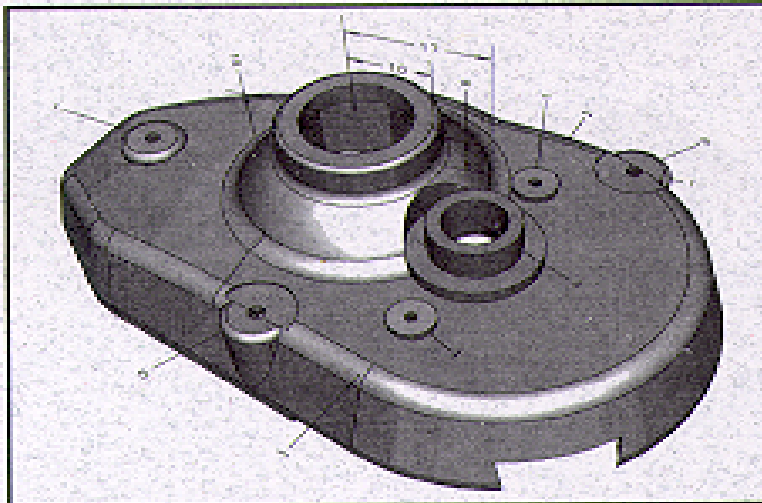
October 7, 2003

# Outlines

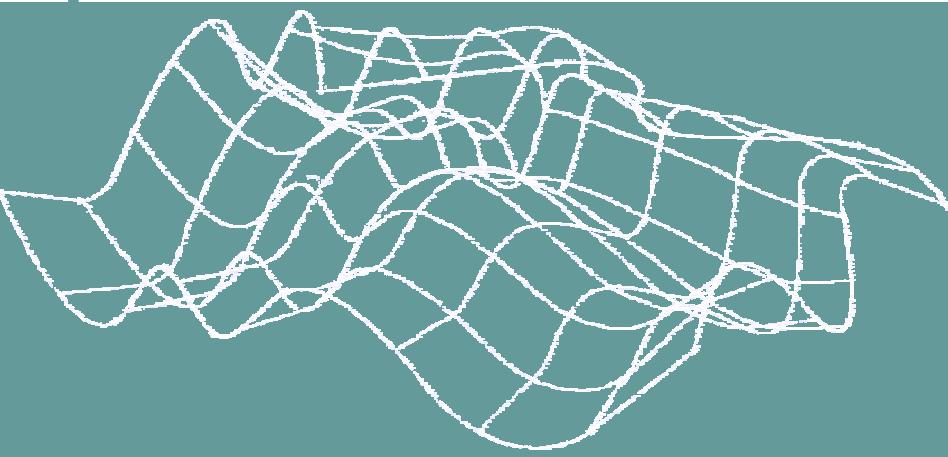
- Introduction of Sculptured Surface Machining (SSM)
- CAD/CAM for Polyhedral Model Machining
- 5-Axis Tool Path Generation in CAD/CAM
- Machining Potential Field (MPF) for Complex Surface Manufacturing
- High Speed Machining (HSM) of Sculptured Surfaces
- Constant Material Removal Rate for HSM
- Adaptive Feedrate Scheduling for HSM
- Conclusions

# **1. Introduction of Sculptured Surface Machining (SSM)**

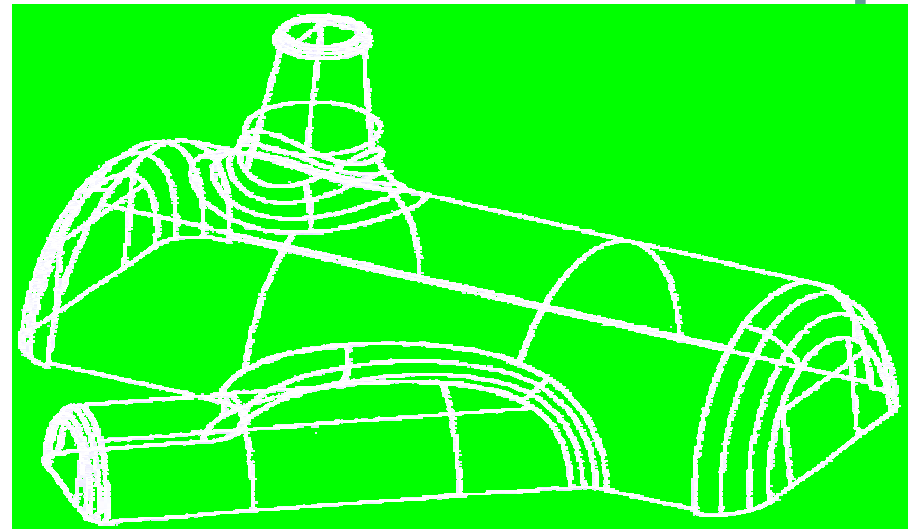
# Product Design with Sculptured Surfaces



# NURBS Surface and Applications



*The NURBS surface interpolating four boundary curves.*

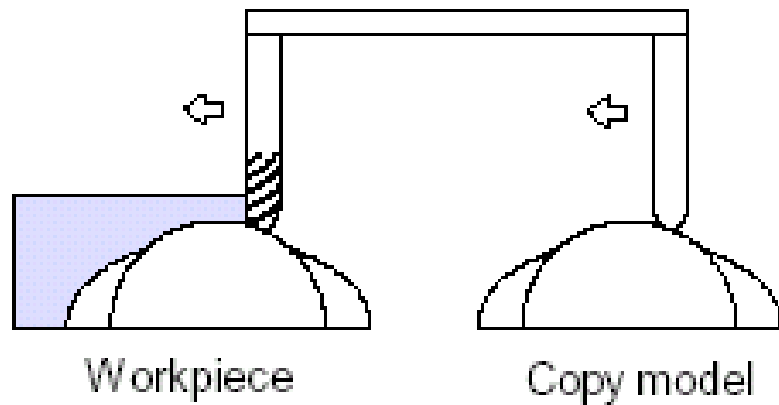


*NURBS surface of the core pattern*

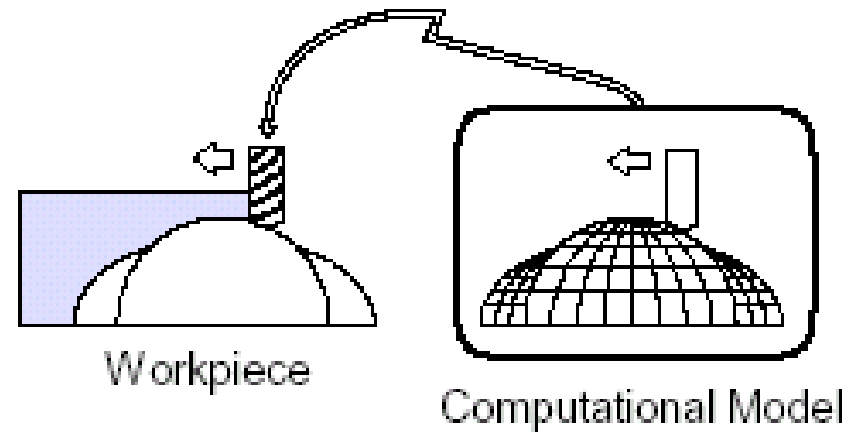
# Product Geometric Modeling and Manufacturing

- **Conceptual model:**
- **Physical model: clay model**
- **Descriptive model : engineering drawing**
- **Mathematical model:**
- **Computational model:**
  - Wireframe model**
  - Surface model**
  - Solid mode**
  - Non-manifold model**

# Introduction to Sculptured Surface Machining (SMM)



Copy milling



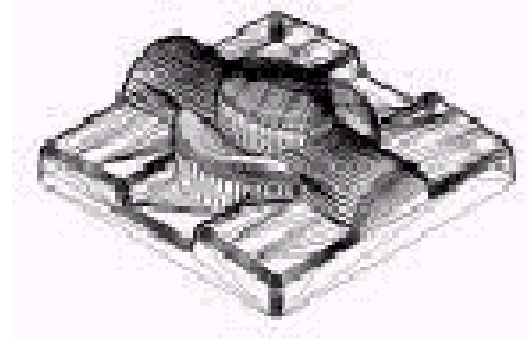
NC milling

## **2. CAD/CAM for Polyhedral Model Machining**



# Polyhedral Models and NC Machining

Polyhedral model



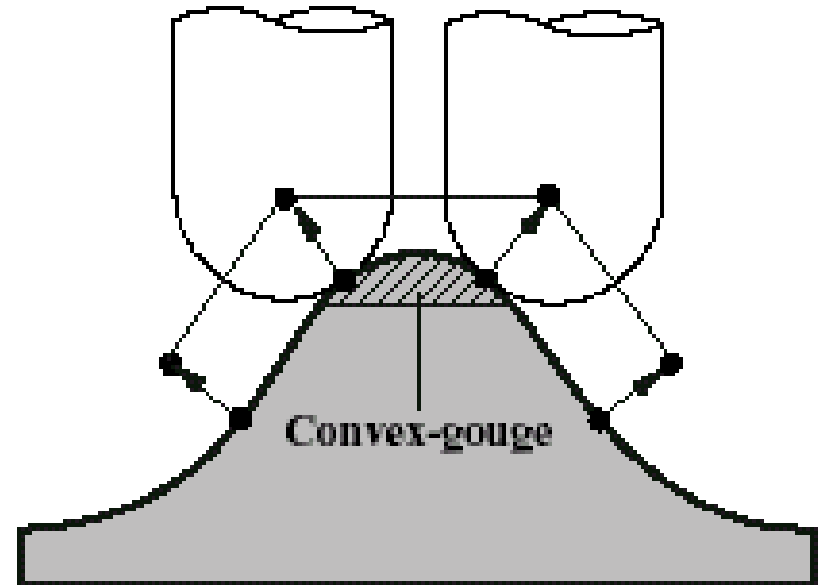
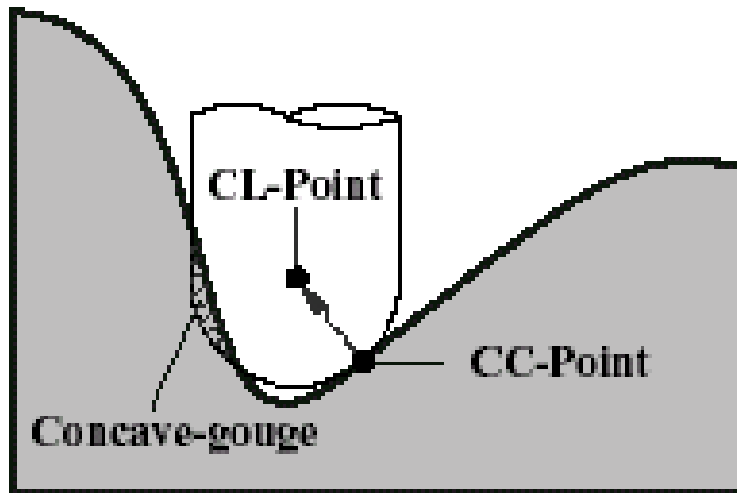
A new curve-based  
polyhedral machining approach



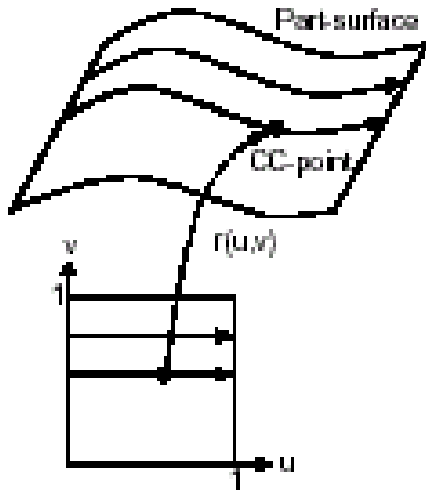
Gouge-free tool paths



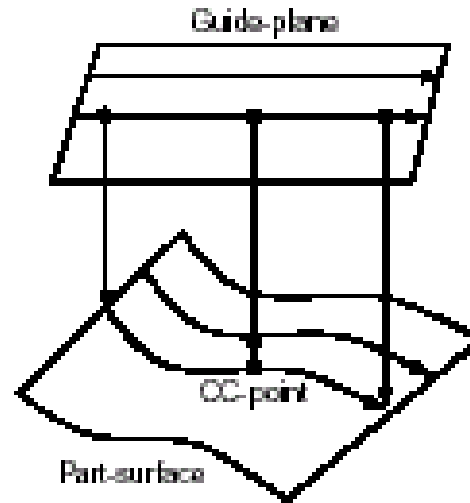
# Cutter Gouging Problems in Sculptured Surface Machining



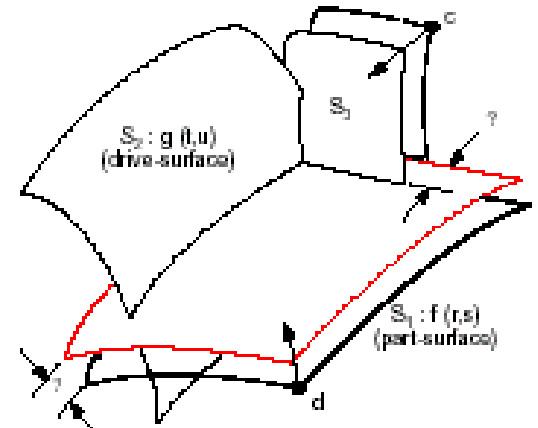
# NC Cutter Path Generation Methods



1. CC-based path  
(Iso-parametric)

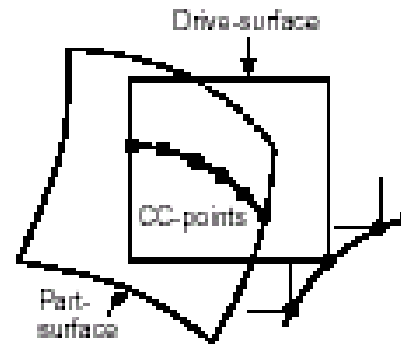


2. CC-Cartesian path



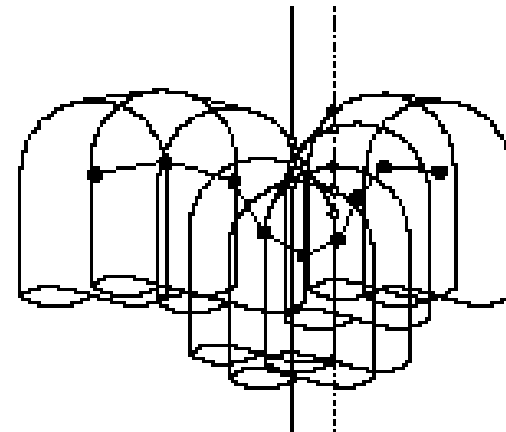
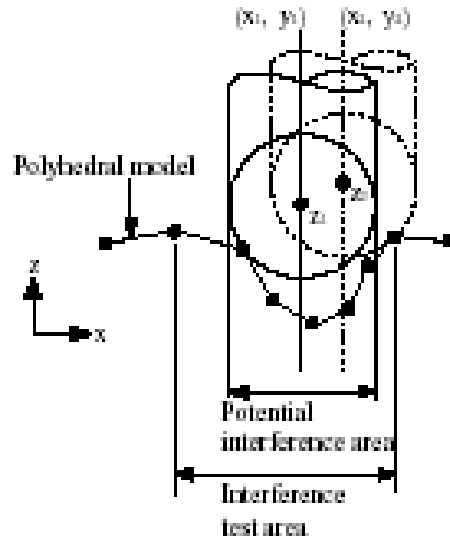
3. CL-based path (offset)

