Shortest Paths and Probability — A Game to get Students Started

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After attending the Leadership Program in the summer of 1998, I was searching for a way to integrate shortest path problems into our algebra curriculum. After many attempts an idea began to form. I would design a game board in which the object of the game was to get from one side of the board to the other, with the students choosing their own paths. My reasoning was that in the context of a game, students would try to win and would try to find the shortest path.

I knew, however, that there had to be more to the game than that. Years ago I had attended a workshop that focused on using games in math. They emphasized that an element of chance often helps to make it more interesting, so I decided to introduce rolling dice into the game. Eureka! I could include probability in the game and bring two important discrete math topics into the same activity.

The design and production of the game took a long time. It was not as polished as I would have liked, but I decided to give it a trial run in my classroom. I planned several activities using the game board, but wanted to begin with just a three-day lesson.

The game board included several paths from the bottom left corner of the board to the top right corner. Some of the paths were gray (light rectangles in the figure below) and some of the paths were brown (dark in the figure.) At first I had planned to use use a theme with cute, clever pictures at each vertex, but for the time being, I used letters instead and intended to replace them once I found a theme that I wanted to use.

Day 1

The game board was 11 by 17 and two students competed against each other. They started at point A and had to go to point J. They took turns rolling two dice; if the sum was even, they moved two spaces, if the sum was odd, they moved three spaces. We called this game *The Sum Game*. I set up this first game to deliberately bias the students towards making certain choices that would not be good choices in the second game.

Students played this game twice and I watched their strategies. Many of them didn't think about the shortest path. A minority of them did seem to be thinking about it, and there was some improvement when they played again.

After they finished two games, I demonstrated on the overhead how to translate the game board into a graph. I drew the game board with the vertices and edges on the overhead projector and counted the spaces from point A to B. I had the students draw their own graphs, label the length of the edges, and then determine the shortest path from A to J (the ending point). We discussed game strategy and what path they would take if they were to play the game again. For homework, the students worked two shortest path problems I had created.

Day 2

The next day we played a new game called *The Product Game*. On the game board some of the paths were now gray