4. APT-type path

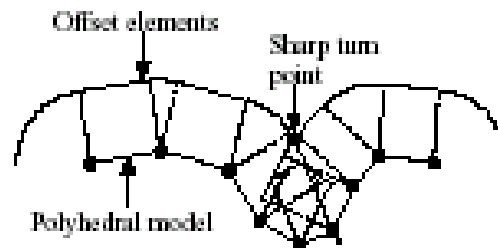


# Offset of Polygon for Cutter Location (CL)

## Point-based approach

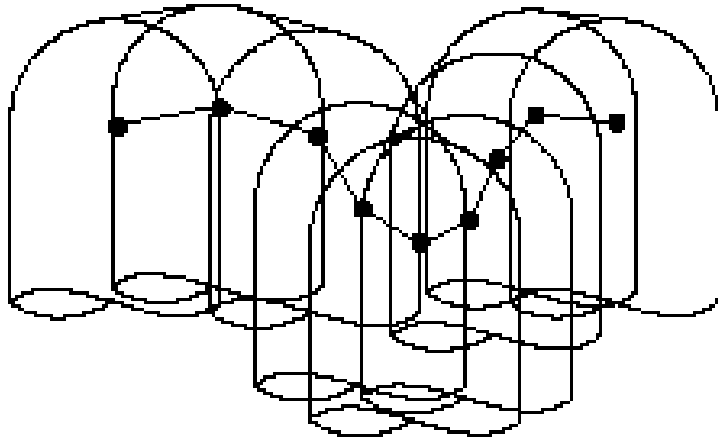


## Curve-based approach

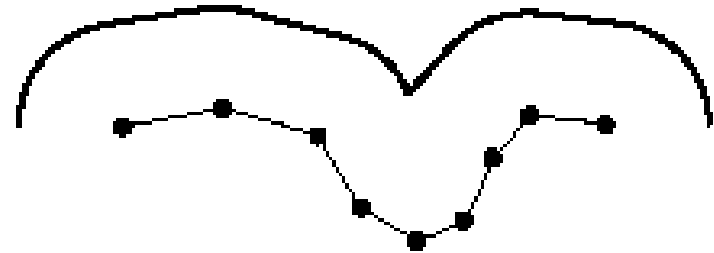


# Three Schemes of Polyhedral Offsetting

? Individual offsetting scheme



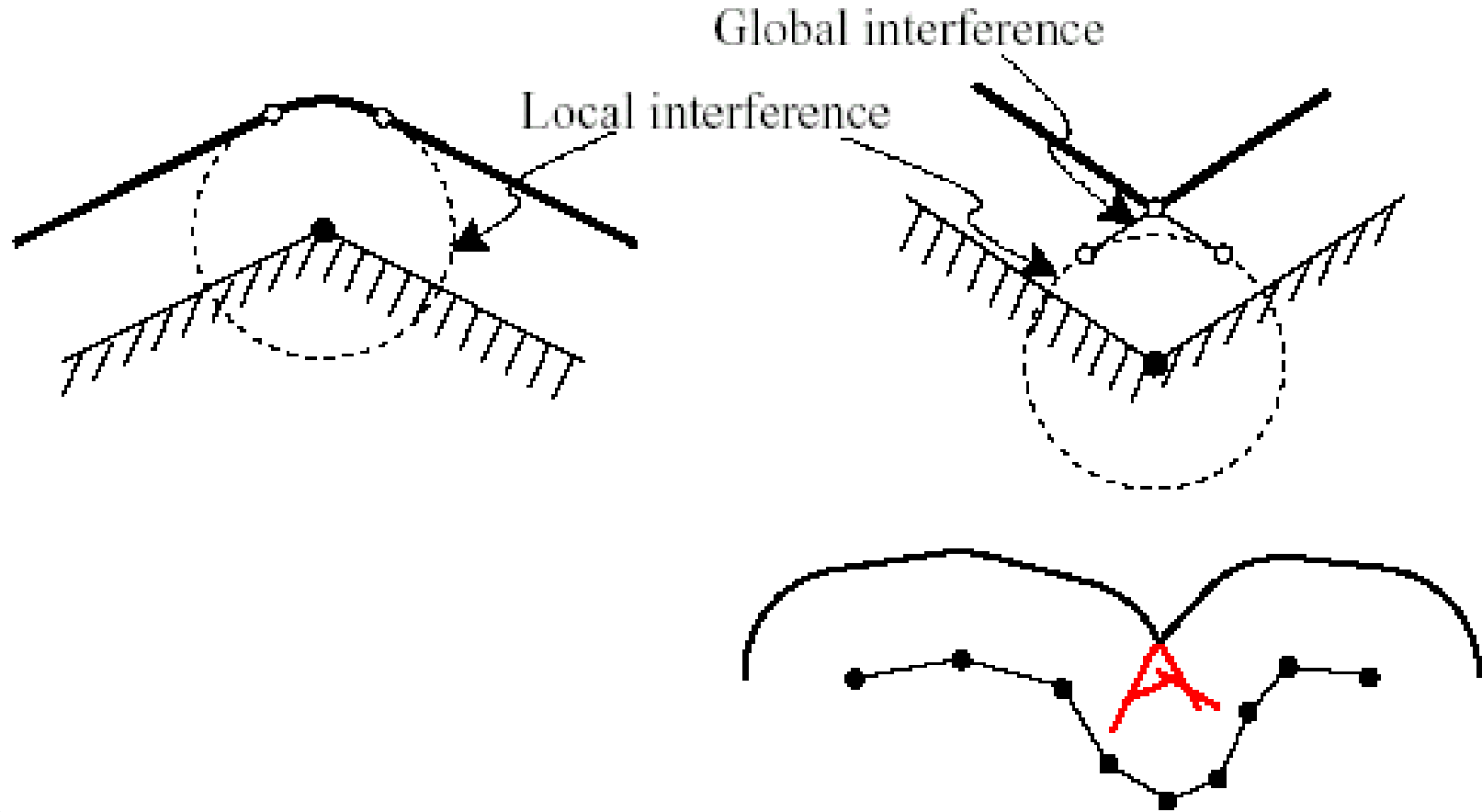
? Global-offsetting scheme



? Local-offsetting scheme

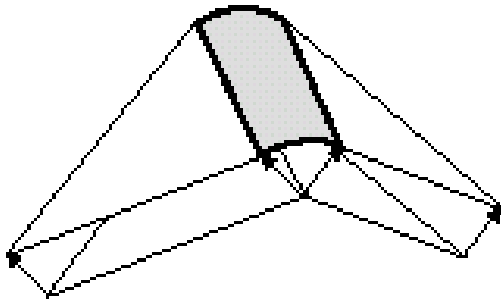


# Deleting Interference to Avoid Gouging

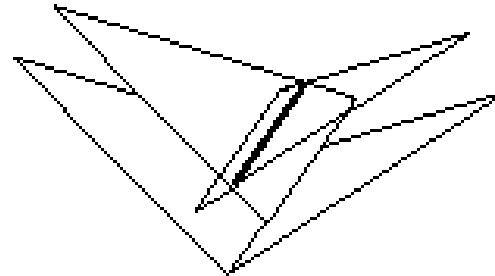


# Offset of Triangles and Edges of Polyhedral Models

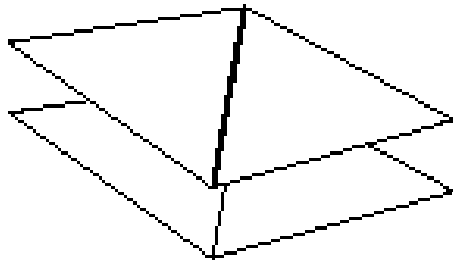
? Convex edge



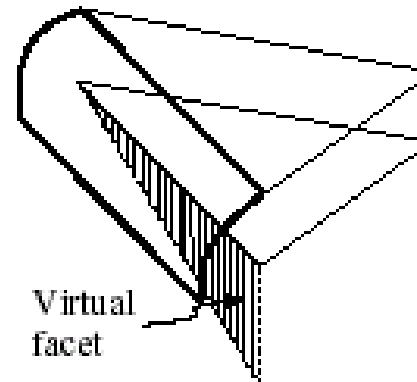
? Concave edge



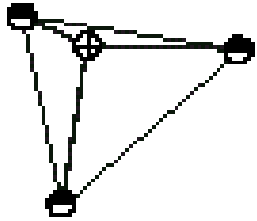
? Flat edge



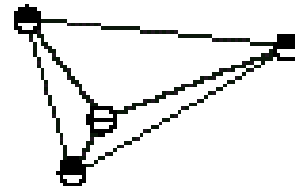
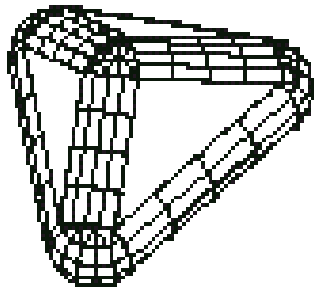
? For a boundary edge



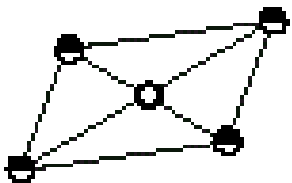
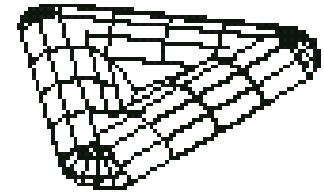
# Offset of Vertex in Polyhedral Models



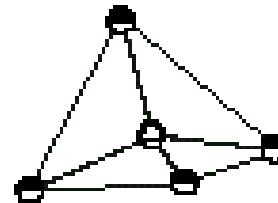
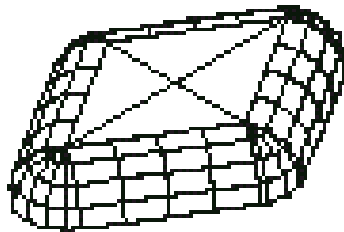
CONVEX



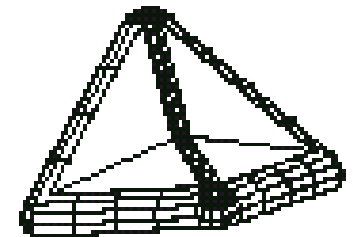
CONCAVE



FLAT

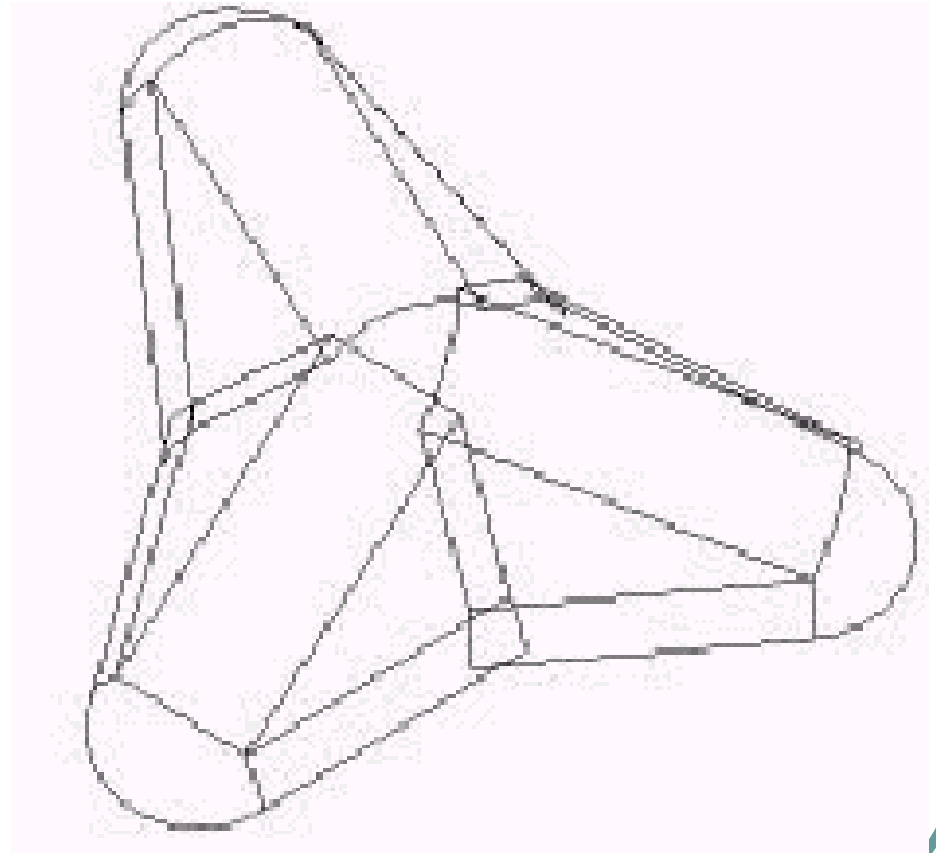
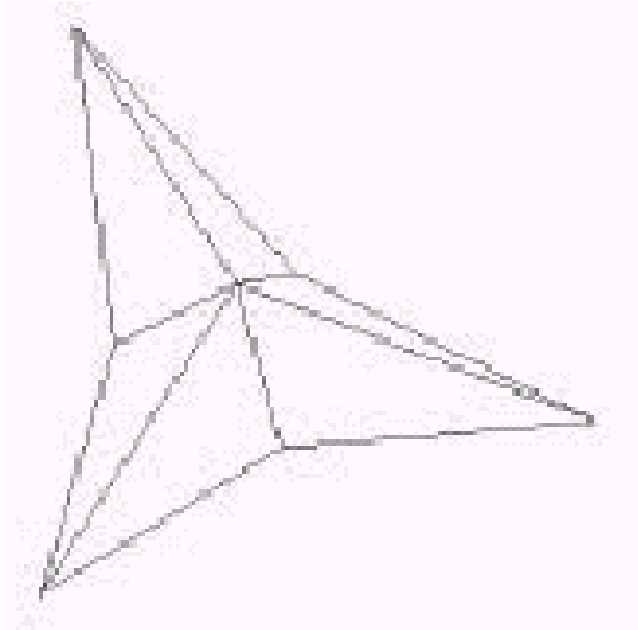


SADDLE

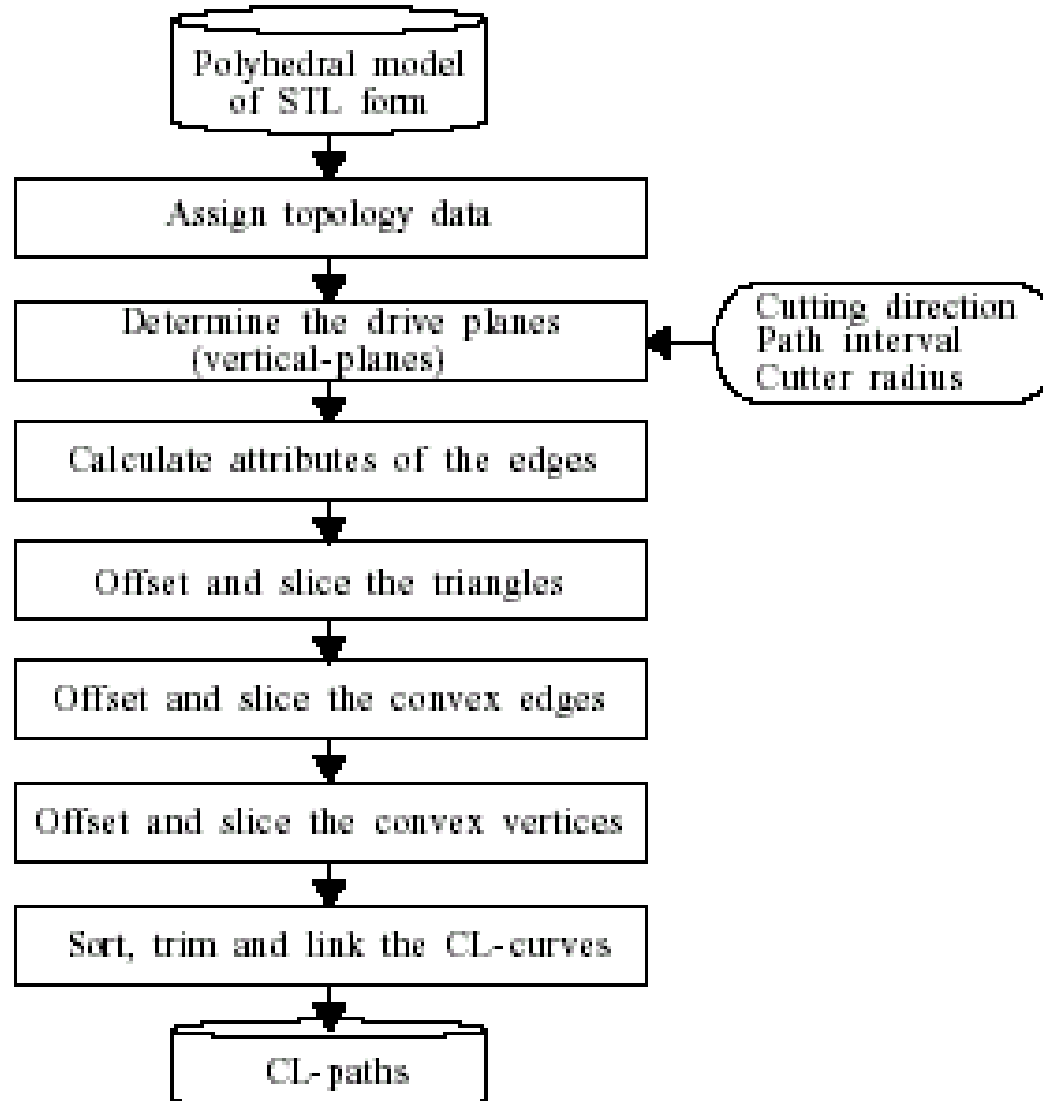




# Local Offset Example



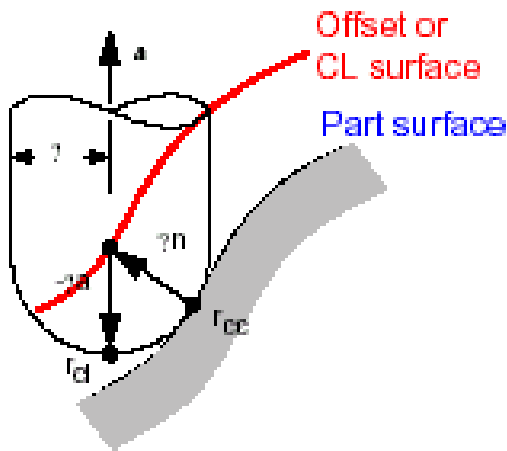
# Tool Path Generation for Polyhedral Machining



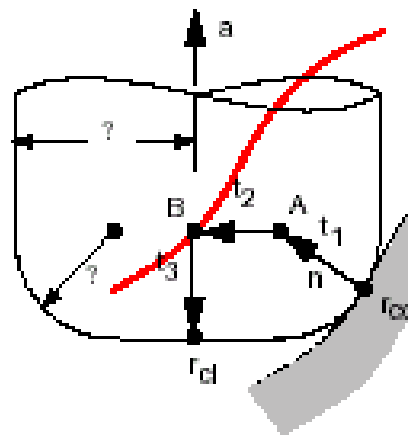
# Cutter Path Generation for NC Machining

CC Point: Cutter contact point

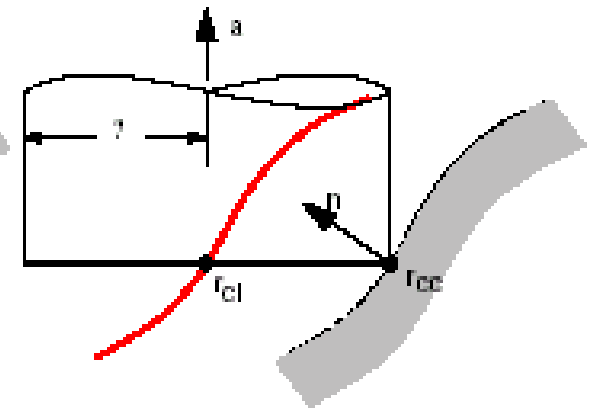
CL Point: Cutter location point



Ball-endmill

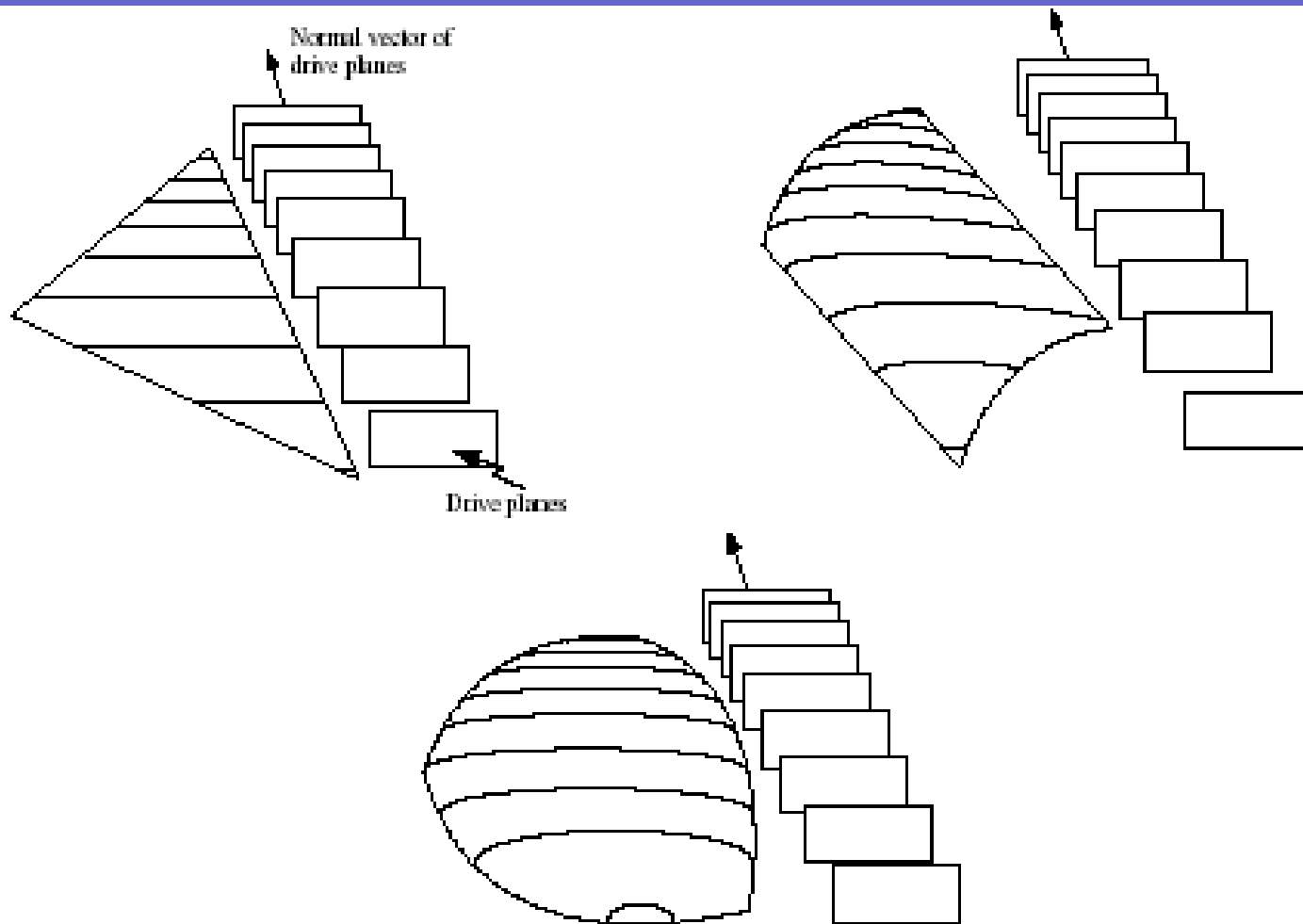


Filleted-endmill

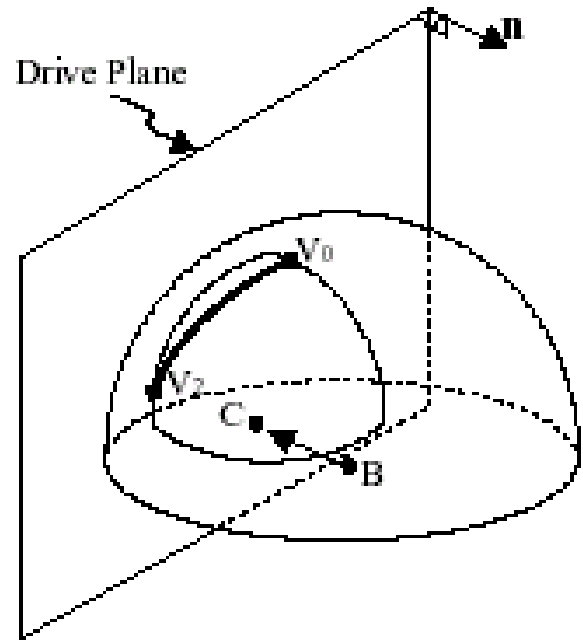
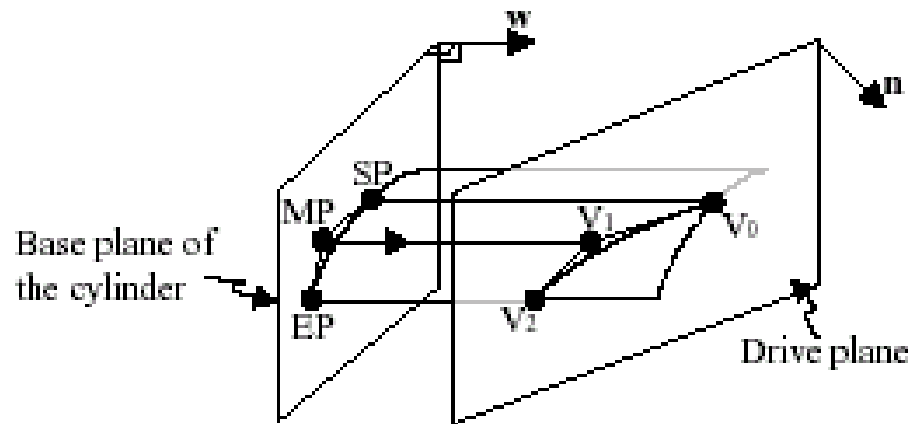


Flat-endmill

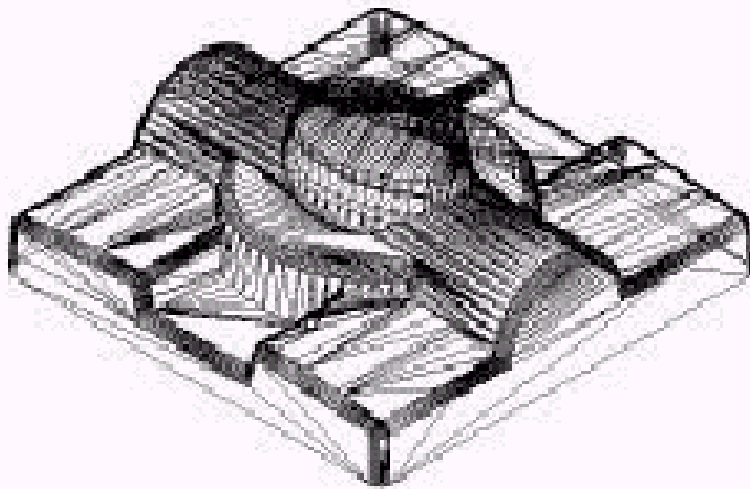
# Slicing of Offset Elements for Tool Path Generation



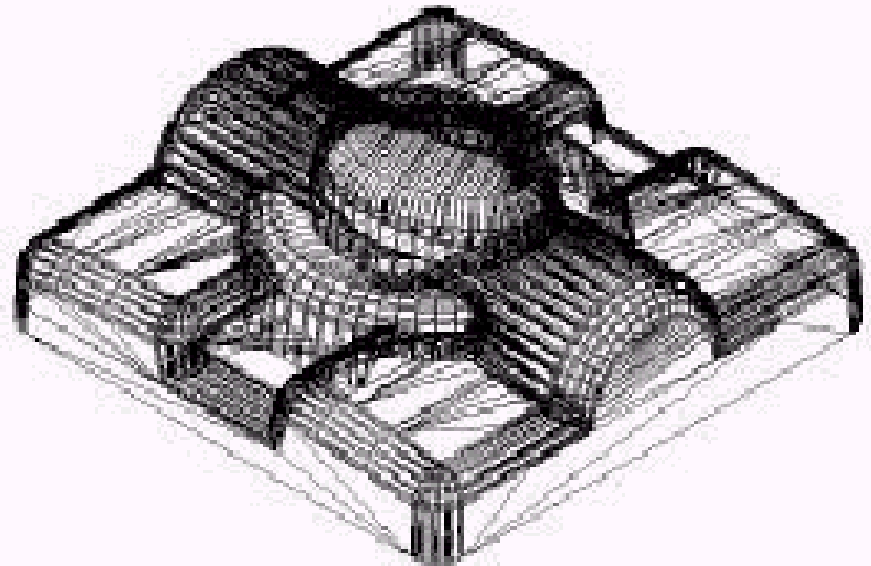
# Slicing the Spherical and Cylindrical Surfaces for Polyhedral Machining



# Example of Polyhedral Model Machining

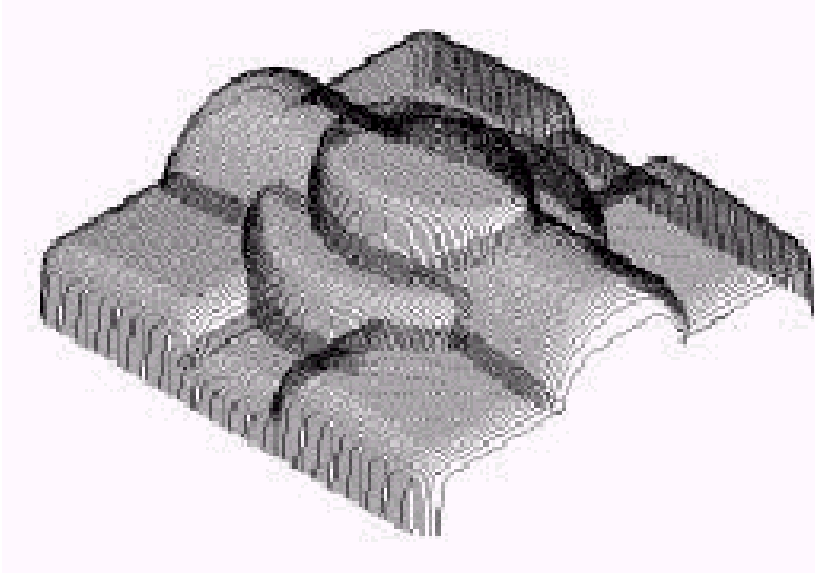


Polyhedral model

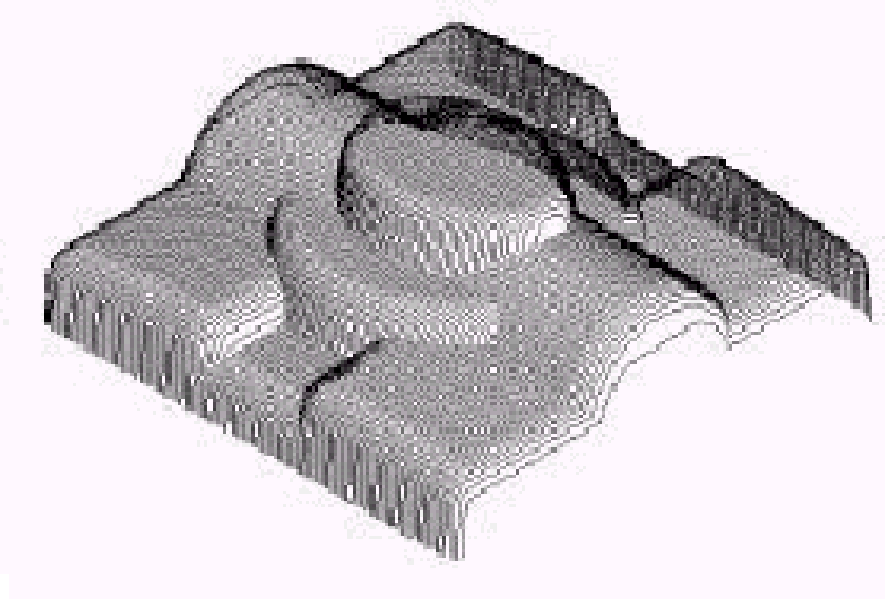


Offset model

# Tool Path Generation for Machining of Example 1



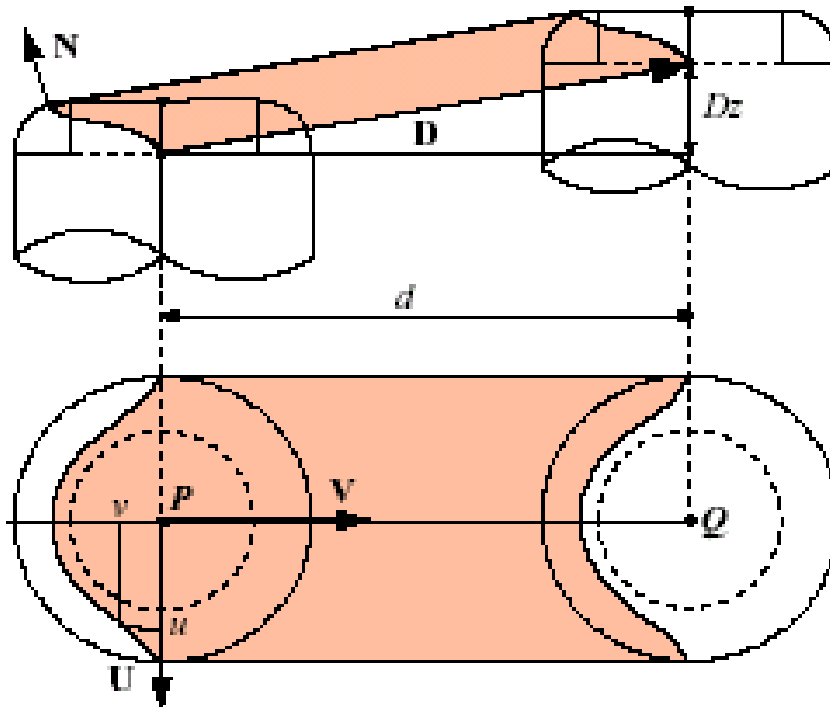
CL-curves



CL-paths

# Polyhedral Machining with Fillet-Endmills

## Offset and Slicing of Convex Edges with Fillet Endmills



ITSS: Inverse Tool Swept Surface

Silhouette curve:  $\mathbf{N} \cdot \mathbf{D} = 0$

$\mathbf{D}$  : tool movement vector

$d = D_{xy}$

$s$  : slope of tool movement  $(D_z / d)$

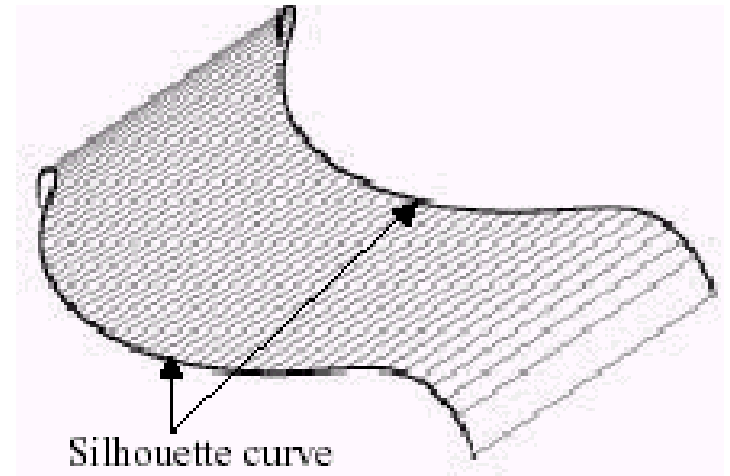
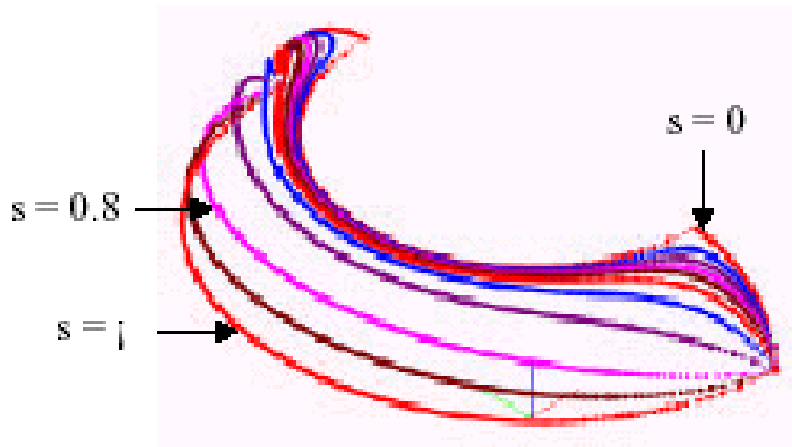
$\mathbf{V} = (D_x, D_y, 0) / d$

$\mathbf{U} = (V_y, -V_x, 0)$

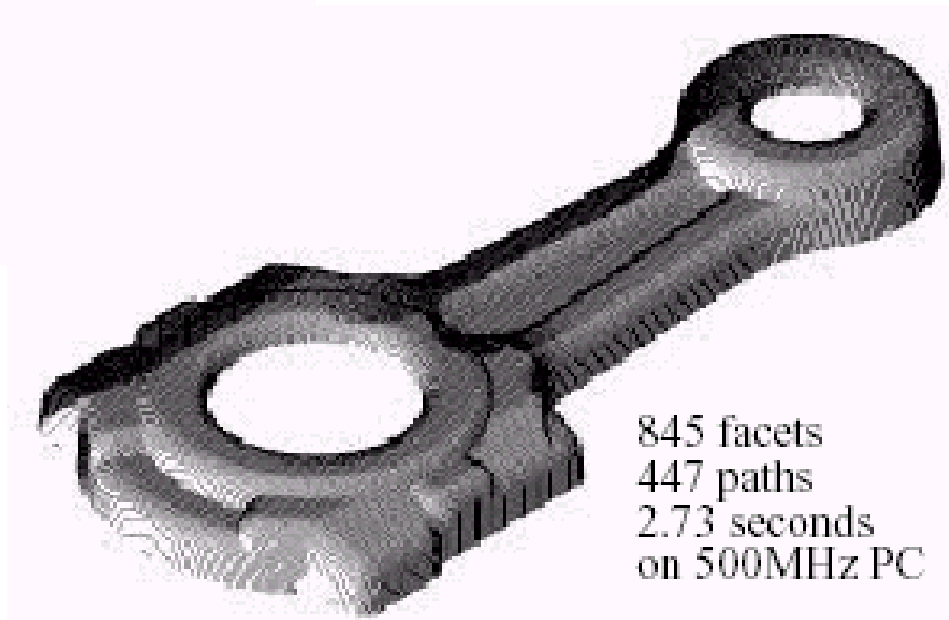
$\mathbf{N}(u, v)$  : normal vector of the ITSS



# Effective Cutting Shapes of Fillet-Endmills

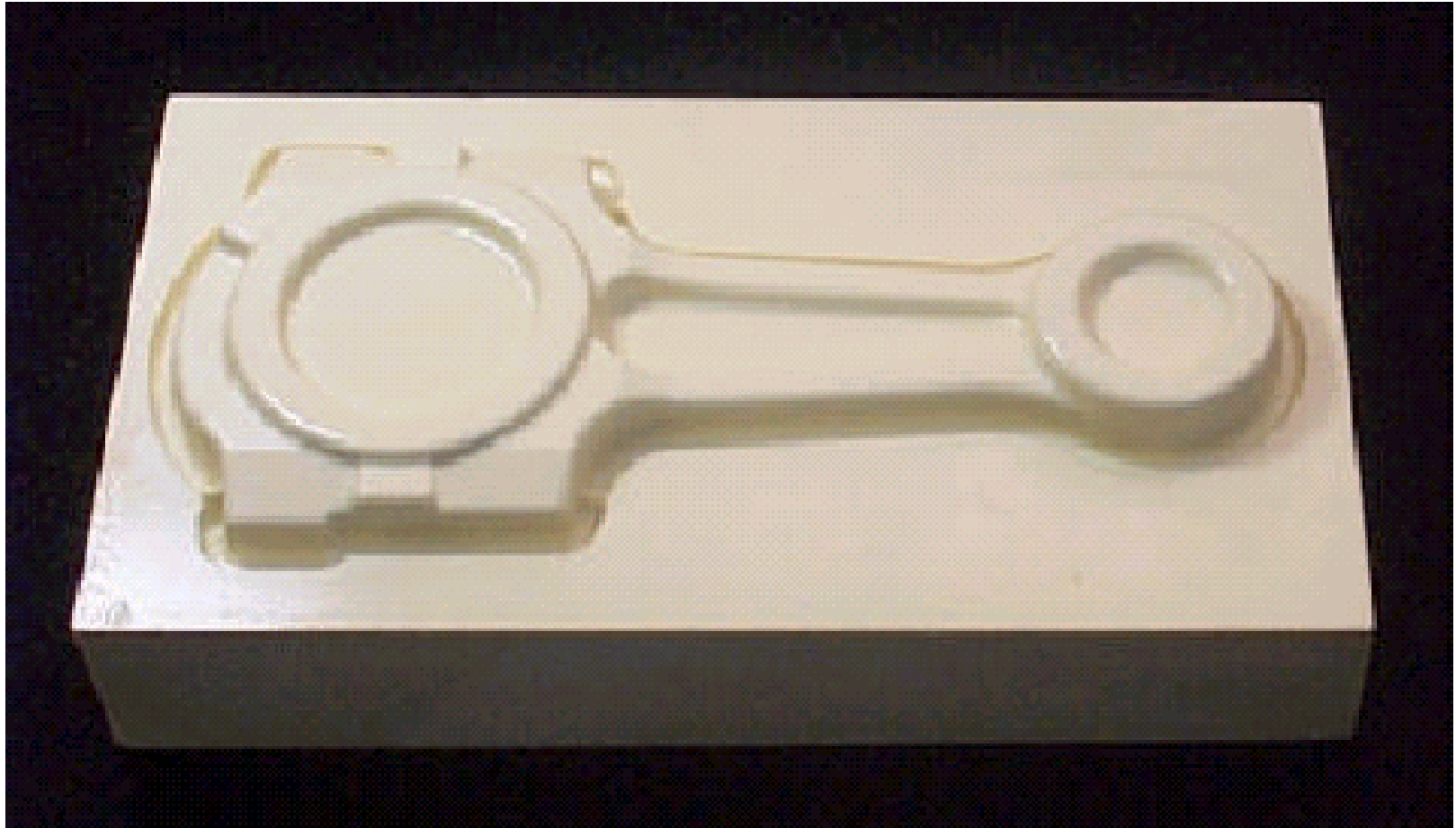


# Example 2 of Polyhedral Machining



845 facets  
447 paths  
2.73 seconds  
on 500MHz PC

# Example 2 of Polyhedral Machining

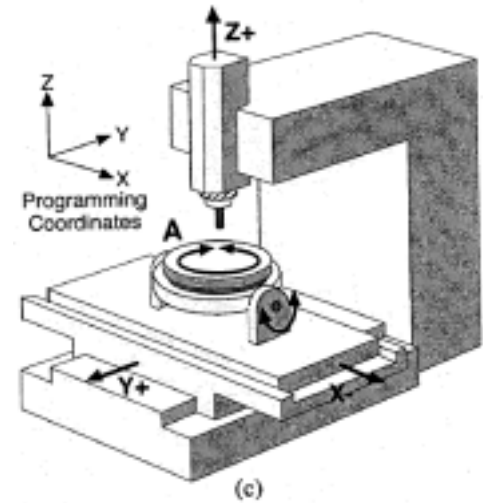
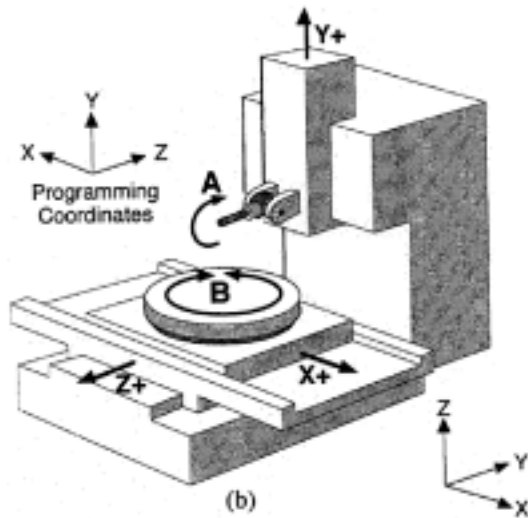
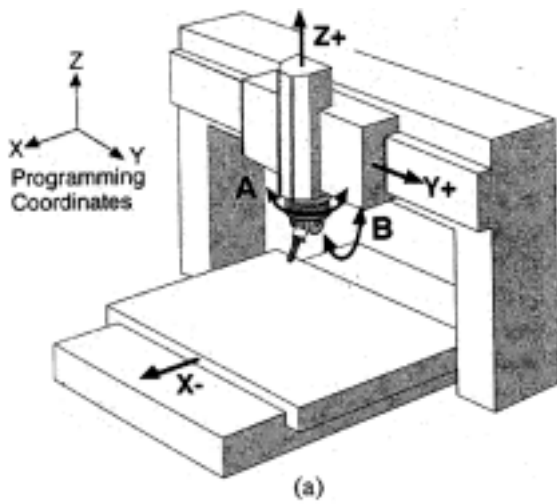


# Computation Time for Machining Examples

Mode	Tool diameter (mm)	Path interval (mm)	Computational time (seconds)	
			A curve based	A point based
Connecting rod (850 facets)	10	0.5	2.73	20.74
	20	0.5	4.36	50.94
	50	0.5	10.95	325.69
	100	0.5	19.63	1413.03
	10	1.0	1.47	10.23
	10	2.0	0.89	5.13
Face 226,440 facets)	3	0.5	233.08	1596.68

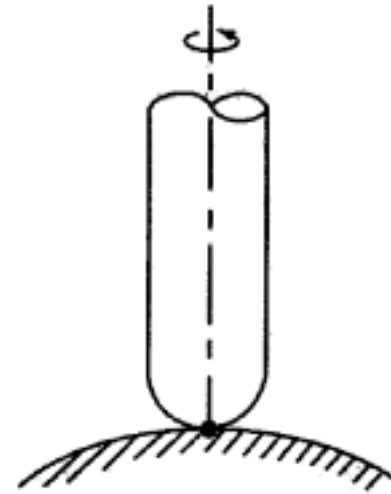
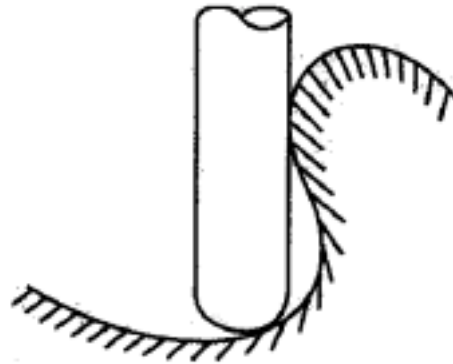
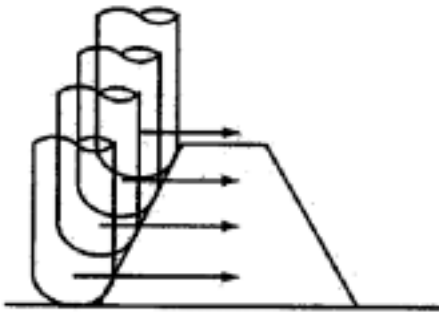
### **3. 5-Axis Tool Path Generation for Sculptured Surface Machining (SSM)**

# 5-Axis NC Machine Tools

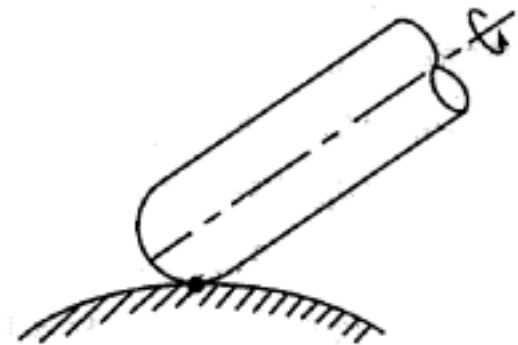
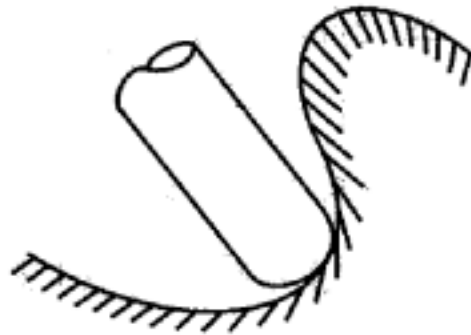
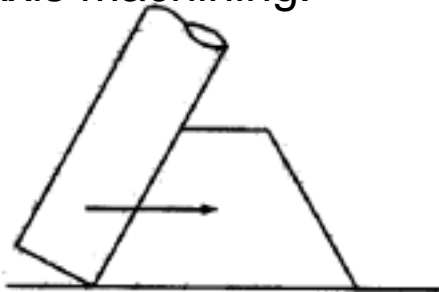


# 5-Axis Machining v.s. 3-Axis Machining (1)

3-Axis machining:



5-Axis machining:

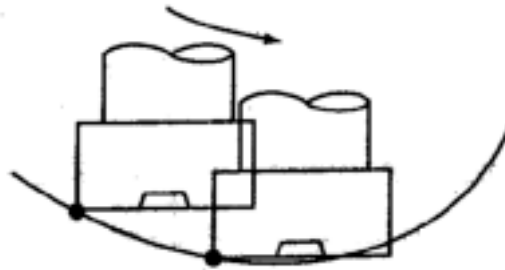
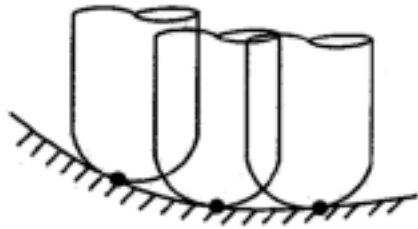


Efficient in machining

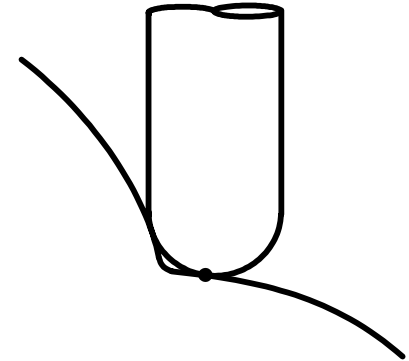
Tool accessibility

# 5-Axis Machining v.s. 3-Axis Machining (2)

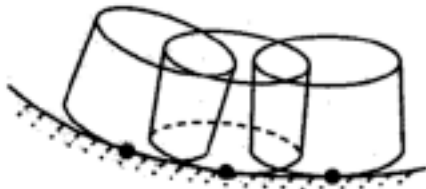
3-Axis machining:



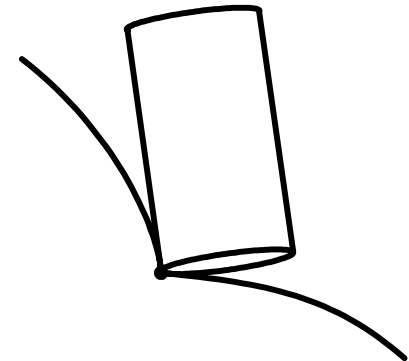
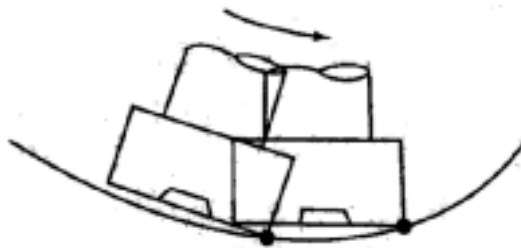
Cutter gouge



5-Axis machining:



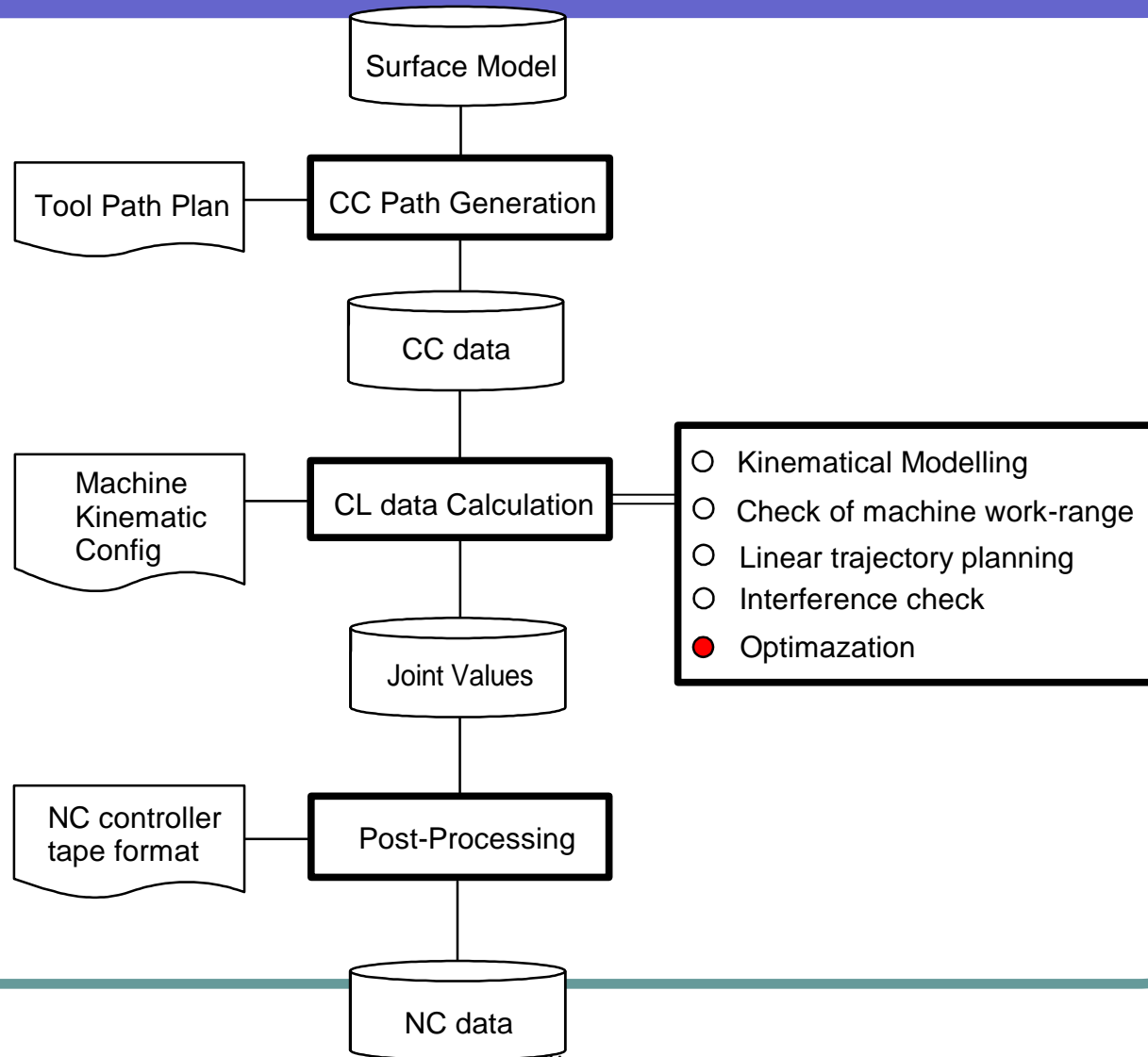
Improved surface finish



Clean-cut

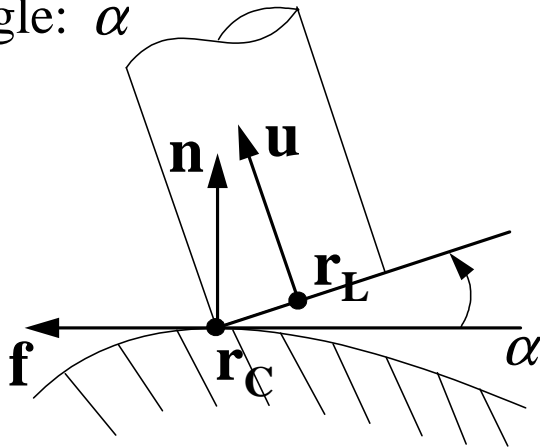


# Procedure of 5-Axis Tool Path Generation



# Definition of Tool Orientation in 5-Axis Machining

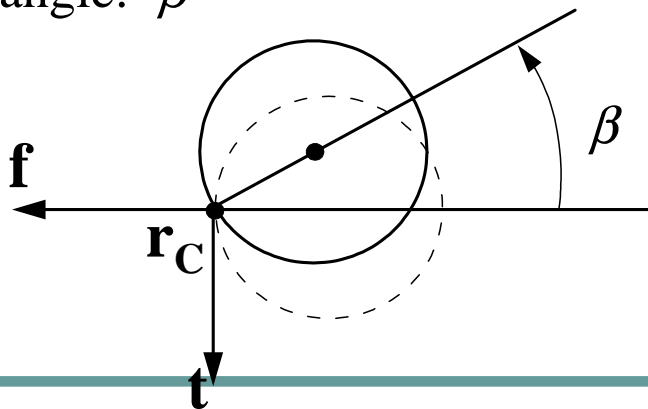
- Tilt angle:  $\alpha$



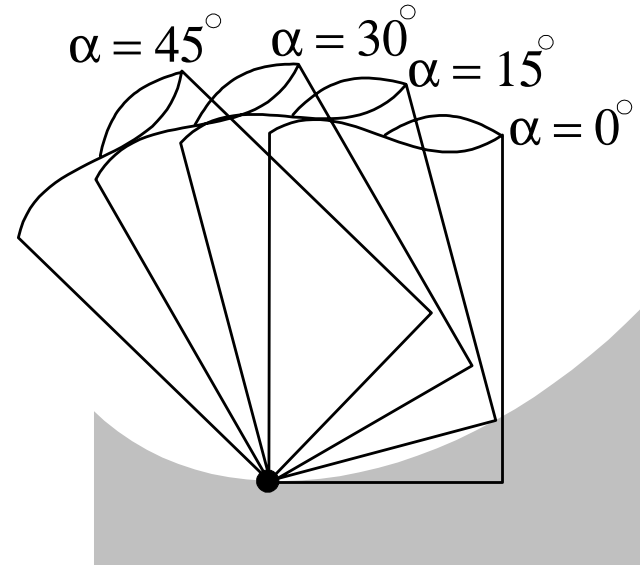
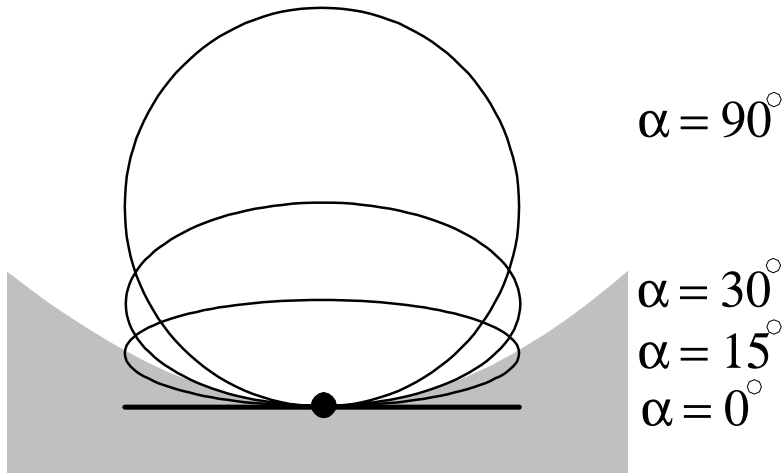
where,

- |                                |   |         |
|--------------------------------|---|---------|
| $r_L$ : cutter location point  | } | CL data |
| $u$ : cutter axis vector       |   |         |
| $r_C$ : cutter contact point   | } | CC data |
| $n$ : normal vector of surface |   |         |
| $f$ : a cutter feed vector     |   |         |
| $t$ : $n \times f$             |   |         |

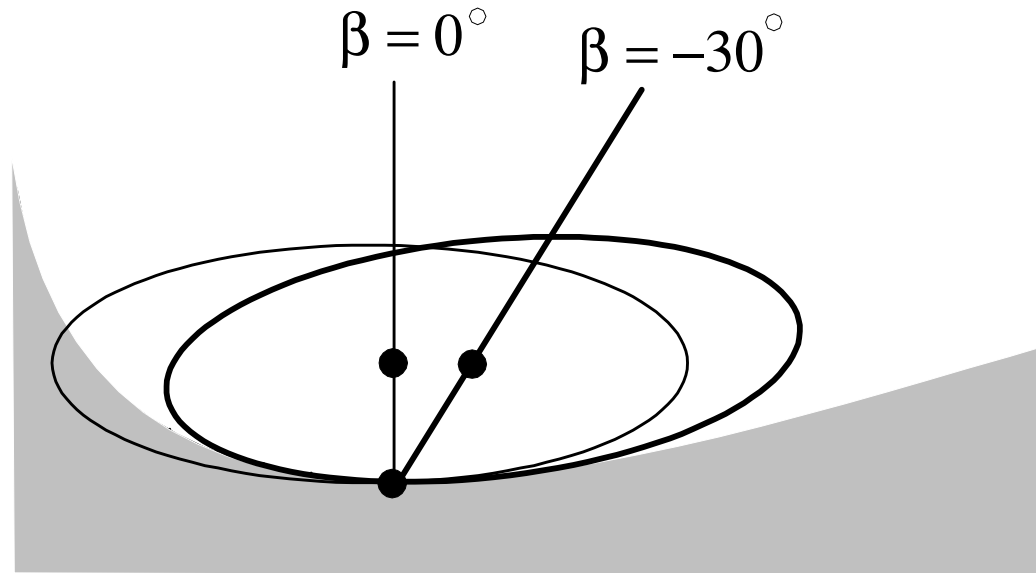
- Yaw angle:  $\beta$



# Effect of Tool Inclination Angle in 5-Axis Machining

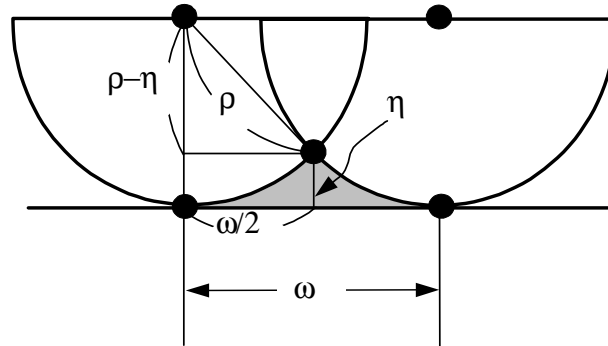


# Effect of Tool Yaw Angle in 5-Axis Machining

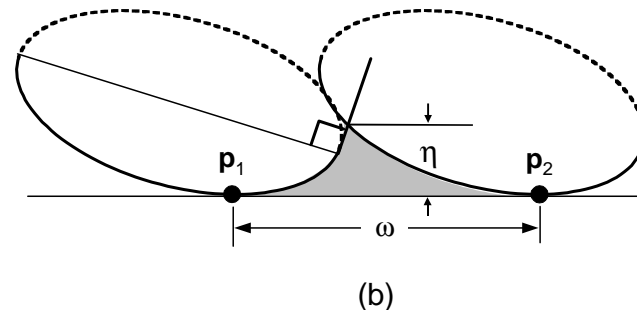
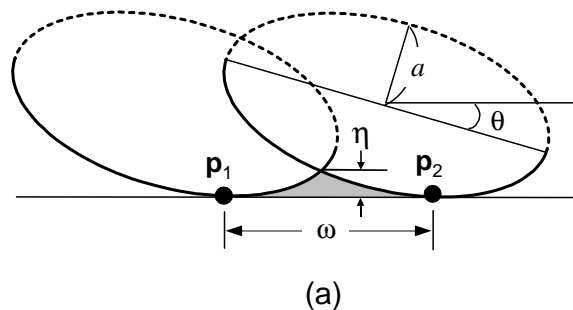
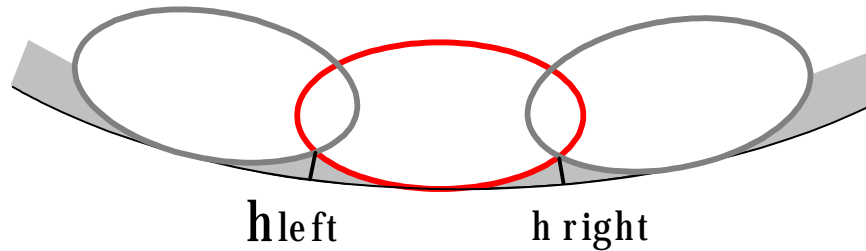


# Cusp Height Errors in Sculptured Surface Machining

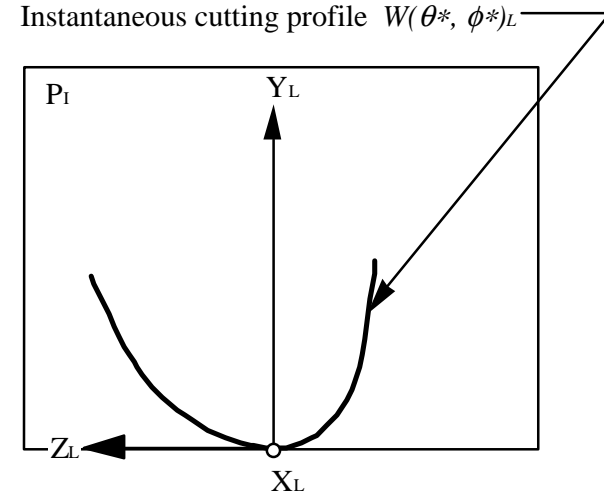
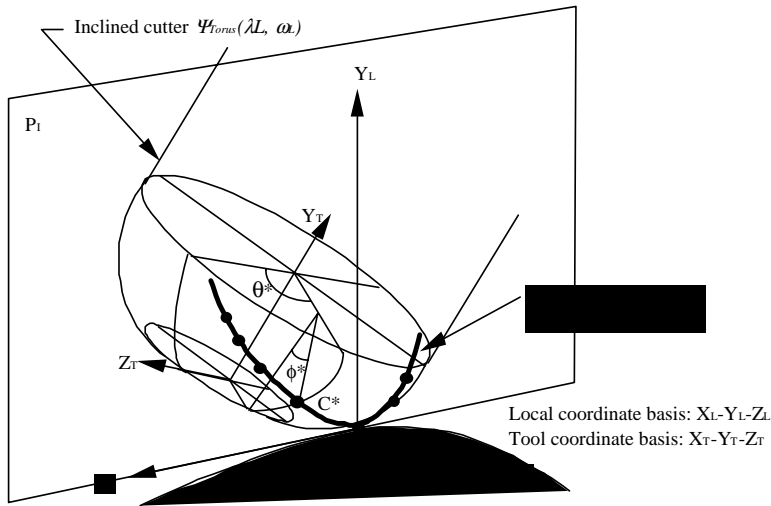
3-axis machining:



5-axis machining:



# Finding Effective Cutting Shape in 5-Axis Machining



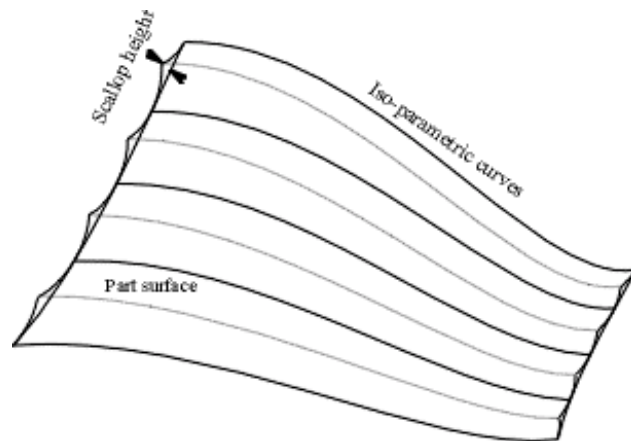
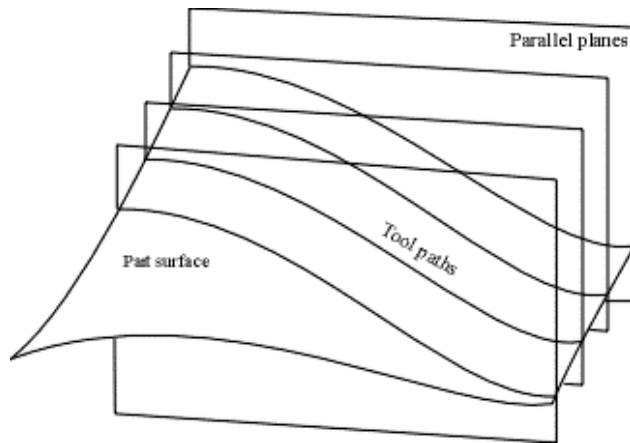
Effective cutting shape can be found as follows:

$$W(\theta, \phi)_L = \begin{pmatrix} x_L = 0 \\ y_L \\ z_L \end{pmatrix}_{\Psi, L}^G = \Psi(\theta, \phi, \lambda_L, \omega_L)_{L, x_L=0}^G$$

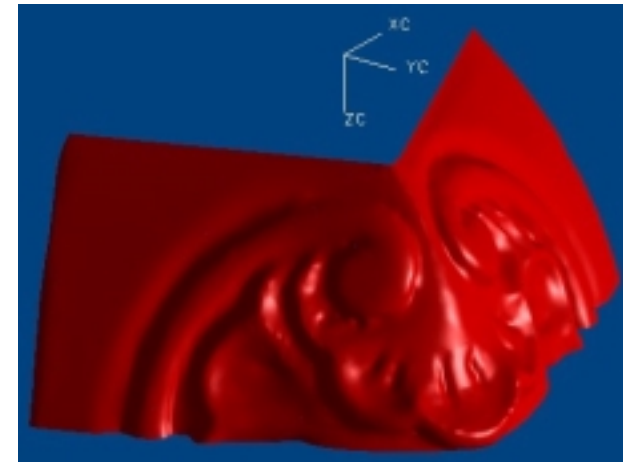
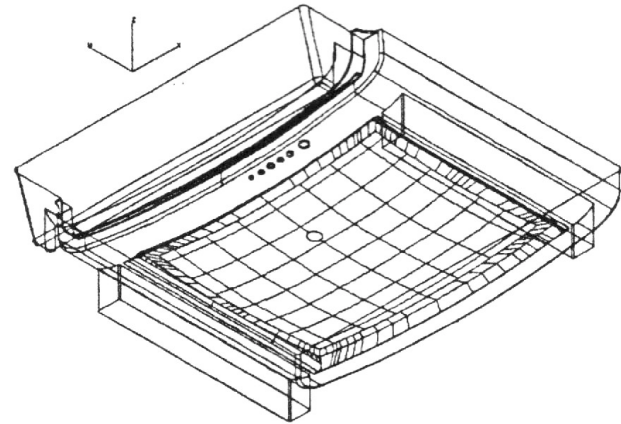
$$W(\theta^*, \phi^*)_L = \begin{pmatrix} x^*_L \\ y^*_L \\ z^*_L \end{pmatrix}_{\Psi, L} = \begin{pmatrix} 0 \\ m_7 \sin \theta^* \sin \phi^* + m_8 \sin \theta^* + m_9 \cos \phi^* + m_{10} \\ m_{11} \sin \phi^* \sin \theta^* + m_{12} \sin \phi^* \cos \theta^* + m_{13} \sin \theta^* + m_{14} \cos \theta^* + m_{15} \cos \phi^* + m_{16} \end{pmatrix}_L$$

### **3. Optimizing Tool Path Generation for CAD/CAM Systems**

# Machining of Sculptured Surfaces



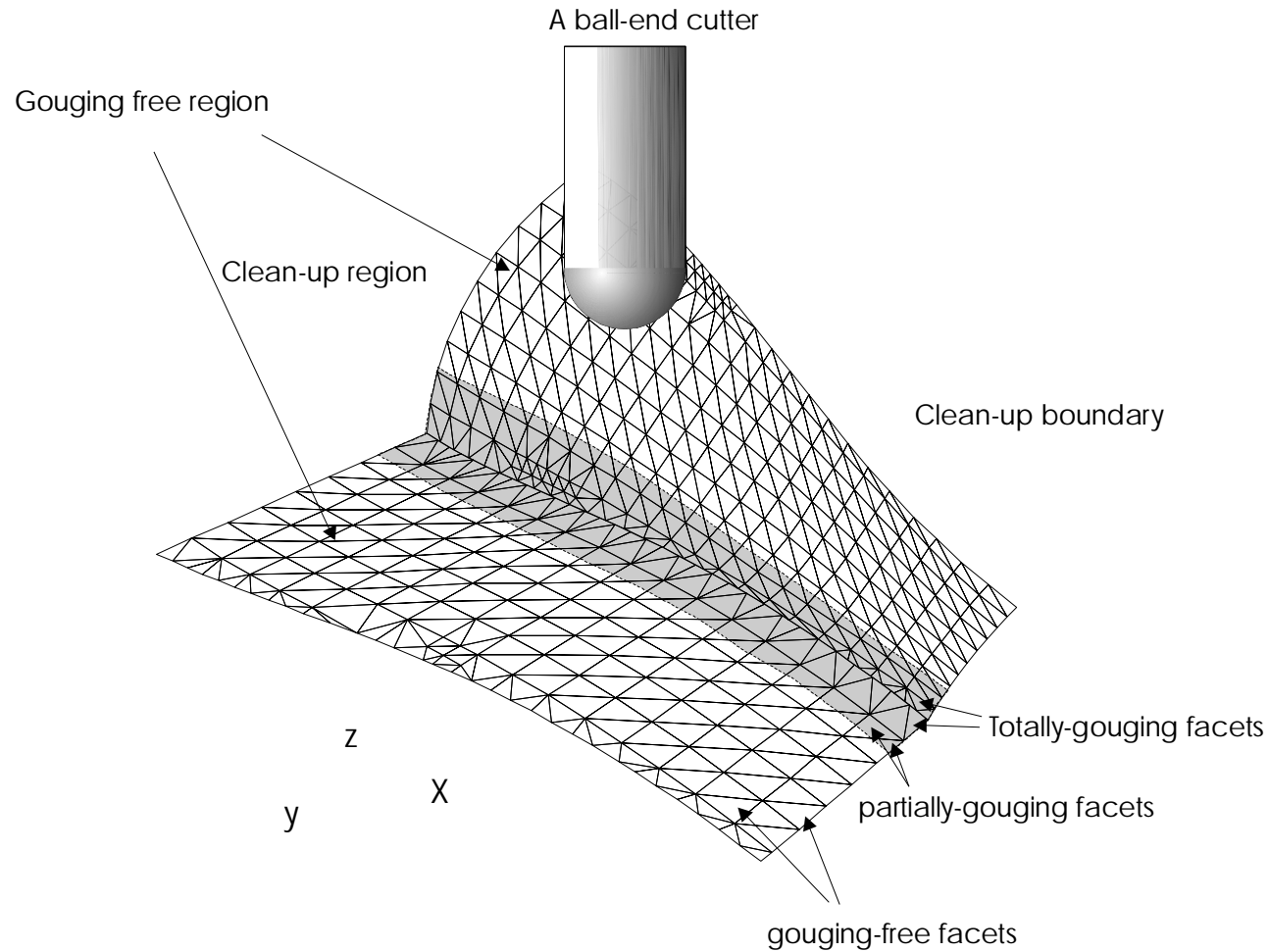
Traditional machining planning



3D path planning

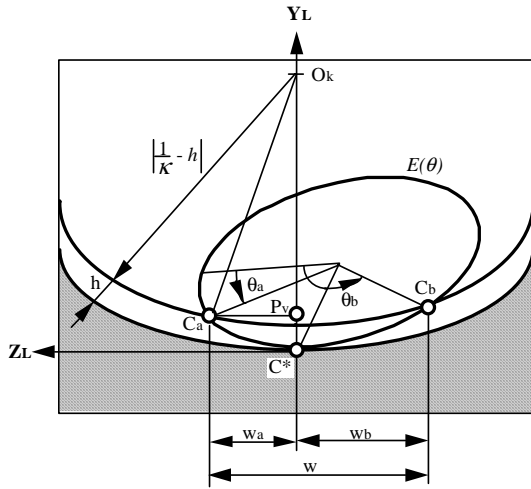


# Rolling-Ball Method for Extracting Clear-Cut Regions

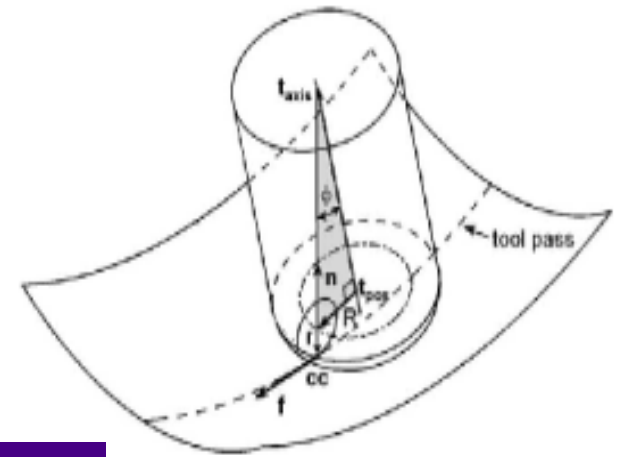


# Finding the Optimal Tool Orientation for 5-Axis Surface Machining

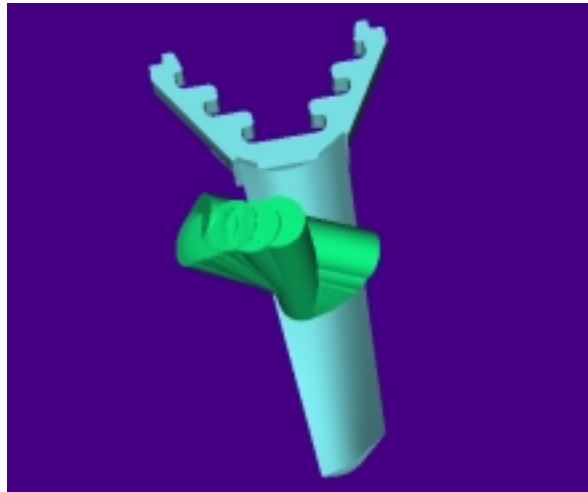
## Fitting cutting shape on local part surface



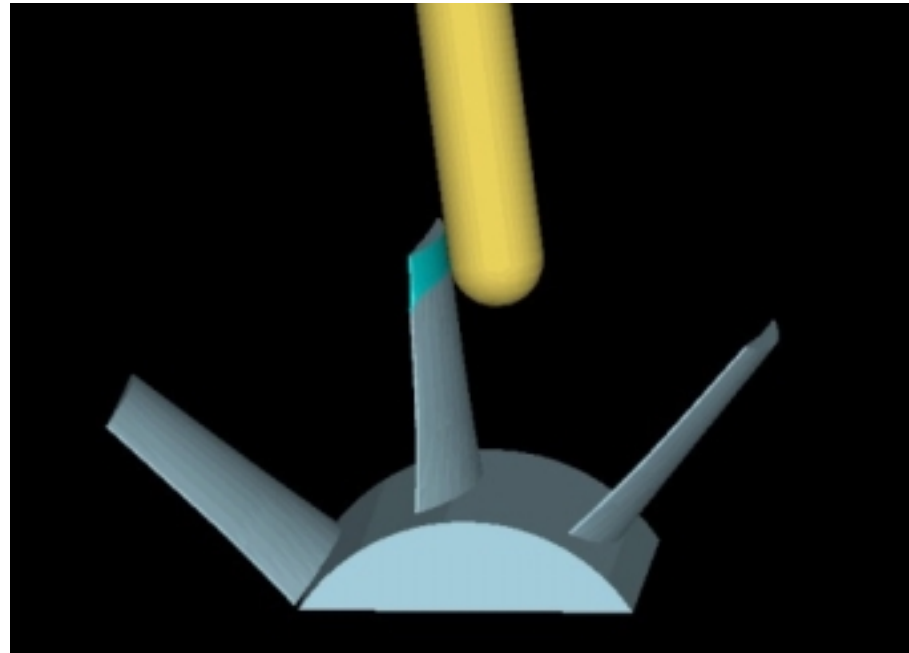
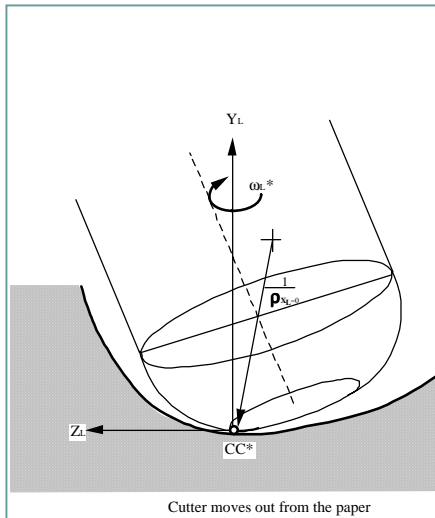
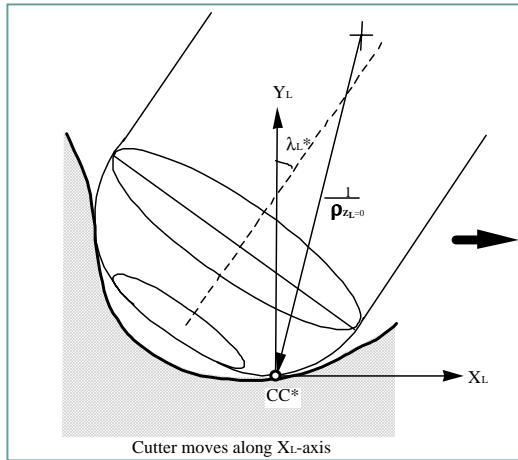
Cutting direction ( $\mathbf{X}_L$ ) out from the paper



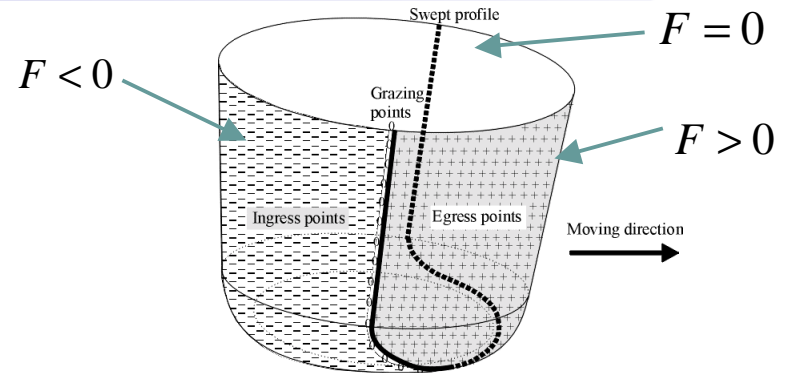
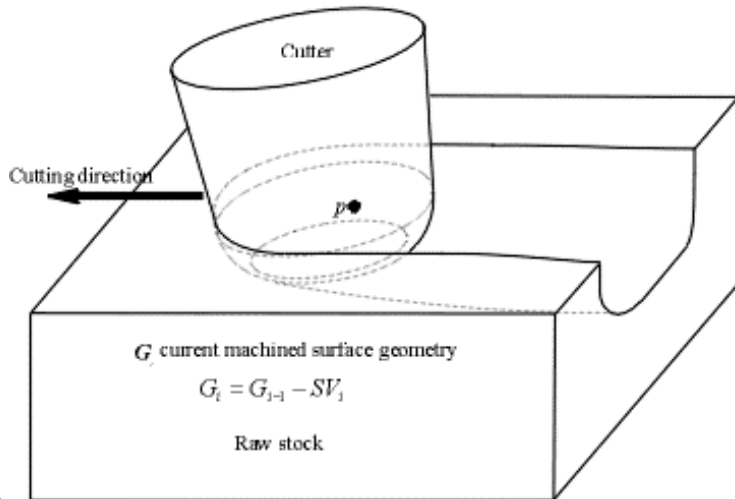
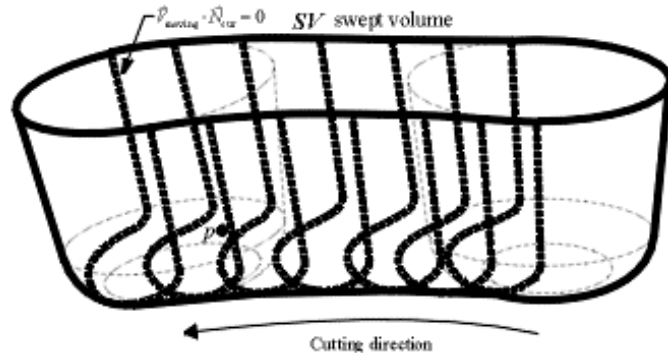
Using surface curvatures for optimal tool orientation



# Tool Collision and Gouging Avoidance in 5-Axis Machining

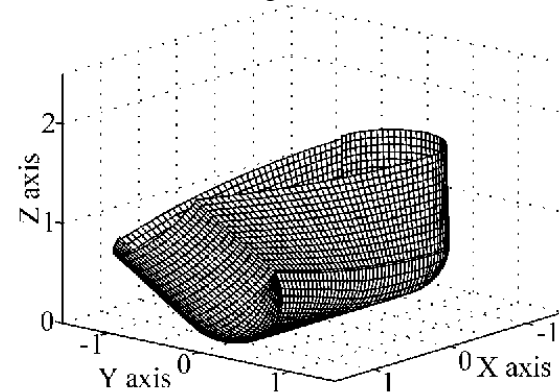


# Material Removal Rate (MRR) Analysis for 5-Axis High Speed Machining



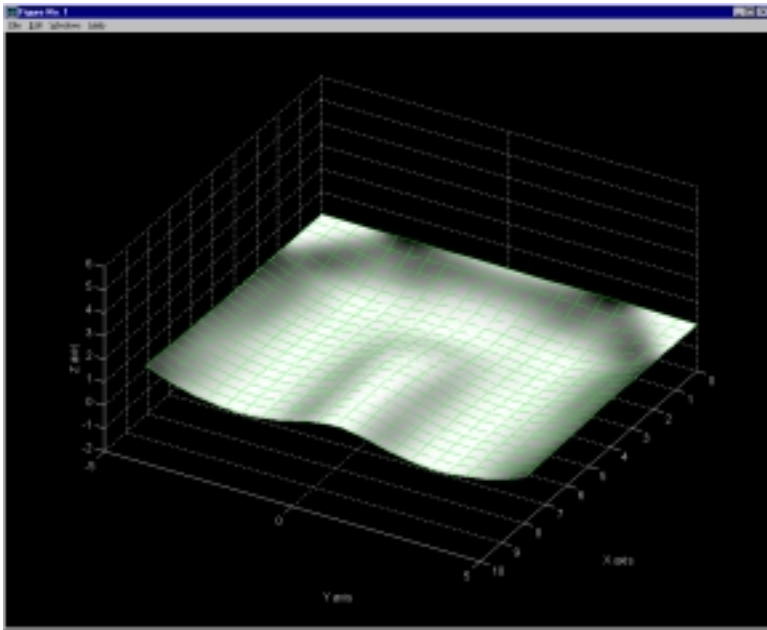
$$\vec{V}_{moving} = \vec{V}_{translation} + \vec{\Theta}_{rot} \cdot \vec{D}_{dis}$$

$$F = \vec{V}_{moving} \cdot \vec{N}_{sur} = 0$$

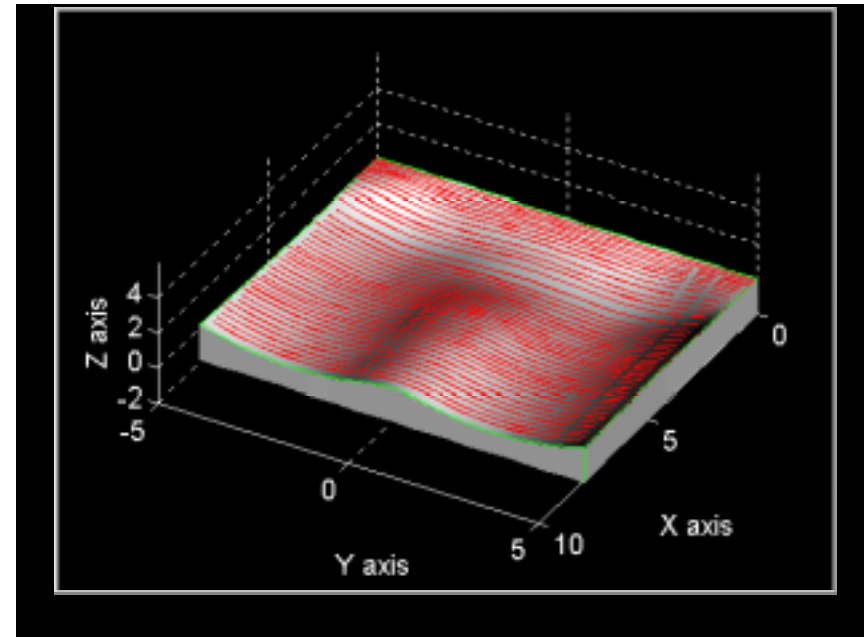


# Optimizing 5-Axis Tool Path Generation in CAD/CAM

**Q: Is it possible to find the best path distribution for SSM?**



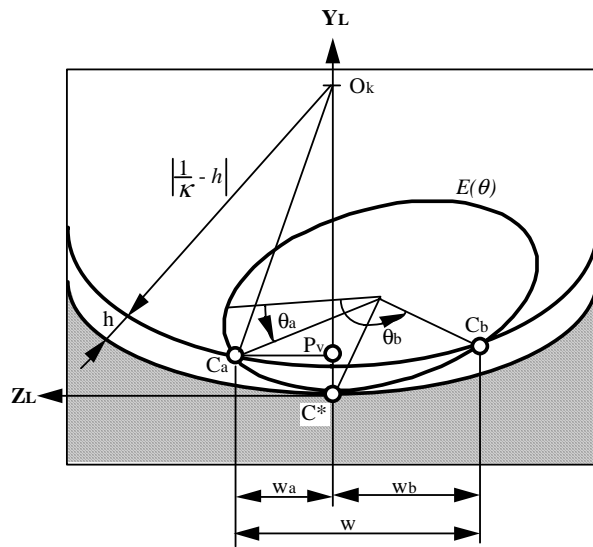
Sculptured surface design



Traditional tool path planning

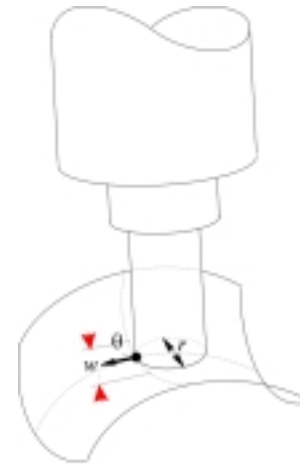
# Optimizing 5-Axis Tool Path Generation

- What is the best cutting direction?

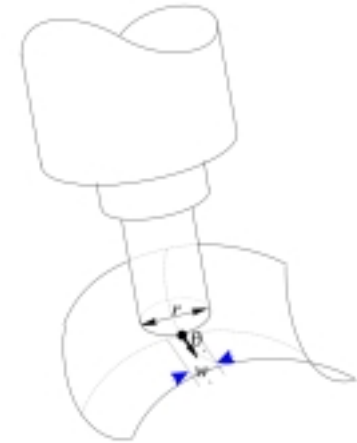


Cutting direction ( $\mathbf{X}_L$ ) out from the paper

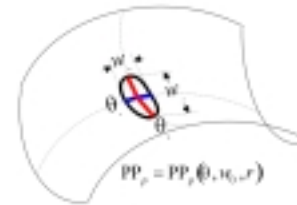
Machining strip width  
(dependent of  $\lambda$ ,  $\omega$ )



(a) cutting direction along  $\theta$



(b) cutting direction along  $\phi$



(c) potential patch (PP)

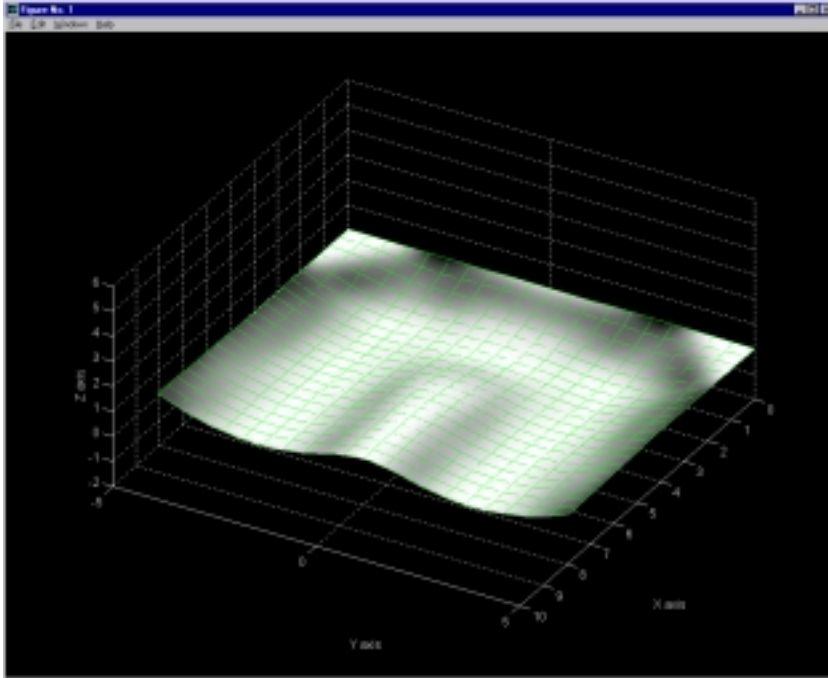


(d) optimal direction (OD)

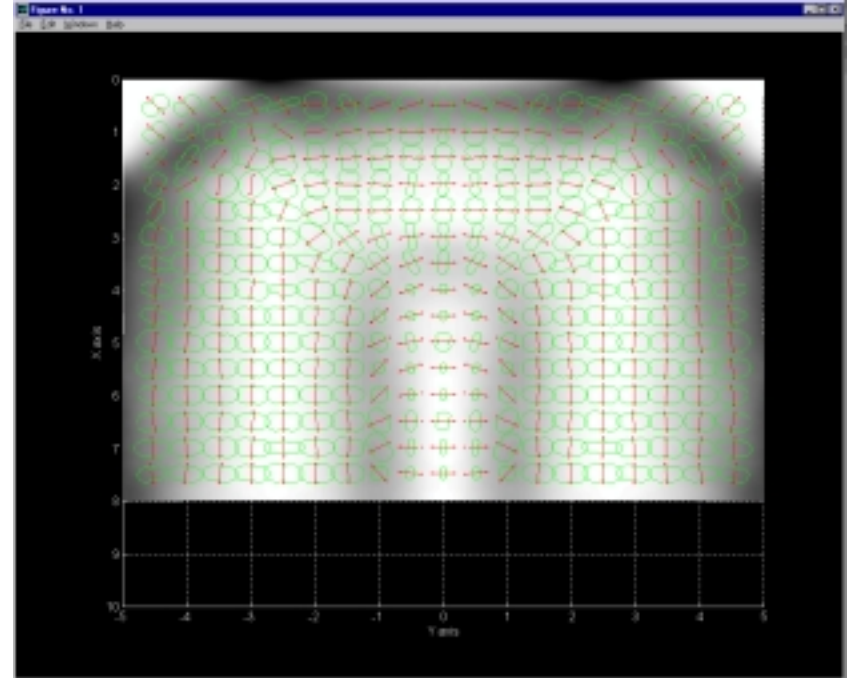
Optimal cutting direction

# Machining Potential Field (MPF) for Sculptured Surface Machining

**Q: Is it possible to find the best path distribution for SSM?**



Sculptured surface design



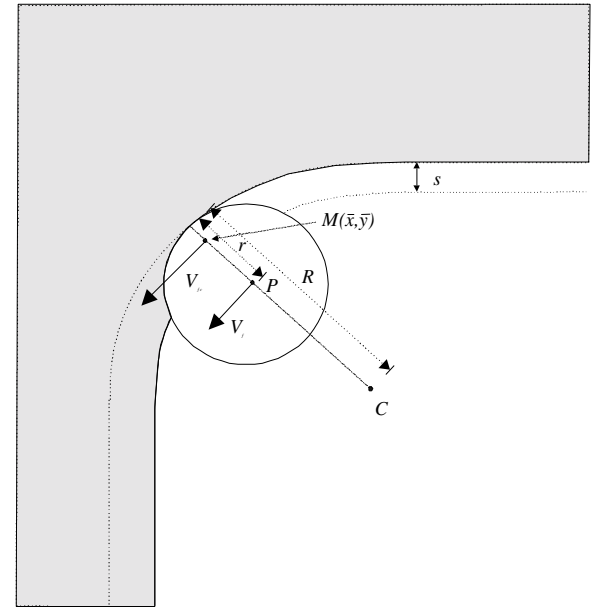
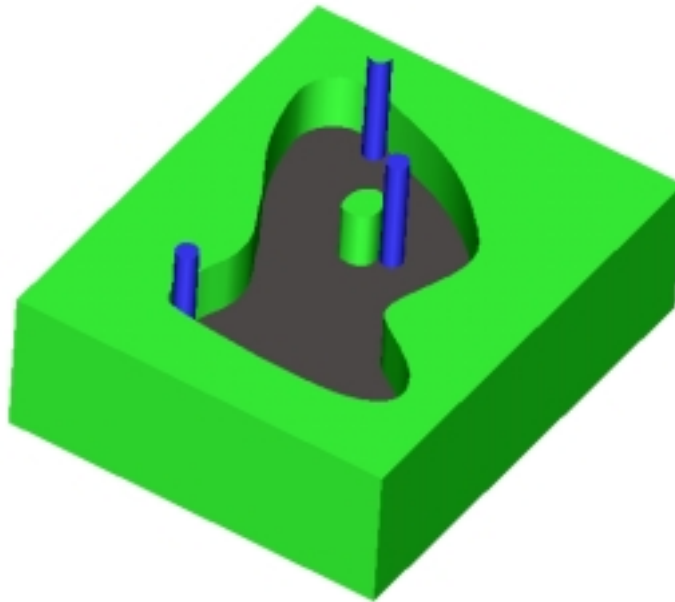
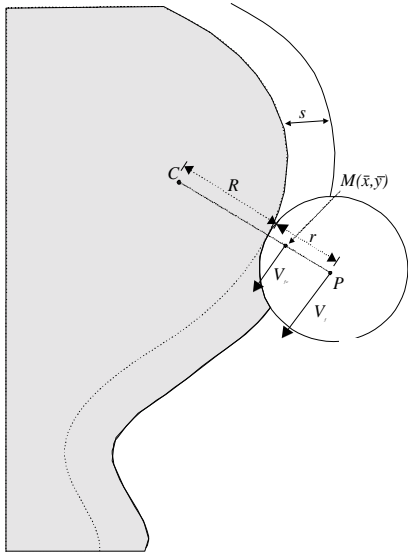
Machining potential patches

## **4. Adaptive Feed Scheduling for High Speed Machining (HSM) of Complex Surfaces**

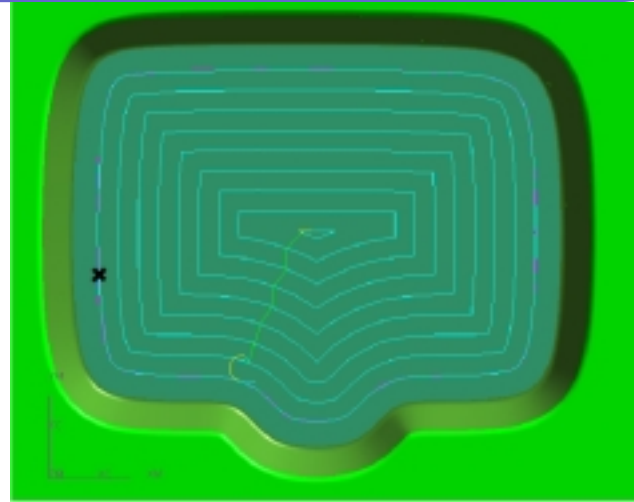
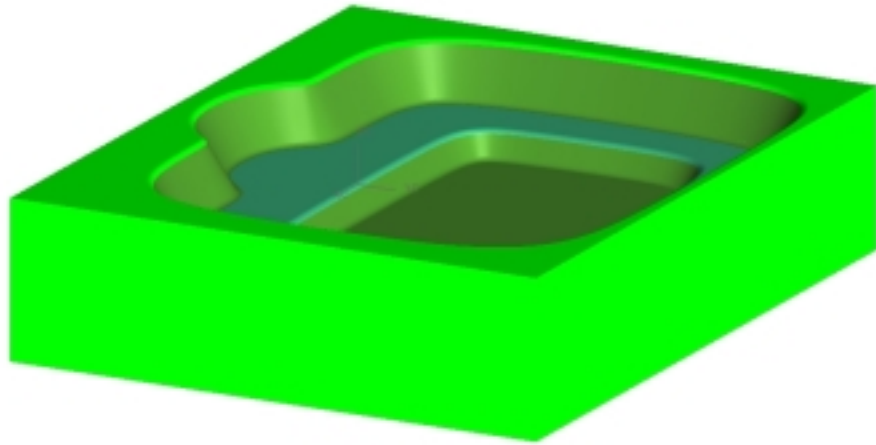


# Change of Material Engagement for High Speed Machining (HSM)

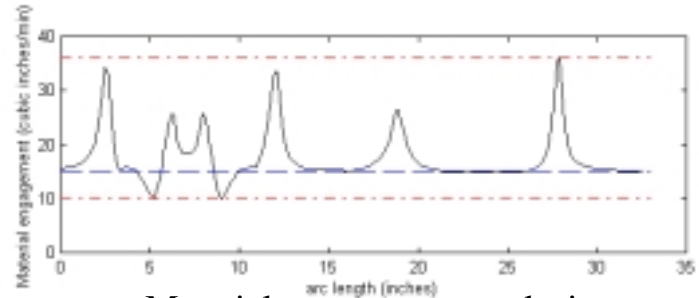
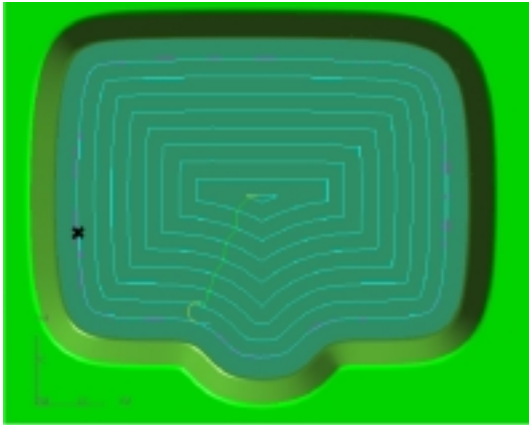
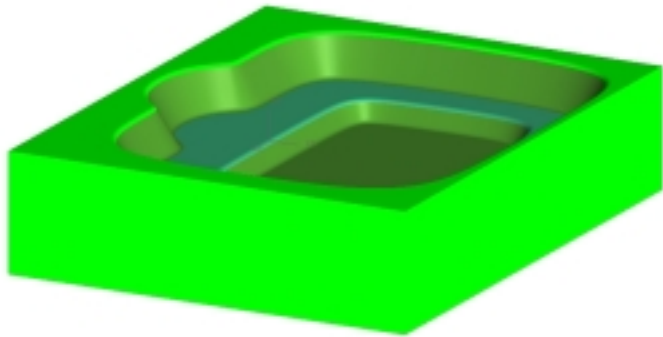
$C$ : center of circular arc  
 $R$ : radius of circular arc  
 $P$ : cutter tip.



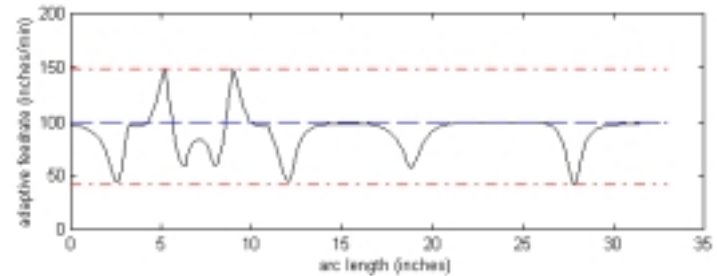
# Adaptive Feed Scheduling For High Speed Machining (HSM)



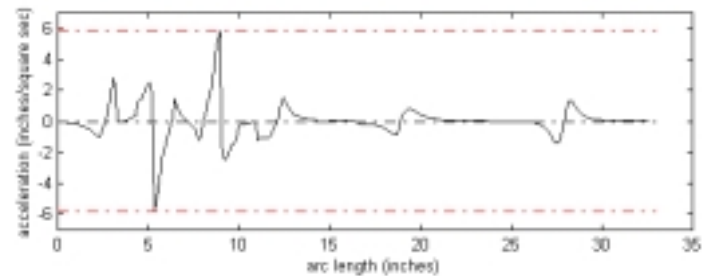
# Adaptive Feed Scheduling for High Speed Machining (HSM)



Material engagement analysis



Adaptive feedrate scheduling



Machine acceleration analysis

# Conclusions

- Modeling of complex surfaces for product development
- CAD/CAM for polyhedral model machining
- 5-Axis machining of sculptured surfaces
- High Speed Machining (HSM) can greatly benefit manufacturing process by shortening the machining time and reducing the manufacturing cost.
- HSM CAD/CAM shares an increasing market in recent years and the trend will continue.

**Thank you !!**

**Any Question ?**